Elastic scattering and total reaction cross sections: Refractive scattering or strong absorption

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The trend of empirical interaction barriers and related parameters is explored for various heavy ion reaction systems for $100 < Z_1Z_2 < 1600$. The empirical radius parameters from reaction cross sections differ significantly from those from elastic scattering quarter points for $Z_1Z_2 < 1000$, but seem to approach one another for $Z_1Z_2 > 1000$. This behavior is shown to be consistent with predominantly refractive scattering for low Z_1Z_2 and dominance for surface absorption for $Z_1Z_2 > 1000$.

[NUCLEAR REACTIONS Semiclassical analysis of elastic scattering and reaction cross sections of heavy ions; predominantly refraction for $Z_1Z_2 < 500$ and strong absorption for $Z_1Z_2 > 1000$.]

I. INTRODUCTION

A puzzle of long standing in heavy ion reactions is related to the fact that very different nuclear potentials predict very similar patterns for elastic scattering and total reaction cross sections.[1] Some workers have favored a dominant role for the real nuclear potential and have used strong refraction (and small absorption) models to interpret data from elastic scattering.[2,3] Others have favored the dominance of the complex (or absorptive) potential and have interpreted the same kinds of data in terms of models with strong surface absorption.[4] Current understanding is that optical model fits to elastic scattering are primarily sensitive to the real potential over a very narrow radial region.[5,6,7] There is some sensitivity to the imaginary potential also, but it is often difficult to exploit due to large demands for energy resolution in the experimental measurements. Nevertheless, optical potentials from elastic scattering (or even one parameter characterizations such as the Blair radii[8] have provided reasonable estimates of reaction cross sections at energies well above the barrier. However if we look at reaction cross section data at low energies we find that they are often inconsistent with potential parameters taken from elastic scattering. These reaction cross sections near the barrier put quite a different burden on the mix of real vs imaginary potential.[9] In the preceding paper[9] we examined these separate demands for the system $^{16}O + ^{208}Pb$. Few if any systems have been studied experimentally as completely as this.[10] Therefore, in the present paper we resort to the trends of the cross section data on reactions[11] and scattering[12,13] to search for trends concerning the strength of surface absorption.

In the preceding paper[9] we explored the relationship between measurable quantities calculated via an optical model and those calculated via certain classical or semiclassical approximations. In particular, for $^{16}O + ^{208}Pb$ we calculated reaction cross sections versus incident energy for the limiting situations of weak and strong surface absorption. We then analyzed these calculated reaction cross sections in the simple classical way as if they represented experimental data. This analysis gave back an "empirical effective barrier", E_0 close to the real potential maximum for the case of weak surface absorption. For the case of strong absorption the empirical effective barrier was close to the real potential at the strong absorption distance (see Table II in Ref. 9). Similarly the quarter-point angle for elastic scattering was calculated from an optical potential with weak surface absorption and by a semiclassical approximation. For energies where semiclassical approximations are applicable (energies comfortably above the barrier) these values of $\theta_{1/4}$ were very close even though the more detailed shapes of the elastic scattering clearly depend on the different methods of treatment for absorption.

These comparisons suggest to us that an examination of the systematic trends of empirical "barrier parameters" (obtained by approximate semiclassical equations) may reveal trends in the underlying complex potential. In particular we have previously noted curious features in the empirical dependence of some of these parameters on the charge product Z_1Z_2.[11,13] In this paper we draw inferences from these trends concerning the strength of the surface absorption.

The starting point for our analysis is an empirical study of the body of data on total reaction cross sections.[11] The two most important parameters in this study are the height of the empirical interaction barrier E_0 and the radius R_0 of this barrier. These parameters were determined from many experimental results by fits to the Wong formula.[14] This equation uses Hill-Wheeler transmission coefficients with effective barrier height and radius independent of l and energy.[14,15] The intent of the study in Ref. 11 was to provide an empirical data base for estimating total reaction cross sections in situations where no measurements were available. The resulting empirical parameters (E_0, R_0) would reflect the maximum height of the real s-wave potential barrier only if the absorptive potential were small for distances outside this barrier. This is also a sufficient condition for application of a simple rainbow scattering model to elastic scattering of heavy ions. On the contrary, if the absorptive potential is large in the nuclear surface then the empirical values of E_0 and R_0 would not reflect the real potential maximum but rather an effective barrier near the mean absorption distance.[9] In the limit of complete control by such absorption, E_0 will approach the Coulomb potential at R_0 and the distance parameter $D_{1/4}$ from Blair's quarter-point recipe[8] will also approach the empirical value of R_0.[12]

If nature were kind enough to provide reaction systems in either of these limiting regions, then the information content of elastic scattering and total reaction cross sections would be focused on either the real or the imaginary potential. If both real and imaginary potentials always play comparable roles, then the problems of many needed parameters and their redundancy will be with us for years to come.[5] The purpose of this paper is to examine the overall trends of the data from elastic scattering quarter points and total reaction cross sections. We search for indications that signal the relative dominance of the real or the imaginary potential. We use rainbow model formulas[16-19] and effective potentials from the empirical study of total reaction cross sections.[11] The magnitudes of the empirical interaction barriers and the values of l_r and $l_{1/2}$ (rainbow l and l for $\frac{1}{2}$ absorption, respectively) obtained in this study clearly signal a transition from refractive scattering to strong surface absorption near $Z_1Z_2 \approx 1000$.

II. EMPIRICAL BARRIER PARAMETERS

A. Equations used to fix various empirical radius parameters

1. Classical and semiclassical parametrization of reaction cross sections

The classical cross section for all touching collisions between hard black spheres can be written

$$\sigma_R(E) = \pi R_{\rm slope}^2[1-(E_{\rm int}/E)]\,, \qquad (1)$$

where $R_{\rm slope}$ is the sum of the two radii and $E_{\rm int}$ is the potential energy (nuclear plus Coulomb) at contact. Cross sections for complete fusion (or for all reactions) have often been parametrized by plotting σ vs E^{-1} and identifying the intercept with $E_{\rm int}$ and the slope with $-\pi R_{\rm slope}^2 E_{\rm int}$.[20] Wong extended this approach[14] by deriving an analytic equation which includes penetrability via the Hill-Wheeler formula,[15]

$$T_l(E) = \{1+\exp[2\pi/\hbar\omega_0)(E_l - E)]\}^{-1}\,, \qquad (2)$$

where the l dependent barrier height E_l is given in terms of the effective s-wave barrier E_0' (or r_e),

$$E_l = E_0' + (l+\tfrac{1}{2})^2\hbar^2/(2\mu R_0'^2)\,. \qquad (3)$$

It is convenient to define two radius parameters r_0 and r_e as follows:

$$R_0 = r_0(A_1^{1/3} + A_2^{1/3}) \qquad (4)$$

and

$$r_e = Z_1Z_2\, e^2/E_0(A_1^{1/3} + A_2^{1/3})\,. \qquad (5)$$

(R_0 and E_0 are the values of empirically determined parameters, primes denote variables in the equations.) The extent of the penetration probability is controlled by the magnitude of $\hbar\omega_0$ in Eq. (2). Wong's final equation is[14]

$$\sigma_R(E) = \frac{R_0'^2}{2}\;\frac{\hbar\omega_0}{E}\cdot \ln\{1+\exp[2\pi(E-E_0')/\hbar\omega_0]\}\,. \qquad (6)$$

This equation approaches Eq. (1) rather rapidly as E exceeds E_0' and analyses by either can give effective radius and barrier parameters for reaction cross sections (either complete fusion, total reaction, or all reactions except inelastic scattering). The essential approximation for both Eqs. (1) and (6) is embodied in the assumption that R_0' is independent of l, not in the parabolic barrier from which Eq. (6) arises. The Hill-Wheeler formula is simply a convenient way to parametrize the penetrabilities which are only important for $E \lesssim E_0'$.

Commonly used nuclear plus Coulomb plus centrifugal potentials exhibit maxima that shrink in radial extent with increasing l. Thus, if one considers complete fusion as a classical friction free passage over the real potential maximum then one should write

$$\sigma_{cf}(E)=\pi[R_m(E)]^2\{1-[E_{lm}(E)/E]\}, \qquad (7)$$

where R_m and E_{lm} correspond to maxima in the l dependent real barrier. Alternatively, one could calculate transmission coefficients from Eq. (2) with R_m, E_{lm}, and $\hbar\omega_m$ each being a function of l for the assumed potential. Then the reaction cross section for complete fusion follows from the well known relationship

$$\sigma_{cf}(E)=\pi ƛ^2\sum_{l=0}^{\infty}(2+1)T_l(E). \qquad (8)$$

This latter approximation has been shown[9] to give results very close to those from an optical model with only interior absorption for $E \leq E_{lm}^{max}$. (E_{lm}^{max} is the energy above which the effective real potential loses its pocket.)

2. Blair quarter-point parametrization of elastic scattering

Many years ago Blair[8] formulated the sharp cutoff approximation as a simple model for correlating elastic scattering cross sections. (More recently Frahn[4] has developed the same general theme with a smooth cutoff.) In this model all partial waves with $l<l_{max}$ are completely absorbed; one distance parameter r_{0B} appears that can be written in terms of the classical distance of closest approach for a Coulomb trajectory leading to the angle $\theta_{1/4}$ (that angle where the elastic scattering cross section is $\frac{1}{4}$ of the Rutherford value),

$$l_{max}=\eta\cot(\tfrac{1}{2}\theta_{1/4}), \qquad (9)$$

$$r_{0B}(A_1^{1/3}+A_2^{1/3})=(Z_1Z_2e^2/2E)[1+\csc(\tfrac{1}{2}\theta_{1/4})], \qquad (10)$$

and

$$\sigma_R(E)=\pi ƛ^2(l_{max}+1)^2. \qquad (11)$$

This distance parameter r_{0B} is often called the strong absorption radius.

B. Systematics of the empirical radius parameters

It is clear from Eqs. (1) and/or (6) that measurements of reaction cross sections at high energy ($E \gg E_0'$) can fix values of R_0' rather unambiguously. In Ref. 11 we showed that these values, as taken from experimental data, vary only slowly with Z_1Z_2 and an average value of 1.416 fm was adopted for the parameter r_0. Similarly many studies have shown that the values of E_0' (or r_e) are very sensitive to the cross sections near the barrier, and that these effective barrier parameters can be extracted rather well for a number of cases. In contrast to the very slow Z_1Z_2 dependence of r_0, we find a rapid dependence of r_e on Z_1Z_2 [the effective barrier parameter from Eq. (6)],[13]

$$r_e=1.951-0.164\log_{10}(Z_1Z_2), \quad \text{for } Z_1Z_2<1700. \qquad (12)$$

The values of r_e and r_0 seem to be converging for $Z_1Z_2\approx 2000$. In Refs. 12 and 13 compilations are given for the Blair radii r_{0B}. The values of r_{0B} vary only slowly for $Z_1Z_2>1000$ and seem to approach a value of ≈ 1.38 fm. They increase rather decidedly for lower Z_1Z_2 and almost track Eq. (12) for r_e. [Recent data from the Oak Ridge group and from Scobel *et al.*[21] strengthen this correlation for $300<Z_1Z_2<1000$ whereas older data[13] indicated many values of r_{0B} lower than those given by Eq. (12).]

What can the trends of these empirical parameters tell us about the trends of the complex potential? If the surface absorption is so strong as to mask completely the Coulomb rainbow, then the real nuclear potential can be neglected with respect to the Coulomb potential. For this limiting situation one expects very simple connections between the various parameters that describe reaction cross sections and the angle $\theta_{1/4}$ for elastic scattering. For the angle $\theta_{1/4}$ one can use the Coulomb trajectory equations [Eqs. (9) and (10)] to identify both l_{max} and the distance of closest approach $R_{0B}\equiv r_{0B}(A_1^{1/3}+A_2^{1/3})$. Also the interaction barrier would be very nearly equal to the Coulomb potential V_c evaluated at R_{0B}. Thus the total reaction cross section σ_R could be approximated by the classical relation, $\sigma_R(E)=\pi R_{0B}^2[1-V_c(R_{0B})/E]$. The fusion barrier would, in turn, be larger than the interaction barrier as it would occur where the nuclear attractive force is equal to the Coulomb repulsive force.[22,23] Empirical correlations have shown that these simple connections are not obeyed for reaction systems with $Z_1Z_2<1000$.[11,13,20] Thus the nuclear potential must be significant at R_{0B} for these cases. However, as shown in Refs. 11–13 the empirical values of r_{0B}, r_e, and r_0 do all seem to converge as Z_1Z_2 approaches 1000, and thus we seem to realize the limit of strong surface absorption for high Z_1Z_2.

At the other extreme, if the surface absorption is very weak then rainbow scattering will occur and the relationships above should not apply[8,16-19] Simple relationships that replace them have not been worked out. Instead we have many parameter sets from optical model analyses for particular cases. Global sets of these parameters are also available but they are often poor predictors of σ_R for near barrier energies. For the case $^{16}O+^{208}Pb$ we have explored this problem via "computer experiments"[9] and have shown that simple semiclassical equations and approximations can be used to correlate several quantities: (1) the values of σ_{cf} with the height of the real potential barrier, (2) the values of σ_R with the real potential at the mean absorption distance, and (3) the values of $\theta_{1/4}$ with the real potential at the rainbow distance.

This suggests a simple means of testing the applicability of (or approach to) the limiting case of weak surface absorption. We use simple semiclassical equations to obtain from $\theta_{1/4}$ and σ_R the parameters that describe the effective real barriers and radii. (For the strong absorption limit we expect Coulomb trajectory equations to relate these parameters.) For weak surface absorptions we search for consistency in the empirical barrier parameters that result from fits to σ_R and $\theta_{1/4}$. In the next section are given the semiclassical approximations that we use to fit σ_{el} near $\theta_{1/4}$ and thus to extract rainbow distances (and barrier heights) for a potential with weak surface absorption.

III. FORMULAS FOR RAINBOW SCATTERING

In the simplest rainbow scattering model one attributes total control of elastic scattering and reaction probability to the real potential.[16-19] This is the effect of the equations given above for the real barrier and the transmission coefficients. The rainbow scattering that results was first formulated by Ford and Wheeler[16] and developed by Berry and Mount,[17,18] Broglia and Winther,[24] and others.[19,24,25]

In the uniform approximation[18] the elastic scattering cross section $\sigma(\theta)$, for angles less than the rainbow θ_r, is given by

$$\sigma(\theta)=\pi(\{\sigma_1(\theta)+\sigma_2(\theta)+2[\sigma_1(\theta)\sigma_2(\theta)]^{1/2}\}\xi^{1/2}Ai^2(-\xi)+\{\sigma_1(\theta)+\sigma_2(\theta)-2[\sigma_1(\theta)\sigma_2(\theta)]^{1/2}\}\xi^{-1/2}Ai'^2(-\xi)). \quad (13)$$

The classical cross sections $\sigma_1(\theta)$ and $\sigma_2(\theta)$ are obtained from branches 1 and 2 of the deflection function $\theta(l)$, and

$$\sigma_i(l)=\lambda\!\!\!^-{}^2(l+\tfrac{1}{2})[\sin\theta\,|d\theta(l)/dl|]^{-1}[1-T_l(E)], \quad (14)$$

where

$$\theta(l)=2d\delta_l/dl)=\pi-2\int_D^\infty (b/r^2)[1-(b/r)^2-V(r)/E]^{-1/2}dr. \quad (15)$$

The turning point D is the solution of the equation

$$1-(b/D)^2-V(D)/E=0, \quad (16)$$

with b the impact parameter corresponding to l, E the incident energy in the center of mass frame, and δ_l the JWKB phase shift. The argument of the Airy function Ai is given by

$$\xi=\{|(\tfrac{3}{4})[2\delta_{l_2}-2\delta_{l_1}-(l_2-l_1)\theta]|\}^{2/3}. \quad (17)$$

Only two branches of $\theta(l)$ have been included as it is expected that other contributions are strongly reduced by large values of the transmission coefficient $T_l(E)$ and/or slope $|d\theta(l)/dl|$.

For angles greater than the rainbow angle we have used the form given by Da Silveira,[19]

$$\sigma(\theta)=\sigma_-(\theta)+\sigma_r(\theta)+2[\sigma_-(\theta)\sigma_r(\theta)]^{1/2}\cos\alpha, \quad (18)$$

where the phase α is

$$\alpha=2\delta_{l_-}-2\delta_{l_r}+(l_-+l_r)\theta-\pi/4. \quad (19)$$

The classical cross section for the negative branch $\sigma_-(\theta)$ is very small in comparison to $\sigma_r(\theta)$ as given by the Airy approximation,

$$\sigma_r(\theta)=2\pi\lambda\!\!\!^-{}^2(l_r+\tfrac{1}{2})(\sin\theta_q{}^{2/3})^{-1}\mathrm{Ai}^2[(\theta-\theta_r)/q^{1/3}] \quad (20)$$

and

$$q=(\tfrac{1}{2})\,|d^2\theta/dl^2|_{l_r}. \quad (21)$$

The quantity q is actually calculated from the equation that results from differentiating Eq. (15) twice.

For the real s-wave potential (Coulomb plus nuclear) we use the following representation:

$$V(r)=E_0-[(\hbar\omega_0)^2/(2\hbar^2/\mu)](r-R_0)^2, \quad \text{for } r\le R_0 \quad (22)$$

and

$$V(r)=V_C(r)+V_N(r)=V_C(r)+V_N(R_0)\exp[-(r-R_0)/d], \quad \text{for } r\ge R_0. \quad (23)$$

Continuity of $V(r)$ at R_0 requires that d be given as

$$d=-R_0V_N(R_0)/V_C(R_0). \quad (24)$$

For transmission coefficients we use Eqs. (2) and (3). The parameters $\hbar\omega_0$ and r_0 were fixed at 4 MeV and 1.42 fm, respectively. Values of E_0 were initially taken from Eqs. (5) and (12) and then varied slightly as required to achieve a fit to the elastic scattering.

Absorption is included by multiplying each classical cross section by a penetrability factor $[1-T_l(E)]$ from Eqs. (2) and (3). There are many more sophisticated ways of including absorption in a rainbow model,[24-26] but we wish to restrict the analysis to one free parameter E_0 and then look at the trends. Now it becomes evident that even though the equations we use are from the rainbow model, the transmission coefficients can dictate results from a nuclear potential that is strongly absorptive in the surface region. If the empirical nuclear potential $V_N(R_0)$ becomes very small then the rainbow trajectory will approach the Coulomb trajectory and we again approach the Blair analysis.[8] Thus we know from the convergence of r_0, r_e, and r_{0B} at $Z_1Z_2>1000$ that the rainbow becomes masked. Our search then is for the self-consistency of the rainbow model analysis of σ_R and $\theta_{1/4}$ for $Z_1Z_2<1000$.

IV. DISCUSSION

In this section is described the use of the semiclassical equations from Sec. III. From the empirical trends of r_e and r_0 [Eq. (12) and Ref. 11], empirical real potentials were built by Eqs. (22)–(24). Then elastic scattering cross sections were calculated and a fit to experiment was sought in the region of angles near $\theta_{1/4}$. The sensitivity of these fits to values of r_e and r_0 is explored. Modifications in the empirical potentials (via changes in r_e) were made to fit the observed scattering near $\theta_{1/4}$, and these modifications are tabulated and discussed. The trend of the results with Z_1Z_2 is examined and comparisons are made to other systematic studies. Finally the nature and uncertainties of the original measurements is discussed as it relates to the meaning of the empirical parameters and to our conclusions.

A. Sensitivity of the calculation to E_0', R_0', and $\hbar\omega_0$

A brief outline of the semiclassical calculation of σ_{el} is now given for the system 113 MeV ^{14}N + ^{108}Ag.[27] We first obtained the potential [Eqs. (22)–(24)] from the Wong effective barrier parameters E_0, r_0, and $\hbar\omega_0$ from the systematics of total reaction cross sections.[11] With this potential we calculated the deflection function, phase shifts, and elastic scattering cross section. This procedure was repeated for several values of E_0', r_0', and $\hbar\omega_0$ in order to test the sensitivity of the calculation to each parameter.

In Fig. 1 we show calculated curves compared to the experimental results of Galin *et al.*[27] First we confine our attention to the angles less than the quarter-point angle $\theta_{1/4}$. For energies not too close to the barrier, the rainbow angle is entirely

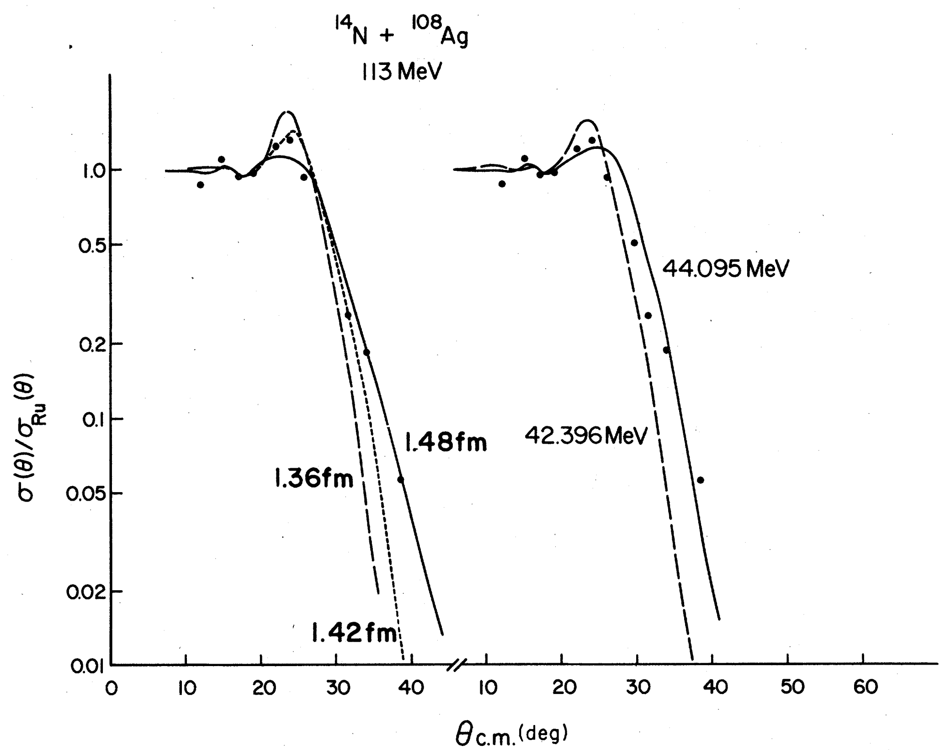

FIG. 1. Calculated and measured elastic scattering cross sections for various values of the parameters; (a) r_0 (with E_0 and $\hbar\omega_0$ fixed at 43.23 MeV and 4.0 MeV, respectively) and (b) E_0 (or r_0 and $\hbar\omega_0$ fixed at 1.42 fm and 4.0 MeV, respectively). Data points are from Ref. 27.

TABLE I. Systematics of barrier parameters and radii from elastic scattering.

Reaction	Z_1Z_2	E_{lab} (MeV)	r_{0B} [a] (fm)	r_e (fm)	r_r [b] (fm)	$V_N(R_r)$ (MeV)	$V_N(R_0)$ (MeV)	d (fm)	l_r ($\hbar$)	$l_{1/2}$ ($\hbar$)	Ref.
$^{12}C + ^{24}Mg$	72	24	1.68	1.65(+0.00) [c]	1.60	0.78	1.95	1.02	10.6	9.0	29
$^{16}O + ^{28}Si$	112	38	1.62	1.60(−0.02)	1.58	0.86	2.30	0.89	15.8	13.5	30
$^{16}O + ^{28}Si$	112	81	1.57	1.60(−0.02)	1.65	0.55	2.30	0.89	37.5	31.8	30
$^{13}C + ^{40}Ca$	120	40	1.62	1.60(−0.01)	1.61	0.71	2.37	0.92	22.4	19.1	2
$^{16}O + ^{52}Cr$	192	60	1.57	1.56(−0.02)	1.56	0.84	2.70	0.77	32.1	28.4	31
$^{32}S + ^{24}Mg$	192	110	1.58	1.57(−0.01)	1.58	0.90	2.96	0.79	33.8	29.6	32
$^{32}S + ^{27}Al$	208	110	1.56	1.55(−0.03)	1.57	0.76	2.76	0.71	36.4	32.0	32
$^{16}O + ^{60}Ni$	224	61.4	1.54	1.53(−0.04)	1.54	0.77	2.50	0.65	31.4	28.0	33
$^{20}Ne + ^{72}Ge$	320	95	1.57	1.54(+0.0)	1.56	1.06	3.68	0.76	52.8	46.8	34
$^{14}N + ^{108}Ag$	329	113	1.53	1.53(−0.01)	1.57	0.75	3.29	0.72	66.5	59.0	27
$^{16}O + ^{122}Sn$	400	73.9	1.54	1.52(−0.01)	1.52	1.23	3.57	0.70	37.4	33.7	33
$^{11}B + ^{208}Pb$	410	72.2	1.50	1.50(−0.02)	1.51	0.82	2.72	0.62	39.8	36.8	40
$^{11}B + ^{209}Bi$	415	74.6	1.51	1.50(−0.02)	1.51	0.81	2.69	0.60	41.9	38.5	41
$^{12}C + ^{181}Ta$	438	124.6	1.44	1.47(−0.05)	1.51	0.31	1.90	0.38	70.6	65.7	38
$^{12}C + ^{197}Au$	474	124	1.43	1.45(−0.06)	1.48	0.18	1.23	0.24	68.2	64.7	39
$^{18}O + ^{142}Nd$	480	98	1.58	1.54(+0.03)	1.55	1.47	4.84	0.87	59.8	53.0	35
$^{13}C + ^{207}Pb$	492	86.1	1.51	1.50(−0.02)	1.51	0.82	3.03	0.59	47.4	43.6	41
$^{12}C + ^{208}Pb$	492	96	1.50	1.50(−0.02)	1.52	0.74	3.05	0.59	54.3	49.6	41
$^{12}C + ^{208}Pb$	492	116.4	1.50	1.50(−0.02)	1.52	0.74	3.05	0.59	67.7	61.5	36
$^{14}N + ^{208}Pb$	574	147	1.46	1.47(−0.03)	1.50	0.42	2.38	0.40	84.4	78.6	38
$^{19}F + ^{159}Tb$	585	160	1.50	1.50(0.00)	1.53	0.83	3.68	0.58	96.3	88.0	37
$^{16}O + ^{197}Au$	632	164	1.48	1.48(−0.01)	1.49	0.90	3.12	0.48	93.3	87.9	39
$^{16}O + ^{208}Pb$	656	170	1.49	1.49(0.00)	1.52	0.73	3.60	0.58	100.1	92.0	38
$^{16}O + ^{208}Pb$	656	191.9	1.50	1.50(+0.01)	1.54	0.78	3.95	0.60	112.6	103.0	37
$^{16}O + ^{209}Bi$	664	170	1.48	1.48(−0.01)	1.51	0.66	3.23	0.49	98.6	91.4	38
$^{20}Ne + ^{208}Pb$	820	161.2	1.52	1.50(+0.03)	1.50	1.69	5.13	0.65	93.1	85.0	7
$^{20}Ne + ^{209}Bi$	830	207.6	1.45	1.46(−0.01)	1.48	0.40	2.34	0.30	117.6	111.6	39
$^{40}Ar + ^{109}Ag$	846	236	1.47	1.47(−0.01)	1.48	0.73	3.21	0.36	123.3	116.0	42
$^{40}Ar + ^{109}Ag$	846	337	1.47	1.47(−0.01)	1.50	0.55	3.21	0.36	176.6	165.0	42
$^{84}Kr + ^{65}Cu$	1044	494	1.44	1.44(−0.02)	1.46	0.32	1.75	0.17	157.7	151.0	42
$^{84}Kr + ^{65}Cu$	1044	604	1.43	1.44(−0.02)	1.47	0.17	1.75	0.17	194.0	187.0	42
$^{40}Ar + ^{209}Bi$	1494	286	1.42	1.43(0.00)	1.44	0.13	1.13	0.09	153.6	149.7	43
$^{40}Ar + ^{209}Bi$	1494	340	1.41	1.43(0.00)	1.45	0.07	1.13	0.09	193.0	188.0	43
$^{40}Ar + ^{238}U$	1656	286	1.42	1.42(0.00)	1.44	0.03	0.50	0.04	148.3	145.2	43
$^{40}Ar + ^{238}U$	1656	340	1.42	1.42(0.00)	1.44	0.02	0.50	0.04	190.9	187.8	43

[a] r_{0B} is the radius parameter corresponding to the Blair quarter-point radius for our calculation.

[b] r_r is the radius parameter corresponding to the classical rainbow radius R_r.

[c] In parentheses we give the difference between the value of r_e given here for the fit to elastic scattering and that from Eq. (12) (from reaction cross sections).

dependent on the relative energy of the colliding ions and the surface potential[8,28] and is independent of the inner curvature of the barrier $\hbar\omega_0$. At these large distances the critical ray giving rise to the rainbow feels only the weak nuclear attraction [$V_N(r)$ parametrized by Eqs. (23) and (24)]. Our calculations show that rather wide variations in curvature ($1 < \hbar\omega_0 < 7$ MeV) have practically no effect on the calculated value of $\theta_{1/4}$. The absorption effect on and near this ray is negligible. The small sensitivity to curvature is also characteristic of total reaction cross sections[11] at energies well above the barrier.

The sensitivity of $\theta_{1/4}$ to r_0' is of greater importance [Fig. 1(a)] and that to r_e' (or E_0') [Fig. 1(b)] is quite large. The effect on $\sigma_{el}(\theta)$ due to variation in r_0' is similar to that for variation in r_e'. The variation of either gives a variation in the nuclear potential. Therefore, one cannot expect to obtain a unique set of potential parameters from elastic scattering alone. We choose to fix r_0 at 1.42 fm to be consistent with measurements of σ_R. Then we vary r_e' to achieve a fit to elastic scattering for $\theta \lesssim \theta_{1/4}$. As we see in Table I, the values of r_e (column 5) from these fits are generally in each case within ±2% of those from Eq. (12) (column 5) and thus reasonable consistency is demonstrated between $V_N(R_0)$ and $V_N(R_r)$ in Eq. (23).

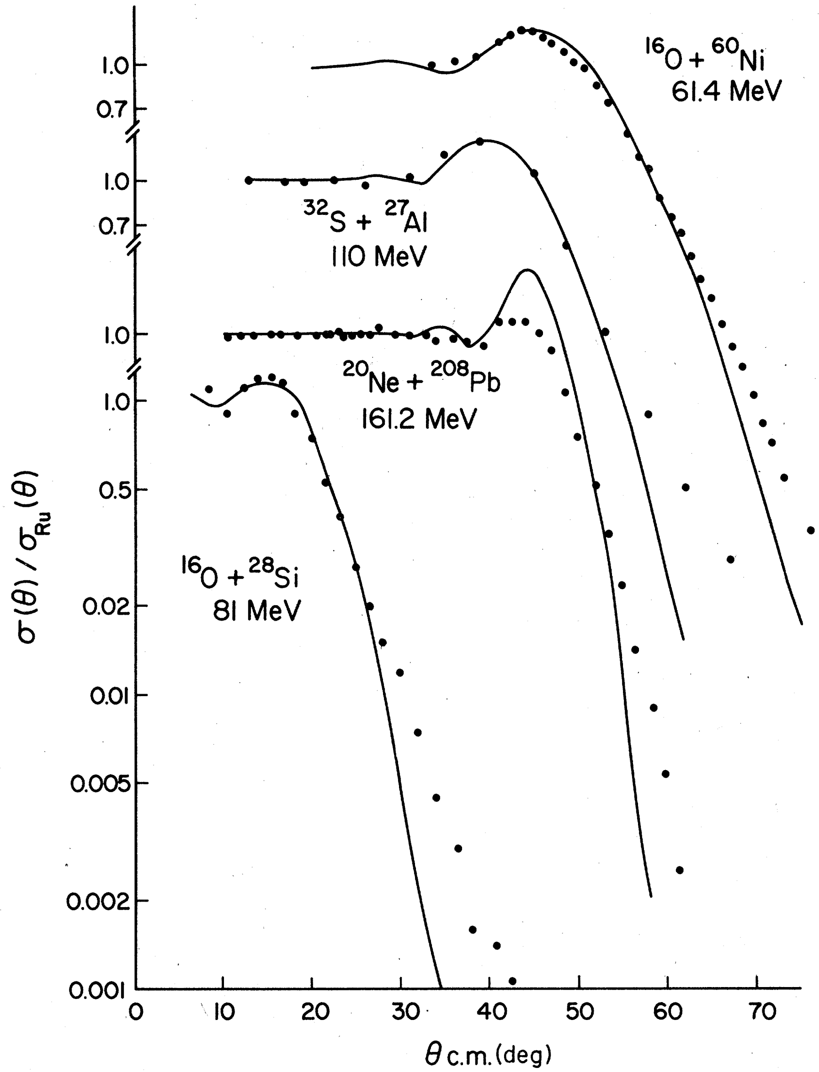

FIG. 2. Angular distributions for elastic scattering of ^{16}O by ^{60}Ni at 61.4 MeV (Ref. 33); ^{32}S by ^{27}Al at 110 MeV (Ref. 32); ^{20}Ne by ^{208}Pb at 161.2 MeV (Ref. 7) and O^{16} by ^{28}Si at 81 MeV (Ref. 30) compared with rainbow model calculations described in the text. The best fit potential parameters are shown in Table I.

B. Trend of results vs Z_1Z_2 and an interpretation of the empirical parameters

In Fig. 2 we show fits to elastic scattering data for a number of systems and in Table I we give the various relevant parameters for reaction systems analyzed.[2,7,27,29-43] The fits have focused on only those angles near $\theta_{1/4}$. There has been only one free parameter (E_0' or r_e') and thus it is not possible to get a close fit as is often achieved by multiparameter formulations. In each case the calculated cross sections for $\theta > \theta_{1/4}$ decrease more rapidly with angle than the measurements. Also the fits to oscillations in $\sigma(\theta)/\sigma_{Ru}(\theta)$ are far from optimal. These features are particularly sensitive to small variations in the transmission coefficients [or the complex potential $W(r)$] which we have not explored.[7] Our goal, however, is to look at the trend with Z_1Z_2 of the empirical values of R_0, $V_N(R_0)$, and $l_{1/2}$ as well as the rainbow parameters R_r, $V_N(R_r)$, and l_r to see if they have a message. In this context we have only allowed one parameter (E_0') to vary as we have sought a reasonable fit to the cutoff of elastic scattering near $\theta_{1/4}$.

First, as noted earlier the values of r_e that give a fit to the elastic scattering are generally within <2% of the starting values from empirical interaction barriers [Eq. (12)]. This indicates an overall consistency of the empirical parametrization of elastic scattering and reaction cross sections. Second, for $Z_1Z_2<1000$, the values of $V_N(R_r)$ (the nuclear potential at the rainbow radius) are consistent with those of Christensen and Winther as obtained from cross over points for real optical potentials.[6] This is also true for the values of l_r, (the rainbow l).

In Table I we have also listed the values of the Blair parameter r_{0B} (column 4) that result from our calculations of elastic scattering via the rainbow model [Eqs. (13) and (18)]. It turns out accidentally that these values are rather close to the empirically obtained r_e parameters that characterize the interaction barrier E_0 [Eq. (12)]. This statement seems to hold for all systems with both low and high Z_1Z_2. In Ref. 13 we have collected many of the measured values of r_{0B} and have compared them to the empirical values of r_e from reaction cross sections σ_R [Eqs. (5), (6), and (12)]. These data also show a close correlation between the Blair radii from scattering and the r_e parameters from reaction cross sections. We conclude that the term strong absorption radius for r_{0B} may be a misleading one for low Z_1Z_2 where the rainbow model seems to apply.[2,3] In the empirical approach used here and in Ref. 11 the mean absorption radius is approximated by R_0 which is smaller than r_{0B} $(A_1^{1/3}+A_2^{1/3})$ for $Z_1Z_2<1000$. Estimation of total reaction cross sections from measured values of $\theta_{1/4}$ does depend on this distinction between radius parameters; Eqs. (6) and (11) do not in general give the same result. This distinction was alluded to by Blair in 1957.

For $Z_1Z_2 \geq 1000$ a dramatic change sets in. The values of the empirical nuclear potential $V_N(R_0)$ and $V_N(R_r)$ (columns 7 and 8 in Table I) become quite small; the fractional differences diminish between l values for the rainbow trajectory l_r and for half absorption, $l_{1/2}$. This is the result of E_0 approaching $V_C(R_0)$ in the empirical real potentials, as also is the decrease of the apparent diffuseness parameter d to <0.3 fm (column 9). These results signal the increasing dominance by absorption as mentioned before.

Our interpretation of these results rests on the meaning of the empirical values of E_0. For $Z_1Z_2 \lesssim 500$ the empirical values of E_0 give good fits to scattering data and reasonable values of the real nuclear potentials and diffuseness parameters. Also the barriers obtained from reaction cross sections are very close to those from fusion cross sections. There is no difficulty in attributing to E_0 the significance of the height of the real barrier. Alternatively, one could say that the simplest rainbow model with little or no surface absorption provides a consistent picture. For $Z_1Z_2 \geq 1000$ the values of the nuclear potential $V_N(R_0)$ and $V_N(R_r)$ are quite small. The fit to $\theta_{1/4}$ is actually achieved by the transmission coefficients. Our choice of the parabolic real barrier in Eq. (22) does indeed force a rainbow angle on the calculated deflection function but the transmission coefficients force $l_{1/2}$ to be so close to l_r that rainbow and absorption effects become indistinguishable. One could say that E_0 is essentially a Coulomb potential at the mean absorption distance. For $500<Z_1Z_2<1000$ there is no gross problem with the rainbow model but absorptive effects seem to be appearing as signaled by decreases in the empirical values of d. It is clear that the very small values of d in Table I are simply an artifact of the empirically built potentials. How small should the empirical values of d become to signify totally dominant surface absorption? One can only answer by a more detailed study of the interplay of real and imaginary potentials as in Figs. 3 and 4 of the preceding paper.[9] Such studies require more experimental data than are generally available.

A comparison with the work of Ball *et al.*[7] is of particular interest in this transition region of $500<Z_1Z_2<1000$. They have made very careful multiparameter optical model fits to elastic scattering of ^{11}B, ^{12}C, ^{16}O, and ^{20}Ne from ^{208}Pb. Their best fits require a gradual increase in the ratio of imaginary to real potential from $^{11}\text{B}(Z_1Z_2=410)$ to $^{20}\text{Ne}(Z_1Z_2=820)$. Similarly they infer real barriers of 48.2 MeV at 11.4 fm for ^{11}B and 95.5 MeV at 11.35 fm for ^{20}Ne. In Table I we give values of 48.3 MeV at 11.6 fm for ^{11}B and 91.1 MeV at 12.3 fm for ^{20}Ne. We are very close for ^{11}B but significantly lower for ^{20}Ne. We believe that this difference is signaling the transition from weak surface absorption for $Z_1Z_2 \lesssim 500$ to strong surface absorption for $Z_1Z_2 \gtrsim 1000$.

There is an interesting divergence of perspective for high Z_1Z_2 between our study and that of Christensen and Winther.[6] The reaction systems treated are often the same and as mentioned earlier give consistent values of l_r for $Z_1Z_2<1000$. However, for 340 MeV ^{40}Ar on ^{209}Bi and ^{238}U our empirical values of E_0 give $l_{1/2}$ values of 193 and 191, respectively, while they give l_r values of 175 and 171. Their treatment would require values of total reaction cross sections $\approx 25\%$ lower than ours. This difference could be tested by careful experiments.

This point now brings us back to the original data used to fix the empirical values of E_0 and R_0 from reaction cross sections. A major question is: "What were the effective boundaries between scattering and reaction? Also from the theoretical side, what parts of the potential do these boundaries reflect?" Total reaction cross sections at energies well over the barrier give the mean interaction radius R_0 mostly obtained from the work of Wilkins and Igo.[44] Reaction cross sections as a function of energy give the mean interaction barrier E_0, a major source is the work of Viola and Sikkeland.[45] The trend of R_0 and E_0 values from these works gives us values of E_0, R_0 for ^{40}Ar + ^{209}Bi and ^{238}U that are consistent with the values of $\theta_{1/4}$ from the elastic scattering data of Birkelund *et al.*[43] (see Table I). Christensen and Winther[6] find that the real potentials $V_N(R_r)$ from the data of Birkelund *et al.*[43] "fall conspicuously outside the systematics" of comparisons to various ion-ion potentials. We conclude that the reason for this effect is the essentially complete blackening of the rainbow scattering by absorption. This conclusion is consistent with the small nuclear potentials favored by these authors and in other similar works.[43,46] In addition, Huizenga *et al.*[12] find a continuation of this trend for $1600 < Z_1Z_2 < 4800$ ($r_{0B} \approx 1.39$).

It should be emphasized that the body of data used both for reaction cross sections and elastic scattering does not have perfect energy resolution. In general inelastic (Coulomb and nuclear) scattering was not distinguished from elastic scattering unless the Q value was several MeV. That is to say that the effective boundary between reaction and scattering corresponded to Q values of several MeV and/or mass transfer of $\gtrsim 1$ u. We will discuss this point further in the next section.

The major point here seems to us to lie in the information content of these two separate kinds of measurements, elastic and reaction cross sections. Both seem to be useful and independent tools to probe the ion-ion potential.[44] In the rainbow scattering region the elastic data probe $V_N(R_r)$ (Ref. 6) while the fusion and reaction cross sections probe $V_N(R_0)$ and R_0.[9,14,47,48] It appears that our knowledge of the ion-ion potentials could be increased significantly by more of these very difficult measurements. However, this approach would not appear to be very fruitful for high Z_1Z_2 in the region that seems to be ruled by strong surface absorption.[4,43] Here the real nuclear potential seems to be heavily masked and only a mean absorption can be determined from either kind of measurement.

C. Inelastic scattering, transfer reactions, and fusion

Recently Thorn *et al.* have made elastic scattering measurements with very good energy resolution.[49] They show that Coulomb excitation drives the true elastic scattering cross sections away from the Rutherford values even for angles much less than $\theta_{1/4}$. In the context of relating nuclear reaction and scattering cross sections it is clearly better to include the coulomb exciting collisions with the truly elastic collisions. As the perturbation of the classical trajectory is generally quite small, this should not be a serious approximation.

Soft reactive collisions due to nuclear interactions are a matter of serious concern both for experiment and theory.[1,6,12] In each experimental study there is a different criterion to separate reaction from scattering. For elastic scattering of high energy heavy ions there is the well known problem of energy resolution for the beam and the detectors.[43] For total reaction cross section measurements there is of course, the same kind of problem.[44]

As elastic scattering measurements have been made more carefully and more soft reactions excluded from detection, the empirical values of the Blair quarter-point radius r_{0B} have increased.[13,21,43] For beam and detector energy resolutions currently available this time dependence continues to be a problem.[11,12,43] It should be emphasized that all the data we cite as evidence for strong surface absorption are derived from experiments using beams of ^{40}Ar and heavier ions from the Berkeley SuperHilac. If these data have significant errors due to energy resolution our conclusions could, of course, be altered.

Recent work by Videbaek *et al.*[10] is important here. They have measured cross sections for elastic scattering, soft reactive collisions, and fission for ^{16}O + ^{208}Pb and ^{16}O + ^{181}Ta. In the former they observe very large cross sections for ^{16}O in the exit channels, while in the latter reaction a much smaller contribution is observed. They feel that this difference may be due simply to their energy resolution and the number of low-lying states in the target. This problem is, of course, one that is inclusive of all the data used in this context, both for elastic scattering and reaction cross sections. It is probably true that in almost all cases most low Q value inelastic collisions with no mass transfer are included with the elastic scattering. Thus it is clear that an implicit assumption in any correlation of these data is that σ_R must exclude these very soft reactive collisions.

The most extensive set of data used in Ref. 11 for obtaining interaction barriers is that for fission cross sections.[45] Fission cross sections are much easier to measure than true total reaction cross sections and are thus appealing. (The fact that ^{238}U is statically deformed seems to have a very very small effect on empirical barrier parameters.[50]) The assumption is made that heavy ions only rarely undergo reactive collisions with ^{238}U that do not lead to fission. Clearly those reactions are excluded which deposit less energy than the fission barrier. As emphasized above essentially all the reaction cross section data available to us will also classify such soft inelastic collisions along with the true elastic scattering. Thus the implicit assumption is that the effective reaction cross section excludes inelastic scattering of Q values less than the typical experimental resolution. This assumption leaves us with a fuzzy boundary between scattering and reaction. This seems undesirable at first glance, but actually may be quite useful in removing much of the nuclear individuality associated with the differences in the low-lying level structure. Here we are searching for the overall trend with Z_1Z_2 and we wish to suppress this individuality.

In the text above we have shown for $100 < Z_1Z_2 < 1000$ the consistency of a simple rainbow model for elastic scattering and reaction cross sections. How do transfer reactions enter this picture and what effect do they have on the meaning of the empirical values of E_0, R_0? If transfer reactions are rare then the fusion barrier and the interaction barrier will coincide. If transfer reaction probability depends only on the distance of approach and not on l or E, then the interaction barriers will reflect the real potential at the mean radius for onset of all reactions.[9]

The recent extensive study of Scobel *et al.*[51] is interesting in this context. They measure fusion cross sections and obtain values of E_{int} and R_{slope} from Eq. (1). In some cases they compare values of R_{slope} from Eq. (1) with values of $R_{1/2}$ which they have deduced from elastic scattering. As R_{slope} is typically $\approx 15\%$ less than $R_{1/2}$ they conclude that $\approx 30\%$ of the reaction cross section goes to transfer reactions with closest interaction distances greater than $R_m(l=0)$. We question this conclusion; consider the following two points regarding the details of obtaining radius parameters: (1) As discussed above the calculation of the reaction cross section is strongly model dependent and the use of Eq. (11) or a strongly absorptive optical potential can lead to large errors.[10,11,13] For $Z_1Z_2 < 500$ the apparent difference between $R_{1/2}$, $D_{1/2}$ from Ref. 51 and R_{slope} may be largely due to this source. (2) Eq. (1) depends on the assumption that fusion occurs at a fixed distance, R_{slope}. The assumption that it is cut off by the top of the real barrier [Eqs. (7), (8)] can lead to a significant change in the interpretation of empirical values of R_{slope} obtained from Eq. (1) (see Ref 9). The barrier height parameters seem to us to be more indicative; it appears that the fusion barriers are indistinguishable from the empirical reaction barriers [E_0 in Eq. (6)] for $Z_1Z_2 \lesssim 500$ and one may therefore infer that the interaction radius is nearly indistinguishable from the s-wave fusion radius. However, for $Z_1Z_2 > 2000$ the fusion cross sections are much less than the reaction cross sections and one may say that the interaction barrier is much lower than the fusion barrier. We conclude that empirical interaction barriers E_0, R_0 reflect the real potential near to its maximum for low Z_1Z_2 and near to the strong absorption radius for high Z_1Z_2.

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