PHYSICAL REVIEW C VOLUME 18, NUMBER 1 JULY 1978

Reduced normalizations from (d,p) reactions and from elastic proton scattering on isobaric analog resonances

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(Received 1 March 1978)

The determination of reduced normalizations by measuring (d,p) reactions in the energy region of sub-Coulomb and quasi-Coulomb stripping with an experimental accuracy which has not been attained in former investigations allows reasonable comparisons with corresponding reduced normalizations of analog states obtained by elastic proton scattering via isobaric analog resonances. This comparison includes 38 states with known spins and parities formed by adding a further nucleon to the nuclei ^{124}Sn, ^{130}Te, ^{138}Ba, ^{140}Ce, ^{142}Nd, and ^{208}Pb. The reduced normalizations of the proton analog states are based on the experimental partial widths given by other authors and on own calculations according to two different shell model theories of isobaric analog resonances. The uncertainties still remaining in the theoretical description of some effects inherent in the direct reaction mechanism of transfer reactions as well as in the shell model reaction theories do not permit a unique decision concerning the validity of the different theoretical approaches in determining the decay widths of single particle states.

[NUCLEAR REACTIONS ^{124}Sn, ^{130}Te, ^{138}Ba, ^{140}Ce, ^{142}Nd, ^{208}Pb(d,p), (p,p) calculated reduced normalizations of 38 parent and analog states, respectively, based on experimental data as well as on DWBA and two shell model reaction approaches; comparison and discussion of results.]

I. INTRODUCTION

The spectroscopic factors (SF) defined for the bound neutron states and the proton analog scattering states should be equal in those cases where the isospin is a good quantum number.[1,2] This is still a good assumption for the low-lying states of medium heavy and heavy nuclei with a high single particle fraction. But numerous comparisons especially of SF attained from (d,p) reactions and from elastic proton scattering via isobaric analog resonances provided results differing to a considerable amount.[3-6] These discrepancies are caused not only by experimental errors but also by theoretical uncertainties in the description of the reaction mechanisms involved.

In recent years the extraction of SF from transfer reactions performed in the sub-Coulomb and quasi-Coulomb energy region became more important, because in this case the theoretical uncertainties of the scattering wave functions needed in the distorted-wave Born approximation (DWBA) formalism could be reduced significantly. Moreover, with the definition of the reduced normalizations (RN) the unsatisfactory dependence of the SF on the parameters of the single particle bound state potential was overcome.[7-12]

Clarkson *et al.*[13,14] have shown that in the analysis of the elastic proton scattering at isobaric analog resonances a corresponding RN can be defined which also depends much less on the single particle potential chosen than the SF of the particular state. However, the RN obtained are still based on the theoretical assumptions in calculating the single particle decay widths and still depend on the parameters of the optical potentials determining the scattering wave functions of entrance and exit channels. Because the channel energies of the resonant proton scattering are fixed, it is impossible to reduce the influence of the nuclear potentials by decreasing the incident energy as in the case of (d,p) reactions (see e.g. Ref. 12).

The objective of this work was to recognize systematic deviations of the results of (d,p) measurements from those of elastic proton scattering. The measurements of (d,p) reactions in the energy region of sub-Coulomb and quasi-Coulomb stripping provided RN of the parent states in ^{125}Sn, ^{131}Te, ^{139}Ba, ^{141}Ce, ^{143}Nd, and ^{209}Pb with an accuracy which has not been attained in former analysis.[12]

The spectroscopic quantities of the analog proton states in ^{125}Sb, ^{131}I, ^{139}La, ^{141}Pr, ^{143}Pm, and ^{209}Bi are based on the experimental partial widths reported by other authors[4-6,15-29] and on own calculations of the theoretical decay widths with the shell model reaction approaches of Zaidi, Darmodjo, and Harney (ZDH theory)[3,30,31] and of Mekjian and MacDonald (MM theory)[32] according to the interpretations and methods given by Harney and Weidenmüller[33] and by Clarkson *et al.*[13,14] The comparisons are restricted to the states with known j^{π} values and with sufficiently large single

particle fractions. Up to now, comparisons of this kind only exist for low-lying parent states in nuclei near mass numbers 90 and 140 (Refs. 34 and 35). But because of the high accuracy of the RN extracted from own (d,p) measurements,[12] it is worthwhile to continue and to extend these comparisons.

In the next section the fundamental formulas for the determination of the SF and RN in the case of elastic proton scattering at isobaric analog resonances are given. Section III provides the results of numerical calculations for the states of interest. The comparison with the neutron RN obtained from (d,p) reactions in the sub-Coulomb and quasi-Coulomb region is performed in Sec. IV. Finally, the results and conclusions are discussed in regarding particularly the theoretical uncertainties still remaining.

II. THEORY

A summary of the theoretical basis for the analysis of (d,p) reactions in the energy region of sub-Coulomb and quasi-Coulomb stripping and the definitions of the neutron SF S_{lj}^{n} and the neutron RN Λ_{lj}^{n} (according to Refs. 7–11) were given in a former paper.[12] There the advantages of the neutron RN were demonstrated extensively.

Clarkson *et al.*[13,14] have shown that it is possible to define a corresponding proton RN in order to describe the single particle fraction of isobaric analog states. It seems useful to present a short survey on the essential formulas leading to the definition of the proton RN in order to understand the difficulties discussed later on.

The analysis of the elastic proton scattering at isobaric analog resonances is commonly based on an energy averaged S-matrix element. Independent of the applied reaction theory, i.e., the R-matrix theory,[36,37] the shell model theory,[32,38] or the projection method,[39,40] the S-matrix element for the elastic proton scattering through an isolated analog resonance superimposed on a smooth background is, taking into account certain assumptions, given by

$$S_{lj,lj} = \exp[2(i\delta_{lj} - \eta_{lj})] \times \left[1 - i\frac{\exp(2i\phi_{lj})\tilde{\Gamma}_{lj}}{E - E_R - i\Gamma/2}\right], \tag{1}$$

where $\exp(2i\delta_{lj} - 2\eta_{lj})$ describes the nonresonant, weakly energy dependent scattering term, ϕ_{lj} is the resonance mixing phase, $\tilde{\Gamma}_{lj}$ is the partial width including the absorptive part $\exp(-2\eta_{lj})$, E is the incident energy, E_R the resonance energy, and Γ the total width of the resonance.

The assumptions referring to the residual interaction and the statistical distribution of the decay widths of the more complicated states in the shell model reaction theory have been discussed in detail by Harney and Weidenmüller[33] and by Mahaux.[41]

The nonresonant background is described either by an appropriate optical potential or in a model independent way using simply a polynomial. In both cases the parameters are extracted by fitting the theoretical distribution to the experimental data.

The relevant spectroscopic quantity is the partial width Γ_{lj}. Γ_{lj} and ϕ_{lj} are defined by the complex transition amplitude γ_{lj} between the analog state and the continuum state $\psi_{E,lj}$

$$\exp(2i\phi_{lj})\Gamma_{lj} = 2\pi\gamma_{lj}^{2}. \tag{2}$$

Generally, γ_{lj} depends on the isospin violating parts of the Hamiltonian operator

$$\gamma_{lj} = \langle\psi_{E,lj}|[H, T^{-}]|\psi_{lj}^{\mathrm{PS}}\rangle, \tag{3}$$

where the parent state ψ_{lj}^{PS} is a solution of the eigenvalue equation $H\psi_i^{\mathrm{PS}} = E_i\psi_i^{\mathrm{PS}}$. The continuum wave function $\psi_{E,lj}$ is assumed to be orthogonal to the wave function of the analog state ψ^{A}. The transition amplitude γ_{lj} may be split into a direct part $\gamma_{lj}^{\mathrm{dir}}$ and an amplitude describing the interaction via a compound nucleus $\gamma_{lj}^{\mathrm{comp}}$. Auerbach *et al.*[39] have shown that in most cases the assumption $\gamma_{lj}^{\mathrm{comp}} \ll \gamma_{lj}^{\mathrm{dir}}$ is justified. Then the interaction potential in (3) is dominated by the Coulomb potential.

Regarding the parent state as formed by a single particle coupled to a 0^{+} core one obtains the single particle decay width $\Gamma_{lj}^{\mathrm{sp}}$,

$$(\Gamma_{lj}^{\mathrm{sp}})^{1/2} = \left(\frac{2\pi}{2T+1}\right)^{1/2} e^{-i\phi_{lj}} \times \int_0^{\infty} \tilde{\psi}_{E,lj}(r)\, V_c(r)\, \tilde{\varphi}_{lj}(r)\, r^2\, dr, \tag{4}$$

where $\tilde{\psi}_{E,lj}(r)$ is the radial part of the proton scattering wave function which is orthogonal to the analog state ψ^{A}, $\tilde{\varphi}_{lj}(r)$ is the radial part of the single particle neutron wave function, and $V_c(r)$ is the average Coulomb potential of the proton in the analog state.

In this work the theoretical widths are defined in a way that allows a direct comparison with the widths extracted from experimental data, i.e., the factor $\exp(-2\eta_{lj})$ is included in the given $\Gamma_{lj}^{\mathrm{sp}}$.

Even though the neutron of the parent state is coupled to a 0^{+} core nucleus, this state is actually not a pure single particle state due to the influence of the residual interaction. This fact is taken into account by introducing the spectroscopic factor S_{lj}^{p} for the proton decay width of the isobaric analog state into Eq. (4):

$$(\Gamma_{lj})^{1/2}=\left(\frac{2\pi}{2T+1}\right)^{1/2}(S_{lj}^{p})^{1/2}e^{-i\phi_{lj}} \times\int_0^\infty \tilde{\psi}_{E,lj}(r)\,V_c(r)\varphi_{lj}(r)r^2dr. \quad (5)$$

Because the experimental partial width $\Gamma_{lj}^{\exp}$ should be equal to this Γ_{lj}, the SF S_{lj}^{p} is simply given by the relation

$$S_{lj}^{p}=\Gamma_{lj}^{\exp}/\Gamma_{lj}^{sp}. \quad (6)$$

Harney and Weidenmüller[33] have shown that in the framework of the ZDH theory Eq. (4) is equivalent to

$$(\Gamma_{lj}^{sp})^{1/2}=\left(\frac{2\pi}{2T+1}\right)^{1/2}e^{-i\phi_{lj}}\langle\psi_{E,lj}|\,V^n-V^p-iW|\varphi_{lj}\rangle\,, \quad (7)$$

while the method of MM in considering the terms of the absorptive potential to the first order yields

$$(\Gamma_{lj}^{sp})^{1/2}=\left(\frac{2\pi}{2T+1}\right)^{1/2}e^{-i\phi_{lj}}\langle\psi_{E,lj}|\,V^n-V^p|\varphi_{lj}\rangle\,. \quad (8)$$

In Eqs. (7) and (8) the scattering wave function $\psi_{E,lj}$ is the solution of the radial Schrödinger equation

$$\left\{-\frac{\hbar^2}{2m_v}\left[\frac{d^2}{dr^2}-\frac{l(l+1)}{r^2}\right]+V_c(r)+V^p(r) +iW(r)-E_p\right\}r\psi_{E,lj}(r)=0\,, \quad (9)$$

where m_v is the reduced mass of the system (p +target), $V_c(r)$ is the Coulomb potential, $V^p(r)$ is the real part of the optical scattering potential consisting of a volume and a spin-orbit term, $W(r)$ is the imaginary part of the optical potential, and E_p is the proton energy in the elastic channel specified by the quantum numbers (l,j).

$\varphi_{lj}(r)$ is the radial solution of the Schrödinger equation of the bound system (n+residual nucleus):

$$\left\{-\frac{\hbar^2}{2m_\mu}\left[\frac{d^2}{dr^2}-\frac{l(l+1)}{r^2}\right]+V^n(r)+E_B\right\}r\varphi_{lj}(r)=0\,, \quad (10)$$

where m_μ is the reduced mass of the system (n +residual nucleus), $V^n(r)$ is the real nuclear potential consisting of a volume and a spin-orbit term, E_B is the binding energy of the neutron in the state (l,j).

The proton scattering wave function $\psi_{E,lj}$ is not orthogonal to the analog state wave function. For that reason the form equivalent to the ZDH matrix element is not only Eq. (7) but also

$$(\Gamma_{lj}^{sp})^{1/2}=\left(\frac{2\pi}{2T+1}\right)^{1/2}e^{-i\phi_{lj}}\langle\psi_{E,lj}|\,V_c-\Delta^C|\varphi_{lj}\rangle\,, \quad (11)$$

where Δ^C is the Coulomb energy difference between the parent state and the analog state.

In both methods, i.e., according to the approaches of ZDH and MM, respectively, the results of numerical calculations depend on the parameters of the potentials used. Unfortunately, these parameters cannot be determined uniquely. But Clarkson *et al.*[13,14] have shown that the dependences on the potential parameters can be reduced if one restricts the calculations to asymptotic quantities.

For the scattering wave the phase shift $\delta_{lj}+i\eta_{lj}$ is the most important asymptotic quantity, while the bound neutron wave function may be approximated in the asymptotic region by the Hankel function of the first order. The latter approximation permits a factorization of the width Γ_{lj}^{sp} in the reduced single particle normalization Λ_{lj}^{sp} and the reduced matrix element G_{lj}^{sp},

$$\Gamma_{lj}^{sp}=\Lambda_{lj}^{sp}G_{lj}^{sp}. \quad (12)$$

The reduced normalization of the actual proton analog state is defined as

$$\Lambda_{lj}^{p}=\Lambda_{lj}^{sp}S_{lj}^{p} \quad (13)$$

and corresponds to the reduced normalization Λ_{lj}^{n} of the parent state which may be e.g. extracted from sub-Coulomb and quasi-Coulomb transfer reactions.

The RN Λ_{lj}^{p} are determined according to Eqs. (6) and (13) by using the experimental partial widths $\Gamma_{lj}^{\exp}$ and the theoretical values of Γ_{lj}^{sp} and Λ_{lj}^{sp} which are calculated according to a particular theory.

The dependence of the Λ_{lj}^{p} on the parameters of the bound state potential is not reduced as drastically as in the case of (d,p) stripping because the energy of the incident proton is fixed and is in most cases near but above the Coulomb barrier. But there is still an advantage using the RN. This is demonstrated in investigating the dependence of the spectroscopic quantities on the single particle potential at the five low-lying analog states in ^{139}La (see Sec. III).

III. REDUCED NORMALIZATIONS FROM ELASTIC PROTON SCATTERING THROUGH ISOBARIC ANALOG RESONANCES

The experimental partial widths used in this work have been reported by other authors,[4-6,15-29] while the appropriate single particle decay widths Γ_{lj}^{sp} were obtained by calculations with the computer code BETTINA[42] according to the shell model theories of ZDH and MM, using Eqs. (11) and (8),

respectively. The same program provides the reduced single particle normalizations Λ_{ij}^{sp}.

The potentials needed in these calculations for the determination of the wave functions as well as the interaction potentials are given in the following subsection. Then the results are presented and finally the dependence of the SF and RN on the radius of the single particle bound state potential is studied.

A. Potentials

The nuclear potentials required in the formulas of Sec. II are chosen to have the following form:

$$V^{p,n}(E_{p,n}, r) = V_V^{p,n}(E_{p,n}, r) + V_S(r) \quad \text{real potential,}$$

$$W(E_p, r) = 4a_W U_W(E_p) \frac{d}{dr} f(r, R_W, a_W)$$

imaginary potential, (14)

$$V_C(r) = \begin{cases} \dfrac{Ze^2}{2R_C}\left[3 - \left(\dfrac{r}{R_C}\right)^2\right], & r \leq R_C \\ \dfrac{Ze^2}{r}, & r \geq R_C \end{cases} \quad \text{Coulomb potential,}$$

with

$$V_V^p(E_p, r) = U_V^p(E_p) f(r, R_V, a_V) \quad \text{volume potential,}$$

$$V_V^n(E_n, r) = U_V^n(E_n) f(r, R_V, a_V) \quad \text{volume potential,}$$

$$V_S(r) = U_S^0 \left(\frac{\hbar}{m_\pi c}\right)^2 (\vec{\sigma} \cdot \vec{l}) \frac{1}{r} \frac{d}{dr} f(r, R_S, a_S)$$

spin-orbit potential,

and

$$U_V^p(E_p) = U_V^0 - \Delta U_V E_p ,$$

$$U_W(E_p) = U_W^0 - \Delta U_W E_p ,$$

where U_V, U_W, and U_S^0 are the potential depths, E_p is the proton energy, ΔU_V and ΔU_W are the slopes of U_V and U_W with respect to E_p, $f(r, R_x, a_x) = [1 + \exp(R - R_x/a_x)]^{-1}$ determines the Woods-Saxon form of the potentials, $R_x = r_x A^{1/3}$ are the radii, and a_x the diffusenesses of the various potentials. Z is the charge number, A is the mass number, $(\vec{\sigma} \cdot \vec{l})$ is the scalar product of the spin operator with the orbital angular momentum operator, and m_π is the pion mass.

The free choice of the parameters is restricted by two conditions in order to reduce the number of parameters:

(a) $R_V = R_W$,

(b) $a_V = a_W = a_S$.

The application of an average proton scattering potential as reported by Perey and Perey[43] or Becchetti and Greenlees[44] cannot be recommended. Using this kind of a potential to describe the background of the reaction ^{140}Ce(p,p_0) in the whole energy region from 9.5 to 12.5 MeV, Schulze-Döbold[25] obtained deviations in the order of 5–30% directed to lower cross sections as compared to the experimental data. Therefore, we applied specific potentials which were fitted by other authors or by our own calculations to the data describing the nonresonant part of reaction cross sections in the particular energy region of analog resonances.

In the analysis of the data of elastic proton scattering at ^{124}Sn, ^{138}Ba, and ^{208}Pb by Darmodjo *et al.*[4] the radii of the real and the imaginary part and the diffusenesses of the various real potential parts have been set equal, respectively. As the differences in the diffusenesses of the real and the imaginary potentials describing the reactions ^{138}Ba(p,p_0) and ^{208}Pb(p,p_0) are very small, they were set equal according to condition (b). In the case of ^{124}Sn the diffusenesses given in Ref. 4 deviate significantly from each other and the fit is unsatisfactory at lab angle $\theta = 90°$. Therefore, we extracted a new scattering potential taking into account the conditions (a) and (b) by fitting the potential parameters of Eq. (14) to the background scattering data of Ref. 4 with the parameter search code MOM3.[45] This code provides more reliable potential parameters since it permits the simultaneous fitting of energy and angular distributions of the experimental data.

The potential given by Hiddleston *et al.*[5] for the elastic proton scattering at ^{130}Te satisfies the condition (b), but with respect to the radii R_V and R_W there exists a slight difference. In this work R_W was set equal to the other radii causing a negligible inaccuracy.

The optical potentials of the reactions ^{140}Ce(p,p_0) and ^{142}Nd(p,p_0) were obtained by fitting the potential parameters of Eq. (14) to the background cross sections reported by Marquardt *et al.*[24] and by Grosse *et al.*,[27] respectively, using the code MOM3.[45] A compilation of the potential parameters used in this work is given in Table I.

The parameters of the single particle neutron potentials were taken from the appropriate proton scattering potentials with the exception of the depths of the volume terms which were adjusted according to the binding energy condition. Hence, the interaction potential in Eq. (8) is approximately of volume type.

The Coulomb energy difference Δ_{ij}^C required in the calculation of the single particle width in the ZDH theory [see Eq. (11)] is the sum of the neutron binding energy and the proton resonance energy. The Coulomb potential in Eq. (11) was used in the

TABLE I. Parameters of the optical potentials according to Eq. (14) as used in this work for describing elastic proton scattering. It is assumed that $\Delta U_W = 0$ and $a_R = a_W = a_S = a_x$.

Target Nucleus	U_V^0 (MeV)	ΔU_V [E_p in (MeV)]	r_V, r_W (fm)	a_x (fm)	U_S^0 (MeV)	r_S (fm)	U_W^0 (MeV)	r_C (fm)	Energy range of fitting (MeV)	Refs.
^{124}Sn	61.6	0.6	1.245	0.700	8.5	1.245	13.2	1.210	8.0-11.0	This work, Data from Ref. 4
^{130}Te	63.0	0.5	1.220	0.670	7.5	1.220	11.0	1.250	7.8-12.0	Ref. 5
^{138}Ba	63.4	0.4	1.230	0.650	5.8	1.230	10.0	1.230	9.5-12.5	Ref. 4
^{140}Ce	64.1	0.6	1.230	0.680	4.4	1.230	8.7	1.230	9.8-11.2	This work, Data from Ref. 2
^{142}Nd	64.1	0.6	1.230	0.650	3.6	1.230	7.1	1.200	9.5-11.1	This work, Data from Ref. 27
^{208}Pb	66.35	0.4	1.190	0.750	5.8	1.190	10.2	1.190	14.5-18.0	Ref. 4

same form as in the scattering potential, i.e., as the potential of a homogeneous charged sphere.

The Coulomb energy condition given by Clarkson *et al.*[13,14] was not used to determine the radius R_C of the Coulomb potential. For this method one requires good knowledge of the neutron bound state potential radius which is unfortunately the most uncertain parameter.

B. Results

The resonance energies, j^π values of resonances, mean values of experimental partial widths as well as the results of our calculations, i.e., the reduced single particle normalizations, single particle partial widths, spectroscopic factors, and reduced normalizations, are presented in Table II. The calculations have been performed using the computer code BETTINA.[42]

The SF and RN were determined according to Eqs. (6) and (13), respectively. The experimental partial widths given are the arithmetic means of those ones published in the literature. Because of the known difficulties in extracting partial widths from the elastic proton scattering data, their total error is in the order of 10 to 30%. This error may be reduced by taking into account the results of inelastic proton scattering or of polarization experiments, as has been done e.g. in the works of Refs. 20, 22, and 25.

TABLE II. Spectroscopic information for elastic proton scattering on target nuclei ^{124}Sn, ^{130}Te, ^{138}Ba, ^{140}Ce, ^{142}Nd, and ^{208}Pb via isobaric analog resonances and analysis according to shell model theories of MM and ZDH (see text).

(a) ^{124}Sn (p, p_0)

E_R^{lab} [a] (MeV)	j^π [b]	Γ_p^{exp} [c] (keV)	Λ_{lj}^{sp}	Γ_{lj}^{sp} (keV) MM	Γ_{lj}^{sp} (keV) ZDH	SF_{lj}^p MM	SF_{lj}^p ZDH	Λ_{lj}^p MM	Λ_{lj}^p ZDH
7.944	$\frac{3}{2}^+$	7.6	145.2	14.9	25.7	0.51	0.30	74.3	42.9
8.130	$\frac{1}{2}^+$	12.7	755.3	41.8	71.7	0.30	0.18	229.3	133.8
10.670	$\frac{7}{2}^-$	23.0	5.027	15.5	49.2	1.49	0.47	7.47	2.35

[a] Reference 4.
[b] References 15 and 16.
[c] Mean values according to Refs. 4, 15, and 16.

TABLE II. (*Continued*)

(b) ^{130}Te (p, p_0)

E_R^{lab} [a] (MeV)	j^π [b]	Γ_p^{exp} [c] (keV)	Λ_{lj}^{sp}	Γ_{lj}^{sp} (keV) MM	Γ_{lj}^{sp} (keV) ZDH	SF_{lj}^p MM	SF_{lj}^p ZDH	Λ_{lj}^p MM	Λ_{lj}^p ZDH
8.039	$\frac{3}{2}^+$	6.17	151.1	17.1	23.5	0.36	0.26	54.5	39.7
8.346	$\frac{1}{2}^+$	10.1	720.8	54.9	76.1	0.18	0.13	132.6	95.7
10.288	$\frac{7}{2}^-$	17.4	9.17	17.8	36.9	0.98	0.47	8.96	4.32
10.538	$\frac{3}{2}^-$	5.48	138.9	23.9	70.5	0.23	0.078	31.8	10.8
10.581	$\frac{3}{2}^-$	13.1	134.7	24.5	71.3	0.53	0.18	72.0	24.7
10.989	$\frac{1}{2}^-$	17.8	92.4	15.8	67.1	1.13	0.27	104.1	24.5

[a] Reference 5.
[b] References 18 and 20.
[c] Mean values according to Refs. 5, 18, 19, and 20.

(c) ^{138}Ba (p, p_0)

E_R^{lab} [a] (MeV)	j^π [b]	Γ_p^{exp} [c] (keV)	Λ_{lj}^{sp}	Γ_{lj}^{sp} (keV) MM	Γ_{lj}^{sp} (keV) ZDH	SF_{lj}^p MM	SF_{lj}^p ZDH	Λ_{lj}^p MM	Λ_{lj}^p ZDH
10.004	$\frac{7}{2}^-$	16.3	25.5	19.1	29.2	0.85	0.56	21.8	14.2
10.631	$\frac{3}{2}^-$	26.3	236.1	46.6	82.8	0.56	0.32	133.2	75.1
11.088	$\frac{1}{2}^-$	21.9	161.7	42.7	83.1	0.51	0.26	82.9	42.6
11.315	$\frac{9}{2}^-$	1.40	0.0268	1.88	2.57	0.74	0.54	0.0200	0.0146
11.427	$\frac{5}{2}^-$	9.88	5.58	25.0	44.6	0.39	0.22	2.20	1.24
11.717	$\frac{5}{2}^-$	6.05	4.07	28.9	50.1	0.21	0.12	0.853	0.492

[a] Reference 4.
[b] References 4, 21, and 22.
[c] Mean values according to Refs. 4, 6, 21, and 22.

(d) ^{140}Ce (p, p_0)

E_R^{lab} [a] (MeV)	j^π [b]	Γ_p^{exp} [c] (keV)	Λ_{lj}^{sp}	Γ_{lj}^{sp} (keV) MM	Γ_{lj}^{sp} (keV) ZDH	SF_{lj}^p MM	SF_{lj}^p ZDH	Λ_{lj}^p MM	Λ_{lj}^p ZDH
9.751	$\frac{7}{2}^-$	11.3	48.9	13.3	21.9	0.85	0.52	41.6	25.2
10.405	$\frac{3}{2}^-$	23.7	402.6	35.5	70.4	0.67	0.34	269.1	135.5
10.882	$\frac{1}{2}^-$	18.9	282.8	33.2	72.9	0.57	0.26	161.1	73.3
11.135	$\frac{9}{2}^-$	1.19	0.0769	1.64	2.29	0.72	0.52	0.0557	0.0400
11.251	$\frac{5}{2}^-$	8.08	12.0	20.8	39.9	0.39	0.20	4.65	2.42
11.536	$\frac{7}{2}^-$	4.38	10.9	38.3	57.2	0.11	0.077	1.24	0.833
11.911	$\frac{5}{2}^-$	3.50	6.37	29.3	52.3	0.12	0.067	0.762	0.427

TABLE II. (*Continued*)

(d) (*Continued*)

E_R^{lab} [a] (MeV)	j^π [b]	Γ_p^{exp} [c] (keV)	Λ_{lj}^{sp}	Γ_{lj}^{sp} (keV) MM	ZDH	SF_{lj}^p MM	ZDH	Λ_{lj}^p MM	ZDH
12.172	$\frac{3}{2}^-$	11.7	113.5	65.7	108.1	0.18	0.11	20.2	12.3
12.217	$\frac{1}{2}^-$	9.55	108.0	54.0	98.4	0.18	0.097	19.1	10.5

[a] References 22, 24, and 25.
[b] References 22, 23, and 25.
[c] Mean values according to Refs. 22-25.

(e) ^{142}Nd (p, p_0)

E_R^{lab} [a] (MeV)	j^π [b]	Γ_p^{exp} [c] (keV)	Λ_{lj}^{sp}	Γ_{lj}^{sp} (keV) MM	ZDH	SF_{lj}^p MM	ZDH	Λ_{lj}^p MM	ZDH
9.495	$\frac{7}{2}^-$	10.5	77.0	11.3	16.3	0.93	0.64	71.5	49.6
10.228	$\frac{3}{2}^-$	23.5	569.1	37.3	65.1	0.63	0.36	358.5	205.4
10.797	$\frac{1}{2}^-$	22.7	389.0	39.1	72.0	0.58	0.32	252.8	122.6
10.922	$\frac{9}{2}^-$	0.78	0.175	1.48	1.78	0.53	0.44	0.0922	0.0767
11.051	$\frac{5}{2}^-$	5.96	21.0	23.3	36.1	0.26	0.17	5.37	3.47
11.395	$\frac{3}{2}^-$	4.60	281.2	61.0	94.5	0.075	0.049	21.2	13.7
11.448	$\frac{5}{2}^-$	7.30	15.4	28.6	43.5	0.26	0.17	3.92	2.58

[a] References 25 and 27.
[b] References 25 and 26.
[c] Mean values according to Refs. 25 and 27.

(f) ^{208}Pb (p, p_0)

E_R^{lab} [a] (MeV)	j^π [b]	Γ_p^{exp} [c] (keV)	Λ_{lj}^{sp}	Γ_{lj}^{sp} (keV) MM	ZDH	SF_{lj}^p MM	ZDH	Λ_{lj}^p MM	ZDH
14.96	$\frac{9}{2}^+$	20.8	5.41	16.3	28.7	1.28	0.72	6.92	3.89
15.72	$\frac{11}{2}^+$	1.95	3.56×10^{-3}	1.25	2.09	1.56	0.93	5.55×10^{-3}	3.32×10^{-3}
16.34	$\frac{15}{2}^-$	0.3	63.9×10^{-6}	0.246	0.988	1.22	0.30	77.9×10^{-6}	19.4×10^{-6}
16.51	$\frac{5}{2}^+$	44.4	35.9	36.8	68.3	1.21	0.65	43.3	23.3
16.96	$\frac{1}{2}^+$	45.2	140.0	37.2	70.6	1.22	0.64	170.1	89.6
17.44	$\frac{7}{2}^+$	29.6	59.0×10^{-3}	21.9	44.1	1.35	0.67	79.7×10^{-3}	39.6×10^{-3}
17.52	$\frac{3}{2}^+$	38.9	7.97	32.5	65.6	1.20	0.59	9.54	4.73

[a] Reference 4.
[b] Reference 28.
[c] Mean values according to Refs. 4, 28, and 29.

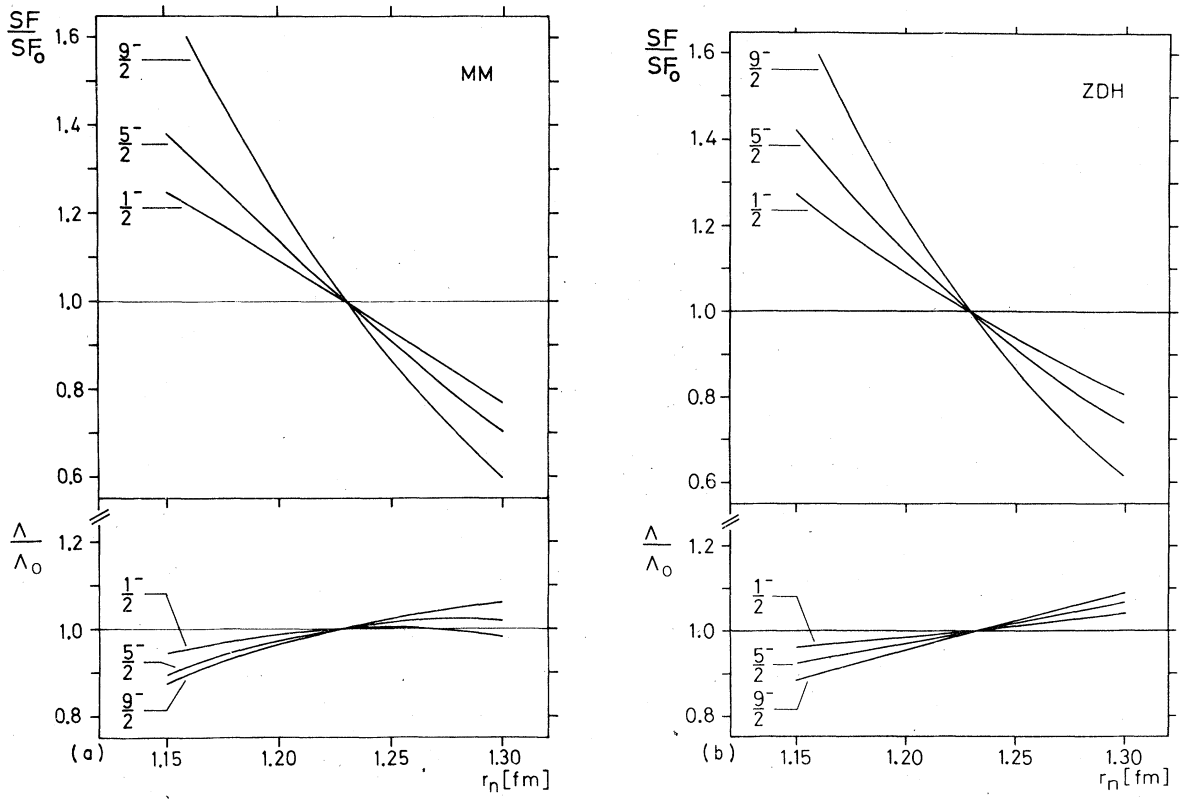

FIG. 1. Dependence of spectroscopic factors SF^{p}_{lj} and reduced normalizations Λ^{p}_{lj} for low-lying states in ^{139}La with $j=l-\frac{1}{2}$ on the bound state potential radius r_n while all the other parameters were held constant to those values given in Table I. Index o denotes results according to $r_n=1.230$ fm used in this work. (a) MM theory, (b) ZDH theory.

C. Dependence of spectroscopic factors and reduced normalizations on the radius of the single particle potential

The dependence of the SF and of the RN on the various potential parameters determining directly or indirectly the spectroscopic quantities derived is studied for the single particle potential only. This restriction is reasonable because the proton scattering potentials are reliable within the limits of the known general uncertainties.

Since in the calculation of the single particle bound state wave function the depth and the radius of the potential are correlated, it is sufficient to vary only one of these parameters. Here the variations of the SF and RN with the potential radius are investigated for the five low-lying analog states in ^{139}La. The results (cf. Figs. 1 and 2) are given as ratios related to the values S_0 and Λ_0 which are obtained with the potential used in the calculations of Sec. III B [cf. Eq. (14) and Table I].

Both shell model theories provide strong dependences of the SF on the neutron potential radius. As compared to this, besides the unimportant inversion of the slopes, the variations of the RN with the radius parameter are much smaller. The reductions become more important with increasing orbital momentum quantum number l. This result is similar to the one in the case of (d,p) stripping in the energy region of sub-Coulomb and quasi-Coulomb stripping (see e.g. Ref. 12). Therefore, the RN are considered to be the proper quantities for comparing the single particle fractions of analog states. This seems to be valid as long as the neutron potential parameters are not fixed either by crucial experiments or by convincing arguments.

IV. COMPARISONS OF REDUCED NORMALIZATIONS FROM (d,p) AND (p,p_0) REACTIONS

Comparisons are presented for those analog states whose parent states were investigated recently via (d,p) reactions in the sub-Coulomb and quasi-Coulomb energy region for ^{124}Sn, ^{130}Te, ^{138}Ba, ^{140}Ce, ^{142}Nd, and ^{208}Pb.[12] Especially the nuclei with magic neutron numbers are well suited to study the agreement expected between results according to the analog resonance approaches of MM and ZDH, respectively, with the results of the DWBA analysis of transfer reactions.

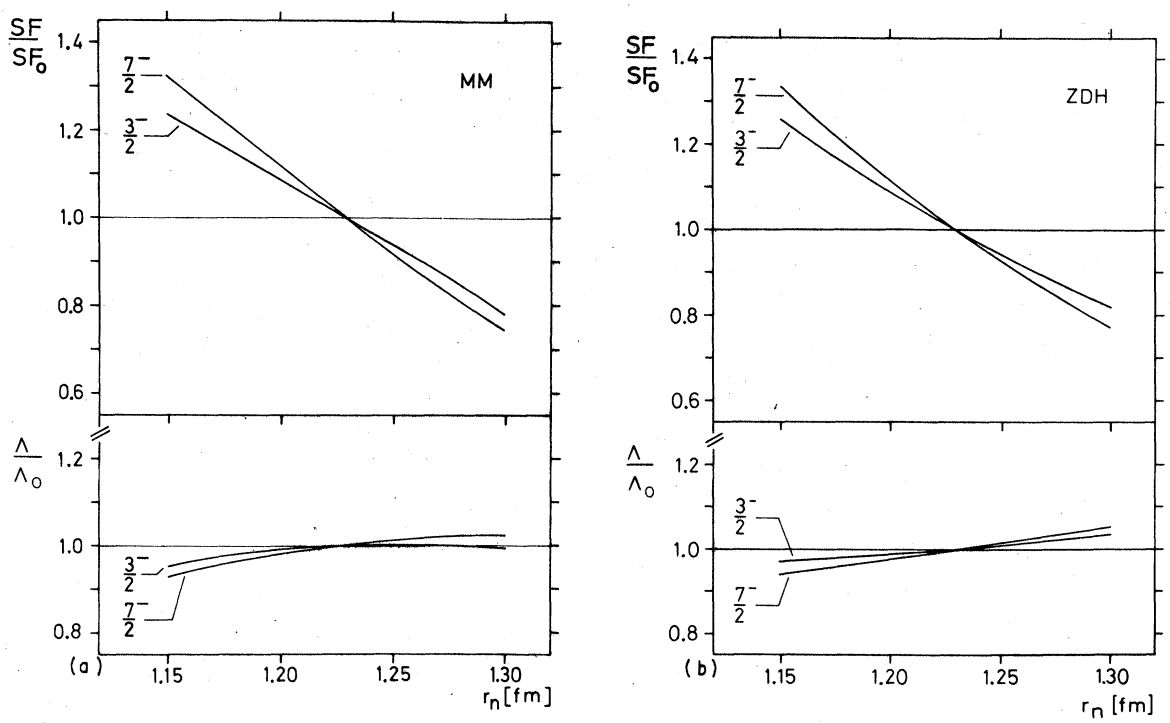

FIG. 2. Dependence of spectroscopic factors SF^{p}_{lj} and reduced normalizations Λ^{p}_{lj} for low-lying states in ^{139}La with $j = l + \frac{1}{2}$ on the bound state potential radius r_n while all the other parameters were held constant to those values given in Table I. Index o denotes results according to $r_n = 1.230$ fm used in this work. (a) MM theory, (b) ZDH theory.

Besides the absolute values of the RN, Table III gives the ratios $\Lambda^{p}_{lj}/\Lambda^{n}_{lj}$ of analog states which as well as the graphical representations in Fig. 3 allow an easier comparison and stress systematic deviations.

For the neutron RN the specific errors are quoted as previously reported,[12] while the errors of the proton RN are assumed to be 15% on the average. The latter ones are only indicated in Fig. 3. In contrast to expectations we had in the beginning,

TABLE III. Comparison of the reduced normalizations Λ^{n}_{lj} and Λ^{p}_{lj} of low-lying analog states formed by (d, p) and (p, p_0) reactions at the nuclei ^{124}Sn, ^{130}Te, ^{138}Ba, ^{140}Ce, ^{142}Nd, and ^{208}Pb.

$E_x^{n\,a}$ (MeV)	j^{π}	$\Lambda^{n\,a}_{lj}$	Λ^{p}_{lj} MM	ZDH	$\Lambda^{p}_{lj}/\Lambda^{n}_{lj}$ MM	ZDH
		(a) ^{125}Sn and ^{125}Sb				
0.029	$\frac{3}{2}^+$	77.2 ± 5.3	74.3	42.9	0.96	0.56
0.219	$\frac{1}{2}^+$	243.1 ± 20.6	229.3	133.8	0.94	0.55
2.788	$\frac{7}{2}^-$	2.61 ± 0.16	7.47	2.35	2.86	0.90
		(b) ^{131}Te and ^{131}I				
0.0	$\frac{3}{2}^+$	55.5 ± 4.1	54.5	39.7	0.98	0.72
0.296	$\frac{1}{2}^+$	167.4 ± 15.1	132.6	95.7	0.79	0.57
2.279	$\frac{7}{2}^-$	5.52 ± 0.39	8.96	4.32	1.62	0.78

TABLE III. (*Continued*)

$E_x^{n\,a}$ (MeV)	j^π	$\Lambda_{lj}^{n\,a}$	Λ_{lj}^{p} MM	Λ_{lj}^{p} ZDH	$\Lambda_{lj}^{p}/\Lambda_{lj}^{n}$ MM	$\Lambda_{lj}^{p}/\Lambda_{lj}^{n}$ ZDH
		(b) (*Continued*)				
2.515	$\frac{3}{2}^-$	8.99 ± 0.65	31.8	10.8	3.54	1.20
2.585	$\frac{3}{2}^-$	41.0 ± 2.7	72.0	24.7	1.76	0.60
3.005	$\frac{1}{2}^-$	26.0 ± 1.9	104.1	24.5	4.00	0.94
		(c) ^{139}Ba and ^{139}La				
0.0	$\frac{7}{2}^-$	22.3 ± 1.6	21.8	14.2	0.98	0.64
0.626	$\frac{3}{2}^-$	123.2 ± 8.1	133.2	75.1	1.08	0.61
1.081	$\frac{1}{2}^-$	68.4 ± 5.3	82.9	42.6	1.21	0.62
1.283	$\frac{9}{2}^-$	0.0167 ± 0.0017	0.0200	0.0146	1.20	0.87
1.419	$\frac{5}{2}^-$	1.58 ± 0.13	2.20	1.24	1.39	0.78
1.697	$\frac{5}{2}^-$	0.678 ± 0.074	0.853	0.492	1.26	0.73
		(d) ^{141}Ce and ^{141}Pr				
0.0	$\frac{7}{2}^-$	42.7 ± 3.3	41.7	25.2	0.97	0.59
0.666	$\frac{3}{2}^-$	199.9 ± 13.8	269.1	135.5	1.35	0.68
1.144	$\frac{1}{2}^-$	116.1 ± 8.9	161.1	73.3	1.39	0.63
1.357	$\frac{9}{2}^-$	0.0437 ± 0.0039	0.0557	0.0400	1.27	0.92
1.505	$\frac{5}{2}^-$	3.13 ± 0.25	4.65	2.42	1.49	0.77
1.748	$\frac{7}{2}^-$	1.07 ± 0.09	1.24	0.833	1.16	0.78
2.129	$\frac{5}{2}^-$	0.620 ± 0.051	0.762	0.427	1.23	0.69
2.421	$\frac{3}{2}^-$	12.4 ± 0.95	20.2	12.3	1.63	0.99
2.438	$\frac{1}{2}^-$	23.6 ± 1.8	19.1	10.5	0.81	0.44
		(e) ^{143}Nd and ^{143}Pm				
0.0	$\frac{7}{2}^-$	65.6 ± 5.5	71.5	49.6	1.09	0.76
0.740	$\frac{3}{2}^-$	288.0 ± 23.6	358.5	205.4	1.24	0.71
1.300	$\frac{1}{2}^-$	172.4 ± 12.9	225.8	122.6	1.31	0.71
1.402	$\frac{9}{2}^-$	0.103 ± 0.010	0.0922	0.0767	0.90	0.74
1.549	$\frac{5}{2}^-$	4.81 ± 0.43	5.37	3.47	1.12	0.72
1.845	$\frac{3}{2}^-$	28.4 ± 2.2	21.2	13.7	0.75	0.48
1.903	$\frac{5}{2}^-$	3.44 ± 0.30	3.92	2.58	1.14	0.75

TABLE III. (*Continued*)

E_x^n [a] (MeV)	j^π	Λ_{lj}^n [a]	Λ_{lj}^p MM	Λ_{lj}^p ZDH	$\Lambda_{lj}^p/\Lambda_{lj}^n$ MM	$\Lambda_{lj}^p/\Lambda_{lj}^n$ ZDH
		(f) ^{209}Pb and ^{209}Bi				
0.0	$\frac{9}{2}^+$	6.56 ± 0.48	6.92	3.89	1.05	0.59
0.781	$\frac{11}{2}^+$	5.59×10^{-3} ± 0.41×10^{-3}	5.55×10^{-3}	3.32×10^{-3}	0.99	0.59
1.427	$\frac{15}{2}^-$	75.9×10^{-6} ± 6.5×10^{-6}	77.9×10^{-6}	19.4×10^{-6}	1.03	0.26
1.570	$\frac{5}{2}^+$	38.7 ± 2.3	43.3	23.3	1.12	0.60
2.036	$\frac{1}{2}^+$	145.6 ± 9.1	170.1	89.6	1.17	0.62
2.496	$\frac{7}{2}^+$	75.0×10^{-3} ± 4.7×10^{-3}	79.7×10^{-3}	39.6×10^{-3}	1.06	0.53
2.541	$\frac{3}{2}^+$	8.81 ± 0.55	9.54	4.73	1.08	0.54

[a] Reference 12.

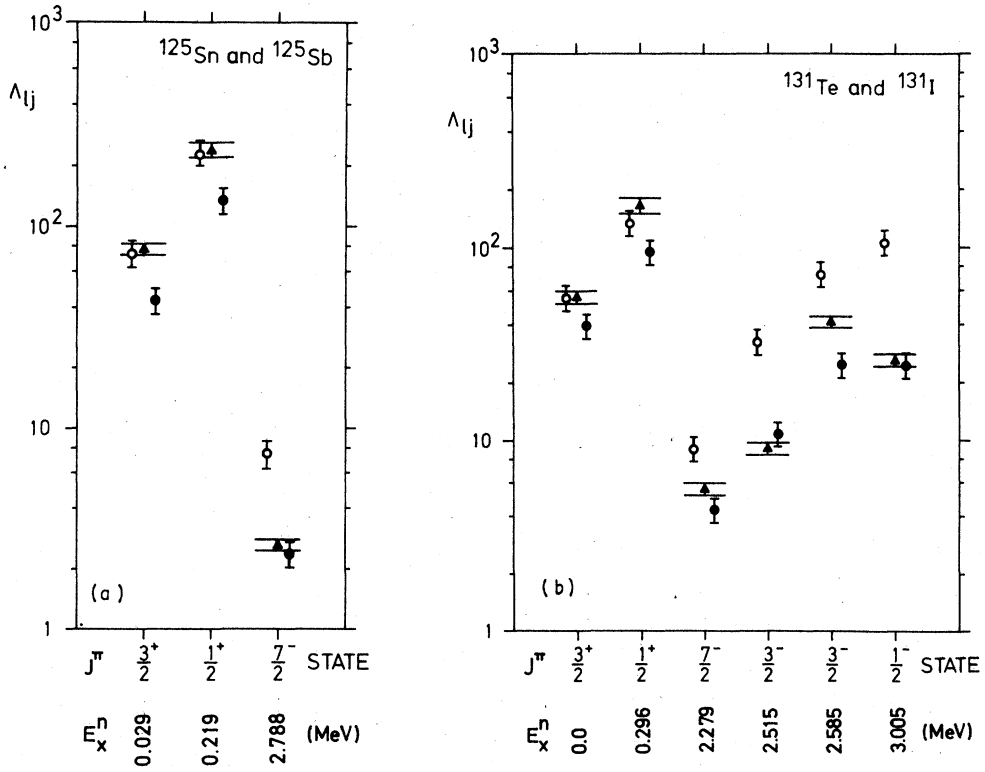

FIG. 3. Comparison of reduced normalizations Λ_{lj}^n and Λ_{lj}^p as resulting from (d,p) reactions and elastic proton scattering via isobaric analog states, respectively. ▲= Λ_{lj}^n extracted from (d,p) reactions in the sub-Coulomb and quasi-Coulomb energy region according to Ref. 12, ○= Λ_{lj}^p analyzed according to MM theory, ●= Λ_{lj}^p analyzed according to ZDH theory. (a) ^{125}Sn and ^{125}Sb, (b) ^{131}Te and ^{131}I, (c) ^{139}Ba and ^{139}La, (d) ^{143}Nd and ^{143}Pm, (e) ^{141}Ce and ^{141}Pr, (f) ^{209}Pb and ^{209}Bi.

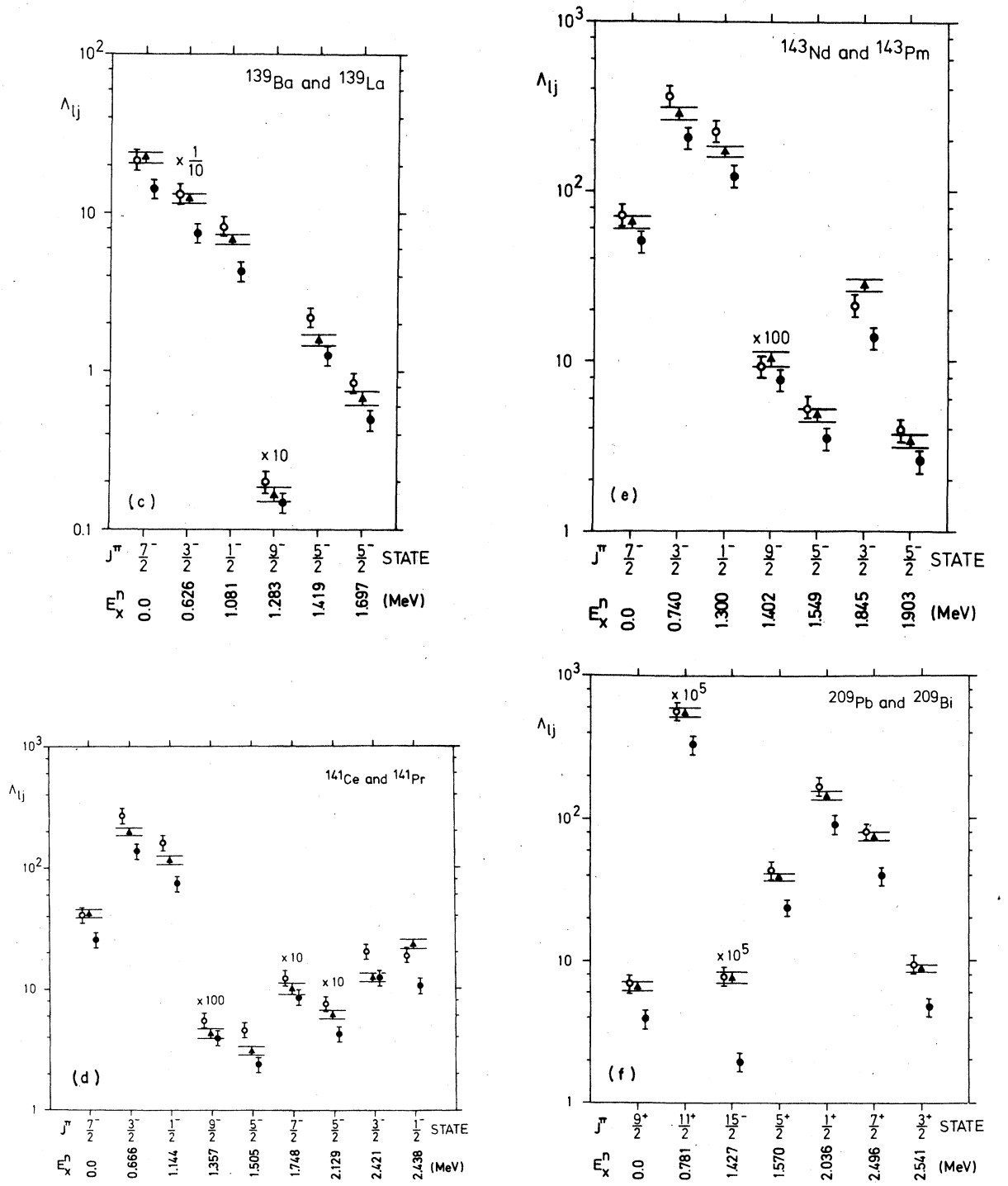

FIG. 3. *(Continued)*

the comparisons do not permit a unique conclusion concerning the validity of one of the shell model approaches discussed. Nevertheless, the results may be summarized by three points:

1. For nuclei with unclosed neutron shells, as ^{124}Sn($N=74$) and ^{130}Te($N=78$), the Λ^p_{lj}(MM) of states with even parity agree well with the corresponding Λ^n_{lj}, while the Λ^p_{lj} (ZDH) are too small as com-

pared to the corresponding neutron RN. For states with odd parity the results according to MM theory tend systematically to larger values, while the Λ^p_{lj} (ZDH) agree well with the (d,p) results.

2. The Λ^p_{lj} (MM) of nuclei with a closed neutron shell at $N=82$ are in good agreement with the (d,p) results for low-lying states with a sufficiently large single particle fraction. The Λ^p_{lj} (ZDH) deviate in this region systematically to smaller values as compared to the corresponding Λ^n_{lj}. Proton resonances at higher energies which have smaller SF and are partly overlapping do not show any tendency.

3. For the double magic core nucleus ^{208}Pb all the states investigated show very good agreement between the Λ^p_{lj} (MM) and the corresponding Λ^n_{lj}, while the Λ^p_{lj} (ZDH) are too small by about 40%.

The arithmetic means and corresponding standard deviations of the ratios $\Lambda^p_{lj}/\Lambda^n_{lj}$ of the 38 states studied in this work are according to the MM theory 1.34 ± 0.11 and according to the ZDH theory 0.69 ± 0.03, respectively. The distinctly smaller deviation of the results according to the ZDH approach from the mean value indicates that there might be a systematic theoretical uncertainty which has so far not been taken into account (see Sec. V).

V. DISCUSSION

Though introducing reduced normalizations as spectroscopic quantities is somewhat disadvantageous because the single particle fractions of the states in question cannot be recognized directly, the comparison with results of neutron transfer reactions is more reasonable even in those cases where calculations are performed with different single particle potentials. The remaining theoretical uncertainties of the RN Λ^n_{lj} were estimated to be about 15% and were directed more probably to lower values.[12] The theoretical accuracy of the Λ^p_{lj} is also limited to a certain amount which will be discussed and quantified in this section.

The equivalence of Eqs. (4) and (11), which allows the calculation of single particle decay widths, was examined by orthogonalizing the proton scattering wave function $\psi^{\rm opt}$ to the neutron wave function φ_{lj}. Using the ansatz

$$\tilde{\psi} = \psi^{\rm opt} - \varphi_{lj}\langle\psi^{\rm opt}|\varphi_{lj}\rangle , \tag{15}$$

one obtains

$$\langle\tilde{\psi}|V_c|\varphi_{lj}\rangle = \langle\psi^{\rm opt}|V_c|\varphi_{lj}\rangle - \langle\varphi_{lj}|V_c|\varphi_{lj}\rangle\langle\psi^{\rm opt}|\varphi_{lj}\rangle . \tag{16}$$

Because of the normalization condition $\langle\varphi_{lj}|\varphi_{lj}\rangle = 1$, the wave functions $\tilde{\psi}$ and φ_{lj} fulfill the equation $\langle\tilde{\psi}|\varphi_{lj}\rangle = 0$.

TABLE IV. Single particle widths of scattering states formed by the reactions ^{138}Ba (p, p_0) and ^{142}Nd (p, p_0) as calculated with code BETTINA (Ref. 42) in using orthogonalized proton scattering wave functions [according to Eq. (4)] and normal optical potential scattering wave functions [according to Eq. (11)], respectively.

	^{138}Ba (p, p_0)			^{142}Nd (p, p_0)	
j^π	$\Gamma^{\rm sp}_{\rm ZDH}$ (keV)	$\Gamma^{\rm sp}_{\rm orth}$ (keV)	j^π	$\Gamma^{\rm sp}_{\rm ZDH}$ (keV)	$\Gamma^{\rm sp}_{\rm orth}$ (keV)
$\frac{7}{2}^-$	29.2	28.4	$\frac{7}{2}^-$	16.3	16.4
$\frac{3}{2}^-$	82.8	80.6	$\frac{3}{2}^-$	65.1	68.3
$\frac{1}{2}^-$	83.1	80.7	$\frac{1}{2}^-$	72.0	76.4
$\frac{9}{2}^-$	2.57	2.53	$\frac{9}{2}^-$	1.78	1.77
$\frac{5}{2}^-$	44.6	43.6	$\frac{5}{2}^-$	36.1	36.8
$\frac{5}{2}^-$	50.1	48.6	$\frac{3}{2}^-$	94.5	99.7
			$\frac{5}{2}^-$	43.5	44.4

The numerical determination of the single particle widths $\Gamma^{\rm sp}_{lj}$ with the computer code BETTINA was changed according to Eq. (16). As an example, resulting widths are presented for some states in ^{139}La and ^{143}Pm in Table IV. They prove that single particle widths according to Eq. (4) are approximately equivalent to the widths according to the ZDH matrix element (11).

Corrections which might be necessary for theoretical predictions of widths were discussed extensively by Auerbach *et al.*[39] Therefrom, the most important uncertainties are due to the charge exchange caused by Coulomb forces, the finite proton size, and the charge dependence as well as charge asymmetry of the nuclear forces. Their whole contribution was estimated and calculated to be in the order of some percent for nuclei in the mass region of $A > 100$. The influence of "rearrangement effects" which increases with decreasing SF is estimated less accurately and may also be in the order of some percent.

The most significant uncertainty is due to the coupling of the analog state to the continuum states via compound states. The statistical assumption of a neglectability of the contributions on the average is not justified generally.

There are states with simple structures similar to the ones of the analog state in question which influence the decay considerably by the isospin mixing. Primarily, the existence of those "door-

way states" has been claimed in order to reduce the discrepancies between the experimental and theoretical total widths of the analog states.[46]

Doorway states are the so-called configuration states which have the same spatial configuration as the analog state but the isospin quantum number $T_< = T_0 - \frac{1}{2}$ and the collective monopole state which also is characterized by an isospin quantum number one unit lower than the one of the analog state.[47,48] Though the experimental information concerning the excitation energies and the widths of these states is rare, some general assumptions lead to satisfying agreement between theoretical and experimental total widths in various cases.[47]

The dependence of the decay widths on the doorway states varies in the region of mean to heavy nuclei with the mass number A in a different way. Contributions due to the configuration states are in the order of some percent and decrease with increasing A,[39] while the values of the partial widths become smaller by about 10–20% with increasing A by the influence of the monopole state.[49]

The total uncertainty implied is about 15–30% and is directed mainly to smaller theoretical single particle widths and, hence, to greater proton reduced normalizations Λ^p_{lj}. As the neutron RN might be smaller by about 15% at most, the ratios $\Lambda^p_{lj}/\Lambda^n_{lj}$ could be larger by about 30–45% as compared to the ratios given in this work. This uncertainty exceeds by far experimental errors of these ratios which are reduced significantly by the procedure of averaging over numerous states of various nuclei. Therefore, one cannot decide between the two shell model theories of MM and ZDH until the theoretical assumptions possible are verified or refused.

ACKNOWLEDGMENTS

We thank Professor Dr. K. H. Lauterjung for providing excellent research conditions in his institute. We are indebted to Dr. W. Fitz, Universität Hamburg, for making the computer code MOM3 available.

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‡Work supported by Bundesministerium für Forschung und Technologie der Bundesrepublik Deutschland.

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