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$B(E2)$ values in ^{155}Gd, ^{161}Dy, ^{163}Dy, ^{167}Er, and ^{175}Lu

P. F. Brown
Duquesne University, Pittsburgh, Pennsylvania 15219

C. Baktash, J. O'Brien, and J. X. Saladin
University of Pittsburgh, Pittsburgh, Pennsylvania 15260
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The $B(E2;I_{g.s.}\rightarrow I_{g.s.}+1)$ and $B(E2;I_{g.s.}\rightarrow I_{g.s.}+2)$ were determined for the isotopes ^{155}Gd, ^{161,163}Dy, ^{167}Er, and ^{175}Lu by means of Coulomb excitation with α particles. Elastically and inelastically scattered α particles were detected either with a magnetic spectrograph or with surface barrier detectors. The measurements were performed in the energy range from 9.0 to 13.5 MeV. The results are compared with other older measurements and are discussed in terms of the rigid rotor model.

[NUCLEAR REACTIONS ^{155}Gd, ^{161,163}Dy, ^{167}Er, ^{175}Lu (α,α'). $E=9.0$ to 13.5 MeV measured $d\sigma_{el}/d\alpha_{inel}$. Deduced B ($E2$) values, Q_0, validity of rigid rotor model.]

INTRODUCTION

In recent years, the electromagnetic properties of even-even deformed nuclei have been investigated extensively by means of high precision experiments using Coulomb excitation. These experiments have resulted in $E2$ and $E4$ transition moments between low lying states, which in turn have given information concerning the shape of the charge distributions.[1-6]

Another topic of considerable interest concerned the limits of validity of the rigid rotor model. Particularly important has been the investigation of the variation of the moment of inertia as a function of the square of the angular velocity ω. All nuclei show a slight increase of $\mathscr{I}$ with ω even for small values of ω. In some nuclei a very sudden and sharp increase of the moment of inertia takes place around $I=12$ to 16, leading to the so-called back-bending phenomena.[7] Since $\mathscr{I}$ varies with ω (or I), there arises the question to what extent the intrinsic quadrupole moment Q_0 is a constant of the motion. This can be tested by comparing reduced transition probabilities between successive pairs of states within a rotational band. In even-even deformed nuclei such tests have been performed using lifetime measurements.[8,9] For states up to $I\cong 8$ no definite deviations have been found from the predictions of the rigid rotor model. The experimental uncertainties in such experiments are, however, typically of the order of 3 to 6% so that effects of the order of a few % would be missed.

In odd deformed nuclei it is possible to measure the transition moments connecting the two lowest excited states with the ground state, directly, by means of Coulomb excitation. By separating inelastically and elastically scattered particles, the ratio of the cross section $d\sigma(I_0+2)/d\sigma(I_0+1)$ (where I_0 is the ground state spin) can be measured directly and is independent of any knowledge of target thickness, integrated charge, solid angle, etc. The ratio can thus be related in a straightforward manner to the ratio of reduced transition probabilities, which, in the rigid rotor model, depend on the squares of Clebsch-Gordan coefficients.

Most of the earlier Coulomb excitation work in this region was based on measuring γ-ray yields, which is inherently a somewhat less precise method.

The reduced transition probabilities measured here can also be related to the intrinsic quadrupole moment Q_0. It is then possible to compare the Q_0's from the present work with those derived from measurements of the spectroscopic (static) quadrupole moments of the ground states of these nuclei. Such measurements have been performed using atomic beam methods,[10-12] and recently by means of muonic and pionic x-ray spectroscopy.[13,14] This latter method is particularly significant in that its results are, like those of Coulomb excitation, model independent.

The purpose of the present paper is thus threefold:

(i) To obtain a set of precise $B(E2)$ values for the nuclei studied.

(ii) To check the constancy of the intrinsic quadrupole moment Q_0 for the low lying states of the ground state rotational band.

(iii) To compare those Q_0 values obtained from measuring transition moments, with those obtained by other methods, such as atomic-beam and muonic x-ray experiments.

EXPERIMENTAL ARRANGEMENT AND DATA REDUCTION

The experimental arrangement is similar to that described in earlier work.[3] The experiment was performed using α particles from the University of Pittsburgh Van de Graaff accelerator, ranging in energy from 9 to 13.5 MeV. In the cases of Gd, Dy, and Er, the elastically and inelastically scattered α particles were detected by means of an Enge split-pole spectrograph with a position sensitive detector located in the focal plane. These measurements were performed at a scattering angle of 143°. In the case of ^{175}Lu, the elastically and inelastically scattered α particles were detected by an annular surface barrier silicon detector at a mean angle of 173.3° (one case at 171.5°) and two surface barrier silicon detectors located at ±140° with respect to the incident beam direction. The target thicknesses in all cases were of the order of 30 μg/cm^2 evaporated on 10 μg/cm^2 carbon backings.

The experimental quantities of interest are the ratios of the number of inelastically scattered particles corresponding to the excitation of the first or second excited states, divided by the number of elastically scattered particles. Since ratios are taken of particles scattered under the same experimental conditions, normalization for target thickness and solid angle are not necessary. In the cases using the annular detector, the areas were extracted using an iterative approach assuming that all peak shapes are the same. This was done by means of a computer code which also took into account corrections due to elastic and inelastic scattering from impurities. The inelastic peaks were then normalized to the ground state peak.

Data from the split-pole spectrograph were analyzed graphically, since here, due to nonlinearities in the position spectra, the assumption of identical peak shapes is no longer valid. The values obtained for the intensity ratios are given in Table I.

From the ratios of the number of α particles in the various peaks, the reduced transition probabilities were obtained using the semiclassical coupled channels program of Winther and De Boer.[15] In each case the ground state and the first three excited states were considered in the calculations. The reduced $E2$ matrix elements between the ground state and the two first excited states were treated as variable parameters. The other matrix elements entering into the calculations were assumed to be given in terms of the intrinsic quadrupole moment through the rigid rotor relations:

TABLE I. The ratios of cross sections are shown for each nuclide, for each value of energy and scattering angle studied.

Nuclide	I_0	Scattering angle	E_α (MeV)	$\frac{d\sigma_{I_0+1}}{d\sigma_{I_0}}\times 10^2$	$\frac{d\sigma_{I_0+2}}{d\sigma_{I_0}}\times 10^2$
^{155}Gd	$\frac{3}{2}$	143	11.0	2.578 ± 0.055	1.330 ± 0.043
			11.4	2.680 ± 0.042	1.469 ± 0.031
			12.0	3.273 ± 0.040	1.813 ± 0.030
			12.5	3.558 ± 0.078	2.124 ± 0.068
^{161}Dy	$\frac{5}{2}$	143	11.4	2.858 ± 0.031	0.9743 ± 0.034
			12.0	3.457 ± 0.048	1.060 ± 0.025
			12.5	3.982 ± 0.040	1.232 ± 0.020
			12.8	4.004 ± 0.088	1.446 ± 0.049
^{163}Dy	$\frac{5}{2}$	143	11.4	3.207 ± 0.031	1.016 ± 0.017
			12.0	3.712 ± 0.054	1.211 ± 0.031
			12.4	3.899 ± 0.062	1.352 ± 0.037
			12.8	4.242 ± 0.076	1.444 ± 0.044
^{167}Er	$\frac{7}{2}$	143	11.5	2.699 ± 0.033	0.633 ± 0.016
			12.0	3.070 ± 0.034	0.749 ± 0.017
			12.5	3.366 ± 0.047	0.889 ± 0.024
^{175}Lu	$\frac{7}{2}$	173.5	9.0	1.09 ± 0.01	0.198 ± 0.003
			11.0	2.04 ± 0.02	0.449 ± 0.005
			13.0	3.48 ± 0.03	0.839 ± 0.010
		171.5	13.5	3.85 ± 0.03	0.948 ± 0.017
		140	9.0	0.894 ± 0.006	0.169 ± 0.003
			11.0	1.65 ± 0.002	0.376 ± 0.005
			13.0	2.90 ± 0.002	0.677 ± 0.007

$$\langle KI_i \| M(E2) \| KI_f \rangle = (2I_i+1)^{1/2} \langle I_i K 2 0 | I_f K \rangle \left(\frac{5}{16\pi}\right)^{1/2} eQ_0 ,$$

where I_i and I_f are the spins of the initial and final states, respectively, and K is the projection of the total angular momentum on the nuclear symmetry axis. K is equal to I_0 for the ground state rotational band. Several calculations were made including a reasonable value (i.e., interpolated from adjacent even nuclei) for the reduced $E4$ matrix element connecting the ground state and third excited state of ^{163}Dy. All other $E4$ matrix elements were computed from rigid rotor relations. The effect of including the $E4$ matrix elements in the calculation of the $B(E2)$ values was less than 0.5%. Since it is not clear how valid such an extrapolation of $E4$ moments from even to odd nuclei is, we did not include them in the other calculations. The experimental ratios

$$R_1 = \frac{d\sigma_{I_{0+1}}}{d\sigma_{I_0}} \quad \text{and} \quad R_2 = \frac{d\sigma_{I_{0+2}}}{d\sigma_{I_0}}$$

are compared to the ratios calculated from the Winther–De Boer code. The double ratios $R_{i,\mathrm{exp}}/R_{i,\mathrm{DWB}}$ [where $R_{i,\mathrm{DWB}}$ are the ratios corresponding to the final $B(E2)$ values] are shown in Fig. 1 as a function of bombarding energy.

In each case the highest bombarding energy was chosen to be below the onset of Coulomb-nuclear interference effects. Criteria for such safe bombarding energies were obtained from extensive previous studies in even-even rare earth nuclei.[16]

Since the nuclides studied represented slightly different experimental problems a short discussion follows concerning each of them.

^{155}Gd: The targets were isotopically enriched containing 91.8% ^{155}Gd with all even impurities of Gd equal to 7.1% and ^{157}Gd equal to 1.1%. The impurities plus background expressed as a percentage of the true intensity represented approximately 8% of the ground state, 16% of the first excited state, and 8% of the second excited state. The large impurity subtraction and the 60 keV separation between the ground state and first excited state account for the 8% spread in the data shown in Fig. 1. The energy resolution [full width at half maximum] (FWHM) for all Gd data was ~23 keV.

^{161}Dy: The isotopic makeup of the ^{161}Dy targets was 95.9% ^{161}Dy, 0.86% ^{163}Dy, and all even Dy isotopes (160, 162, and 164) were 3.2%. The impurities and background represented on the average 4% of the ground state, 8% of the first excited state, and 16% of the second excited state. The energy resolution obtained in this case was 17 keV

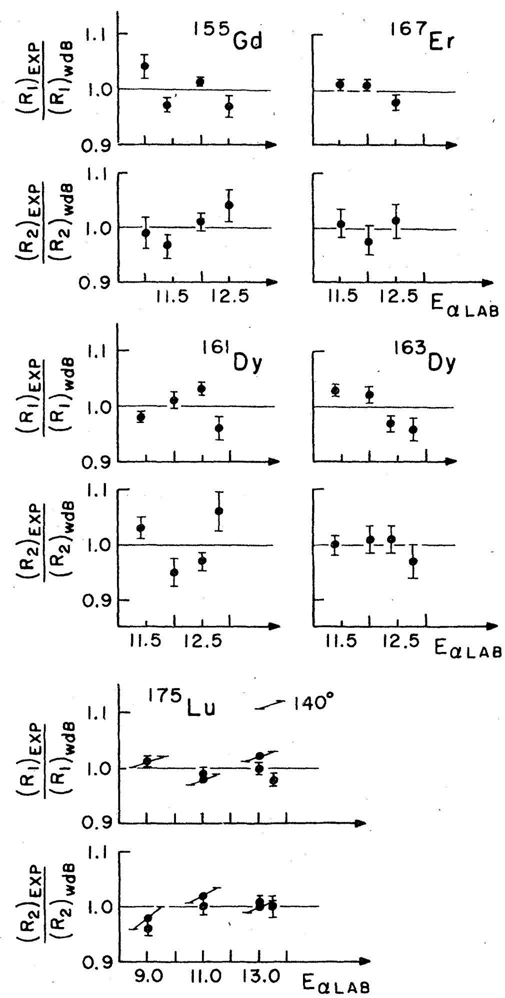

FIG. 1. Double ratios $R_{i,\mathrm{exp}}/R_{i,\mathrm{WDB}}$ as a function of energy.

FWHM. The somewhat larger than expected spread in the data can be attributed to the small energy separation between the states in combination with the substantial impurity subtractions.

^{163}Dy: The isotopically enriched ^{163}Dy targets consisted of 93.1% ^{163}Dy, 0.7% of ^{161}Dy, and all even isotopes (160, 162, and 164) 6.2%. The impurities on the average represented 7% of the ground state and 18% of the first excited state.

^{167}Er: The targets were isotopically enriched and contained 91.5% ^{167}Er, 5.14% of ^{168}Er, and all other even Er isotopes (164, 166, and 170) 3.2%. The impurities and background represented on the average 8.5% of the ground state, 26% of the first excited state, and 11% of the second excited state. The spread in the data here is about 6%. The larger energy separation between the states evidently contributes to a greater precision in extracting the areas of the peaks.

^{175}Lu: The targets were isotopically enriched and contained 99.94% ^{175}Lu and 0.06% ^{176}Lu. The almost complete lack of impurities and wide energy separation between the states allowed a very precise determination of peak areas. The resulting spread in the data is approximately 4%.

The major contributions to the quoted errors are due to statistics and uncertainties associated with impurity and background subtractions.

DISCUSSION

The results of the current investigation are compared to the results of other measurements in Table II. There is in general satisfactory agreement with those measurements which are based on the observation of inelastically scattered particles. The only case in which the measurements do not overlap is that of the $B(E2;\ I_0 \rightarrow I_{0+2})$ value for ^{163}Dy. In general the agreement with values obtained by means of conversion electron-yield determinations or from lifetime measurements is not as good. Most likely this can be attributed to the fact that the methods used in these older experiments are inherently less accurate.

In columns 5 and 6 of Table II a comparison is made with the predictions of the symmetric rigid rotor model (SRRM), according to which the ratio $B(E2;\ I_0 \rightarrow I_0+2)/B(E2;\ I_0 \rightarrow I_0+1)$ is equal to the square of Clebsch-Gordan coefficients. For ^{163}Dy, ^{167}Er, and ^{175}Lu the present data agree to within 2% or better with the SRRM. The results for ^{155}Gd agree to within 3.3% with the SRRM expectation, compared with an overall experimental uncertainty of 7%. A larger discrepancy exists for ^{161}Dy, where the $B(E2)$ ratio differs by 8%. This is barely within the experimental uncertainty which is rather large in this case due to the rather large impurity subtractions. A deviation from the pure SRRM prediction is, however, not unexpected in this nucleus. The configuration of ^{161}Dy can be interpreted as being made up of a $j=\frac{13}{2}$ neutron coupled to a ^{160}Dy core to yield a total spin $I=\frac{5}{2}$ for the ground state (this corresponds to the $|642\rangle$ Nilsson orbital). For such a large j value one expects the Coriolis force to introduce mixing between other nearby Nilsson orbitals (i.e., $|606\rangle$, $|651\rangle$, $|633\rangle$, $|624\rangle$). The energies of the ground band states have been explained satisfactorily on the basis of such a model.[23] These band-mixing effects will of course also influence transition probabilities.

It is, furthermore, interesting to relate the transition moments from this experiment to measurements of the ground state quadrupole moment from atomic beam (hfs) and muonic x-ray experiments. This is best accomplished by means of the intrinsic quadrupole moment Q_0 which can be extracted from the measured $B(E2)$ values. This comparison is made in Table III. In ^{161}Dy and ^{175}Lu there is rather good agreement between the pre-

TABLE II. The values of the transition moments and their ratio obtained by various techniques.

Nuclide	I_0	$B(E2;I_0\rightarrow I_0+1)$ (e^2 b^2)	$B(E2;I_0\rightarrow I_0+2)$ (e^2b^2)	$\frac{B(E2;I_0\rightarrow I_0+2)}{B(E2;I_0\rightarrow I_0+1)}$ Exp.	Rigid rotor	Method	Reference
^{155}Gd	$\frac{3}{2}$	2.049 ± 0.072	1.179 ± 0.041	0.575 ± 0.040	0.556	C.E.(α,α')Inel.	Present
		2.15 ± 0.10	1.12 ± 0.15	0.521 ± 0.093		C.E.$(p,p')(d,d')$Inel.	17
		1.68 ± 0.40				$T_{1/2}$	18
^{161}Dy	$\frac{5}{2}$	2.430 ± 0.073	0.788 ± 0.039	0.324 ± 0.026	0.350	C.E.(α,α')Inel.	Present
		2.54 ± 0.15	0.69 ± 0.10	0.272 ± 0.055		C.E.$(p,p')(d,d')$Inel.	17
		2.36 ± 0.59	0.59 ± 0.14	0.25 ± 0.06		C.E.$(\alpha,\alpha')(p,p')$conv. e	19
		2.74 ± 0.47				$T_{1/2}$	18
^{163}Dy	$\frac{5}{2}$	2.63 ± 0.11	0.900 ± 0.019	0.342 ± 0.021	0.350	C.E.(α,α')Inel.	Present
		2.56 ± 0.15	0.68 ± 0.10	0.266 ± 0.055		C.E.$(p,p')(d,d')$Inel.	17
		1.60 ± 0.32	0.45 ± 0.09	0.28 ± 0.11		C.E.$(\alpha,\alpha')(p,p')$conv. e	19
		1.98 ± 0.12				$T_{1/2}$	18
^{167}Er	$\frac{7}{2}$	2.49 ± 0.10	0.648 ± 0.019	0.260 ± 0.016	0.257	C.E.(α,α')Inel.	Present
		2.607 ± 0.078	0.610 ± 0.040	0.234 ± 0.022		C.E.$(p,p')(d,d')$Inel.	17
		2.34 ± 0.12	0.66 ± 0.40	0.28 ± 0.19		$(d,d')\sigma$	20
		2.9				$T_{1/2}$	21
^{175}Lu	$\frac{7}{2}$	2.283 ± 0.046	0.588 ± 0.015	0.258 ± 0.012	0.257	C.E.(α,α')Inel.	Present
		2.34 ± 0.10	0.57 ± 0.08	0.244 ± 0.046		C.E.$(p,p')(d,d')$Inel.	17
		2.70 ± 0.56	0.59 ± 0.11	0.219 ± 0.081		C.E.$(\alpha,\alpha')(p,p')$conv. e	19
		1.95 ± 0.18				$T_{1/2}$	22

TABLE III. Comparison of intrinsic quadrupole moments from various methods.

Isotope	Transition	$E_f - E_i$ (keV)	$B(E2)$ ($-e$ b)	Q_0 Present	Q_0 Other	Method	Reference
^{155}Gd	$\frac{3}{2}- \to \frac{5}{2}-$	0.0600	2.049±0.072	6.33 ±0.11			
	$\frac{3}{2}- \to \frac{7}{2}-$	0.1460	1.179±0.041	6.44 ±0.11			
	g.s.				7.95 ± 0.80	Atomic hfs	Unsworth (11)
^{161}Dy	$\frac{5}{2}+ \to \frac{7}{2}+$	0.0438	2.430±0.073	7.156±0.11	6.93±0.08		
	$\frac{5}{2}+ \to \frac{9}{2}+$	0.102	0.788±0.039	6.89 ±0.17	6.70±0.16	Muonic x-rays	Powers (14)
	g.s.				6.91±0.08		
					5.64±0.02		Ferch (10)
^{163}Dy	$\frac{5}{2}- \to \frac{7}{2}-$	0.075	2.63 ±0.11	7.45 ±0.16			
	$\frac{5}{2}- \to \frac{9}{2}-$	0.170	0.900±0.019	7.36 ±0.08			
	g.s.				5.96±0.02	Atomic hfs	Ferch (10)
^{167}Er	$\frac{7}{2}+ \to \frac{9}{2}+$	0.07932	2.49 ±0.10	7.68 ±0.15			
	$\frac{7}{2}+ \to \frac{11}{2}+$	0.17795	0.648±0.019	7.73 ±0.11			
	g.s.				6.00±0.11	Atomic	Fuller (12)
^{175}Lu	$\frac{7}{2}+ \to \frac{9}{2}+$	0.11381	2.283±0.046	7.36 ±0.07			
	$\frac{7}{2}+ \to \frac{11}{2}+$	0.2514	0.588±0.015	7.36 ±0.09			
	g.s.				7.48±0.11	Muonic x-rays	Dey (13)
					5.68±0.06	Atomic hfs	Fuller (12)

sent results and those obtained from muonic x-ray studies. There exist, however, significant discrepancies with atomic beam determinations of ground state moments. However, Sternheimer corrections[24] which are known to be significant in this region have not yet been applied to these measurements. This seems to be the most likely explanation for these discrepancies.

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