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Energy-dependent phase shift analysis of pion-nucleon scattering below 400 MeV

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An analytic function of energy is fitted to the available S, P, and D wave πN phase shifts of various goups below 400 MeV. This global average, which reproduces well most of the experiment cross sections, is anticipated to be useful in pion-nucleus and pion-nucleon interaction calculations.

[NUCLEAR REACTIONS $\pi^{\pm}N$; 0–400 MeV; analytic function fitted to tabulated phase shifts.]

The prime source of information on the πN interaction is scattering experiments which provide information on the on-shell scattering amplitude or phase shifts.[1] These energy-dependent phase shifts are a convenient parametrization of the on-shell πN interaction and are employed in numerous pion-nucleon and pion-nucleus interaction calculations. Unfortunately, the emphasis in recent years on extending and intensifying our knowledge of these phase shifts at higher and higher energies has proceeded without accurate determination of the low energy phases. Indeed, the task of accurately measuring the low energy phases in all the partial waves is presently being assumed by the meson factories.

In this work we fit an analytic function of energy to recent S, P, and larger D wave πN phase shifts determined by various groups for energies below 400 MeV. In the past, energy-dependent phase shift analyses have been carried out at low[2,3] and high energies.[4-6] We do not aim to repeat all those analyses, rather we wish to provide a smooth "best" fit to all the modern πN phase shifts which will permit convenient and reliable interpolation in energy, and help indicate where further experimental work is called for. The functions so obtained can then be simply used, e.g., to construct potential or field theoretic models of the πN interaction, to predict pion-nucleus dynamics, and to evaluate dispersion integrals. Indeed, a previous fit of this sort[7] has helped improve the reliability of low energy pion-nucleus calculations and has shown the crucial importance[8] of not using obsolete phases. The present fit incorporates more recent data than in the TRIUMF report[7] and uses improved statistical techniques.

We hasten to repeat that our prime aim is to summarize much data over a wide range of energy; a study of the elementary nature of the πN interaction for its own sake should employ the elementary cross section data with a consistent treatment of Coulomb effects and analyticity. However, in light of the "ancient" fits presently being used in pion-nucleus physics,[8] we feel that this type of global average is necessary.

The pure nuclear phase shifts in each eigenchannel are fitted with an analytic function which incorporates the threshold behavior expected for a finite range interaction plus a term which represents the nearest πN resonance:

$$\frac{\tan\delta_l}{q^{2l+1}} = b + cq^2 + dq^4 + \frac{x\Gamma_0\omega_0 q_0^{-(2l+1)}}{\omega_0^2 - \omega^2}, \tag{1}$$

where q is the πN center-of-mass momentum and ω is the πN c.m. energy. Although Eq. (1) does not represent a fundamental form for the πN interaction it is sufficiently realistic so that values for the real constants b, c, and d could be found which fit the data well for $T_\pi < 400$ MeV. (No fit has been made to the inelasticity parameter η which is >97% for $T_\pi < 400$ MeV.) The resonant parameters x, Γ_0, q_0, and ω_0 were fixed at the values given in the Particle Properties Table[9] and are reproduced in Table I.

TABLE I. Fixed resonance parameters for Eq. (1).

Channel	x	ω_0 (MeV)	q_0 (MeV/c)	Γ_0 (MeV)
S_{11}	0.44	1550	477	105
S_{31}	0.31	1655	550	170
P_{11}	0.61	1435	393	230
P_{13}	0.23	1815	656	255
P_{31}	0.22	1850	678	200
P_{33}	0.99	1233	228	116
D_{13}	0.54	1525	459	125
D_{15}	0.43	1670	560	155

The values of the parameters were determined by performing a least squares fit[10] to $\tan\delta_l/q^{2l+1}$ and the scattering lengths with each datum point having its published, or an assigned, error. The assigned errors were chosen as either the same error as other comparably smooth data points, or (in those instances with large fluctuations) as the standard deviation of the data from a smooth curve. In addition, an error of 2% was assumed for all values of momentum q to account for the energy uncertainty.

At this point we wish to caution the reader that the quality of the data and our limited energy region only permit a truly significant separation of "background" and resonance amplitudes in the P_{33} channel. In all other channels the "resonance" contribution is $\simeq 10\%$ of the total amplitude and thus we are only on the tail of the resonance. In these channels we consider the Breit-Wigner form simply a convenient parametrization of some higher power q dependence and have kept the elasticity parameter x (Γ_{el}/Γ) constant at its on-resonance value. Permitting x to be energy dependent would complicate the parametrization without adding additional physics. Setting $x \equiv 1$ in other channels than P_{33} would be quite reasonable for these low energies—yet this tends to increase the χ^2 by 10–20% and mainly change the least-well-determined parameter "d."

The phase shifts are taken from Refs. 11–17 and the scattering lengths from Ref. 16. These are either recent elastic $\pi^{\pm}p$ experiments or improved analyses of older experiments. The data from Ref. 15 were used for $250 < T_\pi < 400$ MeV to insure a smooth transition to the high energy region. (An average of "Berkeley Path 1," "Berkeley Boone 21," "CERN Theoretical," and "Glasgow Station A" was used, as these formed a consistent set which extrapolated well to lower energies.)

The fitted parameters b, c, and d are given in Table II along with the number of data points, the χ^2, and the deduced scattering length. Some typical fits to the data are shown in Figs. 1 and 2. We see that although the fits appear reasonable in all channels there is considerable deviation of the data from the best fit. This is a prime reason for

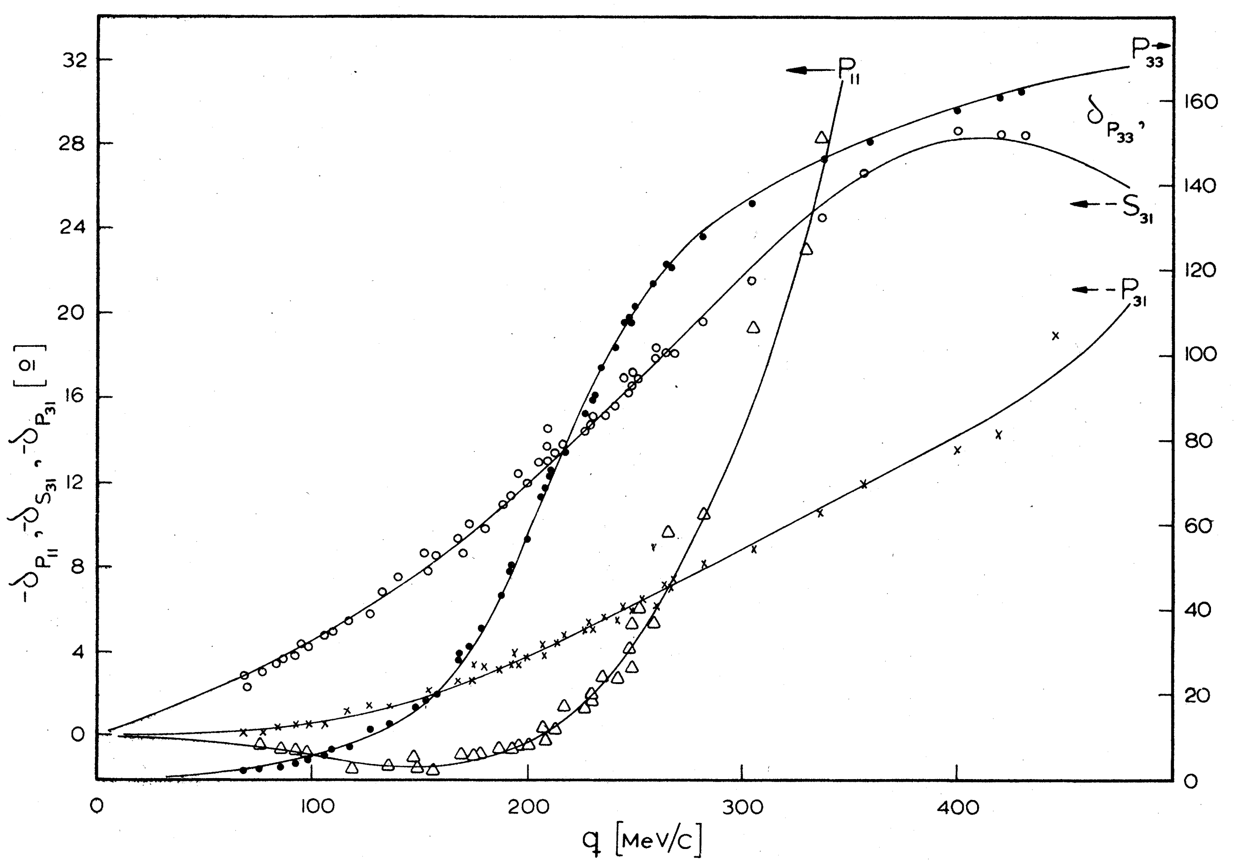

FIG. 1. Phase shifts in degrees for S_{31}, P_{11}, P_{31}, and P_{33} waves. The points have been taken from Refs. 11–17. The full lines are the results of the fits using Eq. (1).

TABLE II. Fitted parameters for Eq. (1).

Channel	b $(10^{-2}m_\pi^{-(2l+1)})$	c $(10^{-3}m_\pi^{-(2l+3)})$	d $(10^{-4}m_\pi^{-(2l+5)})$	N	$\frac{\chi^2}{N-3}$	Scattering length $[m_\pi^{-(2l+1)}]$
S_{11}	16.8 ±0.75	−35.4 ±5.4	27 ±11	38	4.7	0.185 ±0.008
S_{31}	−11.2 ±0.20	−30.7 ±1.1	21 ± 2	54	1.1	−0.098 ±0.003
P_{11}	−5.71 ±0.54	25.4 ±2.1	−29 ± 3	35	1.8	−0.047 ±0.004
P_{13}	−1.31 ±0.08	1.22 ±0.32	−0.4 ± 0.3	40	1.2	−0.013 ±0.002
P_{31}	−2.91 ±0.08	3.45 ±0.27	−1.5 ± 0.2	53	0.6	−0.029 ±0.002
P_{33}	11.4 ±0.30	−15.4 ±2.1	7.2 ± 2.1	49	1.8	0.205 ±0.050
D_{13}	0.109±0.012	−0.031±0.062	0.003± 0.065	54	0.4	0.0013±0.0005
D_{15}	0.112±0.007	−0.270±0.030	0.19 ± 0.02	57	0.7	0.0012±0.0005

the need of such a fit. All χ^2 values indicate reasonable fits with the exception of S_{11}, where there seem to be large systematic errors or an underestimate of statistical errors.

Since we did not fit to the actual cross section data, we have confirmed the significance of our fits by using the fitted function to calculate πN elastic scattering cross sections and compared these to the actual measurements. As we show in Figs. 3 and 4 the agreement is quite respectable for π^+p at all energies, but less good for π^-p—especially at the lower energies. The reason is threefold. First, the smaller $I=\frac{1}{2}$ phases are not determined very well from experiment (as is evident in Fig. 1). Second, low energy π^-p experiments are more difficult than the π^+p ones, and improved π^-p ones are needed. Finally, for $T_\pi < 50$ MeV the radiative capture reaction $\pi^-p \to \gamma n$

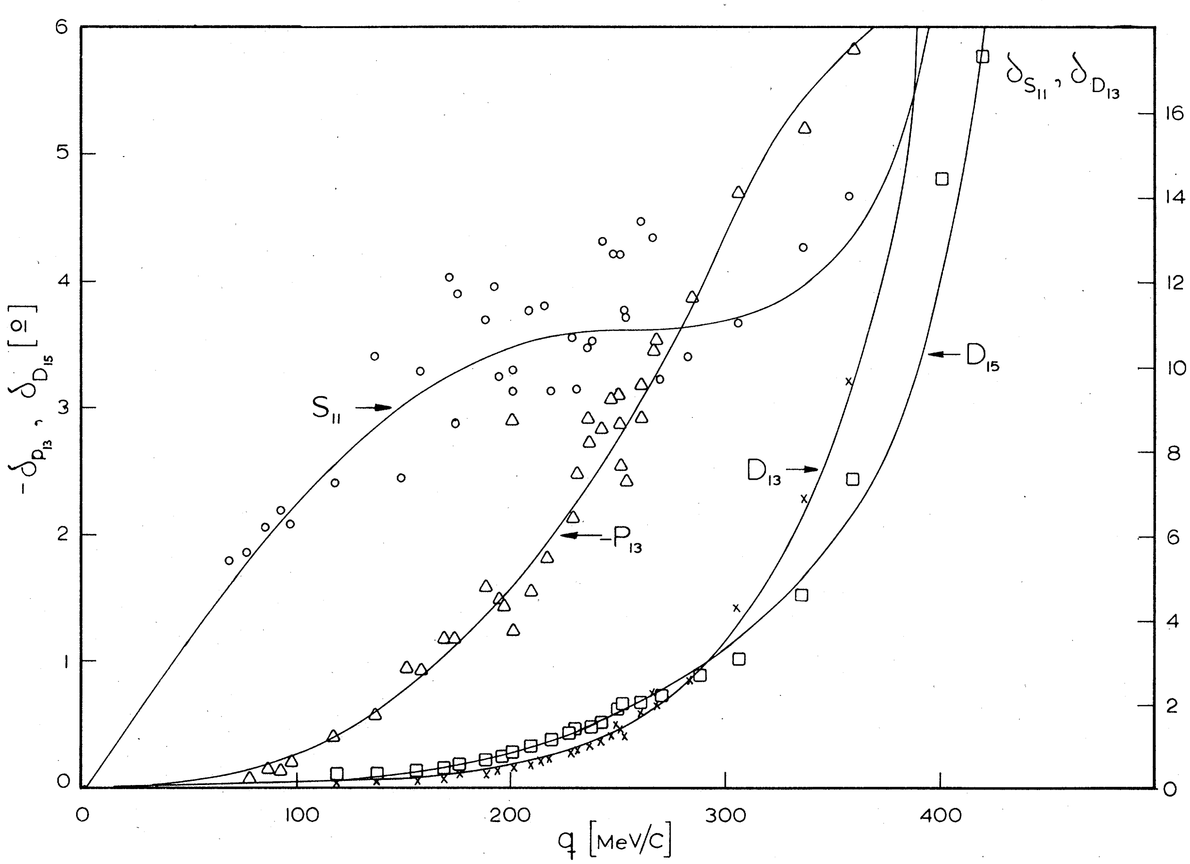

FIG. 2. Phase shifts in degrees for S_{11}, P_{13}, and D_{15} waves.

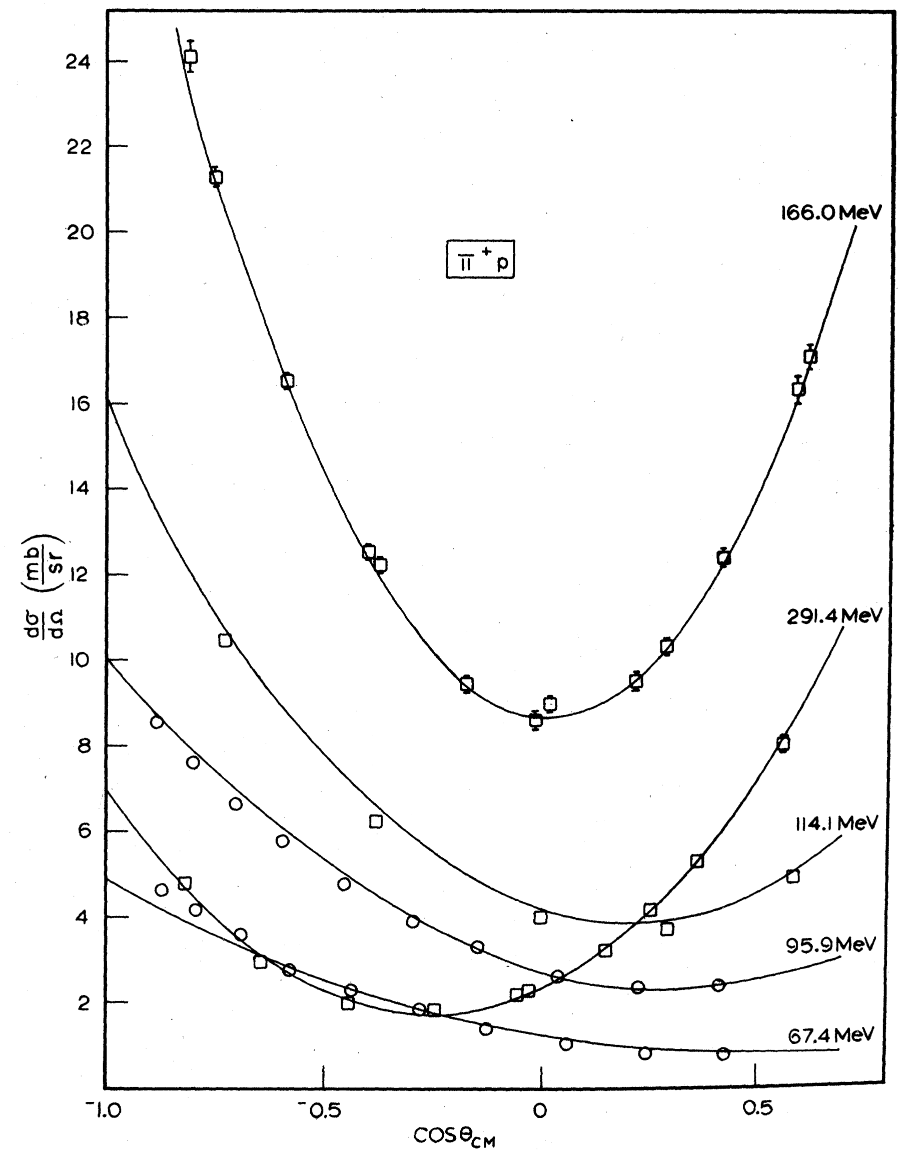

FIG. 3. Comparison of some experimental cross sections with cross sections calculated using our fitted phase shifts for π^+p.

is known to contribute significantly to the total cross section (10–30%), but it has not been included in the original phase shift analysis (which assumed real phases). Thus some caution appears necessary[18] in the use of all low energy phases until this problem is investigated more fully.

In Table II are also listed the deduced values of the scattering lengths for each eigenchannel ($\lim \tan\delta_l/q^{2l+1}$). Our analysis produces a slightly negative isoscalar scattering length $(a_1+2a_3) = (-0.011 \pm 0.008)m_\pi^{-1}$ comparable with the values listed in Ref. 16.

A somewhat different type of fit to the low energy partial waves has been obtained by Nielsen and Oades.[19] In that work, the Almehed-Lovelace[14] and Carter *et al.* phases[11] were used as input to a dispersion relation and amplitudes were determined which had the analyticity of the Mandelstam representation, s-u crossing symmetry, and an imposed (selfconsistent) unitarity. Their tabulated phases generally agree with the output of Eq. (1) to within 10% (with significant increasing discrepancy at lower energies). Since Ref. 19 fits the amplitudes with functions of s, t, and u, it should be possible to numerically project out the s channel behavior of their amplitudes and thus obtain results comparable to Eq. (1). While the amplitudes so determined may be theoretically more sound,

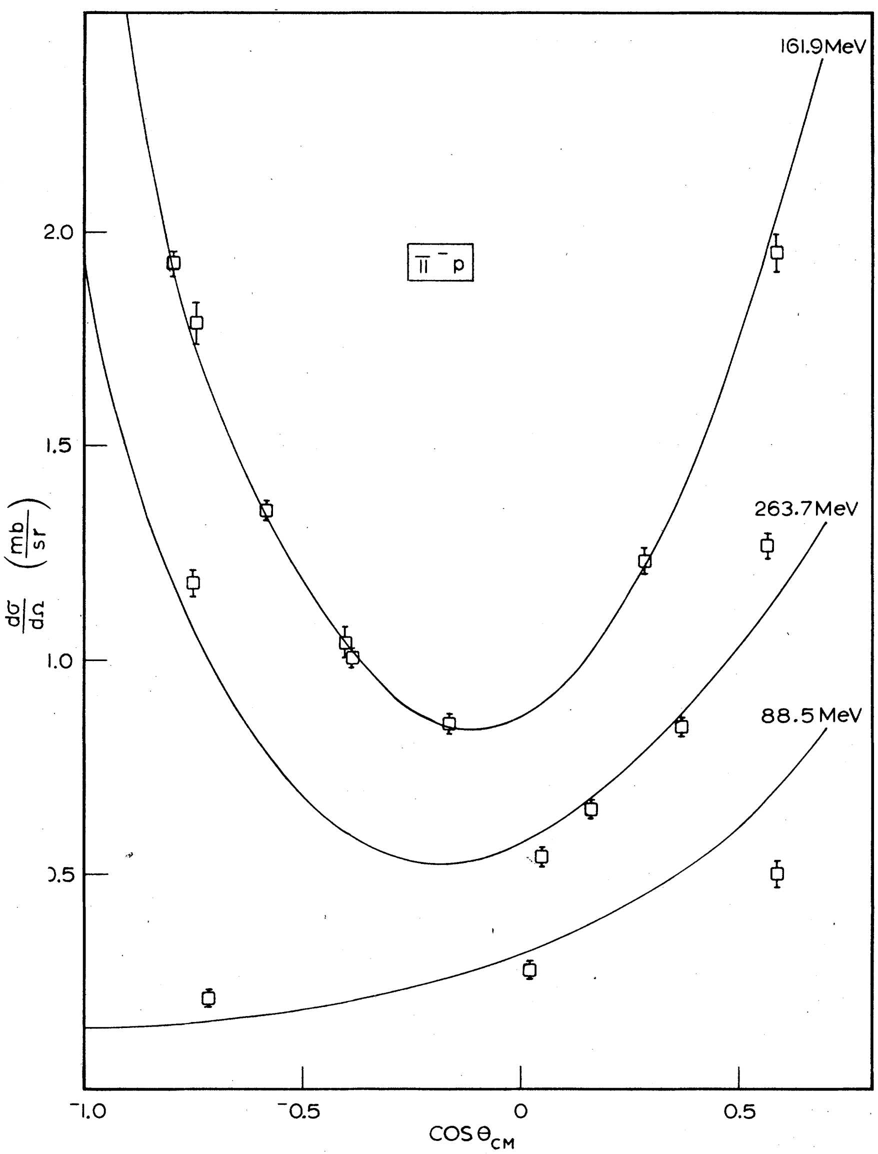

FIG. 4. Cross section comparison for $\pi^- p$.

this procedure is inconvenient, slow, possibly inaccurate, and does not provide the summary of an increasing number of data presented by Eq. (1).

In general, our analysis continuously matches the higher energy analysis of Almehed and Lovelace,[14] but provides a smooth extrapolation to lower energies. For $T_\pi < 100$ MeV our results differ significantly from the CERN theory analysis, particularly in the S wave isoscalar amplitude. These differences have already been shown to be significant for low energy pion-nucleus scattering and would change most of the potential and field theory models for the low energy πN interaction.

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