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Cross sections for trinucleon photoeffect*

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Several new examples of electric dipole transitions to final isospin 3/2 are calculated using the Fabre-Levinger formalism: (i) the Volkov potential (Serber exchange) gives an integrated cross section 4% higher than the O'Connell-Prats sum rule; (ii) the V^x potential (zero force in odd-parity two-body states) gives a ^{3}H photoeffect cross section close to that found by Fabre and Levinger for ^{3}He; (iii) transitions to an uncoupled grand orbital three are several percent of those to grand orbital one. Addition of the cross sections for uncoupled states gives poor agreement with the coupled calculation of Fang *et al.*; (iv) a V^x potential (Wigner-Bartlett exchange) gives an integrated cross section 5% higher than the Thomas-Reiche-Kuhn value. We also calculate for final isospin 1/2, neglecting coupling to two-body breakup. The V^x potential (zero force in odd-parity two-body states) gives an integrated cross section 23% higher than the O'Connell-Prats value. A Wigner-Bartlett mixture gives a 23% disagreement with the Thomas-Reiche-Kuhn sum rule. The calculated cross section (summed for both isospin states) has a higher and narrower peak than that measured by Gorbonov (summed for two-body and three-body breakup).

[NUCLEAR REACTIONS photodisintegration of trinucleon; hyperspherical harmonics.]

I. INTRODUCTION

Expansions in hyperspherical harmonics (h.h.) have been used by a number of workers in the past decade for calculations of the ground state properties of systems of three nucleons[1,2] and of four nucleons[2,3,4]. The convergence of the h.h. expansion is rapid, giving accurate numerical results with truncated expansions.

Delves[5] used a single h.h. for the ground state and continuum wave functions reached in the trinucleon photoeffect. His work was corrected by Fabre and Levinger[6] (designated collectively as FL) and extended to spin-dependent but central two-nucleon potentials. FL calculated for final states of isospin 3/2, to avoid the problem of a final nucleon-deuteron wave function, which poses a severe test of the h.h. expansion[7].

Myers, Fang and Levinger[8] (designated MFL) tested the single-term h.h. expansion for the continuum state by comparison of the integrated cross section with the Thomas-Reiche-Kuhn sum rule for a Volkov potential of Wigner exchange character. They found agreement within 6%.

Recently Fang, Levinger and Fabre[9] (designated FLF) truncated the expansion of the continuum wave function at two h.h. They found an improved agreement with the TRK sum-rule for a potential of Wigner character: the discrepancy was reduced to only 3%. They also calculated with a spin-dependent Volkov potential with Serber exchange character, and found that the total cross section for isospin 3/2 final states was in closer agreement with experiment for three-body break-up than that for a single h.h.

These successes with severely truncated h.h. expansions suggest testing the h.h. technique on other properties of the trinucleon photoeffect, for electric dipole transitions: specifically in this paper the cross section for final states of isospin 1/2 and the total cross section for both isospin states. Our purpose is to find the limits of applicability of h.h. expansions, and to develop an intuitive feeling as to which calculations can be done accurately by this technique.

We shall first review rapidly the concepts and notation involved in calculation of the trinucleon photoeffect (electric dipole transitions to isospin 3/2) using a single h.h. for the final state. We also review sum rule calculations of the integrated cross section. In the next section we apply the FLF formalism to find the total cross section for isospin 3/2 for the ^{3}H photoeffect, for a Wigner-Bartlett exchange mixture and for a Volkov spin independent potential with Serber exchange character. We also examine transitions to grand orbital three in the uncoupled h.h. approximation. In section III we calculate the total cross section for the ^{3}He photoeffect to isospin 1/2 final states. In the last section we summarize the tests of the h.h. expansion by comparison with experiment, and by comparison with sum rules.

We follow FLF notation for the trinucleon photoeffect. We first define Jacobi variables $\vec{\xi}_1$ and $\vec{\xi}_2$ for the trinucleon, in terms of the nucleon coordinates $\vec{r}_1$, $\vec{r}_2$, $\vec{r}_3$ and center of mass coordinate $\vec{R}$.

$$\vec{\xi}_1 = \vec{r}_1 - \vec{r}_2 ; \quad \xi_2 = 3^{\frac{1}{2}}(\vec{r}_3 - \vec{R}). \qquad (1.1)$$

The two vectors in three-dimensional space are combined into a six vector, of length $\xi = (\xi_1 + \xi_2{}^2)^{1/2}$. We denote its five angles collectively by Ω. Two angles give the directions $\omega_1(\theta_1\ \phi_1)$ of the unit vector ξ_1, two others give ω_2 for $\hat{\xi}_2$, the fifth angle is defined as $\tan^{-1}(\xi_1/\xi_2)$.

The kinetic energy operator T is expressed in terms of these six hyperspherical coordinates. If the potential energy were zero and energy $E = \hbar^2k^2/M$, the wave function $\psi(\xi,\Omega)$ obeys the six-dimensional Helmholtz equation, where ∇^2 represents the Laplacian in six-dimensional space,

$$(\nabla^2 + k^2)\psi(\xi,\Omega) = 0 \quad (1.2)$$

The solution of this equation for a single partial wave[10] is a product of a Bessel function of k and a hyperspherical harmonic $H^{m_1,m_2}_{L,\ell_1,\ell_2}(\Omega)$.

$$\psi_L(\xi,\Omega) = (k\xi)^{-2}J_{L+2}(k\xi)H^{m_1,m_2}_{L,\ell_1,\ell_2}(\Omega) \quad (1.3)$$

Here the "grand orbital" $L = \ell_1 + \ell_2 + 2n$, where n is a non-negative integer. The parity is then $(-1)^L$. Normalized h.h. have been discussed by many workers[2]; Fang[11] has recently given explicit equations for a number of h.h.

For the three-nucleon problem, we follow Simonov[1] and Fabre[2] in taking combinations of h.h. which have i) a specified value of the total orbital angular momentum ℓ_{tot}, and ii) a specified symmetry for spatial exchange of a pair of nucleons. We include completely symmetric (denoted by a superscript (0)) and mixed symmetry, either even (+) or odd (-) on exchange of the first and second nucleons. The completely antisymmetric spatial wave function is very small[2,12] and will be neglected. We write these normalized combinations for a specified grand orbital L as $P_L{}^{(0)}(\Omega)$, $P_L{}^{(+)}(\Omega)$, and $P_L{}^{(-)}(\Omega)$. Of course these are combined with appropriate spin-isospin wave functions to give desired values of total spin S and total isospin T, and to obey the Pauli principle.[11,12]

In general the potential energy of the trinucleon $V(\xi,\Omega)$ depends on the angles, thus coupling together different h.h. Fabre truncates the h.h. expansion of the wave function giving a finite number of coupled differential equations, which are solved numerically to give ground state hyperradial functions $u_0(\xi)$, $u_2(\xi)$, $u_4(\xi)$, etc. Here $u_L(\xi) = \xi^{5/2}\psi_L(\xi)$. Since the ground state has even parity, he uses only even value of the grand orbital. L = 0 and L = 4 are completely symmetric h.h., while L = 2 has mixed symmetry.

FLF truncate the partial wave expansion of the 1^-, T = 3/2 final state at L = 3 obtaining coupled differential equations for $u_1(\xi)$ and $u_3(\xi)$. Neglecting the coupling between these two states, the FLF equations reduce to

$$-u_1'' + (35/4\xi^2)u_1 + (M/\hbar^2)U_1{}^{(1)}u_1 = k^2u_1$$
$$-u_3'' + (99/4\xi^2)u_3 + (M/\hbar^2)U_3{}^{(3)}u_3 = k^2u_3 \quad (1.4)$$

The effective potentials $U_1{}^{(1)}(\xi)$ and $U_3{}^{(3)}(\xi)$ are expressed in terms of "hypermultipoles" $V_L{}^{(2S+1)\pm}(\xi)$ of the two-nucleon potential for a state of spin S and parity + or -.

$$U_1{}^{(1)} = (3/2)[V_0{}^{(1+)} + V_0{}^{(3-)} + V_2{}^{(1+)} - V_2{}^{(3-)}]$$
$$U_3{}^{(3)} = (3/2)[V_0{}^{(1+)} + V_0{}^{(3-)} + V_2{}^{(1+)}/3 - V_2{}^{(3-)}/3 - (8/3)V_6{}^{(1+)} + (8/3)V_6{}^{(3-)}] \quad (1.5)$$

The hypermultipole $V_{2K}(\xi)$ for a gaussian potential $V(\xi_1) = v_0 \exp(-\xi_1{}^2/a^2)$ is given by

$$V_{2K}(\xi) = 2v_0 \exp(-x')I_{K+1}(x')/x'$$
$$x' = \tfrac{1}{2}(\xi/2)^2. \quad (1.6)$$

In (1.6) I_{K+1} is a modified Bessel function.

Since the effective potentials are proportional to ξ^{-3} at large hyperradius ξ, we can express the asymptotic solution of Eq. (1.4) for L = 1 as linear combinations of the regular solution given in (1.3) and the irregular solution, proportional to the Neumann function $N_3(k\xi)$. We use the normalization from FLF at large ξ; $u_1(\xi)$ has the asymptotic form

$$u_1(\xi) = (\xi^{1/2}/k^2)\ [\cos\delta_1 J_3(k\xi) - \sin\delta_1 N_3(k\xi)] \quad (1.7)$$

($\delta_1(k)$ is the phase shift for three-body to three-body scattering[11] for grand orbital one, $\ell^\pi_{tot} = 1^-$.)

Considering only transitions to grand orbital one, FLF express the cross section in terms of the hyperradial overlap integral R_{01}

$$R_{01} = \int [u_0(\xi) - u_2(\xi)/2^{1/2}]\xi u_1(\xi)d\xi \quad (1.8)$$

$$\sigma(3/2) = (\pi^2/18)\alpha(M/\hbar^2)(E_\gamma k^4)(R_{01})^2. \quad (1.9)$$

The $u_2(\xi)$ term comes from the mixed symmetry part of the ground state wave function.

We can check our results for the total cross section by using it to evaluate moments $\sigma_p(3/2)$ defined as

$$\sigma_p(3/2) = \int_0^\infty E_\gamma{}^p \sigma(3/2) dE_\gamma . \qquad (1.10)$$

MFL treat a force of pure Wigner character and compare moments from (1.10) for p = -1, 0, and 1 with values found by using sum rules.[13] Below we compare with the O'Connell-Prats[14] (OP) sum rules for potentials with Bartlett, Majorana or Heisenberg exchange; in this case the moments depend on the final isospin.

The choice p = -1 gives $\sigma_{-1}(3/2)$ proportional to the mean square radius of the trinucleon, and independent of the calculation of the continuum wave functions. The integrated cross section, p = 0, provides a test of the truncation used to obtain the continuum wave functions. For a pure Wigner potential, the integrated cross section is given by the Thomas-Reiche-Kuhn (TRK) sum rule, independent of the form of the trinucleon wave function, or the nucleon-nucleon potential. For the spin-independent potential treated by MFL, the cross sections and moments do not depend on the final isospin[14] so MFL calculate the total cross section $\sigma = \sigma(3/2) + \sigma(\frac{1}{2}) = 2\sigma(3/2)$, and compare the integrated total cross section with the TRK sum rule $\sigma_0 = (4\pi^2/3)(e^2\hbar/Mc) = 39.8$ MeV mb. The value found from (1.10) is only 6% higher than the TRK results; while FLF find that the discrepancy is reduced to a mere 3% when two coupled h.h. are used in calculating the continuum wave functions.

For spin-dependent potentials, we compare moments calculated from the cross sections with the OP sum rules. Again the choice p = -1 merely checks numerical accuracy. In our notation

$$\sigma_{-1}(3/2) = \int^\infty [u_0(\xi) - 2^{-\frac{1}{2}} u_2(\xi)]^2 \xi^2 d\xi \qquad (1.11)$$

For $\sigma_{-1}(\frac{1}{2})$, OP change the minus to a plus sign in the integrand.

For p = 0, and only Wigner or Bartlett exchange, the OP sum rules give equality of $\sigma_0(3/2)$ and $\sigma_0(\frac{1}{2})$:

$$\sigma_0(3/2) = \sigma_0(\tfrac{1}{2}) = (2\pi^2/3)(e^2\hbar/Mc) = 19.9 \text{ MeV mb} \qquad (1.12)$$

We use this sum rule below to check the accuracy of our truncation to a single h.h. For a sum of two-nucleon potentials including a fraction x_m of Majorana exchange and a fraction y of isospin exchange, OP find that the integrated cross sections are given by the following equations:

$$\sigma_0(3/2) = (2\pi^2/3)\alpha(\hbar^2/M)[1-(x_m-y)(M/\hbar^2)(\psi_0{}^S, r^2V(r)\psi_0{}^S)]$$
$$\sigma_0(\tfrac{1}{2}) = (2\pi^2/3)\alpha(\hbar^2/M)[1-x_m(M/\hbar^2)(\psi_0{}^S, r^2V(r)\psi_0{}^S)] \qquad (1.13)$$

They have made the approximation of using a completely symmetric ground state wave function $\psi_0{}^S$. In truncating we make the further approximation that $\psi_0{}^S$ can be replaced by the lowest h.h., namely L = 0, which has a weight of some 99% for the examples we consider. We can replace $r^2V(r)$ of OP by $\xi_1{}^2V(\xi_1)$; we expand the latter in h.h. Performing integrations over the five angles, we find

$$(\psi_0{}^S, r^2V(r)\psi_0{}^S) = \tfrac{1}{2}\int_0^\infty [u_0(\xi)]^2 \xi^2 [V_0(\xi) - V_2(\xi)] d\xi \qquad (1.14)$$

Eqs. (1.13) and (1.14) were used by FLF to test the truncation of the continuum wave function to two h.h. for the spin-independent V^X potential with Serber exchange character. A single h.h. gave an integrated cross section very close to the sum rule result, while two h.h. gave an integrated cross section only 2% above the sum rule value. The speed of convergence of the h.h. expansion is similar to that found by MFL for a potential of Volkov shape and pure Wigner exchange character.

II. FINAL ISOSPIN 3/2

We apply the FL formalism outlined in the introduction to several new examples. First we use the Volkov spin-independent potential for the ^{3}H nucleus, considering transition from grand orbital zero to grand orbital one. As discussed above, MFL have calculated in Born approximation, and with assumed pure Wigner forces. We now assume a Serber exchange mixture. We substitute $V_0{}^{(3-)} = V_2{}^{(3-)} = 0$ in Eq. (1.5) for the effective potential $U_1{}^{(1)}$. We evaluate the phase shift δ_1 from (1.7), the cross section $\sigma(3/2)$ from (1.9) and the moments $\sigma_p(3/2)$ from (1.10). Our results are given in Table I. We find large phase shifts, i.e., the Born approximation is invalid for a Serber mixture: compare with MFL for Born approximation for $\sigma(3/2)$. The moment $\sigma_{-1}(3/2)$ is used to check numerical accuracy: it agrees with MFL. We compare the integrated cross section $\sigma_0(3/2)$ with the OP sum rule (1.13) and (1.14). The sum rule gives $\sigma_0(3/2) = 28.1$ MeV mb, only 4% lower than the value 29.3 MeV mb found from $\sigma(3/2)$.

We next use the same Volkov potential for ^{3}H, but this time follow MFL in assuming Wigner forces. We now consider transitions from initial grand orbital four[4] to final grand orbital three. We present in Table II results in Born approximation, and phase shift δ_3 and $\sigma(3/2)$ from Eqs.

(1.5) to (1.9). (In Eq. (1.5) we use $V_L^{(1+)} = V_L^{(3-)}$. In Eq. (1.7) we change δ_1 to δ_3, and $J_3(k\xi)$ and $N_3(k\xi)$ to $J_5(k\xi)$ and $N_5(k\xi)$ respectively.) We see that the phase shift δ_3 is appreciable, and becomes quite large at high energy. Then Born approximation is not accurate.

Transitions to an uncoupled grand orbital three (denoted by primes) are much smaller than those found by MFL to uncoupled grand orbital one. The ratio of $\sigma'_{-1} = 0.0335$ mb to MFL $\sigma_{-1} = 1.44$ mb is 2.3%. The ratio of $\sigma_0' = 1.11$ MeV mb to MFL $\sigma_0 = 21.1$ MeV mb is 5.3%. The ratio of $\sigma'(E_\gamma)/\sigma(E_\gamma)$ increases with photon energy E_γ.

We now consider whether use of two uncoupled partial waves for the final state is a good approximation to FLF's calculation using two coupled partial waves. The use of two uncoupled partial waves is clearly unsatisfactory for finding the integrated cross section. $\sigma_0 + \sigma_0' = 22.2$ MeV mb, while FLF find $\sigma_0 = 20.5$ MeV mb in good agreement with 19.9 MeV mb from the Thomas-Reiche-Kuhn sum rule. FL point out two results of including a second partial wave: i) transitions to the second partial wave increase the cross section; ii) coupling between the two partial waves acts like an increased attractive potential for the dominant partial wave decreasing the cross section at energies past the peak, and decreasing the integrated cross section. We find that the second effect dominates.

FL used the spin-dependent V^x potential, for an uncoupled final state of grand orbital one. They used the ground state Ballot wave functions[4] for ^{3}He, and neglected the Coulomb force in calculating the continuum wave function. We now consider ^{3}H, using Ballot's ground state wave function for this nucleus, and the same wave function used by FL. Table I presents the Born approximation and the numerical calculation of $\sigma(E_\gamma)$. Since we calculate at the same wave number k as used by FL, our phase shifts are the same.

The cross sections and moments should be in reasonable agreement with experimental values on the three-body break-up of ^{3}H. However, we have not found such experiments in the literature; Table I gives predictions to be checked when experiments are completed. We study the importance of coulomb effects on the ground state wave function by comparing Table I with FL. We find that the peak cross section of 0.97 mb is unchanged. Below the peak energy, ^{3}H cross sections are smaller by some 10%; above the peak energy, ^{3}H cross sections are larger by some 10%. (We compare cross sections at the same continuum wave number, or 0.7 MeV higher gamma ray energy for ^{3}H).

In distinction to the Serber mixture for the V^x potential used by FL for ^{3}He, we now use a Wigner-Bartlett mixture. That is, in Eq. (1.5) we assume $V_L^{(3-)} = V_L^{(3+)}$. Our purpose is to check against the OP version of the TRK sum rule: i.e., $\sigma_0 = 19.9$ MeV mb. Table III gives $\sigma(E_\gamma)$, δ_1 and three moments. We see that our value of 21.0 MeV mb for the integrated cross section is 5% higher than the TRK value: the same sort of agreement found by MFL, for a Volkov potential of pure Wigner exchange character.

III. TRANSITIONS TO ISOSPIN ½

As we remarked in the introduction, FL and FLF limited their calculations to final states of isospin 3/2, to avoid the problem of two-body break-up which is difficult to treat in the h.h. formalism.[7] However, if we treat our calculation as a mathematical model, which may or may not be a good approximation to the real world, we can make a simple change in the earlier formalism to calculate transitions to isospin ½ states.

Table I

^{3}H Photoeffect

	Volkov (Serber)		V^x Potential Born	V^x Potential Serber
E_γ(MeV)	$\sigma(3/2)$mb	δ_1(degrees)	$\sigma(3/2)$mb	$\sigma(3/2)$mb
8.84	0.0066	14.4	0.696 E-3	0.186 E-2
9.51	0.108	26.8	0.0113	0.0296
10.61	0.558	42.2	0.0634	0.158
12.33	1.35	60.1	0.204	0.458
15.02	1.70	76.4	0.446	0.828
19.38	1.26	86.7	0.702	0.974
26.99	0.625	89.4	0.792	0.744
41.79	0.208	84.14	0.590	0.354
76.92	0.0345	69.1	0.196	0.083
201.7	0.00017	40.2	0.282 E-2	0.184 E-2
	$\sigma_{-1}(3/2) = 1.43$ mb		1.09 mb	1.09 mb
	$\sigma_0(3/2) = 29.3$ MeV mb		38.4 MeV mb	29.7 MeV mb
	$\sigma_1(3/2) = 783$ MeV^2 mb		1850 MeV^2 mb	1140 MeV^2 mb

Table II

Volkov (Wigner mixture) ^{3}H, L = 3

E_γ(MeV)	$\sigma^B(3/2)$ mb	$\sigma(3/2)$ mb	δ_3(degrees)
8.84	6.75 E-8	1.24 E-7	4.33
9.51	7.53 E-6	1.37 E-5	7.82
10.61	1.50 E-4	2.69 E-4	11.55
12.33	0.00124	0.00217	15.88
15.02	0.00572	0.00955	21.15
19.38	0.0165	0.0251	27.92
26.99	0.0294	0.0363	37.22
41.79	0.0254	0.0178	50.29
76.92	0.00305	8.57 E-6	65.73
201.7	0.00127	7.13 E-4	66.65

$\sigma_{-1}{}^B = 0.0342$ mb $\quad \sigma'_{-1} = 0.0335$ mb

$\sigma_0{}^B = 1.45$ MeV mb $\quad \sigma_0{}' = 1.11$ MeV mb

$\sigma_1{}^B = 113$ MeV^2 mb $\quad \sigma_1{}' = 66.7$ MeV^2 mb

In our mathematical model we truncate the h.h. expansion of the potential energy $V(\xi,\Omega)$ of the three-nucleon system at some finite value of grand orbital. MFL truncated at the lowest term, L = 0; in our current work with a spin-dependent potential we truncate at L = 2. FLF work with two coupled partial waves, and effectively truncate the potential at L = 4. Ballot et al. work with coupled partial waves up to L = 24, and effectively truncate the potential at L ≃ 24. These truncated potentials disagree at large hyperradius ξ with a potential which is the sum of short-range two body potentials since each hypermultipole decreases[2] as ξ^{-3} so any finite sum of hypermultipoles will also decrease as ξ^{-3}. But a short range two-body potential (for example between the first and second nucleons) will give a non-zero contribution to the potential $V(\xi,\Omega)$ for large ξ, provided that ξ_1 is smaller than the range of the two-nucleon force. Hence the truncated expansion is unreliable for large ξ and angle ϕ very near zero.

In assessing the justification of an h.h. expansion of the potential for a specified problem, we must ask, "Do we get a significant contribution from the region of ξ and ϕ for which the truncated expansion is unreliable?" If we want to calculate two-body N-d break-up following the photoeffect we must certainly answer, "Yes, so we cannot use the truncated h.h. expansion". But if we need the final state wave function only for moderate values of the hyperradius, perhaps the truncated h.h. expansion will be satisfactory. Since we use the final state wave function in an overlap integral, the rapid decrease of the ground state wave function at large hyperradius suppresses the influence of the final state wave function at large ξ, so there is hope that the truncated expansion will be a good approximation.

Another reason to hope for some success in this approximation is that we can calculate moments of the cross section $\sigma_p(\frac{1}{2})$ for isospin ½, following OP, using Ballot's ground state wave function and a truncated potential. Since their wave function is reliable, as shown by their calculations of trinucleon form factors, we can expect to calculate reliable moments. But we get similar moments from calculation of the cross section $\sigma(E_\gamma)$, so the cross section curve cannot be completely unreasonable.

On the other hand, it is clear that our truncated expansion must fail in two respects. First, by suppressing two-body break-up we forbid absorption of photons by ^{3}He in the region between the threshold 5.5 MeV for two body break-up and the threshold 7.7 MeV for three-body break-up. Gorbunov[15] finds that the cross section in this 2.2 MeV range is indeed small, and gives a very small contribution to the

Table III

V^x potential, Wigner-Bartlett mixture, ^{3}He

E_γ(MeV)	$\sigma(3/2)$ mb	δ_1(degrees)
8.13	0.278 E-2	8.3
8.81	0.0424	14.8
9.91	0.231	23.0
11.63	0.717	34.4
14.32	1.36	51.5
18.68	1.30	73.8
26.29	0.516	93.1
41.09	0.0859	100.2
76.22	0.390 E-2	92.3
201.0	0.193 E-2	64.1

$\sigma_{-1} = 1.12$ mb

$\sigma_0 = 21.0$ MeV mb

$\sigma_1 = 454$ MeV^2 mb

moments $\sigma_p(\frac{1}{2})$, so this failure should not be serious in getting the overall picture of the photoeffect. Second, we will be unable to calculate the branching of the isospin ½ state for two-body break-up.

The calculation of the cross section for electric dipole transitions to isospin ½ states with grand orbital one uses the FL formalism, with two minor modifications. First, when we evaluate the matrix element for the dipole operator using the spin-isospin wave functions for mixed symmetry final states of isospin ½, Eq. (1.8) for overlap integral R_{01} for transitions to isospin 3/2 is replaced by

$$R'_{01} = \int_0^\infty [u_0(\xi) + u_2(\xi)/2^{\frac{1}{2}}]\xi u'_1(\xi)d\xi \qquad (3.1)$$

That is, we change the sign of the mixed symmetry term $u_2(\xi)/2^{\frac{1}{2}}$ in the ground state wave function. $u'_1(\xi)$ is the continuum radial function for isospin ½. Equation (1.9) holds, replacing R_{01} by R'_{01}.

Second, the effective potential $U_1^{(1)}(\xi)$ given in Eq. (1.5) and used in (1.4) to find the continuum wave function $u_1(\xi)$ is replaced by the new expression

$$U_1^{(1)}(\tfrac{1}{2}) = (3/4)[V_0^{(1+)} + V_0^{(3+)} + V_0^{(1-)} + V_0^{(3-)} + V_2^{(1+)} + V_2^{(3+)} - V_2^{(1-)} - V_2^{(3-)}] \qquad (3.2)$$

We use $U_1^{(1)}(\frac{1}{2})$ to find $u'_1(\xi)$ and δ'_1, the phase shift for isospin ½. Note that all four two-body states now contribute to the effective potential, unlike the case of isospin 3/2 where we have an isospin symmetric wave function and hence only 1+ and 3- two-body states.

If we consider a Volkov potential of Wigner exchange character, as used by MFL, then $u_2(\xi)$ is zero. Further, the effective potentials $U_1^{(1)}$ and $U_1^{(1)}(\frac{1}{2})$ each reduce to 3 $V_0(\xi)$, as used by MFL. For a Volkov-Serber potential, $U_1^{(1)} = U_1^{(1)}(\frac{1}{2}) = (3/2)[V_0^{(1+)} + V_2^{(1+)}]$ as used in Section II. In each case, we obtain the same cross section for final isospin 1/2 or 3/2.

We now apply equations (3.1) and (3.2) using Ballot's ground state wave functions for ^{3}He for the V^x potential. We consider three different effective potentials for the final state: i) Born approximation, ii) Serber exchange with $V_L^{(1-)} = V_L^{(3-)} = 0$, iii) Wigner-Bartlett exchange with $V_L^{(1-)} = V_L^{(1+)}$ and $V_L^{(3-)} = V_L^{(3+)}$. Table IV gives our numerical results for the cross sections for isospin 1/2 states for the three choices. We also present the phase shifts δ_1' for the latter two cases.

The cross sections for Born approximation are very close to 1.4 times those of FL, for final isospin 3/2; the factor 1.4 takes account of the change of sign $u_2(\xi)$ in Eq. (3.1) as compared to (1.8). The phase shifts for a Serber mixture are much larger for isospin 1/2 than those of FL for isospin 3/2, reaching a maximum of 83° for the former and only 49° for the latter. These larger phase shifts are caused by the more attractive effective potential $U_1^{(1)}(\frac{1}{2})$, Eq. (3.2) as compared to $U_1^{(1)}$ of FL for isospin 3/2. The more attrac-

Table IV

V^x Potential, T = ½, ^{3}He

	Born	Serber		Wigner-Bartlett	
E_γ(MeV)	$\sigma^B(\frac{1}{2})$ mb	δ_1'(degrees)	$\sigma(\frac{1}{2})$ mb	δ_1'(degrees)	$\sigma(\frac{1}{2})$ mb
8.13	0.0013	12.7	0.0062	12.9	0.0078
8.81	0.019	23.0	0.093	23.9	0.128
9.91	0.101	35.5	0.472	39.6	9.758
11.63	0.316	50.3	1.22	64.8	2.28
14.32	0.677	65.6	1.75	97.2	2.61
18.68	1.04	77.5	1.46	119.8	1.12
26.29	1.15	83.2	0.765	127.8	0.263
41.09	0.826	91.2	0.251	124.3	0.030
76.22	0.246	69.4	0.039	108.7	0.0004
201.0	0.0020	43.8	0.00026	74.1	0.00009
	σ_{-1}^B = 1.61 mb		1.60 mb		1.6 mb
	σ_0^B = 52.7 MeV mb		32.8 MeV mb		24.5 MeV mb
			Sum rule 26.7 MeV mb		19.9 MeV mb
	σ_1^B = 2350 MeV^2 mb		882 MeV^2 mb		414 MeV^2 mb

tive potential for final isospin ½ states causes the photoeffect cross section to peak at a lower energy for these states, in qualitative agreement with Gorbunov's comparison of $\sigma(2)$ for two-body break-up and $\sigma(3)$ for three-body break-up. Comparisons of the results for two different isospin states for a Wigner-Bartlett force (Tables III and IV) show a similar effect of the more attractive potential for isospin ½ states.

The three values of the moment σ_{-1} given in Table IV are in good agreement with each other, and with the OP sum-rule value. We compare the values of the integrated cross section for a Wigner-Bartlett mixture and for a Serber mixture with those found from the OP calculation. The value σ_o = 24.5 MeV mb for the former is 23% higher than the Thomas-Reiche-Kuhn value of 19.9 MeV mb. For the Serber mixture, the OP equation gives σ_o = 26.7 MeV mb, so our 32.8 MeV mb is 22% higher.

Combining calculations of FL and column 4 of this table gives us an h.h. calculation of the total cross section for the ^{3}He photoeffect, for a V^x potential with a Serber mixture. Figure 1 compares our calculated curve (solid) with Gorbunov's[15] experimental total cross section, which is the sum of two-body and three-body break-up. We also show (dotted) our result for a Born approximation. The agreement between calculations and experiment is only fair. Our calculations for the V^x potential gives too little cross section near threshold, too high a cross section near the peak (12 to 26 MeV) and too low a cross section above 50 MeV. Our previous work using coupled channels for isospin 3/2 (FLF) suggests that a coupled channel calculation for isospin ½ would improve the agreement with experiment both at low energy and above 50 MeV.

We also compare calculated and experimental values for the bremsstrahlung weighted and integrated total cross sections. The three calculated values for σ_{-1} are in good agreement with the experimental σ_{-1} as they must be, since all agree with the rms charge radius of ^{3}He as determined by electron scattering. The value of σ_o for the V^x potential of 61.8 MeV mb based on the cross sections of Table IV is 11% higher than the OP sum rule value of 55.5 MeV mb. Neither is in good agreement with Gorbunov's experimental value of 70 ± MeV mb.

IV. DISCUSSION

In the previous two sections we have applied truncated expansions in h.h. to calculate the trinucleon photoeffect to final isospin 3/2 and 1/2 respectively.

Our work in Section II and Tables I, II and III used the standard formalism for isospin 3/2 of FL applied to several new examples. Table I gives the ^{3}H photoeffect for the Volkov spin-independent potential,

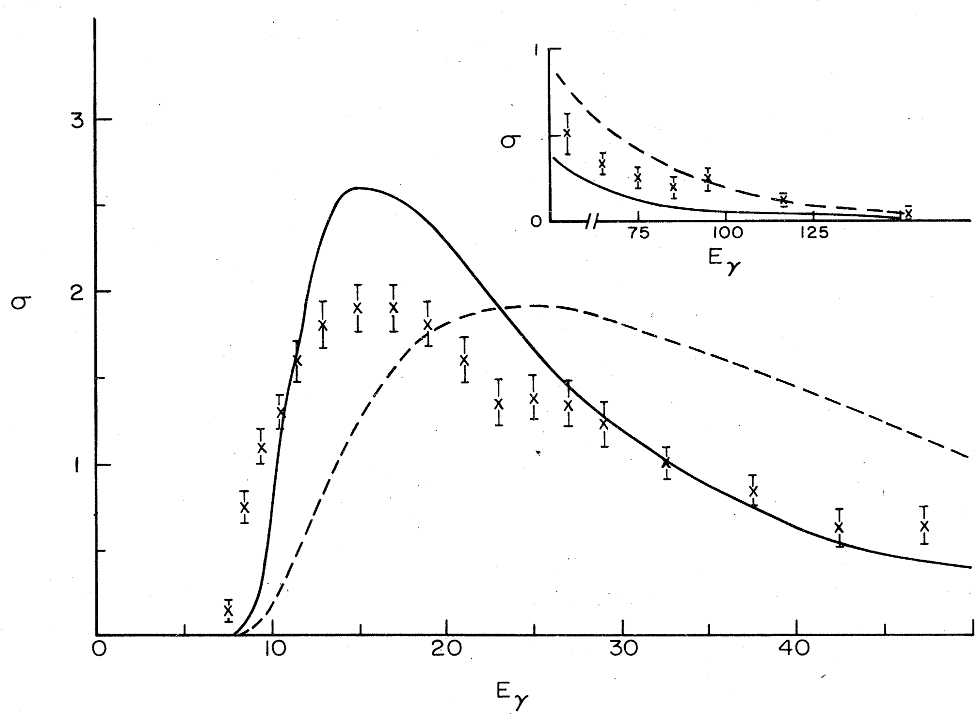

FIG. 1. Total cross section for the ^{3}He photoeffect in mb vs photon energy in MeV. The solid curve shows E1 calculations summed over final isospin states, for a V^x potential, zero in odd-parity two-body states. The dashed curve shows Born approximation. The points with statistical errors show Gorbunov's measurements, summed for two-body and three-body break-up.

with his original Serber exchange mixture, and also the spin-dependent V^x potential, with the same Serber mixture. Both calculations are checked against the OP sum rules. The former disagrees with the OP integrated cross section by only 4%. This excellent agreement is similar to that found by MFL for a Volkov potential of Wigner exchange character. The cross sections for the V^x potential given in Table I are predictions to be checked against future experiments on three-body break-up of ^3H. We note that they are close to the FL results for ^3He for the same nuclear potential: Coulomb effects on the ground state wave function are small.

Table II shows that cross sections to grand orbital three are small, as anticipated. The use of uncoupled grand orbitals is a poor approximation: indeed the use of uncoupled grand orbitals one and three is in general farther from the coupled FLF calculation than the cross sections of MFL for a single uncoupled grand orbital. At high energies where transitions to grand orbital 3 are appreciable more work should be done to study the accuracy of truncation at L = 3, or L = 5, or still higher grand orbitals.

Table III shows that the integrated cross section for the spin-dependent potential is within 5% of the TRK results appropriate to the Wigner-Bartlett mixture assumed.

In Section III we treat final isospin ½, truncating the h.h. expansion of the potential energy of ^3He at grand orbital two and the h.h. expansion of the wave function at the single term with grand orbital one. We present the new expressions for the effective potential and for the overlap integral. For a spin independent potential (with or without Majorana exchange) the photoeffect cross sections are independent of the isospin of the final state. For a Volkov potential with pure Wigner exchange, MFL presented the total cross section, summed over final isospin. In this paper we use a Volkov potential with Serber exchange. The cross sections given in Table I, multiplied by two, give the total cross section. The peak is broader and not as high as for a Wigner mixture, but is still 80% higher than Gorbunov's peak experimental value for the total cross section, summed over both two-body and three-body break-up.

In Table IV we presented cross sections $\sigma(\frac{1}{2})$ for isospin ½ for the V^x potential. The results for the integrated cross potential cross sections are not in close agreement with the approximate OP sum rules for this case. For a Serber mixture, our result is 24% above the sum rule; and for a pure Wigner-Bartlett mixture we are 23% above the TRK value. Presumably the stronger attractive potential in isospin ½ makes the truncation of the final wave function to a single term less accurate than for isospin 3/2.

For the V^x potential with a Serber mixture, the cross section curve $\sigma(\frac{1}{2})$ (Table IV) is narrower and has a higher peak than that found by FL for $\sigma(3/2)$. The sum $\sigma_t = \sigma(\frac{1}{2}) + \sigma(3/2)$ is in better agreement with Gorbunov's total cross section than our calculated σ_t for the Volkov potential (Serber mixture). For instance the 80% discrepancy for the peak cross section is reduced to 35%. But the fit between our calculated σ_t and Gorbunov's values, shown in Fig. 1, is clearly not satisfactory.

We conclude that our calculation for final isospin ½ is in an inconclusive state. A more accurate calculation using coupled h.h. may well improve agreement with experiment. In any event, our calculation of σ_t even with a single h.h. clearly represents an improvement over Born approximation. But, as noted above, any truncated calculation will fail in the energy range between the thresholds for two-body and three-body break-up.

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1. Yu. A. Simonov, Sov. J. Nucl. Phys. (transl.) 3, 461 (1966).
2. M. Fabre de la Ripelle, Proceedings of the International School on Nuclear Theory Physics (Predeal, 1969).
3. V. F. Demin, Yu. F. Pokrovsky and V. D. Efros, Phys. Lett. 44B, 227 (1973).
4. J. L. Ballot, M. Beiner and M. Fabre de la Ripelle, Proceedings of the Symposium on Present Status and Novel Developments in Many-Body Problems, Roma, 1972, edited by V. Calogero and C. Ciofi degli Atti (Bologna, 1974) p. 565.
5. L. M. Delves, Nucl. Phys. 20, 268 (1962).
6. M. Fabre de la Ripelle and J. S. Levinger, Nuov. Cim. 25A, 555 (1975), and Nuov. Cim. Letters 16, 413 (1976).
7. J. Revai and J. Raynal, Nuov. Cim. Letters 9, 461 (1974).
8. Karen J. Myers, K. K. Fang and J. S. Levinger, Phys. Rev. C 15, 1215 (1977).
9. K. K. Fang, J. S. Levinger and M. Fabre de la Ripelle, Phys. Rev. C 17, 24 (1978).
10. P. M. Morse and H. Feshbach, Methods of Theoretical Physics, (McGraw-Hill, New York, 1953), Vol. II, pp. 1730 ff.
11. K. K. Fang, Phys. Rev. C 15, 1204 (1977).
12. M. Verde, Handbook of Physics (Springer-Verlag, New York, 1957),

Vol. 34.
13. J. S. Levinger, Nuclear Photodisintegration (Oxford University Press) 1961.
14. J. S. O'Connell and F. Prats, Phys. Rev. 184, 1007 (1969).
15. A. N. Gorbunov, Photonuclear and Photomesic Processes, Proc. of the P. N. Lebedev Physics Institute, 1974 (Nauka, Moscow), Vol. 71, p. 1 (in Russian); Photonuclear and Photomesic Processes, Trudy, (Editor D. V. Skobel'tsyn), Consultants Bureau, New York, 1976, Vol. 71, p. 1 (transl.).