

## Shell-model studies for the $^{132}\text{Sn}$ region. I. Few proton cases

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The Brueckner  $G$  matrix from the Reid soft-core potential is taken for the leading contribution of the effective two-proton interaction in the  $^{132}\text{Sn}$  region. Phenomenological two-body pairing and multipole forces are added to adjust for all neglected effect. Two model spaces are selected; a small space ( $0g_{7/2}$ ,  $1d_{5/2}$ ) and a large space ( $0g_{7/2}$ ,  $1d_{5/2}$ ,  $0h_{11/2}$ ,  $2s_{1/2}$ ,  $2s_{1/2}$ ). For each space the strengths of the phenomenological terms are adjusted to fit the  $^{134}\text{Te}$  spectrum. It is found that both of the total interactions have a high statistical correlation with the bare  $G$  matrix. The total effective interactions are then used without further adjustment in shell model calculations to obtain good predictions for the spectra of  $^{135}\text{I}$ ,  $^{136}\text{Xe}$ , and  $^{137}\text{Cs}$ .

NUCLEAR STRUCTURE Shell-model structure of  $^{132}\text{Sn}$  core plus valence protons: shell-model effective interactions consisting of Brueckner reaction matrix plus phenomenological multipole corrections.

### I. INTRODUCTION

Several years ago, Wildenthal<sup>1</sup> suggested that the nuclei near  $^{132}\text{Sn}$  might be nearly as suitable for shell-model calculations as those near  $^{208}\text{Pb}$ . Since that time only a few theoretical investigations of the tin region have appeared and they can be conveniently divided into two groups. On the one hand, there are limited shell-model studies (Wildenthal, Newman, and Auble,<sup>2</sup> Wildenthal and Larson<sup>3</sup>). On the other hand, some nuclei have been treated in the quasiparticle approach (Warquier and Heyde,<sup>4</sup> and Miles<sup>5</sup>).

The experimental situation in the tin region is the reverse of that in the lead region. Near  $^{132}\text{Sn}$ , experimental data have been far easier to obtain for nuclei having many valence proton particles or many valence neutron holes than for nuclei with only a few extracore particles or holes. Only in recent years has any significant information been developed for nuclei with few nucleons near the  $^{132}\text{Sn}$  "core."

To date, shell-model calculations for  $^{132}\text{Sn}$  plus a few protons<sup>2,3</sup> have been executed in fairly restricted model spaces and have employed the modified surface  $\delta$  interaction (MSDI).<sup>6</sup> In particular, when the valence nucleons are all protons they have been restricted to the  $0g_{7/2}$  and  $1d_{5/2}$  orbits with perhaps one proton per orbit in the  $1d_{3/2}$  and  $2s_{1/2}$  orbitals. No protons were permitted in the  $0h_{11/2}$  orbit. The parameters of the MSDI and the single-particle splittings were adjusted to yield agreement with available data for  $A = 136-140$  or  $A = 135-145$ .

The degree of success obtained in these phenomenological calculations coupled with recent experimental interest<sup>7</sup> in the nuclei of this region

suggest the need for shell-model studies with more realistic interactions and a larger model space where possible. The goal of the study reported in this paper has been to investigate the level structure of nuclei consisting of  $^{132}\text{Sn}$  plus a few valence protons using a semirealistic interaction. The spectra were calculated using the Rochester-Oak Ridge shell-model codes.<sup>8</sup> The following paper (hereafter referred to as II) continues the study for additional valence protons.

The semirealistic<sup>9</sup> matrix elements used here and in II are composed of Goodman, Vary, and Sorenson's<sup>10</sup> bare reaction matrix<sup>11</sup> elements of the Reid soft core potential<sup>12</sup> plus small phenomenological corrections.<sup>13</sup> In this study the parameters of the corrections are determined by fitting the experimental  $^{134}\text{Te}$  spectrum. As discussed later, the magnitude of the necessary corrections decrease substantially as the model space is increased. This is interpreted as support for the semirealistic philosophy that one can hope to parametrize neglected higher-order effects as "corrections" to a theoretically derived lowest-order effective Hamiltonian.

### II. METHODS AND SHELL-MODEL INGREDIENTS

When possible, the shell-model calculations described in the following sections have been carried out in a small model space ( $0g_{7/2}$ ,  $1d_{5/2}$ ) and also in a large model space ( $0g_{7/2}$ ,  $1d_{5/2}$ ,  $0h_{11/2}$ ,  $1d_{3/2}$ ,  $2s_{1/2}$ ). For all of these calculations the Rochester-Oak Ridge shell-model code (RORSMC)<sup>8</sup> was used. The single proton energies for orbits above  $Z = 50$  are given in Table I. The column labeled Wildenthal and Larson<sup>3</sup> is based on the single-particle splittings they obtained from a

TABLE I. Single-proton energies for the tin region.

Orbit	Experiment (MeV)	Wildenthal and Larson (MeV)
$0g_{7/2}$	0.0	0.0
$1d_{5/2}$	0.963	0.88
$0h_{11/2}$	2.76	
$1d_{3/2}$	2.69	3.12
$2s_{1/2}$	2.99	2.95

fitting procedure for nuclei with  $N=82$  and  $A=136-140$  using a limited model space. Some entries in the experimental column have been extrapolated from other odd  $A$ ,  $N=82$  nuclei as shown in Fig. 1. The experimental single-proton energies were employed in all of the shell-model calculations of the present study. The levels shown

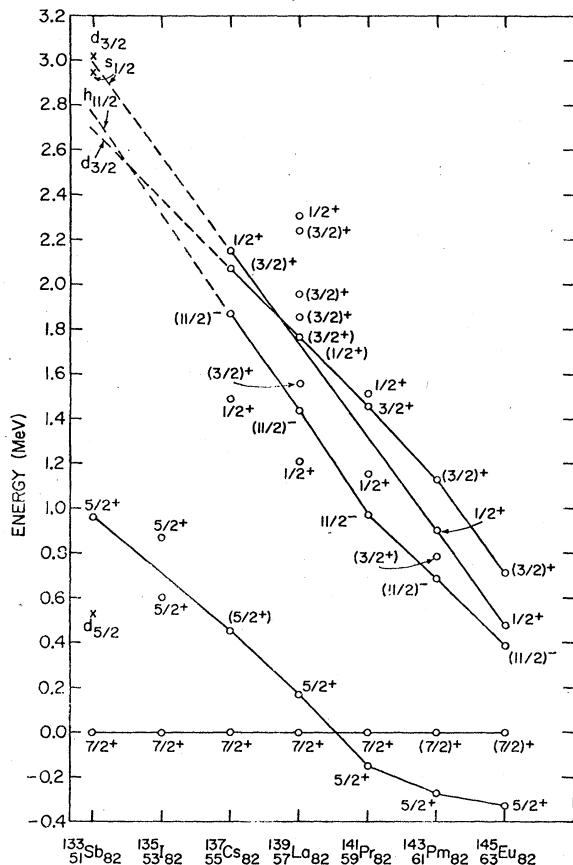


FIG. 1. Extrapolation (dashed lines) to single-proton energies. Open circles depict excitation energies relative to lowest  $\frac{1}{2}^+$  level of low-lying levels of nuclei having  $N=82$  and odd  $Z$  from 51 to 63. The "X's" at  $^{133}\text{Sb}$  indicate values determined by Wildenthal and Larson (Ref. 3).

in Fig. 1 were generally selected on the basis of having moderate to large single-particle spectroscopic factors according to the analysis of Wildenthal, Newman, and Auble.<sup>2</sup> Although these extrapolated values may be subject to considerable uncertainty it seems unlikely that the large gap between the  $1d_{5/2}$  orbit and the next higher orbit would be reduced sufficiently to have any significant effect on the low-lying levels. Further experimental data on the levels of  $^{133}\text{Sb}$  and  $^{135}\text{I}$  would be very useful for evaluating the single-proton energies.

The method<sup>9,13</sup> employed here to determine the semirealistic two-body matrix elements has been used successfully in the lead region where it was shown that the added phenomenological terms in particular are capable of giving a very good approximation to the  $3p-1h$  "core polarization" diagrams.<sup>13</sup> Here, it will suffice to state there are three added terms: pairing, and  $P_2$  and  $P_4$  multipole potentials. The three adjustable strength parameters have been determined by at least squares fit to the relative spacing of the lowest four levels of  $^{134}\text{Te}$  which is treated as two protons outside the  $^{132}\text{Sn}$  core. In this way, separate sets of parameters are obtained for small ( $0g_{7/2}, 1d_{5/2}$ ) and large ( $0g_{7/2}, 1d_{5/2}, 0h_{11/2}, 1d_{3/2}, 2s_{1/2}$ ) model spaces. The resulting two-body matrix elements of the total effective interaction are then used, with no further adjustment, in shell-model calculations for nuclei with three or more valence protons. It is worth noting that only the first four levels of  $^{134}\text{Te}$  have been included in the fitting procedure. The results of the fits are given in Fig. 2 and summarized in Table II. For the large model space, the parameters could be adjusted to give an exceptionally small rms deviation between theory and experiment.

As an indication of the relation between the bare matrix elements and those with corrections included, the correlation between the sets of matrix elements has been computed according to the standard Pearson's correlation formula

$$\text{correlation} = \frac{\sum_i [(X_i - \bar{X})(Y_i - \bar{Y})]}{(\sum_i (X_i - \bar{X})^2 \sum_i (Y_i - \bar{Y})^2)^{1/2}}, \quad (1)$$

where  $X_i$  and  $Y_i$  are member of the respective sets of values. The change in the matrix elements (as measured by the difference from a perfect correlation of 1) for the large model space is only about 3%, while the small model space correction is 7%. Thus, as summarized in Table II, the added terms are small corrections which appear to compensate in part for truncation effects, since they decrease substantially with increasing model

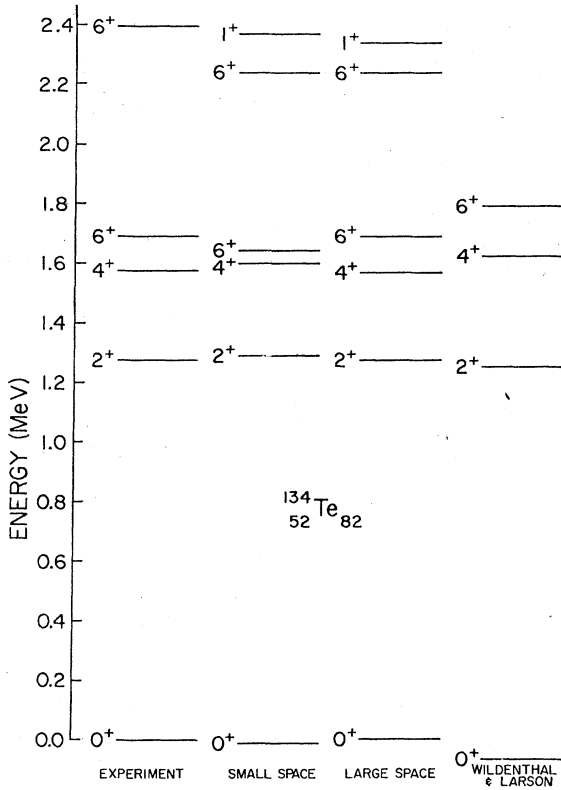


FIG. 2.  $^{134}\text{Te}$  spectra. Experiment is from Nuclear Data Sheets (Ref. 14). Small and large space results are from fitting procedure. The final column gives the results of Ref. 3.

space.

Calculations for  $^{134}\text{Te}$  were also performed for the bare interaction in the small and large spaces. In both cases a lack of  $0_1^+$  to  $2_1^+$  splitting leaves the "bare" spectra rather compressed relative to experiment. The needed splitting is provided by the pairing force which modified only the  $0^+$  ma-

TABLE II. Summary of fitting results. See Ref. 12 for the precise meaning of the strength parameters. The correlation Eq. (1) is between bare and total interaction matrix elements. The rms deviation compares theory and experiment for the four lowest levels of  $^{134}\text{Te}$  after an overall shift of the theoretical spectra to minimize the rms deviation.

	Small space	Large space
Pairing strength	0.180	0.052
$P_2$ strength	0.0031	0.0032
$P_4$ strength	$-0.24 \times 10^{-4}$	$10^{-5}$
Correlation	0.93	0.97
rms deviation (keV)	20	2

TABLE III.  $^{134}\text{Te}$  model wave function amplitudes. S = small space, L = large space, WL = Wildenthal and Larson (Ref. 3). Only amplitudes of magnitude 0.15 or more are listed.

$J^\pi$	Calculation	Model wave function
$0_1^+$	S	$0.903 7/2^2\rangle + 0.430 5/2^2\rangle$
	L	$0.850 7/2^2\rangle + 0.404 5/2^2\rangle - 0.272 11/2^2\rangle + 0.170 3/2^2\rangle$
	WL	$0.90 7/2^2\rangle + 0.43 5/2^2\rangle$
$2_1^+$	S	$0.998 7/2^2\rangle$
	L	$0.990 7/2^2\rangle$
	WL	$0.90 7/2^2\rangle + 0.26 7/2^3\rangle + 0.22 5/2^2\rangle$
$4_1^+$	S	$0.999 7/2^2\rangle$
	L	$0.994 7/2^2\rangle$
	WL	$0.93 7/2^2\rangle - 0.27 7/2^5\rangle$
$6_1^+$	S	$0.996 7/2^2\rangle$
	L	$0.992 7/2^2\rangle$
	WL	$0.93 7/2^2\rangle - 0.37 7/2^5\rangle$
$6_2^+$	S	$0.996 7/2^5\rangle$
	L	$0.992 7/2^5\rangle$

trix elements. As might be anticipated from the correlation values, the large model space "bare" results are not as compressed as those for the small space.

Figure 2 also shows the  $^{134}\text{Te}$  spectrum computed by Wildenthal and Larson<sup>3</sup> using the MSDI. After shifting their spectrum (to minimize the deviation from experiment), an rms deviation of  $\sim 90$  keV can be assigned to their prediction. It is not significant that the current predictions are better for  $^{134}\text{Te}$ , since it alone is fitted here while Wildenthal and Larson achieved an overall fit to several spectra.

Model wave functions for  $^{134}\text{Te}$  are given in Table III. The low-lying levels are seen to be well-described by the small model space with the exception of the ground state which contains significant contributions from the large model space. It is noteworthy that for the  $2_1^+$ ,  $4_1^+$ , and  $6_1^+$  levels, even the large space calculation contains less configuration mixing than found by Wildenthal and Larson.

Although it is pleasing to have such good agreement with experiment for  $^{134}\text{Te}$ , it must be kept in mind that the real test of the method is the nuclei with additional protons.

### III. THREE PROTONS: $^{135}\text{I}$

Experimental data for  $^{135}\text{I}$  are extremely sparse.<sup>15</sup> Theoretical predictions for this nuclide

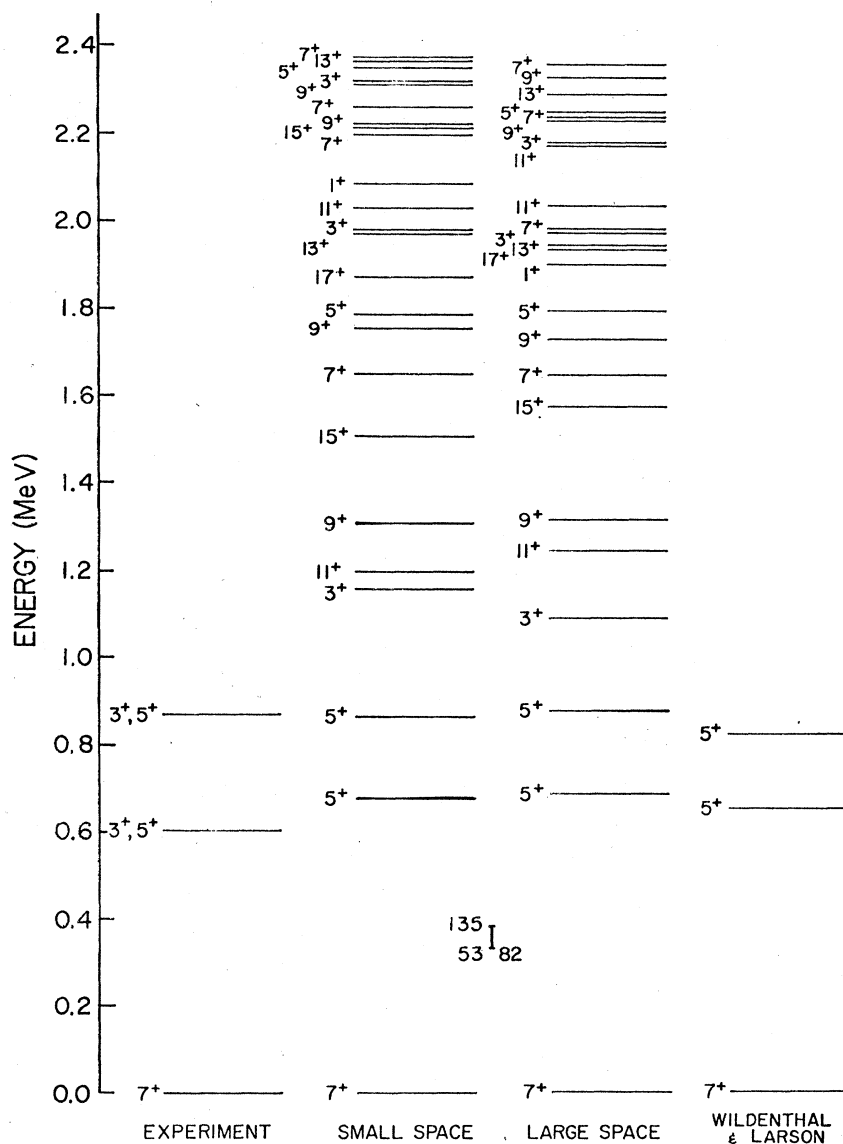


FIG. 3. Shell-model spectra for  $^{135}\text{I}$ . States are labelled by  $2J$ . Here the theoretical spectra are normalized to the experimental (Ref. 15) ground state.

are presented here in the hope that they might stimulate experimental interest and be helpful in the analysis of such experiments.

The two sets of semirealistic two-body matrix elements described in the preceding section were used as input for shell-model calculations in the small and large model spaces. The experimental and theoretical spectra are shown in Fig. 3 with states labeled by  $2J$ . On the basis of the experimental evidence the three shell-model spectra shown are roughly equivalent. All three suggest that the two experimentally known excited states should be  $\frac{5}{2}^+$  levels.

Model wave function information is given in Table IV. There are significant differences between the semirealistic wave functions and those of Wildenthal and Larson<sup>3</sup> for the  $\frac{5}{2}_1^+$  and  $\frac{5}{2}_2^+$  levels. Otherwise, as one would expect, the large model space calculations produce more mixed wave functions than the small space calculations. The low-lying levels are predicted to be well described by the small model space except for 6–7% contributions in the  $\frac{7}{2}_1^+$  and  $\frac{5}{2}_1^+$  levels from configurations having a pair of  $h_{11/2}$  zero-coupled particles with a third proton in the  $g_{7/2}$  or  $d_{5/2}$  orbits, respectively. The first wave function which can be classified as

TABLE IV. Major components of  $^{135}\text{I}$  model wave functions. Only components with amplitudes greater than 0.150 are listed. When necessary to provide an unambiguous description, intermediate angular momenta are specified as  $|a^2(J^*)b\rangle$ , or  $|ab^2(J^*)\rangle$ , or  $|ab(J^*)c\rangle$ . If the seniority is needed to complete the specification it is given as a second superscript,  $|a^{3,1}\rangle$ .

$J^\pi$	Calculation	Model wave function
$\frac{7}{2}^+$	S	$0.891  \frac{7}{2}^3 \rangle - 0.453  \frac{7}{2}^2 \frac{5}{2}^2 (0^+) \rangle$
	L	$0.832  \frac{7}{2}^3 \rangle - 0.436  \frac{7}{2}^2 \frac{5}{2}^2 (0^+) \rangle$ $+ 0.272  \frac{11}{2}^2 (0^+) \frac{7}{2} \rangle - 0.172  \frac{7}{2}^2 \frac{3}{2}^2 (0^+) \rangle$
	WL <sup>3</sup>	$0.90  \frac{7}{2}^3 \rangle - 0.42  \frac{7}{2}^2 \frac{5}{2}^2 (0^+) \rangle$
$\frac{5}{2}^+$	S	$0.902  \frac{7}{2}^2 (0^+) \frac{5}{2} \rangle + 0.326  \frac{5}{2}^3 \rangle$ $+ 0.230  \frac{7}{2}^3 \rangle + 0.165  \frac{7}{2}^2 (2^+) \frac{5}{2} \rangle$
	L	$0.855  \frac{7}{2}^2 (0^+) \frac{5}{2} \rangle + 0.299  \frac{5}{2}^3 \rangle$ $- 0.252  \frac{11}{2}^2 (0^+) \frac{5}{2} \rangle + 0.221  \frac{7}{2}^3 \rangle$ $+ 0.180  \frac{7}{2}^2 (2^+) \frac{5}{2} \rangle$
	WL	$0.84  \frac{7}{2}^2 (0^+) \frac{5}{2} \rangle - 0.40  \frac{7}{2}^3 \rangle + 0.27  \frac{5}{2}^3 \rangle$
$\frac{5}{2}^+$	S	$0.970  \frac{7}{2}^3 \rangle - 0.217  \frac{7}{2}^2 (0^+) \frac{5}{2} \rangle$
	L	$0.953  \frac{7}{2}^3 \rangle - 0.197  \frac{7}{2}^2 (0^+) \frac{5}{2} \rangle$
	WL	$0.80  \frac{7}{2}^3 \rangle + 0.42  \frac{7}{2}^2 (0^+) \frac{5}{2} \rangle + 0.19  \frac{5}{2}^3 \rangle$
$\frac{3}{2}^+$	S	$0.999  \frac{7}{2}^3 \rangle$
	L	$0.976  \frac{7}{2}^3 \rangle$
$\frac{11}{2}^+$	S	$0.995  \frac{7}{2}^3 \rangle$
	L	$0.985  \frac{7}{2}^3 \rangle$
$\frac{9}{2}^+$	S	$0.995  \frac{7}{2}^3 \rangle$
	L	$0.991  \frac{7}{2}^3 \rangle$

extremely mixed is the  $\frac{7}{2}^+$  at nearly 2 MeV of excitation. Several low-lying levels are predicted to be almost pure  $g_{7/2}^3$  states.

#### IV. FOUR PROTONS: $^{136}\text{Xe}$

Western *et al.*<sup>7</sup> recently completed an experimental study of  $^{136}\text{I}$  and established a decay scheme. The lower portion of their resulting  $^{136}\text{Xe}$  level scheme is compared to the 1974 adopted levels<sup>16</sup> in Fig. 4. There are several differences in the two experimentally determined sets of low-lying levels. Western *et al.* did not observe the possible  $2^+$  at 1.920 MeV, although one would expect such a level to be detected by their experiment. The level just above 2.1 MeV, which was previously identified as  $(6^+, 5^-)$ , was tentatively identified by Western *et al.* as  $(3,4)$ . Between 2.2 and 2.7 MeV, Western *et al.* present two new levels, and between 2.8 and 3.0 MeV they have suggested new spin assignments for three levels.

Figure 4 also contains theoretical spectra from several calculations. Above 2.2 MeV the large and small model spaces predict somewhat different

TABLE V. Model wave functions for  $^{136}\text{Xe}$ . See Table IV for comments on notation. When needed to fully describe a configuration, the seniority is given as a second superscript (e.g.,  $\frac{7}{2}^{4,2}$  means four  $\frac{7}{2}$  protons with seniority 2). Only components with amplitudes exceeding 0.15 are listed.

$J^\pi$	Calculation	Model wave function
$0_1^+$	S	$0.767  \frac{7}{2}^4 \rangle + 0.615  \frac{7}{2}^2 (0^+) \frac{5}{2}^2 \rangle$ $+ 0.175  \frac{5}{2}^4 \rangle$
	L	$0.657  \frac{7}{2}^4 \rangle + 0.547  \frac{7}{2}^2 (0^+) \frac{5}{2}^2 \rangle$ $- 0.343  \frac{11}{2}^2 (0^+) \frac{7}{2} \rangle$ $+ 0.211  \frac{7}{2}^2 (0^+) \frac{3}{2}^2 \rangle$ $- 0.172  \frac{11}{2}^2 (0^+) \frac{5}{2}^2 \rangle + 0.151  \frac{5}{2}^4 \rangle$
	WL <sup>3</sup>	$0.80  \frac{7}{2}^4 \rangle + 0.58  \frac{7}{2}^2 (0^+) \frac{5}{2}^2 \rangle$ $+ 0.16  \frac{5}{2}^4 \rangle$
$2_1^+$	S	$0.861  \frac{7}{2}^{4,2} \rangle + 0.486  \frac{7}{2}^2 \frac{5}{2}^2 (0^+) \rangle$ $+ 0.123  \frac{7}{2}^2 (0^+) \frac{5}{2}^2 \rangle$
	L	$0.762  \frac{7}{2}^{4,2} \rangle + 0.481  \frac{7}{2}^2 \frac{5}{2}^2 (0^+) \rangle$ $- 0.263  \frac{11}{2}^2 (0^+) \frac{7}{2} \rangle + 0.174  \frac{7}{2}^2 \frac{3}{2}^2 (0^+) \rangle$ $+ 0.157  \frac{7}{2}^2 (0^+) \frac{5}{2}^2 \rangle$
	WL	$0.80  \frac{7}{2}^{4,2} \rangle + 0.37  \frac{7}{2}^2 \frac{5}{2}^2 (0^+) \rangle$ $+ 0.24  \frac{7}{2}^2 (0^+) \frac{5}{2}^2 \rangle$
$4_1^+$	S	$0.876  \frac{7}{2}^{4,2} \rangle + 0.479  \frac{7}{2}^2 \frac{5}{2}^2 (0^+) \rangle$
	L	$0.791  \frac{7}{2}^{4,2} \rangle - 0.266  \frac{11}{2}^2 (0^+) \frac{7}{2} \rangle$ $+ 0.174  \frac{7}{2}^2 \frac{5}{2}^2 (0^+) \rangle$
	WL	$0.82  \frac{7}{2}^{4,2} \rangle + 0.38  \frac{7}{2}^2 \frac{5}{2}^2 (0^+) \rangle$ $+ 0.26  \frac{7}{2}^3 (\frac{7}{2}^+) \frac{5}{2} \rangle$
$6_1^+$	S	$0.875  \frac{7}{2}^4 \rangle - 0.478  \frac{7}{2}^2 \frac{5}{2}^2 (0^+) \rangle$
	L	$0.805  \frac{7}{2}^4 \rangle - 0.478  \frac{7}{2}^2 \frac{5}{2}^2 (0^+) \rangle$ $+ 0.269  \frac{11}{2}^2 (0^+) \frac{7}{2} \rangle - 0.175  \frac{7}{2}^2 \frac{3}{2}^2 (0^+) \rangle$
	WL	$0.791  \frac{7}{2}^4 \rangle - 0.44  \frac{7}{2}^3 (\frac{7}{2}^+) \frac{5}{2} \rangle$ $- 0.37  \frac{7}{2}^2 \frac{5}{2}^2 (0^+) \rangle$
$6_2^+$	S	$0.923  \frac{7}{2}^3 (\frac{7}{2}^+) \frac{5}{2} \rangle - 0.361  \frac{7}{2}^2 \frac{5}{2}^3 (\frac{5}{2}^+) \rangle$
	L	$0.867  \frac{7}{2}^3 (\frac{7}{2}^+) \frac{5}{2} \rangle - 0.336  \frac{7}{2}^2 \frac{5}{2}^3 (\frac{5}{2}^+) \rangle$ $+ 0.262  \frac{11}{2}^2 (0^+) \frac{7}{2} \frac{5}{2} \rangle$ $- 0.151  \frac{7}{2}^2 \frac{5}{2}^3 (0^+) \rangle$
	WL	$0.80  \frac{7}{2}^3 (\frac{7}{2}^+) \frac{5}{2} \rangle + 0.44  \frac{7}{2}^4 \rangle$ $- 0.30  \frac{7}{2}^2 \frac{5}{2}^3 (\frac{5}{2}^+) \rangle$

orderings and spacings but the overall agreement is reasonable. These results are considered to support the semirealistic approach, especially since both model spaces yield satisfactory low-lying spectra.

The spectrum generated by Wildenthal and Larson<sup>3</sup> is much like the large space calculation up to 2 MeV. Above 2 MeV they predict a presumably correct  $4_2^+, 6_2^+$  ordering as opposed to the inverted ordering predicted by the large space calculation. Otherwise, the experimental uncertainties in spin

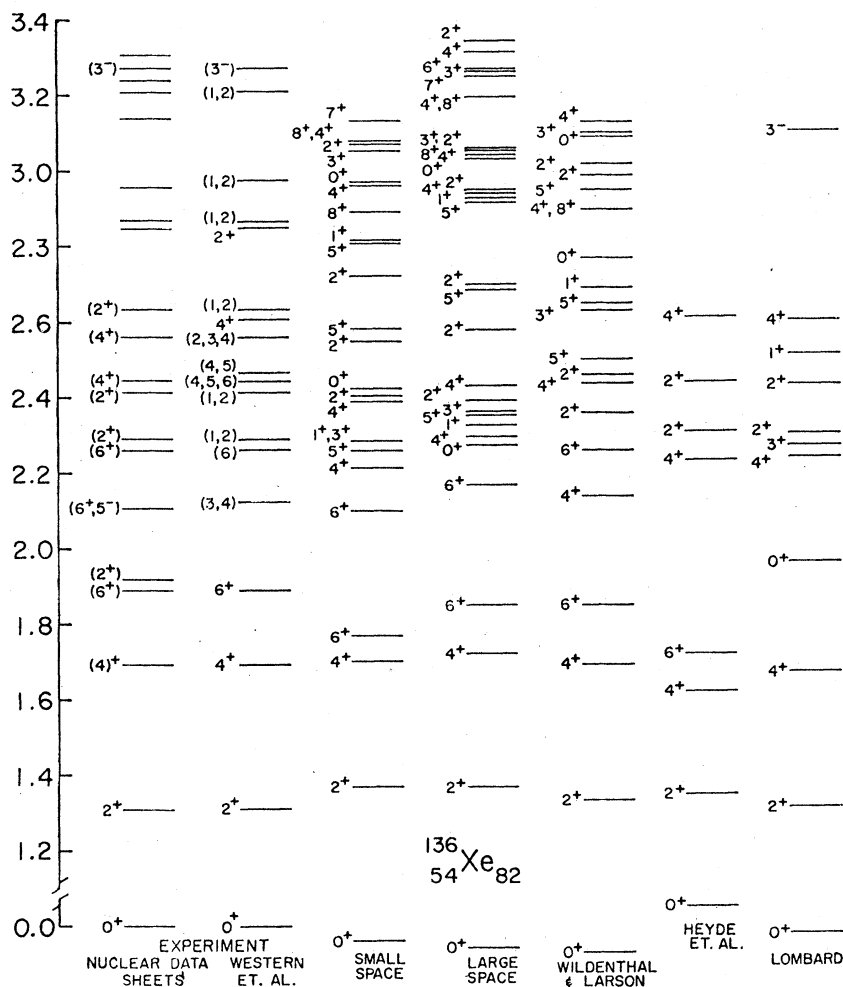


FIG. 4. Experimental and theoretical level schemes for  $^{136}\text{Xe}$ . See text for comments and discussion.

assignments make it difficult to choose between their more uniformly spread spectrum and the large model space results which indicate groupings of levels. The large space does, however, predict the number of  $2^+$  states below 2.8 MeV in better agreement with experiment than did Wildenthal and Larson.

The other two theoretical spectra are from calculations by Heyde *et al.*<sup>17</sup> with quasiparticles in the  $0g_{7/2}$ ,  $1d_{5/2}$  orbitals and by Lombard<sup>18</sup> with quasiparticles in the  $0g_{7/2}$ ,  $1d_{5/2}$ ,  $0h_{11/2}$ ,  $1d_{3/2}$ , and  $2s_{1/2}$  orbitals. The quasiparticle calculations are in less satisfactory agreement with experiment than the results of the shell-model calculations.

In Table V, wave functions for the large and small spaces are presented along with those of Wildenthal and Larson for the low-lying levels. It is evident that when compared with the two- and three-proton cases, these four-proton wave func-

tions involve greater configuration mixing and an increased role for configurations added by the large model space. General features of Wildenthal and Larson's wave functions are mostly similar or intermediate to the small and large space results.

Based on results obtained in a shell-model study of the lead region, McGrory and Kuø<sup>19</sup> suggested a seniority-based truncation scheme for even-even nuclei. In the cases they investigated, for each spin no level below the third was eliminated by truncating according to seniority. However, an exception to this scheme is obtained here since the  $4_2^+$  level at 2.3 MeV is predicted to be more than 50% seniority 4.

#### V. FIVE PROTONS: $^{137}\text{Cs}$

Five valence protons in the large model space would require what must, at the present time, be

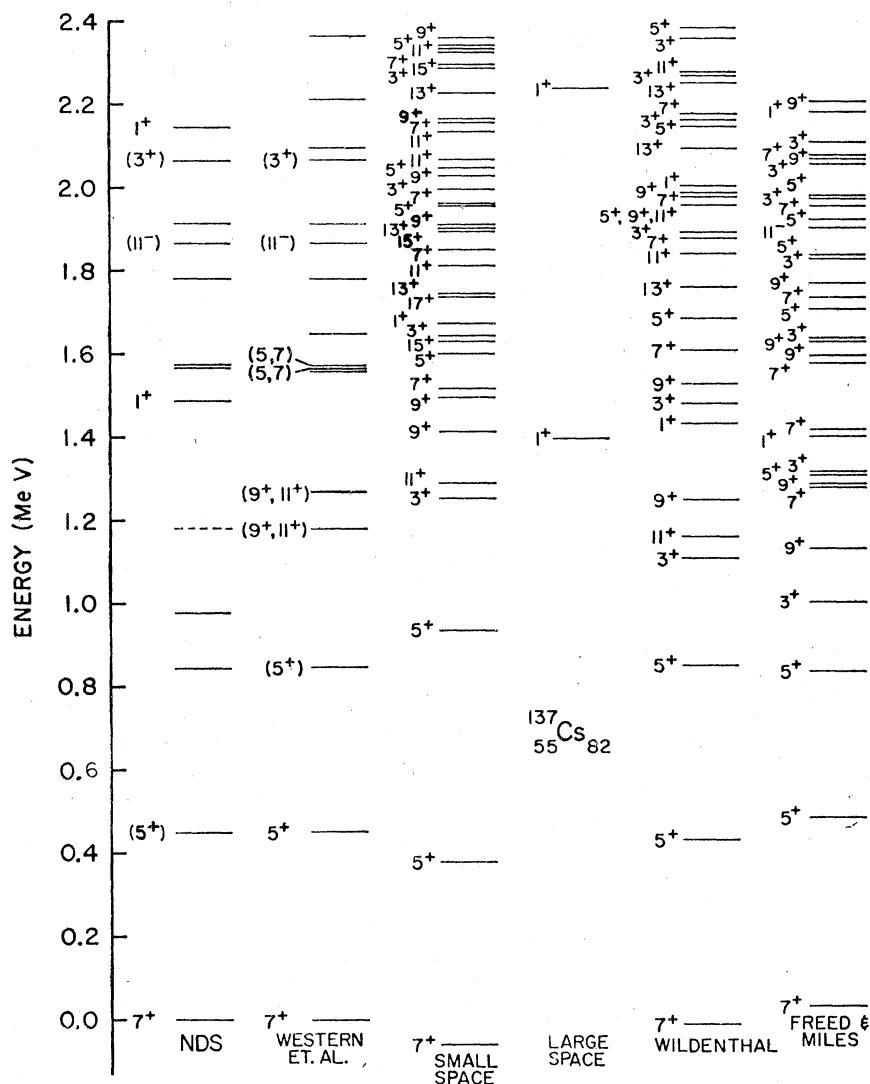


FIG. 5. Experimental and theoretical spectra for  $^{137}\text{Cs}$ . NDS (Ref. 20) and Western *et al.* (Ref. 7) are experiment. Small and large space columns are results of this study. Wildenthal (Ref. 21) is a shell-model study using the SDI. Freed and Miles (Ref. 5) is a quasiparticle Tamm-Dancoff approximation calculation. All levels are labeled by  $2J$ .

considered excessive computer resources. Except for the minor condition that no more than four protons were allowed in the  $h_{11/2}$  orbit, cases involving large space matrices smaller than  $300 \times 300$  have been treated, but were found to contribute few levels to the low-lying spectrum. Since only a few  $^{137}\text{Cs}$  matrices could be treated in the large space, the emphasis is on the small space results and a comparison with the recent experimental data of Western *et al.*<sup>7</sup>

Figure 5 depicts the experimental and theoretical spectra for  $^{137}\text{Cs}$ . The present experimental situation is best interpreted as a blend of the first two columns of Fig. 5. On the one hand,

Western *et al.*<sup>7</sup> made several tentative spin assignments not given in Ref. 20, confirmed the existence of a level at 1.185 MeV, and deduced previously unreported levels at 1.273, 1.651, 2.099, 2.217, and 2.368 MeV. On the other hand,  $\gamma$  rays that might be interpreted as feeding the 0.981 MeV level and the  $\frac{1}{2}^+$  states at 1.490 and 2.150 MeV, in Ref. 20, were placed elsewhere.

The overspreading of the small space results invites speculation on the need for fine tuning the parameters of the additive terms of the effective interaction as one moves further into the open shell. This is in accord with a similar speculation in Ref. 13 regarding semirealistic calcula-

TABLE VI. Model  $^{137}\text{Cs}$  wave functions for small space calculation. See Table V for notation comments.

$\frac{7}{2}^+$	$0.726  \frac{7}{2}^5 \rangle + 0.654  \frac{7}{2}^3 \frac{5}{2}^2 (0^+) \rangle - 0.202  \frac{7}{2}^5 \frac{5}{2}^4 (0^+) \rangle$
$\frac{5}{2}^+$	$0.839  \frac{7}{2}^4 (0^+) \frac{5}{2} \rangle + 0.511  \frac{7}{2}^2 (0^+) \frac{5}{2}^3 \rangle$
$\frac{5}{2}^+$	$0.826  \frac{7}{2}^5 \rangle + 0.528  \frac{7}{2}^3 \frac{5}{2}^2 (0^+) \rangle$
	$- 0.168  \frac{7}{2}^3 (\frac{7}{2}^+ \frac{5}{2}^2 (2^+) \rangle$
$\frac{3}{2}^+$	$0.840  \frac{7}{2}^5 \rangle + 0.510  \frac{7}{2}^3 \frac{5}{2}^2 (0^+) \rangle$
$\frac{1}{2}^+$	$0.829  \frac{7}{2}^5 \rangle + 0.518  \frac{7}{2}^3 \frac{5}{2}^2 (0^+) \rangle$
	$+ 0.176  \frac{7}{2}^3 (\frac{7}{2}^+ \frac{5}{2}^2 (2^+) \rangle$
$\frac{9}{2}^+$	$0.841  \frac{7}{2}^5 \rangle + 0.511  \frac{7}{2}^3 \frac{5}{2}^2 (0^+) \rangle$

tions for six neutron holes in  $^{208}\text{Pb}$ .

The large model space yielded matrices with dimensions as large as 850. Only the  $J = \frac{1}{2}$  and  $J \geq \frac{23}{2}$  cases had matrices smaller than  $300 \times 300$ . As the  $\frac{7}{2}^+$  ground state could not be computed, the two  $\frac{1}{2}^+$  levels shown in the large space column of Fig. 5 were simply positioned to agree as well as possible with the pair of experimental  $\frac{1}{2}^+$  levels. On this basis, one could guess that the lowest  $\frac{7}{2}^+$  eigenvalue (not excitation energy) would be about  $-3.5 \pm 0.1$  MeV. This value is consistent with the lowest small space  $\frac{7}{2}^+$  eigenvalue and small space to large space trends seen in the cases with fewer valence particles. Such consistency implies that the  $\frac{1}{2}^+$  levels are positioned in roughly their proper location.

The remaining theoretical spectra by Wildenthal<sup>21</sup> and by Freed and Miles<sup>5</sup> yield somewhat better agreement with experiment than our small space calculation. It is quite likely that the large model space would yield results approximately comparable to their work. Note that the quasi-particle calculations are in much better agreement with experiment here than in  $^{136}\text{Xe}$ . All calculations shown indicate the sequence of the low-lying levels to be  $\frac{7}{2}^+$ ,  $\frac{5}{2}^+$ ,  $\frac{5}{2}^+$ ,  $\frac{3}{2}^+$ ,  $\frac{11}{2}^+$ , and  $\frac{9}{2}^+$ .

The model wave functions for small space calculation are given in Table VI. With the exception of the  $\frac{5}{2}^+$  level, the levels up to and including the  $\frac{9}{2}^+$  are predominantly  $\frac{7}{2}^5$  states with appreciable admixtures of  $| \frac{7}{2}^3 \frac{5}{2}^2 (0^+) \rangle$  components. Unlike  $^{136}\text{Xe}$  it is seen here, at least for the small model space, that higher seniority states do not play such a significant role in any of the low-lying levels. Thus a seniority truncation<sup>19</sup> scheme appears to be suitable for this case even though McGrory and Kuo proposed it for even-even cases.

## VI. SUMMARY AND CONCLUSIONS

The results presented here support the philosophy of the semirealistic shell-model studies wherein

one employs a realistic Brueckner reaction matrix as the lowest-order effective Hamiltonian and corrects by the addition of pairing,  $P_2$ , and  $P_4$  terms with phenomenologically determined strengths. Such additions appear rather adequate as parametrizations of higher-order effects. The present efforts may be interpreted as an extension of the "Skyrme" philosophy<sup>22</sup> which has been employed for Hartree-Fock studies of the ground state properties of nuclei. There, the phenomenological Skyrme parameters have been derived by a density matrix expansion of the Brueckner reaction matrix.<sup>23</sup> Here, however, the Brueckner reaction matrix is employed directly with small corrections rather than attempting to parametrize the entire reaction matrix.

With respect to the semirealistic philosophy employed in the present studies, one can conclude that when phenomenological modifications are included as described, large and small model space results for three and four valence protons are roughly equivalent. It is, in general, preferable to compute in the large model space when feasible for two reasons: First, the spectra seem to be in better agreement with experimental information, and in this region the  $h_{11/2}$  orbital seems to be important for some low-lying states. Second, the phenomenological corrections deduced for the large model space are significantly smaller. Thus, in the large model space, the microscopic theory from the free nucleon-nucleon interaction is a more dominating aspect of the theoretical Hamiltonian.

In this specific application of the semirealistic approach, shell-model calculations for nuclei consisting of a  $^{132}\text{Sn}$  core plus two through five valence protons have been carried out in a small model space, and, where feasible, in a large model space which includes an entire major shell of proton orbits. The effective two-body matrix elements were constructed of two parts, bare realistic reaction matrix elements plus small corrections. The strengths of the pairing  $P_2$ , and  $P_4$  forces were adjusted to produce agreement with the lowest-lying levels of  $^{134}\text{Te}$ . To allow for truncation effects as much as possible, separate sets of parameters were determined for the small and large model spaces. Although these simulated corrections have a very pronounced effect on the ordering and spacing of the low-lying levels, the calculated correlation between bare and corrected matrix elements in large and small model spaces were 0.97 and 0.93, respectively. These correlation values clearly indicate that the needed corrections to the bare matrix elements are generally small, and depend significantly on truncation of the low-lying single-particle space.



The predicted spectra for  $^{135}\text{I}$  and  $^{136}\text{Xe}$  indicate that the large space calculations are in good agreement with the currently available experimental data. The small space predictions show less detailed agreement, but do reproduce the general features of the empirical data.

It is clear that these results support Wildenthal's speculation that the tin region should be regarded as nearly as good a shell-model region as the lead region. It appears that a full scale experimental and theoretical effort in the tin region is warranted. Experimentally, there is a special need for information on nuclei differing from  $^{132}\text{Sn}$  by one or two nucleons. This data would establish the single-particle energies and would provide the two-body spectra to determine the parameters of the correction terms. Theoretically, there is strong

motivation for study of nuclei with additional valence protons. This is pursued in II.

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