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Theory of radiative muon capture by ^{12}C[†]

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The theory of radiative muon capture, as formulated in a previous paper on the basis of the conservation of the hadronic electromagnetic current, the conservation of the hadronic weak polar current, the partial conservation of the hadronic weak axial-vector current, the SU(2) × SU(2) current algebra for the various hadronic currents, and the "linearity hypothesis," is applied to the process $\mu^- \, ^{12}\mathrm{C} \rightarrow \nu_\mu \, ^{12}\mathrm{B}\gamma$. The resultant total transition amplitude is worked out explicitly and used to calculate various observable quantities.

[RADIOACTIVITY $\mu^- \, ^{12}\mathrm{C} \rightarrow \nu_\mu \, ^{12}\mathrm{B}\gamma$; application of a general theory of radiative muon capture to nuclear spin and isospin $[0^+, 0] \rightarrow [1^+, 1]$ transitions.]

INTRODUCTION

A general theory of radiative muon capture with application to the processes $\mu^- p \rightarrow \nu_\mu n\gamma$ and $\mu^- \, ^3\mathrm{He} \rightarrow \nu_\mu \, ^3\mathrm{H}\gamma$ has been treated in detail in the preceding paper.[1] The present paper addresses itself to the further application of this general theory to the process $\mu^- \, ^{12}\mathrm{C} \rightarrow \nu_\mu \, ^{12}\mathrm{B}\gamma$ [$^{12}\mathrm{B} \equiv {}^{12}\mathrm{B}$ (ground state)]. The process $\mu^- \, ^{12}\mathrm{C} \rightarrow \nu_\mu \, ^{12}\mathrm{B}\gamma$ has also been studied by means of the impulse approximation and a nuclear model.[1a,1b]

FORMULATION

We adopt the following definitions for the various weak nonradiative nuclear form factors, assuming the absence of the second-class currents[2]:

$$\langle ^{12}\mathrm{B}(p^{(f)}, \xi) | V_\lambda(0) | ^{12}\mathrm{C}(p^{(i)}) \rangle = -\sqrt{2}\, \epsilon_{\lambda\kappa\rho\eta} \xi_\kappa^* \frac{q_\rho}{2m_p} \frac{Q_\eta}{2M} F_M(q^2) , \quad (1a)$$

$$\langle ^{12}\mathrm{B}(p^{(f)}, \xi) | A_\lambda(0) | ^{12}\mathrm{C}(p^{(i)}) \rangle = \sqrt{2} \left(\xi_\lambda^* F_A(q^2) + q_\lambda \frac{q \cdot \xi^*}{m_\pi^2} F_P(q^2) - \frac{Q_\lambda}{2M} \frac{q \cdot \xi^*}{2m_p} F_E(q^2) \right) , \quad (1b)$$

$$\langle ^{12}\mathrm{B}(p^{(f)}, \xi) | \partial_\lambda A_\lambda(0) | ^{12}\mathrm{C}(p^{(i)}) \rangle = -i\sqrt{2}\, q \cdot \xi^* \frac{F_D(q^2)}{1 + q^2/m_\pi^2} , \quad (1c)$$

where $q_\lambda \equiv (p^{(f)} - p^{(i)})_\lambda$, $Q_\lambda \equiv (p^{(f)} + p^{(i)})_\lambda$, $M \equiv \frac{1}{2}(M_i + M_f) = \frac{1}{2}[M(^{12}\mathrm{C}) + M(^{12}\mathrm{B})]$, $M_{f,i} = [-(p^{(f),(i)})^2]^{1/2}$, $\Delta \equiv M(^{12}\mathrm{B}) - M(^{12}\mathrm{C}) = 13.881$ MeV, $\xi^* \equiv (\vec{\xi}^*, i\xi_0^*)$ with $\xi \equiv (\vec{\xi}, i\xi_0)$ the polarization four-vector of the spin-one ^{12}B nucleus, and $F_{M,A,P,E}(q^2)$ and $F_D(q^2)/(1 + q^2/m_\pi^2)$ are, respectively, the weak magnetism, axial-vector, pseudoscalar, weak electricity (or pseudotensor), and axial-divergence nuclear form factors.

The transition amplitude for radiative muon capture

$$\mu^-(p^{(\mu)}, s^{(\mu)}) + {}^{12}\mathrm{C}(p^{(i)}) \rightarrow \nu_\mu(p^{(\nu)}, s^{(\nu)}) + {}^{12}\mathrm{B}(p^{(f)}, \xi) + \gamma(k, \epsilon) ,$$

$$p^{(\mu)} + p^{(i)} = p^{(\nu)} + p^{(f)} + k = p^{(\nu)} + p^{(i)} + q + k = p^{(\nu)} - p^{(i)} + Q + k , \quad (2)$$

is decomposed into two parts[1]:

$$\mathcal{T} = \mathcal{T}^{(l)} + \mathcal{T}^{(h)} , \quad (3)$$

where

$$\mathcal{T}^{(l)} = -\frac{Ge}{\sqrt{2}} \langle ^{12}\mathrm{B}(p^{(f)}, \xi) | [V_\lambda(0) + A_\lambda(0)] | ^{12}\mathrm{C}(p^{(i)}) \rangle \frac{1}{(2k_0)^{1/2}} \bar{u}^{(\nu)}(p^{(\nu)}, s^{(\nu)}) \gamma_\lambda (1 + \gamma_5) i \times \frac{m_\mu - i(\not{p}^{(\mu)} - \not{k})}{m_\mu^2 + (p^{(\mu)} - k)^2} \not{\epsilon}^* u^{(\mu)}(p^{(\mu)}, s^{(\mu)}) , \quad (4)$$

$$\mathcal{T}^{(h)} = -\frac{Ge}{\sqrt{2}} \bar{u}^{(\nu)}(p^{(\nu)}, s^{(\nu)}) \gamma_\lambda (1 + \gamma_5) u^{(\mu)}(p^{(\mu)}, s^{(\mu)}) \frac{1}{m_p} \frac{\epsilon^*_\mu}{(2k_0)^{1/2}} [V_{\mu\lambda}(k, q, Q) + A_{\mu\lambda}(k, q, Q)] , \quad (5)$$

$$V_{\mu\lambda}(k, q, Q) \equiv -im_p \int d^4x\, e^{-ik \cdot x} \langle ^{12}\mathrm{B}(p^{(f)}, \xi) | T(\mathcal{J}_\mu(x) V_\lambda(0)) | ^{12}\mathrm{C}(p^{(i)}) \rangle , \quad (5a)$$

$$A_{\mu\lambda}(k,q,Q) \equiv -im_p \int d^4x\, e^{-ik\cdot x} \langle {}^{12}\mathrm{B}(p^{(f)},\xi)\,|\,T(J_\mu(x)A_\lambda(0))\,|\,{}^{12}\mathrm{C}(p^{(i)})\rangle \tag{5b}$$

with $\bar{u} \equiv iu^\dagger\gamma_4$, $\epsilon^* = (\vec{\epsilon}^*, i\epsilon_0^*)$, $J_\mu(x)$ the hadronic electromagnetic current, and

$$T(J_\mu(x)K_\lambda(y)) \equiv J_\mu(x)K_\lambda(y):\ x_0 > y_0,$$
$$\equiv K_\lambda(y)J_\mu(x):\ x_0 < y_0.$$

Here $\mathcal{T}^{(l)}$ describes the contribution to $\mathcal{T}$ arising from radiation by the muon while $\mathcal{T}^{(h)}$ describes the contribution to $\mathcal{T}$ arising from radiation by the initial and final nuclei, by the intermediate hadrons, and by any charged particle ($W^\pm$) that transmits the weak interaction. The Lorentz gauge is used so that

$$\epsilon_\mu^* k_\mu = k_0(\vec{\epsilon}^*\cdot\hat{k} - \epsilon_0^*) = 0\,. \tag{5c}$$

With the SU(2) × SU(2) current algebra for the various hadronic currents (CA), the conservation of the hadronic electromagnetic current (CEC), the conservation of the hadronic weak polar current (CVC), and the partial conservation of the hadronic weak axial-vector current (PCAC) yield the following constraints on $V_{\mu\lambda}(k,q,Q)$ and $A_{\mu\lambda}(k,q,Q)$[1]:

CEC:

$$\frac{k_\mu}{m_p} V_{\mu\lambda}(k,q,Q) = \langle {}^{12}\mathrm{B}(p^{(f)},\xi)\,|\,V_\lambda(0)\,|\,{}^{12}\mathrm{C}(p^{(i)})\rangle\,, \tag{6a}$$

$$\frac{k_\mu}{m_p} A_{\mu\lambda}(k,q,Q) = \langle {}^{12}\mathrm{B}(p^{(f)},\xi)\,|\,A_\lambda(0)\,|\,{}^{12}\mathrm{C}(p^{(i)})\rangle\,, \tag{6b}$$

CVC:

$$\frac{(k_\lambda+q_\lambda)}{m_p} V_{\mu\lambda}(k,q,Q) = \langle {}^{12}\mathrm{B}(p^{(f)},\xi)\,|\,V_\mu(0)\,|\,{}^{12}\mathrm{C}(p^{(i)})\rangle\,, \tag{7}$$

PCAC:

$$\frac{(k_\lambda+q_\lambda)}{m_p} A_{\mu\lambda}(k,q,Q) = \langle {}^{12}\mathrm{B}(p^{(f)},\xi)\,|\,A_\mu(0)\,|\,{}^{12}\mathrm{C}(p^{(i)})\rangle + D_\mu(k,q,Q) \tag{8}$$

with

$$D_\mu(k,q,Q) = \int d^4x\, e^{-ik\cdot x} \langle {}^{12}\mathrm{B}(p^{(f)},\xi) \times |\,T(J_\mu(x)\partial_\chi A_\lambda(0))\,|\,{}^{12}\mathrm{C}(p^{(i)})\rangle \tag{8a}$$

and

$$k_\mu D_\mu(k,q,Q) = i\langle {}^{12}\mathrm{B}(p^{(f)},\xi)\,|\,\partial_\chi A_\lambda(0)\,|\,{}^{12}\mathrm{C}(p^{(i)})\rangle\,. \tag{8b}$$

We proceed to construct the general Lorentz-covariant expressions for $V_{\mu\lambda}(k,q,Q)$, $A_{\mu\lambda}(k,q,Q)$, and $D_\mu(k,q,Q)$ from which approximate expressions for $\epsilon_\mu^* V_{\mu\lambda}(k,q,Q)$ and $\epsilon_\mu^* A_{\mu\lambda}(k,q,Q)$ will be obtained with the aid of the "linearity hypothesis" (LH)[1] and the CEC, CVC, and PCAC constraint equations. We have

$$\begin{aligned}
V_{\mu\lambda}(k,q,Q) = {} & \epsilon_{\mu\alpha\beta\gamma}\frac{\xi_\alpha^*}{m_p^3}\,[Q_\beta q_\gamma(F_{11}^{(a)}k_\lambda + F_{12}^{(a)}Q_\lambda + F_{13}^{(a)}q_\lambda) + q_\beta k_\gamma(F_{21}^{(a)}k_\lambda + F_{22}^{(a)}Q_\lambda + F_{23}^{(a)}q_\lambda) \\
& + k_\beta Q_\gamma(F_{31}^{(a)}k_\lambda + F_{32}^{(a)}Q_\lambda + F_{33}^{(a)}q_\lambda)] \\
& + \epsilon_{\lambda\alpha\beta\gamma}\frac{\xi_\alpha^*}{m_p^3}\,[Q_\beta q_\gamma(F_{11}^{(b)}k_\mu + F_{12}^{(b)}Q_\mu + F_{13}^{(b)}q_\mu) + q_\beta k_\gamma(F_{21}^{(b)}k_\mu + F_{22}^{(b)}Q_\mu + F_{23}^{(b)}q_\mu) \\
& + k_\beta Q_\gamma(F_{31}^{(b)}k_\mu + F_{32}^{(b)}Q_\mu + F_{33}^{(b)}q_\mu)] \\
& + \epsilon_{\mu\alpha\beta\gamma}\frac{k_\alpha Q_\beta q_\gamma}{m_p^3}\left(F_{00}^{(c)}\xi_\lambda^* + \frac{k\cdot\xi^*}{m_p^2}(F_{11}^{(c)}k_\lambda + F_{12}^{(c)}Q_\lambda + F_{13}^{(c)}q_\lambda) + \frac{q\cdot\xi^*}{m_p^2}(F_{21}^{(c)}k_\lambda + F_{22}^{(c)}Q_\lambda + F_{23}^{(c)}q_\lambda)\right) \\
& + \epsilon_{\lambda\alpha\beta\gamma}\frac{k_\alpha Q_\beta q_\gamma}{m_p^3}\left(F_{00}^{(d)}\xi_\mu^* + \frac{k\cdot\xi^*}{m_p^2}(F_{11}^{(d)}k_\mu + F_{12}^{(d)}Q_\mu + F_{13}^{(d)}q_\mu) + \frac{q\cdot\xi^*}{m_p^2}(F_{21}^{(d)}k_\mu + F_{22}^{(d)}Q_\mu + F_{23}^{(d)}q_\mu)\right) \\
& + \epsilon_{\mu\lambda\alpha\beta}\left(\frac{\xi_\alpha^*}{m_p}(F_{01}^{(e)}k_\beta + F_{02}^{(e)}Q_\beta + F_{03}^{(e)}q_\beta) + \frac{k\cdot\xi^*}{m_p^3}(F_{11}^{(e)}Q_\alpha q_\beta + F_{12}^{(e)}q_\alpha k_\beta + F_{13}^{(e)}k_\alpha Q_\beta)\right. \\
& \left. + \frac{q\cdot\xi^*}{m_p^3}(F_{21}^{(e)}Q_\alpha q_\beta + F_{22}^{(e)}q_\alpha k_\beta + F_{23}^{(e)}k_\alpha Q_\beta)\right)\,,
\end{aligned} \tag{9}$$

$$A_{\mu\lambda}(k,q,Q)=\frac{\xi^*_\mu}{m_p}(G_{01}^{(a)}k_\lambda+G_{02}^{(a)}Q_\lambda+G_{03}^{(a)}q_\lambda)+\frac{\xi^*_\lambda}{m_p}(G_{01}^{(b)}k_\mu+G_{02}^{(b)}Q_\mu+G_{03}^{(b)}q_\mu)$$
$$+\frac{k\cdot\xi^*}{m_p}\left(G_{00}^{(a)}\delta_{\mu\lambda}+\frac{k_\mu}{m_p^2}(G_{11}^{(a)}k_\lambda+G_{12}^{(a)}Q_\lambda+G_{13}^{(a)}q_\lambda)+\frac{Q_\mu}{m_p^2}(G_{21}^{(a)}k_\lambda+G_{22}^{(a)}Q_\lambda+G_{23}^{(a)}q_\lambda)\right.$$
$$\left.+\frac{q_\mu}{m_p^2}(G_{31}^{(a)}k_\lambda+G_{32}^{(a)}Q_\lambda+G_{33}^{(a)}q_\lambda)\right)$$
$$+\frac{q\cdot\xi^*}{m_p}\left(G_{00}^{(b)}\delta_{\mu\lambda}+\frac{k_\mu}{m_p^2}(G_{11}^{(b)}k_\lambda+G_{12}^{(b)}Q_\lambda+G_{13}^{(b)}q_\lambda)+\frac{Q_\mu}{m_p^2}(G_{21}^{(b)}k_\lambda+G_{22}^{(b)}Q_\lambda+G_{23}^{(b)}q_\lambda)\right.$$
$$\left.+\frac{q_\mu}{m_p^2}(G_{31}^{(b)}k_\lambda+G_{32}^{(b)}Q_\lambda+G_{33}^{(b)}q_\lambda)\right)\,, \tag{10}$$

$$D_\mu(k,q,Q)=\sqrt{2}\,\xi^*_\mu f_A+\sqrt{2}q\cdot\xi^*\left(\frac{q_\mu}{m_\pi^2}f_P+\frac{k_\mu}{m_\pi^2}f_Q-\frac{Q_\mu}{2M\cdot 2m_p}f_E\right)+\sqrt{2}k\cdot\xi^*\left(\frac{q_\mu}{m_\pi^2}\tilde{f}_P+\frac{k_\mu}{m_\pi^2}\tilde{f}_Q-\frac{Q_\mu}{2M\cdot 2m_p}\tilde{f}_E\right) \tag{11}$$

Here, each of the weak radiative nuclear form factors $R\equiv(F_{ij}^{(a),(b),(c),(d),(e)},G_{ij}^{(a),(b)},f_{A,P,Q,E}$, and $\tilde{f}_{P,Q,E})$ is, in general, a function of the three Lorentz invariants q^2, $Q\cdot k$, and $q\cdot k$.

We now apply CEC, as described by Eqs. (6a), (6b), and (1a), (1b), to $V_{\mu\lambda}(k,q,Q)$ and $A_{\mu\lambda}(k,q,Q)$ as given by Eqs. (9) and (10). This yields

$$F_{1j}^{(a)}=0,\quad j=1,2,3, \tag{12a}$$
$$F_{12}^{(b)}Q\cdot k+F_{13}^{(b)}q\cdot k=\frac{m_p^2}{2}\frac{1}{2M}\sqrt{2}F_M(q^2)\,, \tag{12b}$$
$$F_{22}^{(b)}Q\cdot k+F_{23}^{(b)}q\cdot k=m_p^2F_{03}^{(e)}\,, \tag{12c}$$
$$F_{32}^{(b)}Q\cdot k+F_{33}^{(b)}q\cdot k=-m_p^2F_{02}^{(e)}\,, \tag{12d}$$
$$F_{12}^{(d)}Q\cdot k+F_{13}^{(d)}q\cdot k=m_p^2(-F_{00}^{(d)}+F_{11}^{(e)})\,, \tag{12e}$$
$$F_{22}^{(d)}Q\cdot k+F_{23}^{(d)}q\cdot k=m_p^2F_{21}^{(e)}\,, \tag{12f}$$
$$G_{02}^{(b)}Q\cdot k+G_{03}^{(b)}q\cdot k=m_p^2\sqrt{2}F_A(q^2)\,, \tag{13a}$$
$$G_{21}^{(a)}Q\cdot k+G_{31}^{(a)}q\cdot k=-m_p^2(G_{01}^{(a)}+G_{00}^{(a)})\,, \tag{13b}$$
$$G_{22}^{(a)}Q\cdot k+G_{32}^{(a)}q\cdot k=-m_p^2G_{02}^{(a)}\,, \tag{13c}$$
$$G_{23}^{(a)}Q\cdot k+G_{33}^{(a)}q\cdot k=-m_p^2G_{03}^{(a)}\,, \tag{13d}$$
$$G_{21}^{(b)}Q\cdot k+G_{31}^{(b)}q\cdot k=-m_p^2G_{00}^{(b)}\,, \tag{13e}$$
$$G_{22}^{(b)}Q\cdot k+G_{32}^{(b)}q\cdot k=-\frac{m_p^3}{2}\frac{1}{2M}\sqrt{2}F_E(q^2)\,, \tag{13f}$$
$$G_{23}^{(b)}Q\cdot k+G_{33}^{(b)}q\cdot k=\frac{m_p^4}{m_\pi^2}\sqrt{2}F_P(q^2)\,. \tag{13g}$$

Next, we apply CVC, as described by Eqs. (7) and (1a) to $V_{\mu\lambda}(k,q,Q)$ as given by Eq. (9). This yields

$$F_{11}^{(a)}q\cdot k+F_{12}^{(a)}Q\cdot(q+k)+F_{13}^{(a)}q\cdot(q+k)+m_p^2F_{02}^{(e)}=\frac{m_p^3}{2}\frac{1}{2M}\sqrt{2}F_M(q^2)\,, \tag{14a}$$
$$F_{21}^{(a)}q\cdot k+F_{22}^{(a)}Q\cdot(q+k)+F_{23}^{(a)}q\cdot(q+k)-m_p^2F_{01}^{(e)}+m_p^2F_{03}^{(e)}=0\,, \tag{14b}$$
$$F_{31}^{(a)}q\cdot k+F_{32}^{(a)}Q\cdot(q+k)+F_{33}^{(a)}q\cdot(q+k)-m_p^2F_{02}^{(e)}=0\,, \tag{14c}$$
$$F_{1j}^{(b)}+F_{3j}^{(b)}=0,\quad j=1,2,3\,, \tag{14d}$$
$$F_{11}^{(c)}q\cdot k+F_{12}^{(c)}Q\cdot(q+k)+F_{13}^{(c)}q\cdot(q+k)+m_p^2(F_{00}^{(c)}+F_{11}^{(e)}+F_{13}^{(e)})=0\,, \tag{14e}$$
$$F_{21}^{(c)}q\cdot k+F_{22}^{(c)}Q\cdot(q+k)+F_{23}^{(c)}q\cdot(q+k)+m_p^2(F_{00}^{(c)}+F_{21}^{(e)}+F_{23}^{(e)})=0\,. \tag{14f}$$

Finally, we apply PCAC, as described by Eqs. (8), (8a), (8b), and (1b), to $A_{\mu\lambda}(k,q,Q)$ and $D_\mu(k,q,Q)$ as given by Eqs. (10) and (11). This yields

$$G_{01}^{(a)}q\cdot k+G_{02}^{(a)}Q\cdot(q+k)+G_{03}^{(a)}q\cdot(q+k)=m_p^2\sqrt{2}[F_A(q^2)+f_A]\,, \tag{15a}$$
$$G_{11}^{(b)}q\cdot k+G_{12}^{(b)}Q\cdot(q+k)+G_{13}^{(b)}q\cdot(q+k)+m_p^2(G_{01}^{(b)}+G_{00}^{(b)})=\frac{m_p^4}{m_\pi^2}\sqrt{2}f_Q\,, \tag{15b}$$
$$G_{21}^{(b)}q\cdot k+G_{22}^{(b)}Q\cdot(q+k)+G_{23}^{(b)}q\cdot(q+k)+m_p^2G_{02}^{(b)}=-\frac{m_p^3}{2}\frac{1}{2M}\sqrt{2}[F_E(q^2)+f_E]\,, \tag{15c}$$
$$G_{31}^{(b)}q\cdot k+G_{32}^{(b)}Q\cdot(q+k)+G_{33}^{(b)}q\cdot(q+k)+m_p^2(G_{03}^{(b)}+G_{00}^{(b)})=\frac{m_p^4}{m_\pi^2}\sqrt{2}[F_P(q^2)+f_P]\,, \tag{15d}$$
$$G_{11}^{(a)}q\cdot k+G_{12}^{(a)}Q\cdot(q+k)+G_{13}^{(a)}q\cdot(q+k)+m_p^2(G_{01}^{(b)}+G_{00}^{(a)})=\frac{m_p^4}{m_\pi^2}\sqrt{2}\tilde{f}_Q\,, \tag{15e}$$

$$G_{21}^{(a)}q\cdot k+G_{22}^{(a)}Q\cdot(q+k)+G_{23}^{(a)}q\cdot(q+k)+m_p^2G_{02}^{(b)}=-\frac{m_p^3}{2}\frac{1}{2M}\sqrt{2}\tilde{f}_E\,, \quad (15f)$$

$$G_{31}^{(a)}q\cdot k+G_{32}^{(a)}Q\cdot(q+k)+G_{33}^{(a)}q\cdot(q+k)+m_p^2(G_{03}^{(b)}+G_{00}^{(a)})=\frac{m_p^4}{m_\pi^2}\sqrt{2}\tilde{f}_P\,, \quad (15g)$$

$$-\frac{f_E}{2M\cdot 2m_p}Q\cdot k+\frac{f_P}{m_\pi^2}q\cdot k=\frac{F_D(q^2)}{1+q^2/m_\pi^2}\,, \quad (15h)$$

$$-\frac{\tilde{f}_E}{2M\cdot 2m_p}Q\cdot k+\frac{\tilde{f}_P}{m_\pi^2}q\cdot k=-f_A \quad (15i)$$

and completes the derivation of the CEC, CVC, and PCAC constraint equations.

We go on to calculate explicitly all of the "relevant" $R(q^2,Q\cdot k,q\cdot k)$, i.e., all of the $R(q^2,Q\cdot k,q\cdot k)$ which contribute to $\mathcal{T}^{(h)}$; these are all of the $F_{ij}^{(a),(b),(c),(d),(e)}(q^2,Q\cdot k,q\cdot k)$ and $G_{ij}^{(a),(b)}(q^2,Q\cdot k,q\cdot k)$ except for $F_{i1}^{(b)}(q^2,Q\cdot k,q\cdot k)$, $F_{i1}^{(d)}(q^2,Q\cdot k,q\cdot k)$, $G_{1j}^{(a)}(q^2,Q\cdot k,q\cdot k)$, $G_{01}^{(b)}(q^2,Q\cdot k,q\cdot k)$, and $G_{1j}^{(b)}(q^2,Q\cdot k,q\cdot k)$ which, in view of Eqs. (9), (10), and (5c), do not contribute to $\epsilon_\mu^* V_{\mu\lambda}(k,q,Q)$ and $\epsilon_\mu^* A_{\mu\lambda}(k,q,Q)$ and so do not contribute to $\mathcal{T}^{(h)}$ [Eq. (5)]. Adopting exactly the same procedure as that used in Ref. 1, we first work out an "appropriate" set of relevant $R(q^2,Q\cdot k,q\cdot k)$, i.e., a set of relevant $R(q^2,Q\cdot k,q\cdot k)$ whose specification via perturbation theory in an approximation described just below is not only consistent with the "linearity hypothesis" (LH) as given in Eq. (A1) of Appendix A but is, in addition, sufficient to determine all the other relevant $R(q^2,Q\cdot k,q\cdot k)$ by means of the CEC, CVC, and PCAC constraint equations [Eqs. (12a)–(12f), (13a)–(13g), (14a)–(14f), and (15a)–(15i)] and the assumption that these other relevant R are also of the form required by LH. The approximation in question is [compared Eq. (23) of Ref. 1]

$$F_{M,A,P,E}((q+k)^2,(p^{(i)}-k)^2,-M_f^2)\cong F_{M,A,P,E}((q+k)^2,-M_i^2,(p^{(f)}+k)^2)\cong F_{M,A,P,E}(q^2),$$

$$F_{M,A,P,E}(q^2)\gg F_{M,A,P,E}^{(X\to{}^{12}\mathrm{B})}((q+k)^2,(p^{(i)}-k)^2,-M_f^2)\quad (X\neq{}^{12}\mathrm{C}),$$

$$F_{M,A,P,E}(q^2)\gg F_{M,A,P,E}^{({}^{12}\mathrm{C}\to Y)}((q+k)^2,-M_i^2,(p^{(f)}+k)^2)\quad (Y\neq{}^{12}\mathrm{B}),$$

$$F_M(q^2)\gg[F_{ij}^{(a),(b),(c),(d),(e)}(q^2,Q\cdot k,q\cdot k)]_{\mathrm{BD}}\,,$$

$$F_{A,P,E}(q^2)\gg[G_{ij}^{(a),(b)}(q^2,Q\cdot k,q\cdot k)]_{\mathrm{BD}}\,,$$

$$e_i(k^2,(p^{(i)})^2,(p^{(i)}-k)^2)=e_i(0,-M_i^2,(p^{(i)}-k)^2)\cong e_i(0,-M_i^2,-M_i^2)\equiv e_i=\text{electric charge of }{}^{12}\mathrm{C}=6\,,$$

$$e_f(k^2,(p^{(f)}+k)^2,(p^{(f)})^2)=e_f(0,(p^{(f)}+k)^2,-M_f^2)\cong e_f(0,-M_f^2,-M_f^2)\equiv e_f=\text{electric charge of }{}^{12}\mathrm{B}=5\,,$$

$$\mu_i(k^2,(p^{(i)})^2,(p^{(i)}-k)^2)=\mu_i(0,-M_i^2,(p^{(i)}-k)^2)\cong\mu_i(0,-M_i^2,-M_i^2)\equiv\mu_i=\text{magnetic moment of }{}^{12}\mathrm{C}=0\,,$$

$$\mu_f(k^2,(p^{(f)}+k)^2,(p^{(f)})^2)=\mu_f(0,(p^{(f)}+k)^2,-M_f^2)\cong\mu_f(0,-M_f^2,-M_f^2)\equiv\mu_f=\text{magnetic moment of }{}^{12}\mathrm{B}=\pm 1.002\,, \quad (16)$$

where $X(Y)$ are intermediate hadrons with the same electric charge as ${}^{12}\mathrm{C}({}^{12}\mathrm{B})$ and BD refers to the "box" diagram corresponding to $\mu^-\,{}^{12}\mathrm{C}\to\nu_\mu\,{}^{12}\mathrm{B}\gamma$ where the photon and neutrino are emitted "simultaneously."[1] We also neglect terms of relative magnitude μ_f/e_f and Δ/m_μ; the last neglect allows us to drop the $Q\cdot q$ terms in Eqs. (14a)–(14f) and (15a)–(15g).

Thus, choosing $F_{23}^{(a)}$, $F_{33}^{(a)}$, $F_{00}^{(c)}$, $F_{13}^{(c)}$, $F_{23}^{(c)}$, $F_{00}^{(d)}$, $F_{01}^{(e)}$, $F_{03}^{(e)}$, $F_{1j}^{(e)}$, $F_{2j}^{(e)}$, and $G_{01}^{(a)}$, which are all *zero* as calculated by perturbation theory in the approximation of Eq. (16), as members of an appropriate set, we determine all the other relevant $F_{ij}^{(a),(b),(c),(d),(e)}$ and $G_{ij}^{(a),(b)}$ by the method outlined in Appendix A. We obtain

$$F_{1j}^{(a)}=0,\ j=1,2,3\,, \quad (17)$$

$$F_{2j}^{(a)}=0,\ j=1,2,3\,, \quad (18)$$

$$F_{31}^{(a)}=-\frac{m_p^3}{2}\frac{1}{2M}\sqrt{2}F_M(q^2)\times\left(\frac{e_i}{(Q-q)\cdot k}+\frac{e_f}{(Q+q)\cdot k}\right)\,, \quad (19a)$$

$$F_{32}^{(a)}=\frac{m_p^3}{2}\frac{1}{2M}\sqrt{2}F_M(q^2)\times\left(\frac{e_i}{(Q-q)\cdot k}-\frac{e_f}{(Q+q)\cdot k}\right)\,, \quad (19b)$$

$$F_{33}^{(a)}=0\,, \quad (19c)$$

$$F_{12}^{(b)}=\frac{m_p^3}{2}\frac{1}{2M}\sqrt{2}F_M(q^2)\times\left(\frac{e_i}{(Q-q)\cdot k}-\frac{e_f}{(Q+q)\cdot k}\right)\,, \quad (20a)$$

$$F_{13}^{(b)} = -\frac{m_p^3}{2}\frac{1}{2M}\sqrt{2}F_M(q^2) \times\left(\frac{e_i}{(Q-q)\cdot k}+\frac{e_f}{(Q+q)\cdot k}\right), \tag{20b}$$

$$F_{2j}^{(b)} = 0, \quad j=2,3, \tag{21}$$

$$F_{32}^{(b)} = -\frac{m_p^3}{2}\frac{1}{2M}\sqrt{2}F_M(q^2) \times\left(\frac{e_i}{(Q-q)\cdot k}-\frac{e_f}{(Q+q)\cdot k}\right), \tag{22a}$$

$$F_{33}^{(b)} = \frac{m_p^3}{2}\frac{1}{2M}\sqrt{2}F_M(q^2) \times\left(\frac{e_i}{(Q-q)\cdot k}+\frac{e_f}{(Q+q)\cdot k}\right), \tag{22b}$$

$$F_{00}^{(c)} = 0, \tag{23}$$

$$F_{1j}^{(c)} = 0, \quad j=1,2,3, \tag{24}$$

$$F_{2j}^{(c)} = 0, \quad j=1,2,3, \tag{25}$$

$$F_{00}^{(d)} = 0, \tag{26}$$

$$F_{1j}^{(d)} = 0, \quad j=2,3, \tag{27}$$

$$F_{2j}^{(d)} = 0, \quad j=2,3, \tag{28}$$

$$F_{01}^{(e)} = 0, \tag{29a}$$

$$F_{02}^{(e)} = \frac{m_p}{2}\frac{1}{2M}\sqrt{2}F_M(q^2), \tag{29b}$$

$$F_{03}^{(e)} = 0, \tag{29c}$$

$$F_{1j}^{(e)} = 0, \quad j=1,2,3, \tag{30}$$

$$F_{2j}^{(e)} = 0, \quad j=1,2,3, \tag{31}$$

$$G_{00}^{(a)} = 0, \tag{32}$$

$$G_{0j}^{(a)} = 0, \quad j=1,2,3, \tag{33}$$

$$G_{2j}^{(a)} = 0, \quad j=1,2,3, \tag{34}$$

$$G_{3j}^{(a)} = 0, \quad j=1,2,3, \tag{35}$$

$$G_{00}^{(b)} = \frac{m_p^2}{m_\pi^2}\sqrt{2}F_P(q^2) - \frac{m_p}{2}\frac{1}{2M}\sqrt{2}F_E(q^2)(e_i+e_f), \tag{36}$$

$$G_{02}^{(b)} = m_p^2\sqrt{2}F_A(q^2)\left(\frac{e_i}{(Q-q)\cdot k}-\frac{e_f}{(Q+q)\cdot k}\right), \tag{37a}$$

$$G_{03}^{(b)} = -m_p^2\sqrt{2}F_A(q^2)\left(\frac{e_i}{(Q-q)\cdot k}+\frac{e_f}{(Q+q)\cdot k}\right), \tag{37b}$$

$$G_{21}^{(b)} = -\frac{m_p^4}{m_\pi^2}\sqrt{2}F_P(q^2)\left(\frac{e_i}{(Q-q)\cdot k}-\frac{e_f}{(Q+q)\cdot k}\right) + \frac{m_p^3}{2}\frac{1}{2M}\sqrt{2}F_E(q^2) \times\left(\frac{e_i}{(Q-q)\cdot k}+\frac{e_f}{(Q+q)\cdot k}\right), \tag{38a}$$

$$G_{22}^{(b)} = -\frac{m_p^3}{2}\frac{1}{2M}\sqrt{2}F_E(q^2) \times\left(\frac{e_i}{(Q-q)\cdot k}-\frac{e_f}{(Q+q)\cdot k}\right), \tag{38b}$$

$$G_{23}^{(b)} = \frac{m_p^4}{m_\pi^2}\sqrt{2}F_P(q^2)\left(\frac{e_i}{(Q-q)\cdot k}-\frac{e_f}{(Q+q)\cdot k}\right), \tag{38c}$$

$$G_{31}^{(b)} = \frac{m_p^4}{m_\pi^2}\sqrt{2}F_P(q^2)\left(\frac{e_i}{(Q-q)\cdot k}+\frac{e_f}{(Q+q)\cdot k}\right) - \frac{m_p^3}{2}\frac{1}{2M}\sqrt{2}F_E(q^2) \times\left(\frac{e_i}{(Q-q)\cdot k}-\frac{e_f}{(Q+q)\cdot k}\right), \tag{39a}$$

$$G_{32}^{(b)} = \frac{m_p^3}{2}\frac{1}{2M}\sqrt{2}F_E(q^2) \times\left(\frac{e_i}{(Q-q)\cdot k}+\frac{e_f}{(Q+q)\cdot k}\right), \tag{39b}$$

$$G_{33}^{(b)} = -\frac{m_p^4}{m_\pi^2}\sqrt{2}F_P(q^2)\left(\frac{e_i}{(Q-q)\cdot k}+\frac{e_f}{(Q+q)\cdot k}\right). \tag{39c}$$

Substitution of these values of the $F_{ij}^{(a),(b),(c),(d),(e)}$ and $G_{ij}^{(a),(b)}$ into Eqs. (9) and (10) for $V_{\mu\lambda}(k,q,Q)$ and $A_{\mu\lambda}(k,q,Q)$ and calculation of the quantities $(1/m_p)\epsilon_\mu^* V_{\mu\lambda}(k,q,Q)$ and $(1/m_p)\epsilon_\mu^* A_{\mu\lambda}(k,q,Q)$ which enter linearly into $\mathcal{T}^{(h)}$ yields the remarkably simple expressions:

$$\frac{1}{m_p}\epsilon_\mu^* V_{\mu\lambda}(k,q,Q) = \left(-\sqrt{2}\epsilon_{\lambda\kappa\rho\eta}\xi_\kappa^*\frac{q_\rho+k_\rho}{2m_p}\frac{Q_\eta-k_\eta}{2M}F_M(q^2)e_i\frac{p^{(i)}\cdot\epsilon^*}{p^{(i)}\cdot k} + \sqrt{2}\epsilon_{\lambda\kappa\rho\eta}\xi_\kappa^*\frac{q_\rho+k_\rho}{2m_p}\frac{Q_\eta+k_\eta}{2M}F_M(q^2)e_f\frac{p^{(f)}\cdot\epsilon^*}{p^{(f)}\cdot k}\right)$$
$$+\left[\sqrt{2}\epsilon_{\lambda\kappa\rho\eta}\xi_\kappa^*\frac{q_\rho}{2m_p}\frac{k_\eta}{2M}F_M(q^2)\left(e_i\frac{p^{(i)}\cdot\epsilon^*}{p^{(i)}\cdot k}+e_f\frac{p^{(f)}\cdot\epsilon^*}{p^{(f)}\cdot k}\right)+\sqrt{2}\epsilon_{\mu\kappa\rho\eta}\epsilon_\mu^*\xi_\kappa^*\frac{k_\rho}{2m_p}\frac{Q_\eta}{2M}F_M(q^2)\right.$$
$$\left.\times\left((Q_\lambda-k_\lambda)\frac{e_i}{p^{(i)}\cdot k}-(Q_\lambda+k_\lambda)\frac{e_f}{p^{(f)}\cdot k}\right)+\sqrt{2}\epsilon_{\mu\lambda\kappa\eta}\epsilon_\mu^*\xi_\kappa^*\frac{Q_\eta}{2M}\frac{1}{2m_p}F_M(q^2)\right], \tag{40a}$$

$$\frac{1}{m_p}\epsilon_\mu^* A_{\mu\lambda}(k,q,Q)=\sqrt{2}\left(\xi_\lambda^* F_A(q^2)+(q_\lambda-k_\lambda)\frac{q\cdot\xi^*}{m_\pi^2}F_P(q^2)-\frac{(Q_\lambda-k_\lambda)}{2M}\frac{q\cdot\xi^*}{2m_p}F_E(q^2)\right)e_i\frac{p^{(i)}\cdot\epsilon^*}{p^{(i)}\cdot k}$$
$$-\sqrt{2}\left(\xi_\lambda^* F_A(q^2)+(q_\lambda-k_\lambda)\frac{q\cdot\xi^*}{m_\pi^2}F_P(q^2)-\frac{(Q_\lambda+k_\lambda)}{2M}\frac{q\cdot\xi^*}{2m_p}F_E(q^2)\right)e_f\frac{p^{(f)}\cdot\epsilon^*}{p^{(f)}\cdot k}$$
$$+\sqrt{2}\epsilon_\lambda^*\left(\frac{q\cdot\xi^*}{m_\pi^2}F_P(q^2)-\frac{q\cdot\xi^*}{2m_p}\frac{1}{2M}F_E(q^2)(e_i+e_f)\right). \tag{40b}$$

As an overall check, we can test our results for gauge invariance (GI) by replacing ϵ_μ^* by k_μ in the $\mathcal{T}$ of Eqs. (3)–(5); this gives zero, as required by GI, if

$$\frac{k_\mu}{m_p}[V_{\mu\lambda}(k,q,Q)+A_{\mu\lambda}(k,q,Q)]=\langle{}^{12}\mathrm{B}(p^{(f)},\xi)|[V_\lambda(0)+A_\lambda(0)]|{}^{12}\mathrm{C}(p^{(i)})\rangle \tag{41}$$

a result which is guaranteed by Eqs. (6a) and (6b). Further, we can perform the GI test on the explicit $\mathcal{T}$ of Eqs. (3)–(5) *and* Eqs. (1a), (1b), (40a), and (40b). Replacement of ϵ_μ^* by k_μ in this $\mathcal{T}$ again gives zero.

RESULTS

With the aid of the transition amplitude $\mathcal{T}=\mathcal{T}^{(l)}+\mathcal{T}^{(h)}$ obtained in the previous section [Eqs. (3)–(5), (1a)–(2), (40a), and (40b)], we proceed to evaluate the various observable quantities associated with $\mu^-\,{}^{12}\mathrm{C}\to\nu_\mu\,{}^{12}\mathrm{B}\gamma$. Our results are obtained using the fact that the numerical values of the nonradiative form factors $|F_M(m_\mu/2m_p)|$, $|F_A|$, $|F_P|$, and $|F_E(m_\mu/2m_p)|$ are all of the same order so that the expansion of expressions for the various observable quantities in powers of m_μ/M is rapidly convergent. Thus, using the gauge $\epsilon_0^*=\vec{\epsilon}^*\cdot\hat{k}=0$ and remembering that $e_i=e_f+1$ and $\vec{p}^{(\mu)}=\vec{p}^{(i)}=0$, we can express the $\mathcal{T}^{(l)}$ and $\mathcal{T}^{(h)}$ of Eqs. (4), (1a), (1b), (40a), and (40b) as

$$\mathcal{T}^{(l)}(\vec{\epsilon}^*,\vec{\xi}^*,s_z^{(\nu)};s_z^{(\mu)})\cong\frac{Ge}{\sqrt{2}}\frac{1}{\sqrt{2k_0}}\frac{1}{2m_\mu}(v^{(\nu)})^\dagger_{s_z^{(\nu)}}(1-\vec{\sigma}\cdot\hat{\nu})$$
$$\times\left[\frac{m_\mu}{2m_p}F_M(q^2)i(\vec{\xi}^*\times\vec{\sigma})\cdot\frac{\vec{p}^{(f)}}{m_\mu}+F_A(q^2)\vec{\xi}^*\cdot\vec{\sigma}\right.$$
$$\left.+\frac{m_\mu^2}{m_\pi^2}F_P(q^2)\vec{\xi}^*\cdot\frac{\vec{p}^{(f)}}{m_\mu}\left(\vec{\sigma}\cdot\frac{\vec{p}^{(f)}}{m_\mu}+\frac{\Delta}{m_\mu}\right)-\frac{m_\mu}{2m_p}F_E(q^2)\vec{\xi}^*\cdot\frac{\vec{p}^{(f)}}{m_\mu}\right]$$
$$\times(1-\vec{\sigma}\cdot\hat{k})\vec{\sigma}\cdot\vec{\epsilon}^*(v^{(\mu)})_{s_z^{(\mu)}}, \tag{42a}$$

$$\mathcal{T}^{(h)}(\vec{\epsilon}^*,\vec{\xi}^*,s_z^{(\nu)};s_z^{(\mu)})\cong\frac{Ge}{\sqrt{2}}\frac{1}{(2k_0)^{1/2}}\frac{1}{2m_\mu}(v^{(\nu)\dagger}_{s_z^{(\nu)}}(1-\vec{\sigma}\cdot\hat{\nu})$$
$$\times\left(\frac{m_\mu}{2m_p}F_M(q^2)2[-i(\vec{\xi}^*\times\vec{\sigma})\cdot\vec{\epsilon}^*+i(\vec{\xi}^*\times\vec{\epsilon}^*)\cdot\hat{k}]+\frac{m_\mu^2}{m_\pi^2}F_P(q^2)2\vec{\xi}^*\cdot\frac{\vec{p}^{(f)}}{m_\mu}\vec{\sigma}\cdot\vec{\epsilon}^*\right)(v^{(\mu)})_{s_z^{(\mu)}}, \tag{42b}$$

where $v^{(\nu)}$, and $v^{(\mu)}$ are two-component Pauli spinors for ν_μ, μ^-, $\vec{\sigma}$ is a two-by-two Pauli matrix to be sandwiched between $v^{(\nu)\dagger}$ and $v^{(\mu)}$, $\vec{p}^{(f)}=-(\vec{p}^{(\nu)}+\vec{k})\cong-[(m_\mu-\Delta-k_0)\hat{\nu}+k_0\hat{k}]$, and a term of relative magnitude

$$\frac{m_\pi^2}{4m_p^2}\frac{e_i+e_f}{(2M/m_p)}\frac{F_E}{F_P}$$

has been neglected.

Assuming that, at the instant of μ^- capture, the probability of finding the $\bar{\mu}\,{}^{12}\mathrm{C}$ in the spin configuration specified by $s_z^{(\mu)}$ is $P(s_z^{(\mu)})$ with $P(\pm\frac{1}{2})=\frac{1}{2}(1\pm P_\mu)$ where $P_\mu\hat{z}$ is the μ^- polarization, we have

$$\langle|\mathcal{T}|^2\rangle \equiv \sum_{s_z^{(\mu)}}\left(\sum_{\vec{\epsilon}^*,\vec{\xi}^*,s_z^{(\nu)}} |\mathcal{T}^{(l)}(\vec{\epsilon}^*,\vec{\xi}^*,s_z^{(\nu)};s_z^{(\mu)})+\mathcal{T}^{(h)}(\vec{\epsilon}^*,\vec{\xi}^*,s_z^{(\nu)};s_z^{(\mu)})|^2\right) P(s_z^{(\mu)})$$

$$= \left(\frac{Ge}{\sqrt{2}}\frac{1}{(2k_0)^{1/2}}\frac{1}{m_\mu}\right)^2 2\{[W_1^{(0)}+W_1^{(1)}\hat{\nu}\cdot\hat{k}+W_1^{(2)}(\hat{\nu}\cdot\hat{k})^2]+P_\mu\hat{k}\cdot\hat{z}[W_2^{(0)}+W_2^{(1)}\hat{\nu}\cdot\hat{k}+W_2^{(2)}(\hat{\nu}\cdot\hat{k})^2] + P_\mu\hat{\nu}\cdot\hat{z}(W_3^{(0)}+W_3^{(1)}\hat{\nu}\cdot\hat{k})\}, \tag{43a}$$

where the $W_i^{(n)}$ are structure functions which are homogeneous and quadratic in the $F_{M,A,P,E}(q^2)$ and which also depend on k_0. All the $W_i^{(n)}$ can be decomposed according to

$$W_i^{(n)} = W_i^{(n);(ll)} + W_i^{(n);(lh)} + W_i^{(n);(hh)} \tag{43b}$$

with the $W_i^{(n);(ll)}$, $W_i^{(n);(lh)}$, and $W_i^{(n);(hh)}$ originating, respectively, from

$$\sum_{\vec{\epsilon}^*,\vec{\xi}^*,s_z^{(\nu)}} |\mathcal{T}^{(l)}(\vec{\epsilon}^*,\vec{\xi}^*,s_z^{(\nu)};s_z^{(\mu)})|^2,$$

$$\sum_{\vec{\epsilon}^*,\vec{\xi}^*,s_z^{(\nu)}} 2\,\mathrm{Re}\,\mathcal{T}^{(l)*}(\vec{\epsilon}^*,\vec{\xi}^*,s_z^{(\nu)};s_z^{(\mu)}) \times\mathcal{T}^{(h)}(\vec{\epsilon}^*,\vec{\xi}^*,s_z^{(\nu)};s_z^{(\mu)}),$$

and

$$\sum_{\vec{\epsilon}^*,\vec{\xi}^*,s_z^{(\nu)}} |\mathcal{T}^{(h)}(\vec{\epsilon}^*,\vec{\xi}^*,s_z^{(\nu)};s_z^{(\mu)})|^2.$$

All of the $W_i^{(n);(ll)}$, $W_i^{(n);(lh)}$, and $W_i^{(n);(hh)}$ are given explicitly in Appendix B (Eqs. (B1)–(B21)). We note, in particular, that

$$W_1^{(0),(1),(2);(\xi)} = W_2^{(0),(1),(2);(\xi)},$$
$$W_3^{(0),(1);(\xi)} = 0:\quad \xi = ll, lh. \tag{43c}$$

As shown in Ref. 1, the $W_i^{(n)}(q^2, k_0/k_m)$ in Eq. (43a) can be approximated by $W_i^{(n)}(\frac{3}{5}k_m^2, k_0/k_m)$ where k_m is the maximum photon energy and is approximately $m_\mu - \Delta = 0.87\, m_\mu$. We note, from Appendix B, that these $W_i^{(n)}(\frac{3}{5}k_m^2, k_0/k_m)$ are simple quadratic functions of $k_0/k_m \equiv x$ with $0 \le x \le 1$. With these $W_i^{(n)}(\frac{3}{5}k_m^2, x) \equiv W_i^{(n)}(x)$ we can immediately proceed to calculate various observable quantities associated with $\mu^- {}^{12}\mathrm{C} \to \nu_\mu {}^{12}\mathrm{B}\gamma$. First of all, the neutrino-photon angular correlation

$$\mathcal{C}(\hat{\nu}\cdot\hat{k}) = \frac{\langle|\mathcal{T}|^2\rangle}{\int\frac{d\Omega^{(\gamma)}}{4\pi}\int\frac{d\Omega^{(\nu)}}{4\pi}\langle|\mathcal{T}|^2\rangle} \tag{44a}$$

is given by Eq. (43a) and corresponds to the forward-backward asymmetry

$$\mathcal{A}_{\hat{\nu},\hat{k}} \equiv \frac{\mathcal{C}(\hat{\nu}\cdot\hat{k}=1)-\mathcal{C}(\hat{\nu}\cdot\hat{k}=-1)}{\mathcal{C}(\hat{\nu}\cdot\hat{k}=1)+\mathcal{C}(\hat{\nu}\cdot\hat{k}=-1)} = \frac{W_1^{(1)}(x)+P_\mu\hat{k}\cdot\hat{z}\,[W_2^{(1)}(x)+W_3^{(0)}(x)]}{[W_1^{(0)}(x)+W_1^{(2)}(x)]+P_\mu\hat{k}\cdot\hat{z}\,[W_2^{(0)}(x)+W_2^{(2)}(x)+W_3^{(1)}(x)]}, \tag{44b}$$

where $\hat{\nu}\cdot\hat{k} = \pm 1$ corresponds to

$$\hat{p}^{(f)}\cdot\hat{k} = -1,\ \frac{m_\mu-\Delta-2k_0}{|m_\mu-\Delta-2k_0|}.$$

Further, the angular correlation between the photon momentum and the polarization of the μ^-

$$\mathcal{C}(\hat{k}\cdot\hat{z}) = \frac{\int\frac{d\Omega^{(\nu)}}{4\pi}\langle|\mathcal{T}|^2\rangle}{\int\frac{d\Omega^{(\gamma)}}{4\pi}\int\frac{d\Omega^{(\nu)}}{4\pi}\langle|\mathcal{T}|^2\rangle} \tag{44c}$$

corresponds to the forward-backward asymmetry

$$\mathcal{A}_{\hat{k},\hat{z}} \equiv \frac{\mathcal{C}(\hat{k}\cdot\hat{z}=1)-\mathcal{C}(\hat{k}\cdot\hat{z}=-1)}{\mathcal{C}(\hat{k}\cdot\hat{z}=1)+\mathcal{C}(\hat{k}\cdot\hat{z}=-1)}$$
$$= \frac{(W_2^{(0)}(x)+\frac{1}{3}W_2^{(2)}(x)+\frac{1}{3}W_3^{(1)}(x))P_\mu}{W_1^{(0)}(x)+\frac{1}{3}W_1^{(2)}(x)}$$
$$= \left(1-\frac{W_1^{(0)}(x)+\frac{1}{3}W_1^{(2)}(x)-W_2^{(0)}(x)-\frac{1}{3}W_2^{(2)}(x)-\frac{1}{3}W_3^{(1)}(x)}{W_1^{(0)}(x)+\frac{1}{3}W_1^{(2)}(x)}\right)P_\mu. \tag{44d}$$

where we have used

$$\int \frac{d\Omega^{(\nu)}}{4\pi}\langle |\mathcal{T}|^2\rangle = \left(\frac{Ge}{\sqrt{2}}\frac{1}{(2k_0)^{1/2}}\frac{1}{m_\mu}\right)^2 2\{[W_1^{(0)}(x)+\tfrac{1}{3}W_1^{(2)}(x)]+P_\mu\hat{k}\cdot\hat{z}[W_2^{(0)}(x)+\tfrac{1}{3}W_2^{(2)}(x)+\tfrac{1}{3}W_3^{(1)}(x)]\} \tag{44e}$$

as obtained from Eq. (43a). Finally, we have from Eq. (44e)

$$\int \frac{d\Omega^{(\gamma)}}{4\pi}\int \frac{d\Omega^{(\nu)}}{4\pi}\langle |\mathcal{T}|^2\rangle = \left(\frac{Ge}{\sqrt{2}}\frac{1}{(2k_0)^{1/2}}\frac{1}{m_\mu}\right)^2 2(W_1^{(0)}(x)+\tfrac{1}{3}W_1^{(2)}(x)) \tag{44f}$$

a quantity which determines the photon energy spectrum $d\Gamma(\mu^- {}^{12}C\to\nu_\mu {}^{12}B\gamma)/dk_0$ and the radiative muon capture rate $\Gamma(\mu^- {}^{12}C\to\nu_\mu {}^{12}B\gamma)$. We have, using Eq. (44f),

$$\frac{d\Gamma(\mu^- {}^{12}C\to\nu_\mu {}^{12}B\gamma)}{dx} \cong \frac{\alpha}{\pi}\Gamma_0\left(\frac{k_m}{m_\mu}\right)^2\left(\frac{k_0}{k_m}\right)^2\left[1-\left(\frac{k_0}{k_m}\right)\right]^2\left(\frac{\int \frac{d\Omega^{(\gamma)}}{4\pi}\int\frac{d\Omega^{(\nu)}}{4\pi}\langle|\mathcal{T}|^2\rangle}{2\left(\frac{Ge}{\sqrt{2}}\frac{1}{\sqrt{2}}\frac{1}{m_\mu^{3/2}}\right)^2}\right)$$

$$= \frac{\alpha}{12\pi}\Gamma_0\left(\frac{k_m}{m_\mu}\right)^2[12x(1-x)^2](W_1^{(0)}(x)+\tfrac{1}{2}W_1^{(2)}(x)), \tag{45}$$

$$\Gamma_0 \equiv \frac{G^2 m_\mu^5}{2\pi}\left(1-\frac{m_\mu}{m_\mu+m_f}\right)C_i\left(e_i\alpha\frac{M_i}{m_\mu+M_i}\right)^3, \quad C_i = 0.841.$$

so that

$$\Gamma(\mu^- {}^{12}C\to\nu_\mu {}^{12}B\gamma) = \int_0^1 \frac{d\Gamma(\mu^- {}^{12}C\to\nu_\mu {}^{12}B\gamma)}{dx}dx$$

$$\cong \frac{\alpha}{12\pi}\Gamma_0\left(\frac{k_m}{m_\mu}\right)^2\int_0^1 12x(1-x)^2[W_1^{(0)}(x)+\tfrac{1}{3}W_1^{(2)}(x)]dx. \tag{46a}$$

It is also convenient to introduce the branching ratio for radiative muon capture

$$R \equiv \frac{\Gamma(\mu^- {}^{12}C\to\nu_\mu {}^{12}B\gamma)}{\Gamma(\mu^- {}^{12}C\to\nu_\mu {}^{12}B)}, \tag{46b}$$

where the nonradiative muon capture rate is given by (see Ref. 2)

$$\Gamma(\mu^- {}^{12}C\to\nu_\mu {}^{12}B) = \Gamma_0\left[2\left(F_A(q_{nr}^2)+F_M(q_{nr}^2)\frac{(m_\mu-\Delta)}{2m_p}\right)^2 + \left(F_A(q_{nr}^2)+F_P(q_{nr}^2)\frac{m_\mu(m_\mu-\Delta)}{m_\pi^2}-F_E(q_{nr}^2)\frac{(m_\mu-\Delta)}{2m_p}\right)^2\right],$$

$$q_{nr}^2 \equiv (p^{(f)}-p^{(i)})^2_{\substack{\text{nonrad.}\\ \text{muon capture}}} = (p^{(\mu)}-p^{(\nu)})^2_{\substack{\text{nonrad.}\\ \text{muon capture}}} = (-m_\mu^2+2E^{(\nu)}m_\mu)_{\substack{\text{nonrad.}\\ \text{muon capture}}} \cong m_\mu^2-2m_\mu\Delta = 0.74m_\mu^2. \tag{46c}$$

We note from Eqs. (43c) and (44d) that, if the term

$$\sum_{\vec{\epsilon}^*,\vec{\xi}^*,s_z^{(\nu)}}|\mathcal{T}^{(h)}(\vec{\epsilon}^*,\vec{\xi}^*,s_z^{(\nu)};s_z^{(\mu)})|^2$$

is neglected so that the $W_i^{(n);(hh)}$ are set equal to zero, we have

$$\mathcal{A}_{\hat{k},\hat{z}} = 1\cdot P_\mu \tag{47}$$

which is a manifestation of a result originally stated by Cutkosky,[3] Huang, Yang, and Lee,[4] and recently elaborated by Fearing.[5] In actuality, however, the $W_i^{(n);(hh)}$ are by no means small compared to the $W_i^{(n);(ll)}$ and $W_i^{(n);(lh)}$ so that deviation of $\mathcal{A}_{\hat{k},\hat{z}}/P_\mu$ from unity is appreciable.

The various observable quantities in $\mu^- {}^{12}C\to\nu_\mu {}^{12}B\gamma$ [Eqs. (44b), (44d), (45), (46a)] can be measured by detection of coincidences between the high energy photons and the recoil ^{12}B nuclei—here the μ^- would stop in a gas, e.g., methane (CH_4), with events of the type $\mu^- {}^{12}C\to\nu_\mu {}^{11}Bn\gamma$ eliminated by anticoincidencing on the n. Alternatively, the μ^- could stop in a solid, e.g., graphite (C), with delayed coincidences detected between the high-energy photons and the ^{12}B-decay electrons at such low μ^- fluxes as to minimize the number of spurious (and unrejected) delayed coincidences where the γ originates from the radiative capture of one μ^- and the e^- from the nonradiative capture of another. Both of these experiments appear to be very difficult with the second probably even more difficult than the first in view of the long running time required (because of the low μ^- flux). Nevertheless, in Appendix C, we describe briefly the evaluation of the polarization and alignment of the recoil ^{12}B nuclei produced in $\mu^-\,{}^{12}\mathrm{C} \to \nu_\mu\,{}^{12}\mathrm{B}\gamma$; these quantities are useful for the study of correlations in the γ-e^- delayed coincidence experiment. In addition we derive in Appendix C formulas for the circular polarization of the emitted γ—the measurement of this quantity is again very difficult.

We proceed to obtain the numerical results for the various observable quantities [Eqs. (44a)–(46c)]. Remembering that $q_{nr}^2 = 0.74 m_\mu^2$ and using Ref. 2, we have

$$F_M(0.74 m_\mu^2) = F_M(0) \times 0.750 = 1.97 \times 0.750,$$

$$F_A(0.74 m_\mu^2) = F_A(0) \times 0.750 = 0.510 \times 0.750,$$

$$F_E(0.74 m_\mu^2) = F_E(0) \times 0.750 = (3.64 \times 0.510) \times 0.750,$$

$$F_P(0.74 m_\mu^2) = -\frac{F_A(0.74 m_\mu^2)}{1 + 0.74 m_\mu^2/m_\pi^2}(1 - 0.15) = -0.228.$$

From these values $F_{M,A,E}(\frac{3}{5}k_m^2) = F_{M,A,E}(0.45 m_\mu^2)$ can be obtained by linear interpolation, while

$$F_P(\tfrac{3}{5}k_m^2) = F_P(0.45 m_\mu^2) \cong -\frac{F_A(0.45 m_\mu^2)}{1 + 0.45 m_\mu^2/m_\pi^2}(1 - 0.15) = -0.292;$$

thus the $F_{M,A,P,E}(\frac{3}{5}k_m^2)$ entering into the $W_i^{(n)}(\frac{3}{5}k_m^2, x) \equiv W_i^{(n)}(x)$ are all numerically determined and the $W_i^{(n)}(x)$ are immediately calculable from Eqs. (B1)–(B21). In this way we obtain

$$W_1^{(0)}(x) = 0.817 - 0.681x + 0.362x^2, \quad (48a)$$

$$W_1^{(1)}(x) = -0.741 + 0.432x - 0.141x^2, \quad (48b)$$

$$W_1^{(2)}(x) = -0.028 + 0.249x - 0.221x^2, \quad (48c)$$

$$W_2^{(0)}(x) = 0.755 - 0.649x + 0.275x^2, \quad (48d)$$

$$W_2^{(1)}(x) = -0.707 + 0.340x + 0.034x^2, \quad (48e)$$

$$W_2^{(2)}(x) = 0.309x - 0.309x^2, \quad (48f)$$

$$W_3^{(0)}(x) = -0.028 + 0.028x, \quad (48g)$$

$$W_3^{(1)}(x) = 0.028 - 0.028x. \quad (48h)$$

Using Eqs. (44b), (44d), (45), (46a)–(46c), and (48a)–(48h), we have

$$\mathfrak{A}_{\hat{\nu},\hat{k}} = \frac{(-0.740 + 0.432x - 0.141x^2) + P_\mu \hat{k}\cdot\hat{z}(-0.735 + 0.368x + 0.034x^2)}{(0.789 - 0.432x + 0.141x^2) + P_\mu \hat{k}\cdot\hat{z}(0.783 - 0.368x - 0.034x^2)} \quad (49a)$$

$$\mathfrak{A}_{\hat{k},\hat{z}} = \left(1 - \frac{0.044 - 0.043x + 0.116x^2}{0.808 - 0.598x + 0.288x^2}\right) P_\mu, \quad (49b)$$

$$\frac{\dfrac{d\Gamma(\mu^-\,{}^{12}\mathrm{C} \to \nu_\mu\,{}^{12}\mathrm{B}\gamma)}{dx}}{\dfrac{\alpha}{12\pi}\Gamma_0\left(\dfrac{k_m}{m_\mu}\right)^2[12x(1-x)^2]} = 0.808 - 0.598x + 0.288x^2, \quad (49c)$$

$$\frac{\Gamma(\mu^-\,{}^{12}\mathrm{C} \to \nu_\mu\,{}^{12}\mathrm{B}\gamma)}{(\alpha/12\pi)\Gamma_0(k_m/m_\mu)^2} = 0.626, \quad (49d)$$

$$\Gamma(\mu^-\,{}^{12}\mathrm{C} \to \nu_\mu\,{}^{12}\mathrm{B})/\Gamma_0 = 0.437, \quad (49e)$$

$$R = \frac{\alpha}{12\pi}\left(\frac{k_m}{m_\mu}\right)^2 \frac{0.626}{0.437} = 2.08 \times 10^{-4}, \quad (49f)$$

with

$$\{P_\mu\}_{\exp} = 0.18 \pm 0.01 \text{ (Ref. 6)}. \quad (49g)$$

Finally, we note that the values of $\mathfrak{A}_{\hat{\nu},\hat{k}}$, $\mathfrak{A}_{\hat{k},\hat{z}}$, $d\Gamma(\mu^-\,{}^{12}\mathrm{C} \to \nu_\mu\,{}^{12}\mathrm{B}\gamma)/dx$, and $\Gamma(\mu^-\,{}^{12}\mathrm{C} \to \nu_\mu\,{}^{12}\mathrm{B}\gamma)$ are all rather sensitive to the value assumed for

$$\frac{F_P(\frac{3}{5}k_m^2)}{-F_A(\frac{3}{5}k_m^2)/[1 + \frac{3}{5}k_m^2/m_\pi^2]} \cong \left[1 + \frac{m_\pi^2}{q^2}\left(1 - \frac{F_D(q^2)}{F_A(q^2)}\right)\right]_{q^2 = (3/5)k_m^2} \equiv \xi \text{ (Ref. 2)},$$

the sensitivity being greatest for $\alpha_{\hat{k},\hat{z}}$. Since the estimate of $F_D(\frac{3}{5}k_m^2)/F_A(\frac{3}{5}k_m^2)$ in a nucleus as large as ^{12}C is somewhat doubtful, the estimate we use for ξ, namely, $\xi=0.85$ (Ref. 7), is correspondingly doubtful and it is interesting to calculate the values of

$$\alpha_{\hat{\nu},\hat{k}},\alpha_{\hat{k},\hat{z}},\ \frac{d\Gamma(\mu^-\,{}^{12}\mathrm{C}\to\nu_\mu\,{}^{12}\mathrm{B}\gamma)}{dx}$$

and

$$\Gamma(\mu^-\,{}^{12}\mathrm{C}\to\nu_\mu\,{}^{12}\mathrm{B}\gamma)$$

for ξ other than 0.85. We therefore record below the values of

$$\alpha_{\hat{\nu},\hat{k}},\alpha_{\hat{k},\hat{z}},\ \frac{d\Gamma(\mu^-\,{}^{12}\mathrm{C}\to\nu_\mu\,{}^{12}\mathrm{B}\gamma)}{dx},$$

and

$$\Gamma(\mu^-\,{}^{12}\mathrm{C}\to\nu_\mu\,{}^{12}\mathrm{B}\gamma)$$

for $\xi=\frac{1}{2}\times 0.85$ and $\xi=2\times 0.85$, and compare these values with the values calculated for $\xi=0.85$ and already given in Eqs. (49a)–(49d). Using Eqs. (44b) (44d), (45), (46a)–(46e), and Eqs. (B1)–(B21), we have

$$\alpha_{\hat{\nu},\hat{k}}=\frac{-0.44-0.41P_\mu\hat{k}\cdot\hat{z}}{0.63+0.60P_\mu\hat{k}\cdot\hat{z}}:\ \xi=\tfrac{1}{2}\times 0.85,\ x=\tfrac{2}{3},\tag{50a}$$

$$\alpha_{\hat{\nu},\hat{k}}=\frac{-0.51-0.47P_\mu\hat{k}\cdot\hat{z}}{0.56+0.52P_\mu\hat{k}\cdot\hat{z}}:\ \xi=0.85,\ x=\tfrac{2}{3}\tag{50b}$$

[Eq. (49a)],

$$\alpha_{\hat{\nu},\hat{k}}=\frac{-0.54-0.46P_\mu\hat{k}\cdot\hat{z}}{0.58+0.50P_\mu\hat{k}\cdot\hat{z}}:\ \xi=2\times 0.85,\ x=\tfrac{2}{3},\tag{50c}$$

$$\alpha_{\hat{k},\hat{z}}=0.94P_\mu:\ \xi=\tfrac{1}{2}\times 0.85,\ x=\tfrac{2}{3},\tag{50d}$$

$$\alpha_{\hat{k},\hat{z}}=0.88P_\mu:\ \xi=0.85,\ x=\tfrac{2}{3}\tag{50e}$$

[Eq. (49b)],

$$\alpha_{\hat{k},\hat{z}}=0.69P_\mu:\ \xi=2\times 0.85,\ x=\tfrac{2}{3},\tag{50f}$$

$$\frac{\dfrac{d\Gamma(\mu^-\,{}^{12}\mathrm{C}\to\nu_\mu\,{}^{12}\mathrm{B}\gamma)}{dx}}{\dfrac{\alpha}{12\pi}\Gamma_0\left(\dfrac{k_m}{m_\mu}\right)^2[12x(1-x)^2]}=0.617:\ \xi=\tfrac{1}{2}\times 0.85,\ x=\tfrac{2}{3},\tag{50g}$$

$$\frac{\dfrac{d\Gamma(\mu^-\,{}^{12}\mathrm{C}\to\nu_\mu\,{}^{12}\mathrm{B}\gamma)}{dx}}{\dfrac{\alpha}{12\pi}\Gamma_0\left(\dfrac{k_m}{m_\mu}\right)^2[12x(1-x)^2]}=0.538:\ \xi=0.85,\ x=\tfrac{2}{3}\tag{50h}$$

[Eq. (49c)],

$$\frac{\dfrac{d\Gamma(\mu^-\,{}^{12}\mathrm{C}\to\nu_\mu\,{}^{12}\mathrm{B}\gamma)}{dx}}{\dfrac{\alpha}{12\pi}\Gamma_0\left(\dfrac{k_m}{m_\mu}\right)^2[12x(1-x)^2]}=0.484:\ \xi=2\times 0.85,\ x=\tfrac{2}{3},\tag{50i}$$

$$\frac{\Gamma(\mu^-\,{}^{12}\mathrm{C}\to\nu_\mu\,{}^{12}\mathrm{B}\gamma)}{\Gamma_0}=1.03\times 10^{-4}:\ \xi=\tfrac{1}{2}\times 0.85,\tag{50j}$$

$$\frac{\Gamma(\mu^-\,{}^{12}\mathrm{C}\to\nu_\mu\,{}^{12}\mathrm{B}\gamma)}{\Gamma_0}=0.91\times 10^{-4}:\ \xi=0.85\tag{50k}$$

[Eq. (49d)],

$$\frac{\Gamma(\mu^-\,{}^{12}\mathrm{C}\to\nu_\mu\,{}^{12}\mathrm{B}\gamma)}{\Gamma_0}=0.84\times 10^{-4}:\ \xi=2\times 0.85.\tag{50l}$$

APPENDIX A

In this appendix, we describe the procedure for the determination of all the relevant $R\equiv F_{ij}^{(a),(b),(c),(d),(e)}$ and $G_{ij}^{(a),(b)}$ by means of the CEC, CVC, and PCAC constraints of Eqs. (12a)–(13g), (14a)–(14f), and (15a)–(15i) and the linearity hypothesis (LH) as described in Eq. (20) of Ref. 1; this equation gives

$$R(q^2,Q\cdot k,q\cdot k)=\frac{R^+(q^2)}{(Q+q)\cdot k}+\frac{R^-(q^2)}{(Q-q)\cdot k}+R^0(q^2),\tag{A1}$$

where $R^+(q^2)$ is linear in $F_{M,A,P,E}(q^2)$ and in e_f, $R^-(q^2)$ is linear in $F_{M,A,P,E}(q^2)$ and in e_i, and $R^{(0)}(q^2)$ is linear in $F_{M,A,P,E}(q^2)$ and in e_i,e_f, terms of relative magnitude μ_f/e_f being neglected. In using the LH of Eq. (A1), we further neglect terms of relative magnitude Δ/m_μ and so drop all the $Q\cdot q$ terms in Eqs. (14a)–(14f) and (15a)–(15g).

We begin by determining all the relevant $F_{ij}^{(a),(b),(c),(d),(e)}$ by means of the CEC and CVC constraints of Eqs. (12a)–(12f) and (14a)–(14f) and the LH of Eq. (A1), starting from an appropriate set of $F_{ij}^{(a),(b),(c),(d),(e)}$. The appropriate set used is that calculated by perturbation theory in the approximation of Eq. (16) and given in the text before Eq. (17), viz:

$$F_{23}^{(a)}=F_{33}^{(a)}=F_{00}^{(c)}=F_{13}^{(c)}=F_{23}^{(c)}=F_{00}^{(d)}=F_{01}^{(e)}=F_{03}^{(e)}=F_{1j}^{(e)}=F_{2j}^{(e)}=0.\tag{A2}$$

Then, with $F_{03}^{(e)}$ given by Eq. (A2), Eq. (12c) together with Eq. (A1) yields

$$F_{22}^{(b)}=F_{23}^{(b)}=0.\tag{A3}$$

Analogously, using Eq. (A2), Eqs. (12e), (12f), (14b), (14e), (14f), and Eq. (A1) lead to

$$F_{12}^{(d)}=F_{13}^{(d)}=F_{22}^{(d)}=F_{23}^{(d)}=F_{21}^{(a)}=F_{22}^{(a)}=F_{11}^{(c)}=F_{12}^{(c)}=F_{21}^{(c)}=F_{22}^{(c)}=0\,. \quad \text{(A4)}$$

Equations (12a) and (14a) give

$$F_{11}^{(a)}=F_{12}^{(a)}=F_{13}^{(a)}=0\,,$$

$$F_{02}^{(e)}=\frac{m_p}{2}\frac{1}{2M}\sqrt{2}\,F_M(q^2)\,. \quad \text{(A5)}$$

Finally, Eqs. (12b), (12d), (14c), with $F_{02}^{(e)}$ and $F_{33}^{(a)}$ given by Eqs. (A5) and (A1), yield

$$F_{12}^{(b)}=-F_{32}^{(b)}=F_{32}^{(a)}=\frac{m_p^3}{2}\frac{1}{2M}\sqrt{2}\,F_M(q^2)\left(\frac{e_i}{(Q-q)\cdot k}-\frac{e_f}{(Q+q)\cdot k}\right),$$

$$F_{13}^{(b)}=-F_{33}^{(b)}=F_{31}^{(a)}=-\frac{m_p^3}{2}\frac{1}{2M}\sqrt{2}\,F_M(q^2)\left(\frac{e_i}{(Q-q)\cdot k}+\frac{e_f}{(Q+q)\cdot k}\right) \quad \text{(A6)}$$

which expressions are consistent with Eq. (14d). We thus complete the determination of all the relevant $F_{ij}^{(a),(b),(c),(d),(e)}$ in Eqs. (17)–(31).

We proceed next to determine all the relevant $G_{ij}^{(a),(b)}$ by means of the CEC and PCAC constraints of Eqs. (13a)–(13g) and (15a)–(15i) and the LH of Eq. (A1), starting from an appropriate set of $G_{ij}^{(a),(b)}$. The appropriate set used is that calculated by perturbation theory in the approximation of Eq. (16) and given in the text before Eq. (17), viz:

$$G_{01}^{(a)}=0\,. \quad \text{(A7)}$$

We first note that Eqs. (13a), (13f), (13g), and (15h), together with Eq. (A1), lead to

$$G_{02}^{(b)}=m_p^2\sqrt{2}\,F_A(q^2)\left(\frac{e_i}{(Q-q)\cdot k}-\frac{e_f}{(Q+q)\cdot k}\right), \quad \text{(A8)}$$

$$G_{03}^{(b)}=-m_p^2\sqrt{2}\,F_A(q^2)\left(\frac{e_i}{(Q-q)\cdot k}+\frac{e_f}{(Q+q)\cdot k}\right), \quad \text{(A9)}$$

$$G_{22}^{(b)}=-\frac{m_p^3}{2}\frac{1}{2M}\sqrt{2}\,F_E(q^2)\times\left(\frac{e_i}{(Q-q)\cdot k}-\frac{e_f}{(Q+q)\cdot k}\right), \quad \text{(A10)}$$

$$G_{32}^{(b)}=\frac{m_p^3}{2}\frac{1}{2M}\sqrt{2}\,F_E(q^2)\times\left(\frac{e_i}{(Q-q)\cdot k}+\frac{e_f}{(Q+q)\cdot k}\right), \quad \text{(A11)}$$

$$G_{23}^{(b)}=\frac{m_p^4}{m_\pi^2}\sqrt{2}\,F_P(q^2)\left(\frac{e_i}{(Q-q)\cdot k}-\frac{e_f}{(Q+q)\cdot k}\right), \quad \text{(A12)}$$

$$G_{33}^{(b)}=-\frac{m_p^4}{m_\pi^2}\sqrt{2}\,F_P(q^2)\left(\frac{e_i}{(Q-q)\cdot k}+\frac{e_f}{(Q+q)\cdot k}\right), \quad \text{(A13)}$$

$$f_E=-4Mm_p\frac{F_D(q^2)}{1+q^2/m_\pi^2}\times\left(\frac{e_i}{(Q-q)\cdot k}-\frac{e_f}{(Q+q)\cdot k}\right), \quad \text{(A14)}$$

$$f_P=-m_\pi^2\frac{F_D(q^2)}{1+q^2/m_\pi^2}\times\left(\frac{e_i}{(Q-q)\cdot k}+\frac{e_f}{(Q+q)\cdot k}\right), \quad \text{(A15)}$$

and, using Eqs. (A10), (A12), (A8), and (A14), Eq. (15c) yields

$$G_{21}^{(b)}=-\frac{m_p^4}{m_\pi^2}\sqrt{2}\,F_P(q^2)\left(\frac{e_i}{(Q-q)\cdot k}-\frac{e_f}{(Q+q)\cdot k}\right)+\frac{m_p^3}{2}\frac{1}{2M}\sqrt{2}\,F_E(q^2)\left(\frac{e_i}{(Q-q)\cdot k}+\frac{e_f}{(Q+q)\cdot k}\right),$$

which is clearly consistent with the LH of Eq. (A1). Equations (13e), (A16), and (A1) determine $G_{31}^{(b)}$ and $G_{00}^{(b)}$:

$$G_{31}^{(b)}=\frac{m_p^4}{m_\pi^2}\sqrt{2}\,F_P(q^2)\left(\frac{e_i}{(Q-q)\cdot k}+\frac{e_f}{(Q+q)\cdot k}\right)-\frac{m_p^3}{2}\frac{1}{2M}\sqrt{2}\,F_E(q^2)\left(\frac{e_i}{(Q-q)\cdot k}-\frac{e_f}{(Q+q)\cdot k}\right), \quad \text{(A17)}$$

$$G_{00}^{(b)}=\frac{m_p^2}{m_\pi^2}\sqrt{2}\,F_P(q^2)-\frac{m_p}{2}\frac{1}{2M}\sqrt{2}\,F_E(q^2)(e_i+e_f)\,. \quad \text{(A18)}$$

We note that Eq. (15d) provides a consistency check for the results obtained in Eqs. (A17), (A11), (A13), (A9), (A18), and (A15) for $G_{31}^{(b)}$, $G_{32}^{(b)}$, $G_{33}^{(b)}$, $G_{03}^{(b)}$, $G_{00}^{(b)}$, and f_P.

Further, Eqs. (13c) and (A1) show that $G_{02}^{(a)}$ depends only on q^2. Similarly, Eqs. (13d), (15i), and (A1) show that both $G_{03}^{(a)}$ and f_A depend only on q^2 so that Eq. (15a) becomes

$$G_{02}^{(a)}Q\cdot k+(G_{01}^{(a)}+G_{03}^{(a)})q\cdot k=-G_{03}^{(a)}q^2+m_p^2\sqrt{2}\,[F_A(q^2)+f_A]=\text{function of } q^2 \text{ only}\,. \quad \text{(A19)}$$

Equations (A19) and (A1), together with the fact that $G_{02}^{(a)}$ depends only on q^2, yield

$$G_{02}^{(a)} = 0\,, \tag{A20}$$

$$G_{01}^{(a)} + G_{13}^{(a)} = 0\,, \tag{A21}$$

$$-G_{03}^{(a)} q^2 + m_p{}^2\sqrt{2}\,[F_A(q^2) + f_A] = 0\,. \tag{A22}$$

Equations (A21) and (A22), together with Eq. (A7), lead to

$$G_{03}^{(a)} = 0\,, \tag{A23}$$

$$f_A = -F_A(q^2)\,. \tag{A24}$$

With $G_{02}^{(a)}$, $G_{03}^{(a)}$, and f_A given by Eqs. (A20), (A23), and (A24), Eqs. (13c), (13d), and (15i) yield

$$G_{22}^{(a)} = G_{32}^{(a)} = 0\,, \tag{A25}$$

$$G_{23}^{(a)} = G_{33}^{(a)} = 0\,, \tag{A26}$$

$$\bar{f}_E = -4Mm_p F_A(q^2)\left(\frac{e_i}{(Q-q)\cdot k} - \frac{e_f}{(Q+q)\cdot k}\right), \tag{A27}$$

$$\bar{f}_P = -m_\pi{}^2 F_A(q^2)\left(\frac{e_i}{(Q-q)\cdot k} + \frac{e_f}{(Q+q)\cdot k}\right) \tag{A28}$$

and, with $G_{22}^{(a)}$, $G_{23}^{(a)}$, $G_{02}^{(b)}$, and $\bar{f}_E$ given by Eqs. (A25), (A26), (A8), and (A27), Eq. (15f) yields

$$G_{21}^{(a)} = 0\,. \tag{A29}$$

Finally, with $G_{21}^{(a)}$, and $G_{01}^{(a)}$ given by Eqs. (A29) and (A7), Eqs. (13b) and (A1) determine $G_{31}^{(a)}$ and $G_{00}^{(a)}$:

$$G_{31}^{(a)} = 0\,, \tag{A30}$$

$$G_{00}^{(a)} = 0\,, \tag{A31}$$

which completes the determination of all the relevant $G_{ij}^{(a),(b)}$ in Eqs. (32)–(39c).

We note that the results of Eqs. (A30), (A25), (A26), (A9), (A31), and (A28) for $G_{31}^{(a)}$, $G_{32}^{(a)}$, $G_{33}^{(a)}$, $G_{03}^{(b)}$, $G_{00}^{(a)}$, and $\bar{f}_P$ satisfy Eq. (15g); this provides a consistency check. We also note that the PCAC constraint equations (15b) and (15e), which relate $G_{01}^{(b)}$, $G_{1j}^{(b)}$, and $G_{1j}^{(a)}$ to $G_{00}^{(b)}$, f_Q and $G_{00}^{(a)}$, $\bar{f}_Q$ need not be used since, as noted in the text [after Eq. (15i)], $G_{01}^{(b)}$, $G_{1j}^{(b)}$, and $G_{1j}^{(a)}$ do not contribute to $\mathcal{T}^{(h)}$.

APPENDIX B

In this appendix, we list the structure functions $W_i^{(n)} = W_i^{(n);(ll)} + W_i^{(n);(lh)} + W_i^{(n);(hh)}$ as derived from Eqs. (42a)–(43b). Using the notation $F_{M,A,P,E} \equiv F_{M,A,P,E}(q^2)$ and $x \equiv k_0/k_m$ (with k_m the maximum photon energy so that $0 \leq x \leq 1$), we have

$$\begin{aligned} W_1^{(0);(ll)} = {} & 3F_A{}^2 + 2F_A\left(F_P\frac{m_\mu k_m}{m_\pi{}^2} - F_E\frac{k_m}{2m_p}\right) + 4F_AF_M\frac{k_m}{2m_p}(1-2x) \\ & + \left(F_P\frac{m_\mu k_m}{m_\pi{}^2} - F_E\frac{k_m}{2m_p}\right)^2(1-2x+2x^2) + 2\left(F_M\frac{k_m}{2m_p}\right)^2(1-3x+3x^2)\,, \end{aligned} \tag{B1}$$

$$\begin{aligned} W_1^{(0);(lh)} = {} & 2F_AF_P\frac{m_\mu k_m}{m_\pi{}^2} + 6F_AF_M\frac{k_m}{2m_p}\frac{m_\mu}{k_m} + 2\left(F_P\frac{m_\mu k_m}{m_\pi{}^2} - F_E\frac{k_m}{2m_p}\right)F_P\frac{m_\mu k_m}{m_\pi{}^2}(1-2x+2x^2) \\ & + 2\left(F_P\frac{m_\mu k_m}{m_\pi{}^2} - F_E\frac{k_m}{2m_p}\right)F_M\frac{k_m}{2m_p}\frac{m_\mu}{k_m} + 4\left(F_M\frac{k_m}{2m_p}\right)^2\frac{m_\mu}{k_m}(1-2x)\,, \end{aligned} \tag{B2}$$

$$\begin{aligned} W_1^{(0);(hh)} = {} & 2\left(F_P\frac{m_\mu k_m}{m_\pi{}^2}\right)^2(1-2x+2x^2) + 2F_P\frac{m_\mu k_m}{m_\pi{}^2}F_M\frac{k_m}{2m_p}\frac{m_\mu}{k_m}(1-x) \\ & + 6\left(F_M\frac{k_m}{2m_p}\right)^2\left(\frac{m_\mu}{k_m}\right)^2, \end{aligned} \tag{B3}$$

$$\begin{aligned} W_1^{(1);(ll)} = {} & -F_A{}^2 + 2F_A\left(F_P\frac{m_\mu k_m}{m_\pi{}^2} - F_E\frac{k_m}{2m_p}\right) - 4F_AF_M\frac{k_m}{2m_p}(1-2x) + \left(F_P\frac{m_\mu k_m}{m_\pi{}^2} - F_E\frac{k_m}{2m_p}\right)^2 \\ & - 2\left(F_M\frac{k_m}{2m_p}\right)^2(1-2x)^2\,, \end{aligned} \tag{B4}$$

$$\begin{aligned} W_1^{(1);(lh)} = {} & 2F_AF_P\frac{m_\mu k_m}{m_\pi{}^2} - 2F_AF_M\frac{k_m}{2m_p}\frac{m_\mu}{k_m} + 2\left(F_P\frac{m_\mu k_m}{m_\pi{}^2} - F_E\frac{k_m}{2m_p}\right)F_P\frac{m_\mu k_m}{m_\pi{}^2} \\ & + 2\left(F_P\frac{m_\mu k_m}{m_\pi{}^2} - F_E\frac{k_m}{2m_p}\right)F_M\frac{k_m}{2m_p}\frac{m_\mu}{k_m} - 4\left(F_M\frac{k_m}{2m_p}\right)^2\frac{m_\mu}{k_m}(1-2x)\,, \end{aligned} \tag{B5}$$

$$\begin{aligned} W_1^{(1);(hh)} = {} & \left(F_P\frac{m_\mu k_m}{m_\pi{}^2}\right)^2 4x(1-x) + F_P\frac{m_\mu k_m}{m_\pi{}^2}F_M\frac{k_m}{2m_p}\frac{m_\mu}{k_m}4x \\ & - 4\left(F_M\frac{k_m}{2m_p}\right)^2\left(\frac{m_\mu}{k_m}\right)^2, \end{aligned} \tag{B6}$$

$$W_1^{(2);(ll)} = \left[\left(F_P \frac{m_\mu k_m}{m_\pi^2} - F_E \frac{k_m}{2m_p}\right)^2 - \left(F_M \frac{k_m}{2m_p}\right)^2\right] 2x(1-x)\,, \quad \text{(B7)}$$

$$W_1^{(2);(lh)} = \left(F_P \frac{m_\mu k_m}{m_\pi^2} - F_E \frac{k_m}{2m_p}\right) \times F_P \frac{m_\mu k_m}{m_\pi^2}\, 4x(1-x)\,, \quad \text{(B8)}$$

$$W_1^{(2);(hh)} = F_P \frac{m_\mu k_m}{m_\pi^2} F_M \frac{k_m}{2m_p} \frac{m_\mu}{k_m}\, 2(1-x)\,, \quad \text{(B9)}$$

$$W_2^{(0);(ll)} = W_1^{(0);(ll)}\,, \quad \text{(B10)}$$

$$W_2^{(0);(lh)} = W_1^{(0);(lh)}\,, \quad \text{(B11)}$$

$$W_2^{(0);(hh)} = 2F_P \frac{m_\mu k_m}{m_\pi^2} F_M \frac{k_m}{2m_p} \frac{m_\mu}{k_m} (1+x) + 4\left(F_M \frac{k_m}{2M_p}\right)^2 \left(\frac{m_\mu}{k_m}\right)^2, \quad \text{(B12)}$$

$$W_2^{(1);(ll)} = W_1^{(1);(ll)}\,, \quad \text{(B13)}$$

$$W_2^{(1);(lh)} = W_1^{(1);(lh)}\,, \quad \text{(B14)}$$

$$W_2^{(1);(hh)} = 2\left(F_P \frac{m_\mu k_m}{m_\pi^2}\right)^2 (1-2x+2x^2) + 2F_P \frac{m_\mu k_m}{m_\pi^2} F_M \frac{k_m}{2m_p} \frac{m_\mu}{k_m}(1-x) - 2\left(F_M \frac{k_m}{2m_p}\right)^2 \left(\frac{m_\mu}{k_m}\right)^2, \quad \text{(B15)}$$

$$W_2^{(2);(ll)} = W_1^{(2);(ll)}\,, \quad \text{(B16)}$$

$$W_2^{(2);(lh)} = W_1^{(2);(lh)}\,, \quad \text{(B17)}$$

$$W_2^{(2);(hh)} = \left(F_P \frac{m_\mu k_m}{m_\pi^2}\right)^2 4x(1-x)\,, \quad \text{(B18)}$$

$$W_3^{(0);(ll)} = W_3^{(1);(ll)} = W_3^{(0);(lh)} = W_3^{(1);(lh)} = 0\,, \quad \text{(B19)}$$

$$W_3^{(0);(hh)} = 2F_P \frac{m_\mu k_m}{m_\pi^2} F_M \frac{k_m}{2m_p} \frac{m_\mu}{k_m}(1-x)\,, \quad \text{(B20)}$$

$$W_3^{(1);(hh)} = -2F_P \frac{m_\mu k_m}{m_\pi^2} F_M \frac{k_m}{2m_p} \frac{m_\mu}{k_m}(1-x)\,. \quad \text{(B21)}$$

APPENDIX C

In this appendix, we present the results of our evaluation of (1) the polarization and alignment of the recoil ^{12}B nuclei from $\mu^- {}^{12}\text{C} \to \nu_\mu {}^{12}\text{B}\gamma$ and (2) the circular polarization of the photons emitted in $\mu^- {}^{12}\text{C} \to \nu_\mu {}^{12}\text{B}\gamma$.

We first describe the evaluation of the polarization P_{av} and alignment A_{av} of the recoil ^{12}B nuclei. These two quantities can be measured in an experiment in which delayed coincidences between the high-energy photons (from $\mu^- {}^{12}\text{C} \to \nu_\mu {}^{12}\text{B}\gamma$) and the ^{12}B-decay electrons (from the subsequent $^{12}\text{B} \to {}^{12}\text{C}\, e^- \bar\nu_e$) are detected (see below). To start the calculation, we have [compare Eqs. (43a) and (44e)]

$$\int \frac{d\Omega^{(\nu)}}{4\pi} \langle |\mathcal{T}|^2; \xi\rangle \equiv \int \frac{d\Omega^{(\nu)}}{4\pi} \sum_{s_z^{(\mu)}} \left(\sum_{\vec\epsilon^*, s_z^{(\nu)}} |\mathcal{T}^{(l)}(\vec\epsilon^*, \vec\xi^*, s_z^{(\nu)}; s_z^{(\mu)}) + \mathcal{T}^{(h)}(\vec\epsilon^*, \vec\xi^*, s_z^{(\nu)}; s_z^{(\mu)})|^2 \right) P(s_z^{(\mu)})$$

$$= \left(\frac{Ge}{\sqrt2} \frac{1}{(2k_0)^{1/2}} \frac{1}{m_\mu}\right)^2 2\,[(1+P_\mu \hat k\cdot\hat z)(F_1^{(+)} + F_2^{(+)} \hat k\cdot\vec\xi \hat k\cdot\vec\xi^* + F_3^{(+)} \hat k\cdot i\vec\xi \times \vec\xi^*) + (1-P_\mu \hat k\cdot\hat z)(F_1^{(-)} + F_2^{(-)} \hat k\cdot\vec\xi \hat k\cdot\vec\xi^* + F_3^{(-)} \hat k\cdot i\vec\xi\times\vec\xi^*) + G_1 P_\mu \hat z\cdot i\vec\xi\times\vec\xi^* + G_2 P_\mu(\hat k\cdot\vec\xi \hat z\cdot\vec\xi^* + \hat k\cdot\vec\xi^* \hat z\cdot\vec\xi)]\,, \quad \text{(C1)}$$

$$F_i^{(\pm)} = F_i^{(\pm);(ll)} + F_i^{(\pm);(lh)} + F_i^{(\pm);(hh)}, \quad i=1,2,3,$$

$$G_i = G_i^{(ll)} + G_i^{(lh)} + G_i^{(hh)}, \quad i=1,2\,, \quad \text{(C2)}$$

where the decomposition in Eq. (C2) is defined in complete analogy to that in Eq. (43b) and where the structure functions $F_i^{(\pm)}, G_i$ are homogeneous and quadratic in the $F_{M,A,P,E}(q^2)$ and also depend on k_0. With the approximation $F_i^{(\pm)}(q^2, k_0/k_m) \cong F_i^{(\pm)}(\frac35 k_m^2, k_0/k_m) \equiv F_i^{(\pm)}(x), G(q^2, k_0/k_m) \cong G_i(\frac35 k_m^2, k_0/k_m) \equiv G_i(x)$ [see text after Eqs. (43c)] and the notation $F_{M,A,P,E} \equiv F_{M,A,P,E}(\frac35 k_m^2)$, $x \equiv k_0/k_m$ $(0 \le x \le 1)$ we have

$$F_1^{(+);(ll)}(x) = F_A^2 + F_A\left(F_P \frac{m_\mu k_m}{m_\pi^2} - F_E \frac{k_m}{2m_p}\right)\tfrac23(1-x) + F_A F_M \frac{k_m}{2m_p}\tfrac23(2-5x) + \left(F_P \frac{m_\mu k_m}{m_\pi^2} - F_E \frac{k_m}{2m_p}\right)^2 \tfrac13(1-x)^2 - F_M \frac{k_m}{2m_p}\left(F_P \frac{m_\mu k_m}{m_\pi^2} - F_E \frac{k_m}{2m_p}\right)\tfrac23 x(1-x) + \left(F_M \frac{k_m}{2m_p}\right)^2 \tfrac13(2-8x+9x^2)\,, \quad \text{(C3)}$$

$$F_1^{(+);(lh)}(x)=F_AF_P\frac{m_\mu k_m}{m_\pi^2}\tfrac{2}{3}(1-x)+2F_AF_M\frac{k_m}{2m_p}\frac{m_\mu}{k_m}+F_P\frac{m_\mu k_m}{m_\pi^2}\left(F_P\frac{m_\mu k_m}{m_\pi^2}-F_E\frac{k_m}{2m_p}\right)\tfrac{2}{3}(1-x)^2$$
$$-F_P\frac{m_\mu k_m}{m_\pi^2}F_M\frac{k_m}{2m_p}\tfrac{2}{3}x(1-x)$$
$$+F_M\frac{k_m}{2m_p}\left(F_P\frac{m_\mu k_m}{m_\pi^2}-F_E\frac{k_m}{2m_p}\right)\frac{m_\mu}{k_m}\tfrac{2}{3}(1-x)+\left(F_M\frac{k_m}{2m_p}\right)^2\frac{m_\mu}{k_m}\tfrac{2}{3}(2-5x), \tag{C4}$$

$$F_1^{(+);(hh)}(x)=\left(F_P\frac{m_\mu k_m}{m_\pi^2}\right)^2\tfrac{1}{3}(1-x)^2+F_P\frac{m_\mu k_m}{m_\pi^2}F_M\frac{k_m}{2m_p}\frac{m_\mu}{k_m}\tfrac{2}{3}(1-x)+2\left(F_M\frac{k_m}{2m_p}\right)^2\left(\frac{m_\mu}{k_m}\right)^2, \tag{C5}$$

$$F_2^{(+);(ll)}(x)=F_A\left(F_P\frac{m_\mu k_m}{m_\pi^2}-F_E\frac{k_m}{2m_p}\right)2x+F_AF_M\frac{k_m}{2m_p}2x+\left(F_P\frac{m_\mu k_m}{m_\pi^2}-F_E\frac{k_m}{2m_p}\right)^2\tfrac{1}{3}x(2+x)$$
$$+F_M\frac{k_m}{2m_p}\left(F_P\frac{m_\mu k_m}{m_\pi^2}-F_E\frac{k_m}{2m_p}\right)2x(1-x)+\left(F_M\frac{k_m}{2m_p}\right)^2\tfrac{1}{3}x(4-7x), \tag{C6}$$

$$F_2^{(+);(lh)}(x)=F_AF_P\frac{m_\mu k_m}{m_\pi^2}2x+F_P\frac{m_\mu k_m}{m_\pi^2}\left(F_P\frac{m_\mu k_m}{m_\pi^2}-F_E\frac{k_m}{2m_p}\right)\tfrac{2}{3}x(2+x)+F_P\frac{m_\mu k_m}{m_\pi^2}F_M\frac{k_m}{2m_p}2x(1-x)$$
$$+F_M\frac{k_m}{2m_p}\left(F_P\frac{m_\mu k_m}{m_\pi^2}-F_E\frac{k_m}{2m_p}\right)\frac{m_\mu}{k_m}2x+\left(F_M\frac{k_m}{2m_p}\right)^2\frac{m_\mu}{k_m'}2x, \tag{C7}$$

$$F_2^{(+);(hh)}(x)=\left(F_P\frac{m_\mu k_m}{m_\pi^2}\right)^2\tfrac{1}{3}x(2+x)+F_P\frac{m_\mu k_m}{m_\pi^2}F_M\frac{k_m}{2m_p}\frac{m_\mu}{k_m}\tfrac{1}{3}(1+2x), \tag{C8}$$

$$F_3^{(+);(ll)}(x)=-F_A^2-F_A\left(F_P\frac{m_\mu k_m}{m_\pi^2}-F_E\frac{k_m}{2m_p}\right)\tfrac{2}{3}(1-x)-F_AF_M\frac{k_m}{2m_p}\tfrac{2}{3}(2-5x)$$
$$-\left(F_P\frac{m_\mu k_m}{m_\pi^2}-F_E\frac{k_m}{2m_p}\right)^2\tfrac{2}{3}(1-x)(1-2x)+\left(F_M\frac{k_m}{2m_p}\right)^2\tfrac{1}{3}(1+2x), \tag{C9}$$

$$F_3^{(+);(lh)}(x)=-F_AF_P\frac{m_\mu k_m}{k_\pi^2}\tfrac{2}{3}(1-x)-2F_AF_M\frac{k_m}{2m_p}\frac{m_\mu}{k_m}-F_P\frac{m_\mu k_m}{m_\pi^2}F_M\frac{k_m}{2m_p}\tfrac{2}{3}(1-x)(1-2x)$$
$$-F_M\frac{k_m}{2m_p}\left(F_P\frac{m_\mu k_m}{m_\pi^2}-F_E\frac{k_m}{2m_p}\right)\tfrac{2}{3}(1-x)-\left(F_M\frac{k_m}{2m_p}\right)^2\frac{m_\mu}{k_m}\tfrac{2}{3}(2-5x), \tag{C10}$$

$$F_3^{(+);(hh)}(x)=-F_P\frac{m_\mu k_m}{m_\pi^2}F_M\frac{k_m}{2m_p}\frac{m_\mu}{k_m}\tfrac{1}{6}(5-8x)-\frac{1}{2}\left(F_M\frac{k_m}{2m_p}\right)^2\left(\frac{m_\mu}{k_m}\right)^2, \tag{C11}$$

$$F_1^{(-);(\xi)}(x)=F_2^{(-);(\xi)}(x)=F_3^{(-);(\xi)}(x)=G_1^{(\xi)}(x)=G_2^{(\xi)}(x)=0;\ \xi=ll,lh, \tag{C12}$$

$$F_1^{(-);(hh)}(x)=\left(F_P\frac{m_\mu k_m}{m_\pi^2}\right)^2\tfrac{1}{3}(1-x)^2, \tag{C13}$$

$$F_2^{(-);(hh)}(x)=\left(F_P\frac{m_\mu k_m}{m_\pi^2}\right)^2(-\tfrac{1}{3})x(2-5x)+F_P\frac{m_\mu k_m}{m_\pi^2}F_M\frac{k_m}{2m_p}\frac{m_\mu}{k_m}\tfrac{1}{3}(1-4x), \tag{C14}$$

$$F_3^{(-);(hh)}(x)=F_P\frac{m_\mu k_m}{m_\pi^2}F_M\frac{k_m}{2m_p}\frac{m_\mu}{k_m}\tfrac{1}{6}(1-4x)+\frac{1}{2}\left(F_M\frac{k_m}{2m_p}\right)^2\left(\frac{m_\mu}{k_m}\right)^2, \tag{C15}$$

$$G_1^{(hh)}(x)=F_P\frac{m_\mu k_m}{m_\pi^2}F_M\frac{k_m}{2m_p}\frac{m_\mu}{k_m}\tfrac{1}{3}(1-4x)+\left(F_M\frac{k_m}{2m_p}\right)^2\left(\frac{m_\mu}{k_m}\right)^2, \tag{C16}$$

$$G_2^{(hh)}(x)=-G_1^{(hh)}(x). \tag{C17}$$

We choose the $\hat{z}'$ axis for the quantization of the spin of the recoil ^{12}B nucleus so that $\vec{\xi}(\pm1)=(\mp1/\sqrt{2})(\hat{x}'\pm i\hat{y}')$, $\vec{\xi}(0)=\hat{z}'$ (with $\hat{x}',\hat{y}',\hat{z}'$ orthogonal) correspond, respectively, to the $s_{z'}^{(^{12}\mathrm{B})}=\pm1,0$ substates. We then determine the three populations $h_{\pm1},h_0$ corresponding to the $s_{z'}^{(^{12}\mathrm{B})}=\pm1,0$ substates by substitution of $\vec{\xi}(\pm1)=(\mp1/\sqrt{2})(\hat{x}'\pm i\hat{y}')$, $\vec{\xi}(0)=\hat{z}'$ into Eq. (C1). In this way, we obtain

$$P_{\mathrm{av}}(x,\hat{k}\cdot\hat{z},\hat{k}\cdot\hat{z}',\hat{z}\cdot\hat{z}')\equiv\frac{h_{+1}-h_{-1}}{h_{+1}+h_{-1}+h_0}=2\frac{\hat{k}\cdot\hat{z}'\{[F_3^{(+)}(x)+F_3^{(-)}(x)]+P_\mu\hat{k}\cdot\hat{z}[F_3^{(+)}(x)-F_3^{(-)}(x)]\}+P_\mu\hat{z}\cdot\hat{z}'G_1(x)}{\{3[F_1^{(+)}(x)+F_1^{(-)}(x)]+[F_2^{(+)}(x)+F_2^{(-)}(x)]\}+P_\mu\hat{k}\cdot\hat{z}\{3[F_1^{(+)}(x)-F_1^{(-)}(x)]+[F_2^{(+)}(x)-F_2^{(-)}(x)]+2G_2(x)\}} \tag{C18}$$

$$A_{\mathrm{av}}(x,\hat{k}\cdot\hat{z},\hat{k}\cdot\hat{z}',\hat{z}\cdot\hat{z}')\equiv\frac{h_{+1}+h_{-1}-2h_0}{h_{+1}+h_{-1}+h_0}=-2\frac{[\frac{3}{2}(\hat{k}\cdot\hat{z}')^2-\frac{1}{2}]\{[F_2^{(+)}(x)+F_2^{(-)}(x)]+P_\mu\hat{k}\cdot\hat{z}[F_2^{(+)}(x)-F_2^{(-)}(x)]\}+[\frac{3}{2}(\hat{k}\cdot\hat{z}')(\hat{z}\cdot\hat{z}')-\frac{1}{2}\hat{k}\cdot\hat{z}]2P_\mu G_2(x)}{\{3[F_1^{(+)}(x)+F_1^{(-)}(x)]+[F_2^{(+)}(x)+F_2^{(-)}(x)]\}+P_\mu\hat{k}\cdot\hat{z}\{3[F_1^{(+)}(x)-F_1^{(-)}(x)]+[F_2^{(+)}(x)-F_2^{(-)}(x)]+2G_2(x)\}}. \tag{C19}$$

In particular, we note, from Eqs. (C3)–(C19), that $P_{\mathrm{av}}(x,\hat{k}\cdot\hat{z},\hat{k}\cdot\hat{z}',\hat{z}\cdot\hat{z}')=-\frac{2}{3}\hat{k}\cdot\hat{z}'$ and $A_{\mathrm{av}}(x,\hat{k}\cdot\hat{z},\hat{k}\cdot\hat{z}',\hat{z}\cdot\hat{z}')=0$ if all the $F_{M,A,P,E}(q^2)$ but $F_A(q^2)$ are set to be zero. We also note that a procedure entirely analogous to the above for the evaluation of the recoil ^{12}B polarization and alignment in the case of the nonradiative muon capture $\mu^-{}^{12}\mathrm{C}\to\nu_\mu{}^{12}\mathrm{B}$ yields[8]

$$P_{\mathrm{av;nonrad.}}(\hat{z}\cdot\hat{z}')=\tfrac{2}{3}\hat{z}\cdot\hat{z}'\,P_\mu\left(1-\frac{\{F_P(q_{nr}^2)[m_\mu(m_\mu-\Delta)/m_\pi^2]-[F_M(q_{nr}^2)+F_E(q_{nr}^2)][(m_\mu-\Delta)/2m_p]\}^2}{2\{F_A(q_{nr}^2)+F_M(q_{nr}^2)[(m_\mu-\Delta)/2m_p]\}^2+\{F_A(q_{nr}^2)+F_P(q_{nr}^2)[m_\mu(m_\mu-\Delta)/m_\pi^2]-F_E(q_{nr}^2)[(m_\mu-\Delta)/2m_p]\}^2}\right), \tag{C20}$$

$$A_{\mathrm{av;nonrad.}}(\hat{z}\cdot\hat{z}')=0 \tag{C21}$$

which is to be compared with Eqs. (C18) and (C19).

As regards the feasibility of measuring $P_{\mathrm{av}}(x,\hat{k}\cdot\hat{z},\hat{k}\cdot\hat{z}',\hat{z}\cdot\hat{z}')$ and $A_{\mathrm{av}}(x,\hat{k}\cdot\hat{z},\hat{k}\cdot\hat{z}',\hat{z}\cdot\hat{z}')$, we consider the delayed coincidence between the photon emitted in the radiative muon capture $\mu^-{}^{12}\mathrm{C}\to\nu_\mu{}^{12}\mathrm{B}\gamma$ and the electron emitted in the subsequent decay of recoil $^{12}\mathrm{B}\to{}^{12}\mathrm{C}e^-\bar{\nu}_e$. Choosing $\hat{z}'=\hat{k}$ so that $\hat{z}'\cdot\hat{z}\equiv\hat{k}\cdot\hat{z}\equiv\cos\theta_{\mu\gamma}$, $\hat{z}'\cdot\hat{k}=\hat{k}\cdot\hat{k}=1$, and $\hat{z}'\cdot\hat{p}_e=\hat{k}\cdot\hat{p}_e\equiv\cos\theta_{e\gamma}$, we note that the number of delayed-coincidence γ-e^- events at an angle $\theta_{e\gamma}$ will be proportional to[2]

$$1-P_{\mathrm{av}}(x;\cos\theta_{\mu\gamma})(1+\alpha_-E_e)\cos\theta_{e\gamma}+A_{\mathrm{av}}(x;\cos\theta_{\mu\gamma})\alpha_-E_e(\tfrac{3}{2}\cos^2\theta_{e\gamma}-\tfrac{1}{2}), \tag{C22}$$

where

$$P_{\mathrm{av}}(x;\cos\theta_{\mu\gamma})\equiv P_{\mathrm{av}}(x,\hat{k}\cdot\hat{z},\hat{k}\cdot\hat{z}',\hat{z}\cdot\hat{z}')|_{\hat{z}'=\hat{k}}=2\frac{[F_3^{(+)}(x)+F_3^{(-)}(x)]+P_\mu\cos\theta_{\mu\gamma}[F_3^{(+)}(x)-F_3^{(-)}(x)+G_1(x)]}{\{3[F_1^{(+)}(x)+F_1^{(-)}(x)]+[F_2^{(+)}(x)+F_2^{(-)}(x)]\}+P_\mu\cos\theta_{\mu\gamma}\{3[F_1^{(+)}(x)-F_1^{(-)}(x)]+[F_2^{(+)}(x)-F_2^{(-)}(x)]+2G_2(x)\}}, \tag{C23}$$

$$A_{\mathrm{av}}(x;\cos\theta_{\mu\gamma})\equiv A_{\mathrm{av}}(x,\hat{k}\cdot\hat{z},\hat{k}\cdot\hat{z}',\hat{z}\cdot\hat{z}')|_{\hat{z}'=\hat{k}}=-2\frac{[F_2^{(+)}(x)+F_2^{(-)}(x)]+P_\mu\cos\theta_{\mu\gamma}[F_2^{(+)}(x)-F_2^{(-)}(x)+2G_2(x)]}{\{3[F_1^{(+)}(x)+F_1^{(-)}(x)]+[F_2^{(+)}(x)+F_2^{(-)}(x)]\}+P_\mu\cos\theta_{\mu\gamma}\{3[F_1^{(+)}(x)-F_1^{(-)}(x)]+[F_2^{(+)}(x)-F_2^{(-)}(x)]+2G_2(x)\}}, \tag{C24}$$

and[2]

$$\alpha_- = \frac{1}{3m_p}\left(\frac{F_M(0)}{F_A(0)} - \frac{F_E(0)}{F_A(0)}\right). \tag{C25}$$

If we now remember that α_- is expected to be quite small [Eq. (47) of Ref. 2], we see from Eq. (C22) that a measurement of the γ-e^- angular correlation will determine $P_{av}(x;\cos\theta_{\mu\gamma})$.

As a numerical example, we obtain

$$\begin{aligned} P_{av}(x=\tfrac{2}{3};\cos\theta_{\mu\gamma}) &= 2\frac{(-0.179)+(-0.179)P_\mu\cos\theta_{\mu\gamma}}{0.538+0.609P_\mu\cos\theta_{\mu\gamma}}, \\ A_{av}(x=\tfrac{2}{3};\cos\theta_{\mu\gamma}) &= -2\frac{(-0.115)+(-0.176)P_\mu\cos\theta_{\mu\gamma}}{0.538+0.609P_\mu\cos\theta_{\mu\gamma}} \end{aligned} \tag{C26}$$

for

$$\left(\frac{F_P(q^2)}{-F_A(q^2)/(1+q^2/m_\pi^2)}\right)_{q^2=(3/5)k_m^2} \cong \left[1+\frac{m_\pi^2}{q^2}\left(1-\frac{F_D(q^2)}{F_A(q^2)}\right)\right]_{q^2=(3/5)k_m^2} \equiv \xi = 0.85$$

and $x=\frac{2}{3}$. As can be seen from Eqs. (C3)–(C17) and (C23), (C24), the values of $P_{av}(x;\cos\theta_{\mu\gamma})$ and $A_{av}(x;\cos\theta_{\mu\gamma})$ are quite sensitive to the value assumed for

$$\left(\frac{F_P(q^2)}{-F_A(q^2)/(1+q^2/m_\pi^2)}\right)_{q^2=(3/5)k_m^2} \cong \left[1+\frac{m_\pi^2}{q^2}\left(1-\frac{F_D(q^2)}{F_A(q^2)}\right)\right]_{q^2=(3/5)k_m^2} \equiv \xi$$

but, unfortunately, and as noted in the text, the measurement of $P_{av}(x;\cos\theta_{\mu\gamma})$ and $A_{av}(x;\cos\theta_{\mu\gamma})$ by the γ-e^- delayed coincidence method appears to be very difficult.

We next proceed to evaluate the circular polarization of the photon emitted in $\mu^{-\,12}\mathrm{C}\to\nu_\mu{}^{12}\mathrm{B}\gamma$. Here we have

$$\begin{aligned} P_{\text{circ.}} &= \frac{\int\frac{d\Omega^{(\gamma)}}{4\pi}\int\frac{d\Omega^{(\nu)}}{4\pi}(\langle|\mathcal{T}|^2\rangle_+ - \langle|\mathcal{T}|^2\rangle_-)}{\int\frac{d\Omega^{(\gamma)}}{4\pi}\int\frac{d\Omega^{(\nu)}}{4\pi}(\langle|\mathcal{T}|^2\rangle_+ + \langle|\mathcal{T}|^2\rangle_-)} \\ &= 1-2\frac{\int\frac{d\Omega^{(\gamma)}}{4\pi}\int\frac{d\Omega^{(\nu)}}{4\pi}\langle|\mathcal{T}|^2\rangle_-}{\int\frac{d\Omega^{(\gamma)}}{4\pi}\int\frac{d\Omega^{(\nu)}}{4\pi}(\langle|\mathcal{T}|^2\rangle_+ + \langle|\mathcal{T}|^2\rangle_-)} \\ &= 1-2\frac{\frac{d\Omega^{(\gamma)}}{4\pi}\;\frac{d\Omega^{(\nu)}}{4\pi}\langle|\mathcal{T}|^2\rangle_-}{\int\frac{d\Omega^{(\gamma)}}{4\pi}\int\frac{d\Omega^{(\nu)}}{4\pi}\langle|\mathcal{T}|^2\rangle}, \end{aligned} \tag{C27}$$

where

$$\langle|\mathcal{T}|^2\rangle_\pm = \sum_{s_z^{(\mu)}}\left(\sum_{\vec{\xi}^*,\,s_z^{(\nu)}} |\mathcal{T}^{(l)}(\vec{\epsilon}_\pm^*,\vec{\xi}^*,s_z^{(\nu)};s_z^{(\mu)}) + \mathcal{T}^{(h)}(\vec{\epsilon}_\pm^*,\vec{\xi}^*,s_z^{(\nu)};s_z^{(\mu)})|^2\right) \times P(s_z^{(\mu)}) \tag{C28}$$

with $\vec{\epsilon}_+^*$ and $\vec{\epsilon}_-^*$ right and left circular polarization vectors, respectively ($\vec{\epsilon}_\pm = (1/\sqrt{2})(\hat{\epsilon}_1 \pm i\hat{\epsilon}_2)$, $\hat{k}=\hat{\epsilon}_3$ with $\hat{\epsilon}_1,\hat{\epsilon}_2,\hat{\epsilon}_3$ mutually orthogonal). Equation (C28) for $\langle|\mathcal{T}|^2\rangle_\pm$ is to be compared to Eq. (43a) for $\langle|\mathcal{T}|^2\rangle = \langle|\mathcal{T}|^2\rangle_+ + \langle|\mathcal{T}|^2\rangle_-$. Evaluating the sums on the right side of Eq. (C28) we have

$$\begin{aligned} \langle|\mathcal{T}|^2\rangle_\pm &= \left(\frac{Ge}{\sqrt{2}}\frac{1}{(2k_0)^{1/2}}\frac{1}{m_\mu}\right)^2 \\ &\quad\times 2\{[U_{1;\pm}^{(0)} + U_{1;\pm}^{(1)}\hat{\nu}\cdot\hat{k} + U_{1;\pm}^{(2)}(\hat{\nu}\cdot\hat{k})^2] \\ &\quad + P_\mu\hat{k}\cdot\hat{z}[U_{2;\pm}^{(0)} + U_{2;\pm}^{(1)}\hat{\nu}\cdot\hat{k} + U_{2;\pm}^{(2)}(\hat{\nu}\cdot\hat{k})^2] \\ &\quad + P_\mu\hat{\nu}\cdot\hat{z}[U_{3;\pm}^{(0)} + U_{3;\pm}^{(1)}\hat{\nu}\cdot\hat{k}]\}, \\ U_{i;\pm}^{(n)} &= U_{i;\pm}^{(n);(ll)} + U_{i;\pm}^{(n);(lh)} + U_{i;\pm}^{(n);(hh)}, \end{aligned} \tag{C29}$$

$$W_i^{(n)} = U_{i;+}^{(n)} + U_{i;-}^{(n)},$$

where the $U_{i;\pm}^{(n)}$ are structure functions which are homogeneous and quadratic in the $F_{M,A,P,E}(q^2)$ and which also depend on k_0. We note that $\mathcal{T}^{(l)}(\vec{\epsilon}_-^*,\vec{\xi}^*,s_z^{(\nu)};s_z^{(\mu)})=0$ [this follows from Eq. (42a) since $(1-\vec{\sigma}\cdot\hat{k})\vec{\sigma}\cdot\vec{\epsilon}_-^*=0$]; thus only $\mathcal{T}^{(h)}(\vec{\epsilon}_-^*,\vec{\xi}^*,s_z^{(\nu)};s_z^{(\mu)})$ contributes to $\langle|\mathcal{T}|^2\rangle_-$ (i.e., $U_{i;-}^{(n)} = U_{i;-}^{(n);(hh)}$) so that $\langle|\mathcal{T}|^2\rangle_-$ depends only on $F_M(q^2)$ and $F_P(q^2)$ [see Eq. (42b)] and $P_{\text{circ.}}=1$ only if the terms involving $F_M(q^2)$ and $F_P(q^2)$ are neglected.

We now approximate [see discussion after Eq. (43c)] $U_{i;-}^{(n)}(q^2;k_0/k_m)$ by $U_{i;-}^{(n)}(\frac{3}{5}k_m^2;k_0/k_m)$, calculate the $U_{i;-}^{(n)}(\frac{3}{5}k_m^2;k_0/k_m)$ from the $F_{M,A,P,E}(\frac{3}{5}k_m^2)$ listed in the text, insert the values obtained in Eq. (C29) to get $\langle|\mathcal{T}|^2\rangle_-$, and integrate over all $\hat{\nu}$ and $\hat{k}$. We then get

$$\int \frac{d\Omega^{(\gamma)}}{4\pi} \int \frac{d\Omega^{(\nu)}}{4\pi} \langle |\mathcal{T}|^2\rangle_- = \left(\frac{Ge}{\sqrt{2}} \frac{1}{(2k_0)^{1/2}} \frac{1}{m_\mu}\right)^2 2 \left[\left(F_P(\tfrac{3}{5}k_m^2)\frac{m_\mu k_m}{m_\pi^2}\right)^2 \tfrac{2}{3}(3-8x+8x^2) + \left(F_M(\tfrac{3}{5}k_m^2)\frac{k_m}{2m_p}\right)\left(F_P(\tfrac{3}{5}k_m^2)\frac{m_\mu k_m}{m_\pi^2}\right)\frac{m_\mu}{k_m}\tfrac{4}{3}(1-4x) + 3\left(F_M(\tfrac{3}{5}k_m^2)\frac{k_m}{2m_p}\right)^2\left(\frac{m_\mu}{k_m}\right)^2\right] \tag{C30}$$

which, together with Eqs. (44f) and (48a)–(48c) for

$$\int \frac{d\Omega^{(\gamma)}}{4\pi} \int \frac{d\Omega^{(\nu)}}{4\pi} \langle |\mathcal{T}|^2\rangle,$$

yields upon substitution in Eq. (C27)

$$P_{\text{circ.}} = 1 - 2\frac{0.052 - 0.043x + 0.117x^2}{0.808 - 0.598x + 0.288x^2} = 0.72 \text{ for } x = \tfrac{2}{3}. \tag{C31}$$

As can be seen from Eqs. (C30) and (C27), the value of $P_{\text{circ.}}$ is also sensitive to the value assumed for

$$\left[\frac{F_P(q^2)}{-F_A(q^2)/(1+q^2/m_\pi^2)}\right]_{q^2=(3/5)k_m^2} \cong \left[1+\frac{m_\pi^2}{q^2}\left(1-\frac{F_D(q^2)}{F_A(q^2)}\right)\right]_{q^2=(3/5)k_m^2} \equiv \xi .$$

But, again, the experimental determination of $P_{\text{circ.}}$ appears to be extremely difficult.

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[1]W-Y. P. Hwang and H. Primakoff, preceding paper, Phys. Rev. C 18, 414 (1978). (a) J. Bernstein, Phys. Rev. 115, 694 (1959); (b) A. Wullschleger, Partial Radiative Muon Capture on ^{12}C, University of Mainz report (unpublished).

[2]W-Y. P. Hwang and H. Primakoff, Phys. Rev. C 16, 397 (1977).

[3]R. E. Cutkosky, Phys. Rev. 107, 330 (1957).

[4]K. Huang, C. N. Yang, and T. D. Lee, Phys. Rev. 108, 1340 (1957).

[5]H. W. Fearing, Phys. Rev. Lett. 35, 79 (1975).

[6]A. Possoz, D. Favart, L. Grenacs, J. Lehmann, P. Macq, D. Meda, and L. Palffy, Phys. Lett. 50B, 438 (1974).

[7]C. W. Kim and J. S. Townsend, Phys. Rev. D 11, 656 (1975); C. W. Kim (unpublished).

[8]B. R. Holstein, Phys. Rev. D 13, 2499 (1976), gives a formula for $P_{\text{av; nonrad}}(\hat{z}\cdot\hat{z}')|_{\hat{z}=\hat{z}}$, which agrees with Eq. (C20).