Super-allowed Fermi β decay: Half-lives of ^{14}O and $^{38}K^m$ †

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The half-lives of the Fermi super-allowed β-ray transitions from ^{14}O and $^{38}K^m$ have been measured by multiscaling γ rays (from ^{14}O) or β rays (from $^{38}K^m$) detected in a plastic scintillator. The activities were made via the reactions $^{12}C(^3He,n)^{14}O$ at $E_{^3He}$ = 2.9 MeV and $^{35}Cl(\alpha,n)\,^{38}K^m$ at E_α = 8.0 MeV respectively. Results for the half-lives were as follows: ^{14}O-70.613 $\pm$ 0.025 sec, and $^{38}K^m$-921.71 $\pm$ 0.65 msec. Recommendations as to the "best" half-life values for the set of eight most accurately measured Fermi super-allowed transitions are presented and comments are made on the present status of knowledge of the nucleon's mean quark charge as extracted from such transitions following acceptance of Cabibbo universality and the gauge theories as exploited by Sirlin. The value recommended for the vector coupling coefficient effective for nucleon β decay (including the "inner" radiative correction) is $g^R_{\beta V} = (1.412\,48 \pm 0.000\,44) \times 10^{-49}$ erg cm^3.

[NUCLEAR REACTIONS Radioactivity ^{14}O, $^{38}K^m$; measured $T_{1/2}$; deduced ft values; compared with systematics.]

I. INTRODUCTION

The pure Fermi super-allowed β transitions of $J^\pi = 0^+ \rightarrow 0^+$ are well known to present the best approach to the determination of the vector coupling constant effective for nuclear β decay. At present the only candidates that seem to offer a precision of 0.1% or better in the squared effective coupling constant are ^{14}O, $^{26}Al^m$, ^{34}Cl, $^{38}K^m$, ^{42}Sc, ^{46}V, ^{50}Mn, and ^{54}Co. Determination of the coupling constant requires accurate knowledge both of the energy release in the β decay and of the half-life. Significant advances, to which we refer later, have recently been made in the determination of the energy release. We have addressed ourselves to the lifetimes and have recently published values for $^{26}Al^m$, ^{34}Cl, $^{38}K^m$, ^{42}Sc, ^{46}V, ^{50}Mn, and ^{54}Co;[1,2] this paper "completes the set" with a measurement of ^{14}O and a remeasurement of $^{38}K^m$.

As before[2] we take the opportunity of presenting what we regard as the "best values" for the eight half-lives in question and of summarizing our present knowledge as to radiative corrections and through commitment to Cabibbo theory and gauge theories of the mean charge of the nucleon's quarks.

II. EXPERIMENTAL PROCEDURES

^{14}O

Previous recent half-life measurements[3-7] on ^{14}O have in all cases been made by detecting the 2.31-MeV γ rays emitted from this activity. The presence of other positron-emitting activities has precluded the use of the ^{14}O positrons for half-life purposes. For example, if the $^{12}C(^3He,n)^{14}O$ reaction is used, as in the present work, the competing reaction $^{12}C(^3He,\alpha)^{11}C$ makes 20-min ^{11}C which has a positron end-point energy of 0.96 MeV. The summing in the detector of these positron pulses with 0.51-MeV annihilation radiation results in a spectrum extending up to 1.47 MeV which cannot be differentiated well enough from the ^{14}O positrons (endpoint 1.81 MeV) to avoid a high long-lived background.

Previous ^{14}O half-life measurements have furthermore employed almost exclusively either NaI(Tl) or Ge(Li) detectors with a pulse-height window set on the 2.31-MeV γ-ray peak. With this arrangement the background counting rate can be made very low. However, the NaI(Tl) system is quite sensitive to the counting-rate gain shifts which tend to afflict this type of detector and which can be of importance for precise half-life measurements even though they may not be great enough to detect as significant peak shifts. Ge(Li) detector systems have the disadvantage of large dead times for which the corrections must be made with great care. On the other hand the plastic scintillators normally used for detecting β rays have extremely short decay constants thereby very considerably reducing the problems with counting-rate gain shifts as well as dead-time effects in the counting electronics It was therefore decided to use such a device to detect the γ rays from ^{14}O. Of course, the pulse-height spectrum due to the 2.31-MeV γ rays in such a scintillator is essentially a broad Compton distribution, but one really needs to discriminate only against 0.51-MeV γ rays by a pulse-height bias in order to eliminate the yields due to known contaminant activities.

The experimental arrangement consisted of a 10-cm diam by 7.5-cm thick Pilot-B scintillation crystal mounted on an RCA 6342A photomultiplier tube and installed inside a massive Pb shield in the accelerator control room. A Pb absorber 1.6 cm thick was placed in front of the crystal to discriminate against 511-keV γ rays as well as to eliminate positrons. Pulses from the photo tube were fed to a fast amplifier and fast discriminator both operating with time constants of 50 nsec, and thence to a gate-and-delay generator set at the minimum pulse width of 0.4 μsec. These pulses were then multi-scaled at 0.7 sec per channel for 2048 channels (20 half-lives) in a Northern Scientific Co. analyzer.

Sources of ^{14}O were made by bombarding samples of reactor-grade graphite with 2.9-MeV ^{3}He ions from the 3.5-MeV Van de Graaff. To ensure the lowest possible background a fresh target was used for each pass, with two to four passes comprising a given run. Samples were removed from the target chamber and carried to the shielded detector for counting.

In order to check the consistency of the method runs were made for various biases over the Compton distribution ranging upwards from 1.4 MeV to the highest bias that gave an adequate counting rate (over 2 MeV). Fifteen runs were made with initial counting rates ranging from 1500 to 3200 per sec. Analysis of the data consisted of first compressing the decay spectra in the computer by a factor of 8 to 5.6 sec per channel. In these 256-channel compressed decay curves, having 12.6 channels per half-life, there were 30 000 to 40 000 counts in the first channel and the long-lived background per channel in the last 50 channels was 0.05–0.08 % of the count in the first channel. Extraction of the half-life was made by a nonlinear least-squares minimization program to fit the data to an exponential plus constant with fits starting in time channels 1, 6, 11, 16, and 30.

$^{38}K^m$

The remeasurement of the $^{38}K^m$ half-life was made, with some improvements in technique, in order to check and improve upon our earlier results.[1] As previously[1] the detector was a 5-cm diam by 2.5-cm thick NE102 plastic scintillator placed ~4 cm from the source so as to reduce the β-γ summing yield of the long-lived (7.6 min) ^{38}K decay. Sources were made via the $^{35}Cl(\alpha,n)^{38}K^m$ reaction at $E_\alpha = 8.0$ MeV using a target of KCl deposited on a 0.0025-cm thick Au backing foil and α-particle beams of 200–350 nA for 1.5 sec from a tandem Van de Graaff. Counts were multiscaled at 0.07 sec per channel for 256 channels (19 half-lives). The main improvement in the present work over our earlier measurement[1] was the use of the fast electronics system described above in the section of ^{14}O.

Ten runs were made on the $^{38}K^m$ half-life at β-ray biases ranging from 2.1 to 3.1 MeV. Counts in the first channel varied from 18 000 to 38 000 and the long-lived background per channel in the last 50 channels was 0.1–0.3 % of the counts in the first channel.

Extraction of the half-life was made both for an exponential plus a constant background and for an exponential plus a background of half-life 7.6 min appropriate to ^{38}K. In both cases analyses were made starting in time channels 1, 14, 27, and 40.

III. ANALYSIS AND RESULTS

Owing to our present use of substantially faster electronic systems than in our earlier work,[1,2] the pileup corrections to the apparent lifetimes[8] are very small for the results reported here; they were made using the polynomial treatment of the pulse spectra as described earlier.[2]

We now make some observations on the individual bodies.

^{14}O. No dependence of apparent half-life on the starting channel of the analysis was discernible. The "pileup" correction over the whole bias range was substantially smaller than the error. We find a half-life of 70.613 ± 0.025 sec with $\chi^2/\nu = 1.00$.

$^{38}K^m$. There was again no dependence of apparent half-life on starting channel. The difference in $^{38}K^m$ lifetime extracted using a time-independent background and one of 7.6 min appropriate to ^{38}K was only about 30% of our final overall error: The result quoted is for the mean of the two treatments of the background (between which χ^2 tests did not help us to discriminate). The pileup correction was very small but was discernible at the two

highest biases. The difference between using all bias values in a pileup analysis and using straight averaging of all values except those for the two highest biases was negligible in relation to the errors. We find from the straight averaging, a half-life of 921.71 ± 0.65 msec with $\chi^2/\nu = 0.91$.

Our present result of 70.613 ± 0.025 sec for ^{14}O compares well with the earlier most precise result[3] of 70.588 ± 0.028 sec. The latter measurements were carried out almost exclusively using NaI(Tl) or Ge(Li) analysis of the ^{14}N γ ray. In view of the different detection systems for the two sets of experiments [and in view of the fact that the earlier experiments[3] used the $^{14}N(p,n)^{14}O$ reaction as the source for over one third of the runs] the close agreement is convincing. Several other measurements of the ^{14}O half-life have been made, but the only other one of accuracy better than 0.1 sec is that from the Lockheed group,[7] namely a half-life of 70.684 ± 0.077 sec which combines acceptably ($P(\chi^2;\nu) = 0.47$) with the other two to give the value that we now accept: 70.606 ± 0.018 sec. This adopted value is not in disagreement with two of the other three recent measurements of 70.48 ± 0.15 sec[4] and 70.43 ± 0.18 sec (Ref. 6) but disagrees with the third, 70.32 ± 0.12 sec.[5] Combination of the first two of these values with our adopted value would give a half-life of 70.603 ± 0.018 sec ($\chi^2/\nu = 0.79$) and inclusion of the third would give 70.597 ± 0.018 sec ($\chi^2/\nu = 1.7$).

Our present result of 921.71 ± 0.65 msec for $^{38}K^m$ compares well with our earlier result[1] of 922.3 ± 1.1 msec and we combine the two to adopt 921.86 ± 0.56 msec.

At this point we summarize for convenience, in Table I, our recommendations as to the half-lives of the eight bodies named in the Introduction. With the exception of ^{14}O and $^{38}K^m$, where we quote the values just presented, the half-lives of Table I are those previously adopted.[2] (Note that the error for ^{46}V was slightly misquoted in Table I of Ref. 2.)

TABLE I. Recommended half-lives, in msec, of the eight accurately measured super-allowed Fermi β emitters. With the exception of ^{14}O and $^{38}K^m$, for which we quote the values adopted in the text, these half-lives are those recommended in our previous survey (Ref. 2).

Body	Recommended
^{14}O	70 606 ±18
$^{26}Al^m$	6 343.9 ±2.8
^{34}Cl	1 525.4 ±1.0
$^{38}K^m$	921.86 ±0.56
^{42}Sc	680.98 ±0.62
^{46}V	422.33 ±0.20
^{50}Mn	282.75 ±0.20
^{54}Co	193.23 ±0.14

IV. VECTOR COUPLING CONSTANT

It is appropriate to review the present status of our knowledge of the vector coupling constant $g_{\beta V}^R$ appropriate to nuclear β decay. The superscript R here reminds us that the coupling constant operative in practice is that which would obtain in the absence of radiative corrections $g_{\beta V}$ corrected for those "inner" radiative corrections, $\Delta_{\beta V}^{\alpha}$, that depend on the anatomy of the β-decay process and of the nucleon but that do not depend on the energy release in the β-decay:

$$g_{\beta V}^R = g_{\beta V}(1 + \tfrac{1}{2}\Delta_{\beta V}^{\alpha}) .$$

In writing down the half-life, t, for the $J^\pi = 0^+ \to 0^+$ $T=1$ super-allowed Fermi transitions in question

$$f^R t = \frac{\pi^3 \hbar^7 \ln 2}{m_e{}^5 c^4 g_{\beta V}^{R\,2}(1-\epsilon)} = \frac{6.1531 \times 10^{-95}}{g_{\beta V}^{R\,2}(1-\epsilon)} \text{ erg}^2\text{ cm}^6\text{ sec} ,$$

we encounter radiative corrections in two further places: (i) in the superscript R on f^R which reminds us that there are "outer" radiative corrections of order $\alpha, Z\alpha^2, Z^2\alpha^3, \ldots$ that do not depend significantly on the anatomies that enter $\Delta_{\beta V}^{\alpha}$ but that may depend on the energy release; (ii) the quantity $(1-\epsilon)$ that represents the change in the squared nuclear matrix element for the transition, the *de facto* mismatch due to Coulomb and other charge-dependent effects between the initial and final nuclear states. The "outer" radiative corrections have been discussed at length[9-11] and appear to be under good control. The mismatch factor ϵ then governs our ability to extract $g_{\beta V}^R$ from the $f^R t$ values. Before discussing this we present a summary of the experimental data in Table II.

The lifetimes used for Table II are the "adopted" values from Table I corrected for electron capture and, in the case of ^{14}O only, for branching. The energy releases E_0 are those adopted in Ref. 12 with the exception of ^{14}O, ^{34}Cl, and $^{38}K^m$ (note that the important Ref. 7 of Ref. 12 has now been published[13]). For ^{14}O we have combined the value of Ref. 12 with the consistent but more accurate value $E_0 = 1808.7 \pm 0.4$ keV;[14] for ^{34}Cl we have replaced the preliminary Auckland value (Ref. 12 of Ref. 12) by its published version[15] and combined it with the other values as used in Ref. 12; for $^{38}K^m$ we have combined the values of Ref. 12 with the new Liverpool value[16] $E_0 = 5020.71 \pm 0.85$ keV.

The f^R values have been derived using standard procedures starting from the parametrization for axial transitions,[17] applying the correction appropriate to the vector case[17] and also the "outer" radiative corrections of order $Z\alpha^2$ and $Z^2\alpha^3$ (that of

TABLE II. Decay properties of the eight accurately measured super-allowed Fermi β decay bodies. The E_0 values are discussed in the text. The t values are the half-lives of Table I corrected for electron capture and, in the case of ^{14}O, for branching. f^R is the phase space factor derived and adjusted for the "outer" radiative correction as described in the text. The figures in parentheses in the t and f^R columns are the respective errors in percent.

Body	E_0 (keV)	t (msec)	f^R	f^Rt (sec)
^{14}O	1808.62 ± 0.35	71146 (0.028)	43.382 (0.083)	3086.5 ± 2.7
$^{26}Al^m$	3210.79 ± 0.37	6349.2 (0.044)	486.10 (0.052)	3086.3 ± 2.1
^{34}Cl	4469.93 ± 0.32	1526.6 (0.066)	2030.70 (0.030)	3100.1 ± 2.2
$^{38}K^m$	5020.86 ± 0.82	922.63 (0.061)	3351.3 (0.074)	3092.0 ± 3.0
^{42}Sc	5401.64 ± 0.39	681.64 (0.091)	4545.6 (0.034)	3098.4 ± 3.0
^{46}V	6028.60 ± 0.56	422.74 (0.049)	7340.3 (0.044)	3103.0 ± 2.0
^{50}Mn	6609.90 ± 0.38	283.04 (0.071)	10943.3 (0.027)	3097.4 ± 2.3
^{54}Co	7219.53 ± 0.56	193.46 (0.073)	16068.0 (0.039)	3108.5 ± 2.6

order α being included in the parametrization). An additional refinement, that we apply here for the first time, is for the effect of the distortion of the outgoing positron wave due to the fact that the nuclear charge distribution is not a uniform sphere but has a smooth edge. This correction is only 0.02% for ^{14}O but rises to 0.1% for the heaviest bodies of Table II and, since the overall errors in f^Rt are somewhat less than 0.1%, this correction is worth applying.[11] (For the bodies of Table II the effect of this shape correction is, roughly, to multiply the phase space factor computed for the uniformly charged sphere by $1+a+bW_0$ where W_0 is the total energy of the positron end point in natural units and where $a=1.8\times10^{-5}\times Z^{1.36}$, $b=-1.2\times10^{-6}Z$; Z here refers to the daughter and is taken as positive.) The f^Rt values of Table II are shown in Fig. 1 where the clear trend with Z, only noticeable after the accurate E_0 values from München[13] became available,[12] is well seen.

We should pause at this point to draw attention to a worry about E_0 for ^{14}O. The new E_0 value $E_0 = 1808.62\pm0.35$ keV, which comes from concordant measurements at München[13] and Auckland[14] disagrees with the older literature value[18] 1809.90 ± 0.35 keV and has been preferred over that older value, which itself derives chiefly from several, mutually consistent, $^{12}C(^3He, n)^{14}O$ threshold measurements, simply because it is newer. However, there has only recently appeared[19] a preliminary value, based on the reaction $^{16}O(p,t)^{14}O$, of $E_0 = 1810.4\pm0.6$ keV, agreeing with the older value and disagreeing with the newer. The seriousness of this disagreement is expressed by noting that $E_0 = 1810$ keV leads to $f^Rt = 3096.6$ sec which, as may be seen from Fig. 1, would spoil the systematics of f^Rt with Z. (We may also note that the new value[16] of E_0 for $^{38}K^m$, which dominates the overall accuracy of that energy release, is based on the lower value of E_0 for ^{14}O.) We shall base our discussion on the E_0 value of Table II but must bear in mind the underlying anxiety which it would be most valuable to resolve.

We now turn to the nuclear mismatch factor ϵ. Two types of approach to ϵ have been extensively discussed: (i) The fitting of the f^Rt values to the form $f^Rt=(f^Rt)_{Z=0}+aZ^x$ where the second term on the right-hand side represents the nuclear mismatch and where, if we could apply directly to our sequence of nuclei the Behrends-Sirlin-Ademollo-

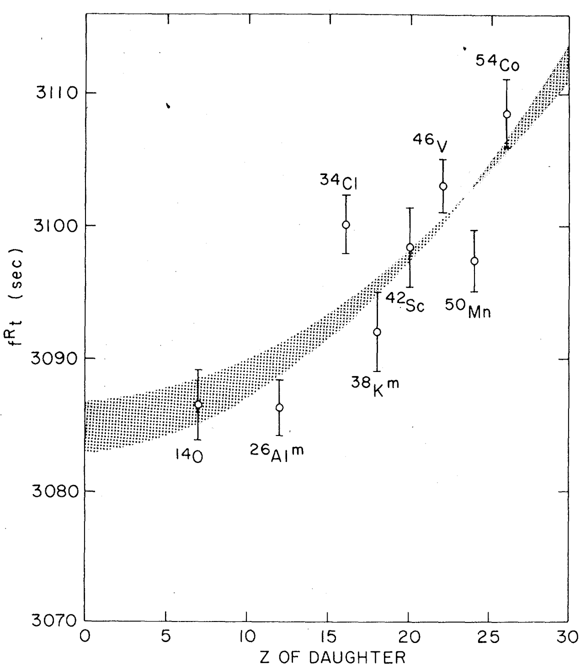

FIG. 1. f^Rt values taken from Table II as a function of the Z value of the daughter nucleus. The stippled area shows the plus and minus one standard deviation fits of the data to the form $f^Rt=(f^Rt)_{Z=0}+aZ^{1.86}$ as described in the text.

Gatto (BSAG) theorem[20] that tells us that the renormalization of coupling constants goes as the squares of the mass splittings within the multiplets, we expect[1] $x \simeq 1.86$; (ii) direct computation of ϵ from nuclear models so that $\mathcal{F}t = f^R t(1-\epsilon)$ should be the same for all transitions. ϵ according to this second approach has two components: $\epsilon = \epsilon_1 + \epsilon_2$ where ϵ_1 is due to the isospin symmetry-breaking effects of the effective overall one-body charge-dependent forces (Coulomb plus nuclear) and ϵ_2 is due to the configuration-dependent two-body charge dependences. ϵ_2 has been estimated systematically only once[21]; we use those values here. ϵ_1 may be estimated through the "giant monopole" approach (call this ϵ_1^0) or through the "parentage expansion" approach (call this ϵ_1^π). We will not describe these approaches in detail here since this has recently been done elsewhere,[11,22] but content ourselves with noting that there are available three estimates of ϵ_1^0, namely, ϵ_1^0 (LM),[23] ϵ_1^0(THH),[24] and ϵ_1^0 (F) (Ref. 25) and two estimates of ϵ_1^π, namely, ϵ_1^π (W)[22] and ϵ_1^π (THH).[24]

We can now try out these various approaches to ϵ but before doing so we notice two things: (i) that the straight mean of the $f^R t$ values of Table II has $\chi^2/\nu = 10.7$; (ii) that because of the undoubted effects associated with shells and with the detailed structure of individual nuclei we should not expect the BSAG approach to give $\chi^2/\nu \simeq 1$ as measurements get better and better but rather that χ^2/ν should rise indefinitely with improving accuracy of the data with a value significantly greater than unity indeed signaling arrival at a data set of sufficient accuracy to put such shell and related effects into evidence. This has been noted before[11,12] as also has the fact that as little as a year ago that point had not been reached, with the straight mean of the then $f^R t$ values showing $\chi^2/\nu = 1.54$ and the BSAG fit $\chi^2/\nu = 1.45$.

Table III shows the estimates of ϵ_2 and the various estimates of ϵ_1. Figure 1 shows the plus-and-minus-one-standard-deviation fits using BSAG with $x = 1.86$ which gives

$$(f^R t)_{Z=0} = 3084.8 \pm 1.9 \text{ sec}, \quad \chi^2/\nu = 3.7.$$

We see the considerable improvement of this χ^2/ν value over that of the straight mean and also that this χ^2/ν, as is to be expected for sufficiently accurate data, is significantly greater than unity. If we permit x in the "power-of-Z" fit to vary we find that χ^2/ν is minimized, but only very shallowly, for $x \simeq 1.1$ and for $(f^R t)_{Z=0} \simeq 3079$ sec. This x value, while not being significantly different on a χ^2 test, is quite far from the BSAG expectation of about 1.9 but this association $(f^R t)$ value is significantly different from that for $x = 1.86$.

We have already noted that the BSAG approach cannot be expected to account for the subtler effects on ϵ associated with the detailed nuclear structure of the states involved. These subtler, wave-function-dependent, effects might be expected to reside chiefly in ϵ_2 which is associated with the details of the relative nucleon-nucleon relationships within the nuclei, although it is clear[22] that ϵ_1 also must to some degree be structure dependent. If, however, the basis of the microscope parentage expansion approach to ϵ_1,[22,24] namely, the assignment of single-particle radial wave functions to the transforming nucleons, is inadequate for a proper treatment of the one-body aspects of the mismatch problem (it manifestly fails, for example, to respect antisymmetrization, a criticism that does not apply to the calculation of ϵ_2) then we might still hope that the BSAG approach might work overall with at least some of the fluctuations of individual cases away from the smooth overall trend of ϵ with Z being accounted for by ϵ_2. We therefore correct the individual $f^R t$ values of Table II by the

TABLE III. Estimates of the nuclear mismatch factor ϵ according to various approaches. ϵ_2 is the two-body contribution (Ref. 21). ϵ_1^0 (LM) (Ref. 23 and private communication from A. M. Lane), ϵ_1^0 (THH) (Ref. 24), and ϵ_1^0 (F) (Ref. 25) are three estimates of the one-body contribution using the giant monopole approach. ϵ_1^π (W) (Ref. 22) and ϵ_1^π (THH) (Ref. 24) are two different versions of the microscopic one-body approach of summing over parent states (all in %).

Body	ϵ_2	ϵ_1^0 (LM)	ϵ_1^0 (THH)	ϵ_1^0 (F)	ϵ_1^π (W)	ϵ_1^π (THH)
^{14}O	0.05	0.04	0.05	0.03	0.41	0.28
^{26}Al^m	0.07	0.11	0.13	0.08	0.35	0.27
^{34}Cl	0.23	0.18	0.20	0.13	0.57	0.62
^{38}K^m	0.16	0.21	0.24	(0.18)	0.34	0.54
^{42}Sc	0.13	0.25	0.37	0.23	0.31	0.35
^{46}V	0.04	0.29	0.40	0.26	0.31	0.36
^{50}Mn	0.03	0.33	0.43	0.31	0.56	0.40
^{54}Co	0.04	0.38	0.47	0.34	0.67	0.56

departure of the individual ϵ_2 values of Table III from $\bar{\epsilon}_2$ and treat the resultant ϵ_2-corrected $f^R t$ values as before. Fitting with $x = 1.86$ yields

$$(f^R t)_{Z=0} = 3083.4 \pm 1.9 \text{ sec}, \quad \chi^2/\nu = 2.6 .$$

We see that indeed χ^2/ν, while still remaining substantially above unity, is considerably improved from the $\chi^2/\nu = 3.7$ of the $x = 1.86$ fit to the uncorrected $f^R t$ values, but that $(f^R t)_{Z=0}$ is not changed outside its error.

If now, as before, we permit x to vary we find that χ^2/ν minimizes, again shallowly, for $x \simeq 2.3$ and for $(f^R t)_{Z=0} \simeq 3083$ sec. In addition to the improvement in χ^2/ν we see that the ϵ_2-corrected $f^R t$ values yield a "best value" of x closer to the BSAG expectation than the uncorrected $f^R t$ values but that, in contradistinction to the case for the uncorrected values, the value of $(f^R t)_{Z=0}$ resulting for the "best value" of x does not differ significantly from that for the BSAG expectation. We turn now to the directly computed ϵ values of Table III and apply $\epsilon^{0,\pi} = \epsilon_1^{0,\pi} + \epsilon_2$ to the $f^R t$ values of Table II to get $\mathfrak{F}t^{0,\pi}$ values and hence the $\overline{\mathfrak{F}t^{0,\pi}}$ values that, if all is well, should agree with the $(f^R t)_{Z=0}$ value just presented. We find

$$\overline{\mathfrak{F}t^0} \text{ LM} = 3087.0 \pm 0.9 \text{ sec}, \quad \chi^2/\nu = 4.9 ,$$

$$\overline{\mathfrak{F}t^0} \text{ THH} = 3085.1 \pm 0.9 \text{ sec}, \quad \chi^2/\nu = 3.4 ,$$

$$\overline{\mathfrak{F}t^0} \text{ (F)} = 3087.9 \pm 0.9 \text{ sec}, \quad \chi^2/\nu = 5.1 ,$$

$$\overline{\mathfrak{F}t^\pi} \text{ (W)} = 3080.3 \pm 0.9 \text{ sec}, \quad \chi^2/\nu = 9.9 ,$$

$$\overline{\mathfrak{F}t^\pi} \text{ (THH)} = 3081.1 \pm 0.9 \text{ sec}, \quad \chi^2/\nu = 10.0 .$$

We see, as has been remarked earlier,[12] that the "giant monopole" approaches (reinforced by ϵ_2) are not in essential disagreement with the BSAG approach, the best of them, $\overline{\mathfrak{F}t^0}$ (THH), which is also the most detailed and thorough in its evaluation, doing just about as well on χ^2/ν as BSAG and being in best agreement with BSAG. On the other hand, the "parentage expansion" approaches scarcely improve χ^2/ν from that of the straight mean and give $\mathfrak{F}t$ values in distinct disagreement with $(f^R t)_{Z=0}$.

As is evident, the conclusions from any such discussion as the present one depend on the way in which the data base is chosen. We have adopted consistently the somewhat eclectic procedure started in our paper of 1976 (Ref. 1), namely, beginning with the latest measurement to work backwards in time, using all data available of accuracy not too much below that of the "best" measurements, if the authors of those data regard them as in final form, rejecting, as we come to them, data that disagreed significantly on the χ^2 test from the mean of the data amassed to that point. This we have done for both the t values of Table I and the E_0 values of Table II. An alternative, rather more liberal, procedure[10] is to take all measurements whose quoted errors are within a factor of 10 of the most precise and then, if necessary, to inflate the overall error in a standard way to cover incompatibility. This procedure leads to somewhat "better" χ^2/ν values than ours on account of the inflated errors and to a less-clear preference for BSAG over the "parentage expansion" approach.

We may note at this point that some check on the reliability of the direct estimates of ϵ could come from differential measurements of individual forbidden Fermi transitions: The strength corresponding to ϵ must reside in these forbidden transitions, most of which will, however, be energetically inaccessible but some of which could be measured either directly if they were of $J^\pi = 0^+ \to 0^+$ type or indirectly by the various techniques for determining Fermi/Gamow-Teller mixtures if not. An extensive survey[26] shows that such forbidden transitions have, broadly speaking, not unreasonable strengths, but there are as yet very few cases available where we may have sufficient confidence in the initial and final state wave functions to establish a meaningful check on the calculated strengths. The only case of a measured $J^\pi = 0^+ \to 0^+$ transition is that of ^{42}Sc to the state of ^{42}Ca at 1.837 MeV (Ref. 27) which corresponds to $\Delta\epsilon = (0.043 \pm 0.018)\%$ and where theoretical estimates (see references in Ref. 27) range from 0.04% to 0.4%.

In view of the uncertainties in the direct computation of ϵ, we feel that at the moment the best estimate of $(f^R t)_{Z=0}$ comes from the BSAG approach; this we may cautiously supplement by the use of the calculated values of ϵ_2 in the way that we have indicated above because such use distinctly improves χ^2/ν as we have seen. However, we have also seen that there is no present firm indication of the reliability of these computed ϵ_2 values so we quote as our recommended value the mean of the straight BSAG approach and the BSAG approach modified by ϵ_2:

$$(f^R t)_{Z=0} = 3084.1 \pm 1.9 \text{ sec}$$

to which corresponds

$$g^R_{\beta\nu} = (1.412\,48 \pm 0.000\,44) \times 10^{-49} \text{ erg cm}^3$$

and to which corresponds the difference between the "inner" radiative corrections for nucleon and muon,[9]

$$\Delta^\alpha_{\beta\nu} - \Delta^\alpha_\mu = (2.12 \pm 0.16)\% ,$$

which is practically the same as the values recommended as the result of recent reviews.[1,2,9]

Although, as we have seen, the conclusion as to $g^R_{\beta V}$ resulting from other treatments of the mismatch problem may perhaps differ significantly from that just presented, the conclusions as to the important difference of the "inner" radiative corrections do not differ significantly; this is on account of the error attaching to the extraction of the Cabibbo angle from strange particle decay (where we have here used our earlier-recommended[1] value $\sin\theta_C = 0.229 \pm 0.003$).

V. DISCUSSION

Since our present results have not sensibly changed $\Delta_{\beta V}^{\alpha} - \Delta_{\mu}^{\alpha}$ from the value on which earlier discussions[1,2,9,11] were based, the conclusion of those earlier discussions as to the mean quark charge of the nucleon, $\overline{Q}$, which is arrived at by a comparison between the above "experimental" value of $\Delta_{\beta V}^{\alpha} - \Delta_{\mu}^{\alpha}$ and the value that arises from gauge theories, will also not be sensibly altered and we shall therefore not repeat those discussions here but merely quote the results:

$$\overline{Q} = 0.17 \pm 0.06.$$

We should, however, repeat the earlier cautions[1,2,9,11] that significant uncertainties in the extraction of $\overline{Q}$ from $\Delta_{\beta V}^{\alpha} - \Delta_{\mu}^{\alpha}$, apart from those associated with the validity of the gauge theories themselves, reside in the "nonasymptotic" piece C of the theoretical nucleon radiative correction and in the SU(3) symmetry-breaking correction to the Cabibbo angle. We should also note, however, the important advance that has been made by Sirlin[28] in our understanding of the nucleon radiative correction through his clarification of the role of the strong interactions. Very briefly, he develops a current-algebra formulation in the framework of quantum chromodynamics and finds that the only change, of marginal significance, that is introduced by the strong interactions and which therefore moderates his treatments[29] on which the earlier analyses were based is of order 0.05% in $\Delta_{\beta V}^{\alpha} - \Delta_{\mu}^{\alpha}$ and is therefore comfortably contained within the present "experimental" error on that quantity even before we begin to concern ourselves with the nonasymptotic piece C and with SU(3) breaking.

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