## Quasiparticle-phonon coupling model for odd-A Ru isotopes

S. Bhattacharya and S. K. Basu\* Saha Institute of Nuclear Physics, Calcutta-700009, India (Received 19 January 1978)

The odd-A Ru isotopes with A = 101-105 are studied in a semimicroscopic model which couples the neutron quasiparticle motion in the N = 50-82 shell to the quadrupole vibrations of the neighbouring Ru core. The agreement between the calculated and available experimental data on energy spectra, spectroscopic factors, moments and transition rates is found to be reasonably good.

NUCLEAR STRUCTURE <sup>101, 103, 105</sup>Ru; calculated levels,  $J, \pi, S, Q, \mu, B(E2), B(M1)$ , quasiparticle-phonon coupling.

## I. INTRODUCTION

The odd-mass ruthenium nuclei have drawn considerable attention in recent years. The latest Nuclear Data compilations<sup>1-5</sup> on these isotopes and the subsequent published reports on their level structure reveal that these nuclei are rather complex and are characterized by both rotational and vibrational types of nuclear excitations. Some of the notable features of the low energy spectra of these nuclei are (1) high density of low-lying "phonon" levels (excited by Coulomb excitation), (2) close occurrence of levels of same i which have completely different characters. (3) a steady decrease in the energy of the first excited  $\frac{3}{2}$ \* state which is very weakly excited in (d, p) reaction and which becomes the ground state in the case of <sup>103,105</sup>Ru, (4) a low-lying  $\frac{11}{2}$  state in neutron rich nuclei which is interpreted as the  $\frac{11}{2}$  [505] Nilsson orbital. Sheline<sup>6</sup> has suggested that the neutron rich nuclei of Mo, Ru, and Pd might have stable quadrupole deformation which was later supported by the fission product decay studies<sup>7</sup> and by the two neutron transfer reaction experiments.<sup>8</sup> Prompted by the success of the Coriolis-coupling rotational model<sup>9</sup> in explaining the so-called "decoupled bands" observed in nuclei belonging to the transitional region, Imanishi et al.<sup>10</sup> calculated the level structure of <sup>103,105</sup>Ru in a similar model. Their calculation, however, appears to be in rather poor agreement with the experimental data. Recently, Rekstad<sup>11</sup> has recalculated the level structure of <sup>103</sup>Ru in the Nilsson model with Coriolis coupling wherein he has treated the "recoil effects" separately in a manner analogous to the treatment of other collective effects. This author has got good agreement with respect to energy spectra, spin parities, and spectroscopic strengths in transfer reactions but has not quoted any values with regard to moments and transition rates. However, Fortune et al.12 in an attempt to study the neutron single-particle strengths in <sup>103</sup>Ru and <sup>105</sup>Ru

by (d, p) reaction, did not get conclusive evidence of any stable deformation in these nuclei. Very recently, Hollas *et al.*<sup>13</sup> also arrived at a similar situation while studying <sup>96,100</sup>Ru(d, p)<sup>97,101</sup>Ru reactions. Both of the above authors observed appreciable splitting in the single-neutron strengths up to 2 MeV in these nuclei, which, as suggested by Mottleson,<sup>14</sup> may result from coupling of the oddneutron motion with the quadrupole vibration.

The quasiparticle-phonon-coupling model has been applied successfully earlier by several workers<sup>15,16</sup> to explain the low energy electromagnetic properties of several odd-proton as well as oddneutron nuclei in the mass-100 region. Goswami and Sherwood<sup>17</sup> calculated the level structure of the nearly spherical Ru nuclei, i.e., 97,99Ru in an extended quasiparticle coupling model where they treated the core vibrations in a microscopic way. The agreement obtained was, however, poor and the ground state was not reproduced at all. In the present work, we have tried to calculate the level structure of some of the neutron rich odd-A Ru isotopes, i.e., <sup>101</sup>Ru, <sup>103</sup>Ru, and <sup>105</sup>Ru, in a quasiparticle-phonon-coupling model which couples the odd-neutron quasiparticle motion in the  $2d_{5/2}$ ,  $3s_{1/2}$ ,  $1g_{7/2}$ ,  $2d_{3/2}$ , and  $1h_{11/2}$  orbitals to the quadrupole vibrations of the corresponding even-Ru core. In doing so, we have used the available experimental data on the (d, p) reaction as a guideline. From energy considerations, core excitations up to two quadrupole phonons have been considered. It has been found that the inclusion of three quadrupole phonons does not significantly alter the spectra. As the model used in the present work has been widely discussed, we will mention only the important features necessary for subsequent discussion

### II. MODEL

The total Hamiltonian of the coupled system is given by

18

where  $H_c$  describes the core vibrations and  $H_{sp}$  is the usual single-particle shell model Hamiltonian. The basis states used are of the type  $|j;NR;IM\rangle$  in which  $H_c + H_{sp}$  is diagonal; j is the particle angular momentum; R is the core angular momentum for a state of N phonons; and I = R + j with the z component equal to M. The eigenfunction of H at an energy  $E_{\alpha}$  can be expanded as

$$\left| E^{\alpha}, IM \right\rangle = \sum_{jNR} C_{\alpha}(j; NR; I) \left| j; NR; IM \right\rangle.$$
<sup>(2)</sup>

$$H_{\rm int} = -\xi \hbar \omega_2 (\pi/5)^{1/2} \sum_{\mu} Q_{2\mu} Y_{2\mu}(\theta, \phi) , \qquad (3)$$

where  $Q_{2\mu}$  is the quadrupole operator for the core and  $Y_{2\mu}(\theta, \phi)$  is the angular part of the quadrupole operator for the particle. The parameter  $\xi$  describes the strength of the coupling. The pairing effects are introduced to the calculations through the interaction Hamiltonian whose matrix elements are

$$\langle j'; N'R'; IM | H_{int} | j; NR; IM \rangle = (-1)^{I+R+1/2} \xi \hbar \omega_2 [\frac{1}{4} (2j+1)(2j'+1)]^{1/2} \begin{pmatrix} j & 2 & j' \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{pmatrix} \\ \times \begin{cases} I & R & j \\ 2 & j' & R' \end{cases} \langle N'R' || Q_2 || NR \rangle (U_j U_{j'} - V_j V_{j'}). \end{cases}$$
(4)

Here  $U_j^2$  and  $V_j^2$  represent the nonoccupation and occupation probabilities in the state *j*, respectively. The reduced matrix elements for the phonons have been calculated following the method of Ford and Levinson.<sup>18</sup> The static electric and magnetic moments of different levels and *E*2 and *M*1 transition rates for several transitions are also calculated. The corresponding expressions are almost identical to those given by Heyde and Brussard,<sup>19</sup> except for a multiplicative factor in the particle part involving  $U_j$  and  $V_j$ . This factor is usually taken to be  $(U_jU'_j \pm V_jV'_j)$  where upper (lower) sign refers to the magnetic (electric) operator.

#### **III. RESULTS AND DISCUSSIONS**

There are several parameters in this calculation, viz., the phonon energy  $\hbar \omega_2$ , the quasiparticle energies  $\epsilon_i$ 's, the nonoccupation probability factors  $U_i^2$ , and the coupling strength  $\xi$ . The phonon energies for the respective cases have been taken from the excitation spectrum of the neighboring  $\frac{A^{-1}X_{N-1}}{Z}$  even core and are kept fixed in the calculation. The trial values for  $U_i^2$  and  $\epsilon_i$ 's are estimated from the available experimental data on (d, p) reactions.<sup>12,13</sup> The Hamiltonian is diagonalized for each isotope for different values of j. The  $\epsilon_i$ 's and  $\xi$  are adjusted to obtain the best fit with the experimental data. A minor adjustment in trial  $U_i^2$  values was necessary. The parameter values for which fairly good agreement with the experimental data has been achieved, are given in Table I. It is clear from Table I that the best fit parameters conform well with the experimental data on transfer reactions within the limits of uncertainty in the experimental measurements. The calculated energy spectra and spectroscopic factors are shown in Figs. 1-3 along with the corresponding data obtained in (d, p) reaction studies. For completeness, the information obtained from other studies and as compiled in the Nuclear Data Sheets are also included in the above figures. From the systematics of the energy spectra, it is seen that there is a low-lying  $\frac{3}{2}$  state in all these nuclei, which is very weakly excited in the (d, p) reaction. This state is believed to be a state with a dominant multiparticle configuration and is not reproduced in our calculation. Excitation energies and spectroscopic factors for the  $\frac{5}{21}$  state, the first few  $\frac{1}{2}{}^{\star}$ states, the  $\frac{7}{21}$  and  $\frac{11}{21}$  states are more or less correctly reproduced in this calculation. The calculated  $\frac{3}{2}$  state is obtained at a lower excitation energy in all these isotopes with a spectroscopic

TABLE I. Parameter values used in the calculation. Experimental values are from Refs. 12 and 13. Quasiparticle energies are relative to the  $2d_{5/2}$  state.

	101	Ru	Nuc 103	leus Ru	<sup>105</sup> Ru		
Parameter	Calc. Exp.		Calc. Exp.		Calc.	Exp.	
ξ	2.75		4.0		4.0		
$\epsilon_{1/2}$ (MeV)	0.500	0.444	0.150	0.258	0.100	0.256	
$\epsilon_{7/2}$ (MeV)	0.600	0.599	0.250	0.227	0.250	0.300	
$\epsilon_{3/2}$ (MeV)	0.830	0.870	0.600	0.780	0.400	0.581	
$\epsilon_{11/2}$ (MeV)	1.690	1.695	0.250	0.237	0.200	0.186	
$U_{5/2}^{2}$	0.36	0.35	0.30	0.29	0.30	0.26	
$U_{1/2}^{2}$	0.77	0.78	0.60	0.59	0.60	0.61	
$U_{7/2}^{7/2}$	0.45	0.60	0.30	0.40	0.23	0.23	
$U_{3/2}^{1/2}$	0.96	0.92	0.72	0.72	0.72	0.75	
$U_{11/2}^{2}$	0.18	0.18	0.30	0.27	0.28	0.28	
$\hbar \omega_2$	0.540	0.540	0.475	0.475	0.360	0.360	



FIG. 1. Calculated and experimental level schemes of  ${}^{101}$ Ru. The excitation energies and spin parities of the levels are shown. The spectroscopic factors are given provided (2J+1)S > 0.01. The experimental data are taken from Refs. 12 and 13.

factor slightly more than that obtained experimentally. The calculated density of levels within the excitation energy under consideration is in good agreement with the experimental data though lack of unique spin-parity assignments forbids proper identification of them.

Static electromagnetic moments for some of the levels and E2 and M1 transition rates for several transitions are also calculated. The magnetic moments and transition rates are calculated with  $g_l = 0$ ,  $(g_S)_{eff} \approx 0.42 (g_S)_{free} = 1.6$ , and  $g_R \approx Z/A$  $\simeq 0.43$ . The  $g_s$  value has been adjusted for all cases to obtain better agreement with the groundstate magnetic moment data. The value turned out to be slightly lower than the suggested<sup>19</sup> value of  $0.58(g_s)_{free}$ . The electric quadrupole moment and transition rates are calculated with  $e_{\text{eff}} = e_p$ ,  $eZ(\hbar\omega_2/2C_2)^{1/2} = Ke_p$ , where K = 4.2, 5.1, and 5.6, respectively. In evaluating K, the stiffness parameter  $C_2$  relating to the core vibrations has been obtained from the tabulations of Wong.<sup>20</sup> The radial matrix elements are calculated in the harmonic oscillator basis, and  $R_0$  is as usual taken to be  $1-2A^{1/3}$  fm. The calculated results are shown in Tables II and III, along with the very little experimental data<sup>21-23</sup> available for <sup>101,103</sup>Ru. We shall now discuss the agreement between the calculated and experimental results for each individual isotope.

#### <sup>101</sup> Ru

This isotope has been studied mainly through the decay of <sup>101</sup>Rb isomers,<sup>24</sup> <sup>101</sup>Tc isomers,<sup>25</sup> by Coulomb excitation,<sup>22</sup> and by  $(\alpha, xn)$  reactions.<sup>26</sup> The results published prior to July, 1973 have been reviewed in the Nuclear Data Sheets by Todd *et al.*<sup>2</sup> Very recently, Hollas *et al.*<sup>13</sup> studied the <sup>100</sup>Ru $(d, p)^{101}$ Ru reaction at  $E_d = 11.5$  MeV and assigned *l* values and spectroscopic factors to 17 states observed below 1.9 MeV. The calculated energy spectrum (Fig. 1) is in fair agreement with the experimental spectrum observed in (d, p) reaction as well as with that given in the Nuclear Data Sheets.<sup>2</sup> The calculated and experimental spectrum for the nuclear Data Sheets.<sup>2</sup> The calculated and experimental spectrum for the nuclear Data Sheets.<sup>2</sup> The calculated and experimental spectrum for the nuclear Data Sheets.<sup>2</sup> The calculated and experimental spectrum for the nuclear Data Sheets.<sup>2</sup> The calculated and experimental spectrum for the nuclear Data Sheets.<sup>2</sup> The calculated and experimental spectrum for the nuclear Data Sheets.<sup>2</sup> The calculated and experimental spectrum for the nuclear Data Sheets.<sup>3</sup> The calculated and experimental spectrum for the nuclear Data Sheets.<sup>3</sup> The calculated and experimental spectrum for the nuclear Data Sheets.<sup>3</sup> The calculated and experimental spectrum for the nuclear Data Sheets.<sup>3</sup> The calculated and experimental spectrum for the nuclear Data Sheets.<sup>3</sup> The calculated and experimental spectrum for the nuclear Data Sheets.<sup>4</sup> The calculated and experimental spectrum for the nuclear Data Sheets.<sup>4</sup> The calculated and experimental spectrum for the nuclear Data Sheets.<sup>4</sup> The calculated and experimental spectrum for the nuclear Data Sheets.<sup>4</sup> The calculated and experimental spectrum for the nuclear Data Sheets.<sup>4</sup> The calculated and experimental spectrum for the nuclear Data Sheets.<sup>4</sup> The calculated and experimental spectrum for the nuclear Data Sheets.<sup>4</sup> The calculated and experimental spectrum for the nuclear Data Sheets.<sup>4</sup> The calculated and exper



FIG. 2. Calculated and experimental level schemes of <sup>103</sup>Ru. The experimental data are taken from Refs. 14 and 12. For other details refer to the caption of Fig. 1.

troscopic factors for the  $\frac{5}{2}$  ground state, the first three  $\frac{1}{2}^{+}$  states, and the  $\frac{11}{2}^{-}$  state are in excellent agreement with one another. The calculated  $\frac{3}{21}$ state at 0.425 MeV having maximum single-neutron strength is slightly lower than the corresponding state observed in (d, p) reaction, though from considerations of energy and B(E2) value this state corresponds well with the 0.422 MeV observed in Coulomb excitation.<sup>22</sup> The (d, p) reaction<sup>13</sup> suggests a state at 0.408 MeV with  $J^{\pi} = \frac{5}{2}^{+}$ ,  $\frac{3}{2}^{+}$  and spectroscopic factors 0.17 and 0.20, respectively, and another state at 0.127 MeV with  $J^{*} = \frac{3}{2}^{+}$  and spectroscopic factor 0.08. This latter state is suggested to be a state with multiparticle configuration and is beyond the scope of the present work. The calculation predicts two  $\frac{7}{2}$  states, one at 0.515 and the other at 0.582 MeV, where the latter is predominantly due to  $1g_{7/2}$  neutron-quasiparticle excitation in agreement with (d, p) data.<sup>13</sup> The lower member is identified with the  $\frac{7}{2}$  state, observed in Coulomb excitation,<sup>22</sup> at 0.545 MeV, on the basis of good agreement in the B(E2) value. Similarly, the calculated  $\frac{9}{2}$  at 0.540 MeV is correlated with the 0.720 MeV  $(\frac{7}{2}, \frac{9}{2})$  state observed in Coulomb excitation.<sup>22</sup> The calculation predicts

a cluster of states, mostly collective in nature between 0.950 and 1.200 MeV with spins ranging from  $\frac{1}{2}$  to  $\frac{11}{2}$ . The low decay energy of the neighboring isobars and their high spin values do not allow these states to be populated in the decay work. The observed density of states at this excitation is quite low, though some of these states may be identified with the calculated ones from energy and spectroscopic factor considerations. The agreement in the calculated B(E2) values for some of the states with those of the corresponding Coulomb excited states seems to be remarkable (cf. Table ш).

# <sup>103</sup>Ru

The experimental data on this isotope have been reviewed in a recent Nuclear Data compilation by Kocher.<sup>4</sup> It has been studied by (d, p) stripping and (d, t) pickup reactions.<sup>12</sup> Further information on its level structure has been obtained from  $(n, \gamma)$ experiment<sup>27</sup> and from the studies of  $^{103}\text{Tc} \rightarrow ^{103}\text{Ru}$ decay.<sup>28</sup> Recently, Klamra and Rekstad<sup>29</sup> studied the  ${}^{100}Mo(\alpha, n){}^{103}Ru$  reaction and made a number of unambiguous spin-parity assignments from  $\gamma$ ray angular distributions. The assignment of  $\frac{3}{2}^{+}$ 



105 44 <sup>R</sup>"61

FIG. 3. Calculated and experimental level schemes of  $^{105}$ Ru. The experimental data are taken from Refs. 15 and 12. For other details refer to the caption of Fig. 1.

to the ground state and  $\frac{5}{2}$  to the 2.7 keV have been proposed in  $(n, \gamma)$  work<sup>27</sup> and is further supported from  $(\alpha, n\gamma)$  experiment.<sup>29</sup> That is why, the calculated and (d, p) excitation energies, displayed in Fig. 2, are shifted by 2.7 keV. The calculated and experimental energy spectra, are in good agreement (cf. Fig. 2), except that the calculation cannot reproduce the complicated  $\frac{3}{2}$  ground state. Within the experimental uncertainties, the agreement between the calculated and experimental spectroscopic factors for most of the low-lying levels seem to be satisfactory. Though the calculated level density is in reasonable agreement with that observed experimentally, the correspondence between levels particularly in the higher-energy part cannot be established due to absence of unique spin assignments. From spectroscopic factor considerations, the observed 0.541 MeV (l=2)level is assigned  $\frac{3}{2}$ \* spin-parity whereas the 0.589 MeV seems to be  $\frac{5}{2}^+$ . In the high energy part, the 1.105 and 1.245 MeV levels are identified with the calculated levels at 1.060 MeV  $(\frac{1}{2}^{+})$  and 1.100 MeV  $(\frac{3}{2})$ , respectively, from similar arguments. The calculation predicts a good number of states between 1.0 and 1.5 MeV with spectroscopic factors

 $\approx$ 0.05, which seem difficult to observe in the (d, p) reaction. The calculated ground-state magnetic moment is in good agreement with the result obtained in a recent measurement with oriented nuclei.<sup>23</sup>

# <sup>105</sup>Ru

This isotope has been recently reviewed in the Nuclear Data Sheets by Bertrand.<sup>5</sup> Information on its level structure comes mainly from (d,p),<sup>12</sup>  $(n, \gamma)$ ,<sup>30</sup> and <sup>105</sup>Tc decay measurements.<sup>31</sup> Very recently, Sümmerer et al.<sup>32</sup> studied the decay of <sup>105</sup>Tc to levels in <sup>105</sup>Ru by  $\beta$ - $\gamma$  spectroscopy and suggested spin-parity assignments to several states on the basis of available information. Hrastnik et al.<sup>30</sup> pointed out that the lowest-energy state observed in (d, p) work<sup>12</sup> corresponds to 20.5 keV in <sup>105</sup>Ru. Therefore, we have added 20.5 keV to the levels observed in (d, p) work in accordance with the Nuclear Data Sheets and shifted our calculated level spectrum to the same extent. The calculated and experimental energy spectra and spectroscopic factors for the  $\frac{5}{2}$ ,  $\frac{1}{2}$ ,  $\frac{11}{2}$ ,  $\frac{7}{2}$ , and  $\frac{3}{2}$  states are in good agreement with one another. From a spectroscopic factor consideration, the

	$E_{\star}$		Q (	<i>e</i> b)	$\mu (\mu_N)$		
Nucleus	(MeV)	$J^{\pi}$	Theo.	Exp.	Theo.	Exp.	
<sup>101</sup> Ru	0.0	$\frac{5}{2}^{+}$	+0.27	0.44(4) <sup>a</sup>	-0.76	$-0.7152(60)^{a}$	
	0.325	$\frac{1}{2}^{+}$	0.0	• • •	-0.72	•••	
	0.625	$\frac{1}{2}^{+}$	0.0	• • •	-0.51	•••	
	0.545	$\frac{7}{2}^{+}$	~0	•••	+1.1	• • •	
	0.599	$\frac{7}{2}^{+}$	+0.08	• • •	+0.58	•••	
	0.422	$\frac{3}{2}^{+}$	-0.35	•••	+0.48	•••	
	0.535	$\frac{3}{2}^{+}$	-0.02	• • •	-0.14	•••	
<sup>103</sup> Ru	0.003	$\frac{5}{2}^{+}$	+0.55	000	-0.71	-0.67 (11) <sup>b</sup>	
	0.177	$\frac{1}{2}^{+}$	0.0	• • •	-0.70	• • •	
	0.435	$\frac{1}{2}^{+}$	0.0		-0.71		
	0.216	$\frac{7}{2}^{+}$	+0.58	• • •	+0.59	•••	
	0.502	$\frac{7}{2}^{+}$	+0.13	• • •	-0.05	•••	
	0.408	$\frac{3}{2}^{+}$	-0.17	• • •	+0.72	• • •	
	0.740	$\frac{3}{2}^{+}$	+0.21		+0.04	•••	
<sup>105</sup> Ru	0.020	<u>5</u> <sup>+</sup> 2	+0.62	•••	-0.71	• • •	

TABLE II. Calculated and experimental Q and  $\mu$  values.

<sup>a</sup> See Ref. 22.

=

0.423 MeV (l=2) state observed in the (d,p) reaction is identified with the calculated 0.422 MeV  $(\frac{5}{2}^{+})$ . The  $\frac{1}{22}^{+}$  and  $\frac{7}{2}^{+}$  states are obtained at a lower excitation though their spectroscopic factors are in reasonable agreement with those obtained in (d,p)reaction. The (d,p) reaction<sup>12</sup> suggests several states with l=2 transfers some of which may be correlated with the calculated ones. For this nu-

÷

 $\frac{1}{2}^{1}$ 

3

0.0

+0.76

-0.26

0.160

0.228

0.465

<sup>b</sup> See Ref. 23.

-0.70

+0.60

+0.77

...

••

. . .

cleus there exist no measurements of moments and transition rates.

. . .

. . .

. . .

#### **IV. CONCLUSION**

The aim of the present calculation has been to show how good the quasiparticle-phonon-coupling model is in describing the odd-A Ru isotopes with

Transition $J_i^{\pi} \rightarrow J_f^{\pi}$	A = 1 calc.	L01 exp. <sup>a</sup>	B(E2) (A = 1) calc.	e <sup>2</sup> b <sup>2</sup> ) 103 exp.	A = 1 calc.	105 exp.	A= calc.	101 exp. <sup>a</sup>	B(M1) ( $A = 1$ calc.	μ <sub>N</sub> ) <sup>2</sup> 103 exp.	A = 1 calc.	L05 exp.
									•			
$\frac{1}{2}^+_1 \rightarrow \frac{5}{2}^+_1$	0.019	0.018	0.015		0.014	•••	•••					
$\frac{1}{2}^+_1 \rightarrow \frac{5}{2}^+_1$	0.102	0.021	0.156	•••	0.200	•••	• • •	•••			o <b>o e</b>	• • •
$\frac{3^+}{2_1} \rightarrow \frac{5^+}{2_1}$	0.040	0.025	0.017	•••	0.006	• • •	0.060	≤0.09	0.044	• • •	0.06	•••
$\frac{3^+}{2_2} \rightarrow \frac{5^+}{2_1}$	0.072	• • •	0.102	•••	• • •	• • •	0.058	•••	0.108	• • •	• • •	
$\frac{7}{2}^+_1 \rightarrow \frac{5}{2}^+_1$	0.084	0.105	0.010		0.018	•••	0.008	0.10	0.002	• • •	0.002	•••
$\frac{7}{2}^+_2 \rightarrow \frac{5}{2}^+_1$	0.028	0.009	0.132	•••	•••	•••	0.003	≤0.02	0.028			•••
$\frac{9^+}{2_1} \rightarrow \frac{5^+}{2_1}$	0.114	0.061		•••	•••	•••	•••	•••	•••	•••		• • • •

TABLE III. Calculated and experimental B(E2) and B(M1) values.

<sup>a</sup> See Ref. 22, experimental B(E2)<sup>†</sup> values are reduced to B(E2)<sup>‡</sup> values.

A = 101 - 105. It is evident from the previous discussions that our calculation is quite successful in reproducing the energy spectra, spectroscopic factors, known electromagnetic moments, and transition rates for these nuclei in spite of the simplicity of the model used. It would have been interesting to compare our calculation with other model calculations for these nuclei. Unfortunately, the work of Goswami and Sherwood<sup>17</sup> and that of Imanishi et al.<sup>10</sup> are qualitative in nature, restricted to the calculation of energy spectra only. Rekstad,<sup>11</sup> also, did not report any data on the calculation of moments and transition rates though he has been successful in explaining other data on <sup>103</sup>Ru. Under these circumstances, we state below some of the important points which seems worth mentioning.

(1) Though we have varied both  $U_j^2$  and  $\epsilon_j$  in order to obtain a better fit with the experimental data, this variation led to values for these parameters which conform well with the physical situation in all the cases studied (cf. Table I).

(2) In the case of <sup>101</sup>Ru, for which a considerable amount of experimental data is available by now, the agreement seems to be remarkable. Besides reproducing the energy spectra and spectroscopic factors, our calculation for this nucleus has been able to reproduce the B(E2) data obtained from Coulomb excitation experiments<sup>22</sup> and the magnetic dipole moment and electric quadrupole moment of  $\frac{5}{2}$ + ground state. Similar agreement has been obtained for <sup>103</sup>Ru though such data for <sup>105</sup>Ru are yet to come.

(3) Our calculation could not reproduce the first excited  $\frac{3}{2}$  state at 0.127 keV in  $^{101}$ Ru which is very weakly excited in (d, p) reaction and this is further depressed in case of  $^{103,105}$ Ru to become the ground state. As mentioned by Hrastnik *et al.*,<sup>30</sup> the experimental data on *M*1 transition rates support an interpretation of these states as states with multiparticle configuration of higher seniority. States

at 0.307 MeV  $(\frac{7}{2}^{*})$ , 0.325 MeV  $(\frac{5}{2}^{*})$  in <sup>101</sup>Ru, 0.090 MeV  $(\frac{5}{2}^{*}, \frac{3}{2}^{*})$  in <sup>103</sup>Ru, and 0.108 MeV  $(\frac{3}{2}^{*}, \frac{5}{2}^{*})$ , 0.164 MeV  $(\frac{3}{2}, \frac{5}{2})$ , 0.246 MeV  $(\frac{7}{2}^{*})$  in <sup>105</sup>Ru, also appear to arise from similar a type of configuration. The inability of our calculation to reproduce these states is a natural consequence of the very restricted configuration used by us.

The depression of the  $\frac{11}{2}$  states in higher mass Ru isotopes and the observation of negative-parity states with  $J^* \ge \frac{11}{2}$  in  $(\alpha, xn)$  reaction with  $\frac{11}{2}$  as the bandhead could not be explained by our calculation. Similar bands have been described by Hagemann and Dönau<sup>33</sup> for Pd nuclei in terms of Coriolis-decoupled band theory. Rekstad<sup>11</sup> also tried to describe these decoupled bands in <sup>103</sup>Ru by his model. However, as observed by him, the poor energy fit to the members of the band suggests that a more realistic approach for the rotational band is needed. From a systematic point of view it will be worthwhile to see how his model is successful in explaining other Ru nuclei, e.g., <sup>101</sup>Ru and <sup>105</sup>Ru. This is more so because of the poor agreement of Imanishi's work. In this respect our calculation has demonstrated the trend in a systematic way in these nuclei.

It is felt that the present calculation may help in stimulating further experimental and theoretical investigations. In particular, the experimental work on measurements of magnetic and quadrupole moments for the low-lying state in these isotopes would be highly desirable.

The authors express their gratitude to Professor A. P. Patro and Professor B. Basu for their kind interest in the work and for necessary facilities. They are grateful to Dr. S. Sen for a critical reading of the manuscript and for many helpful discussions and suggestions during the course of the work. The help of Dr. K. Krishan during computation is also acknowledged with thanks.

- \*Present address: Variable Energy Cyclotron Project, Bhabha Atomic Research Centre, Calcutta-700 064, India.
- <sup>1</sup>L. R. Medsker, Nucl. Data Sheets 10, 1 (1973).
- <sup>2</sup>R. R. Todd, W. H. Kelly, F. M. Bernthal, and W. C.
- McHarris, Nucl. Data Sheets, 10, 47 (1973).
- <sup>3</sup>L. R. Medsker, Nucl. Data Sheets <u>12</u>, 431 (1974).
- <sup>4</sup>D. C. Kocher, Nucl. Data Sheets <u>13</u>, 337 (1974).
- <sup>5</sup>F. E. Bertrand, Nucl. Data Sheets 11, 449 (1974).
- <sup>6</sup>R. K. Sheline, Rev. Mod. Phys. <u>32</u>, 1 (1960).
- <sup>7</sup>E. Cheifetz, R. C. Jared, S. G. Thompson, and J. B.
- Wilhelmy, Phys. Rev. Lett. 25, 38 (1970); E. Cheifetz, J. B. Wilhelmy, R. C. Jared, and S. G. Thompson,

Phys. Rev. C 4, 1913 (1971).

- <sup>8</sup>R. F. Casten, E. R. Flynn, O. Hansen, T. Mulligan, R. K. Sheline, and P. Kienle, Phys. Lett. <u>32</u>B, 45 (1970); H. Taketani, M. Adachi, M. Ogawa, K. Ashibe, and T. Hattori, Phys. Rev. Lett. 27, 520 (1971).
- <sup>9</sup>F. S. Stephens, R. M. Diamond, and S. G. Nilsson, Phys. Lett. <u>44B</u>, 429 (1973); M. A. Deleplanque, C. Gerschel, N. Perrin, and P. Quentin, Phys. Lett. 46B, 317 (1973).
- <sup>10</sup>N. Imanishi, I. Fujiwara, and T. Nishi, Nucl. Phys. A205, 531 (1973).
- <sup>11</sup>J. Rekstad, Nucl. Phys. <u>A247</u>, 7 (1975).
- <sup>12</sup>H. T. Fortune, G. C. Morrison, J. A. Nolen, Jr., and P. Kienle, Phys. Rev. C 3, 337 (1971).

- <sup>13</sup>C. L. Hollas, K. A. Aniol, D. W. Gebbie, M. Borsaru, J. Nurzynski, and L. O. Barbopoulos, Nucl. Phys. A276, 1 (1977).
- <sup>14</sup>B. R. Mottleson, J. Phys. Soc. Jpn. Suppl. 24, 87 (1968).
- <sup>15</sup>K. Krishan, S. K. Basu and S. Sen, Phys. Rev. C <u>13</u>, 2055 (1976); K. Krishan and S. Sen, *ibid*. <u>14</u>, 758 (1976);
- K. Krishan and S. Sen, *ibid*. (to be published). <sup>16</sup>S. K. Basu and S. Sen, Nucl. Phys. <u>A220</u>, 580 (1974);
- J. E. Kitching, Z. Phys. <u>258</u>, 22 (1973).
- <sup>17</sup>A. Goswami and A. I. Sherwood, Phys. Rev. <u>161</u>, 1231 (1967).
- <sup>18</sup>K. W. Ford and C. Levinson, Phys. Rev. <u>137</u>, B793 (1965).
- <sup>19</sup>K. Heyde and P. J. Brussard, Nucl. Phys. <u>A104</u>, 81 (1967).
- <sup>20</sup>C. Y. Wong, Nucl. Data, <u>A4</u>, 271 (1968).
- <sup>21</sup>S. Büttgenbach, R. Dicke, H. Gebaner, and M. Herschel, Z. Phys. A280, 217 (1977).
- <sup>22</sup>O. C. Kistner and A. Schwarzschild, Phys. Rev. <u>146</u>, 869 (1966); K. I. Erokhina, I. Kh. Lemberg, A. S. Mishin, and A. A. Pasternak, Izv. Akad. Nauk SSSR Sec. Fiz. <u>38</u>, 1673 (1974) [Bull. Acad. Sci., USSR, Phys. Ser. <u>38</u>, 97 (1974)].

- <sup>23</sup>J. A. Barclay, S. S. Rosenblum, W. A. Steyert, and K. S. Krane, Phys. Rev. C 14, 1183 (1976).
- <sup>24</sup>N. K. Aras, G. D. O'Kelley, and G. Chilosi, Phys. Rev. 146, 869 (1966).
- <sup>26</sup>B. Siwamogsatham and H. T. Easterday, Nucl. Phys. <u>A162</u>, 42 (1971); N. K. Aras, P. Fellweis, G. Chilosi, and G. D. O'Kelley, *ibid.* A169, 209 (1971).
- <sup>26</sup>C. M. Lederer, J. M. Jaklevic, and J. M. Hollander, Nucl. Phys. A169, 489 (1971).
- <sup>27</sup>W. Delang, P. Göttel, H. H. Guven, A. M. Hassan, B. Hrastnik, and H. Seyfarth, Report No. KFA-IKP-10/73, (unpublished), p. 283.
- <sup>28</sup>W. W. Hein, thesis, Iowa State University, 1970 (unpublished).
- <sup>29</sup>W. Klamra and J. Rekstad, Nucl. Phys. <u>A243</u>, 395 (1975).
- <sup>30</sup>B. Hrastnik, H. Seyfarth, A. M. Hassan, W. Delang, and P. Göttel, Nucl. Phys. A219, 381 (1974).
- <sup>31</sup>P. Kienle, K. Wien, U. Zahn, and B. Weckermann, Z. Phys. 176, 226 (1963).
- <sup>32</sup>K. Sümmerer, N. Kaffrell, and N. Trautmann, Z. Phys. <u>A273</u>, 77 (1975).
- <sup>33</sup>U. Hagemann and F. Dönau, Phys. Lett. 59B, 321 (1975).