Interpretation of the anomalous electron-capture to positron decay ratio in ^{22}Na†

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The impact of second-forbidden corrections is studied in order to relate the ϵ/β^+ ratio, the spectral shape factor, and the β-γ directional correlation measurements in ^{22}Na decay.

[RADIOACTIVITY ^{22}Na; second-forbidden corrections to β-decay calculations.]

I. INTRODUCTION

The electron-capture to positron decay branching ratio (hereafter ϵ/β^+) for ^{22}Na was measured originally in order to test for Fierz interference.[1] The allowed theory of β decay had been well established, and theoretical ϵ/β^+ ratios could be readily calculated. Early experiments indicated that experiment and theory agreed to within several percent,[2] and this was used as evidence against the existence of Fierz terms.

The ^{22}Na ϵ/β^+ ratio has since been remeasured by several groups, leading to five especially precise results,[3-7] which are shown in Table I. The first three results agree quite closely, and, although the fourth result differs from the others, it too is in significant disagreement with theory. The fifth result, although in agreement with theory, is a less direct measurement because it relies on tabulated detector efficiencies. In addition, this result is quite sensitive to small amounts of γ-ray attenuation in the absorbers near the source. Until it is understood why this result differs so drastically, we shall take the point of view that a possible substantial deviation exists, and we shall in the calculations discussed below (which stand apart from the experimental uncertainties) consider the theoretical conclusions implied by such large deviations from allowed theory.

Experimental data also exist on the ^{22}Na β spectral shape and β-γ directional correlation, both of which are sensitive to Fierz interference and/or second-forbidden effects. Several authors have precisely measured the ^{22}Na spectral shape.[8-11] Wenninger, Stiewe, and Leutz[8] published their raw data which we have examined below, while the other authors generally reported a $1/E$ dependence. The resulting linear slopes between 100 keV $\leq E_\beta \leq 400$ keV are presented in Table II. These values are consistent with the slope being near zero. The results were originally analyzed to show the absence of a Fierz term; however, the analysis did not contain second-forbidden form factors, which, in view of the very hindered nature of this decay, could be rather significant.[12] Finally, two precise values of the ^{22}Na β-γ directional correlation coefficient A_{22} have been measured[13,14] and are given in Table III. Although these values are not entirely consistent with each other, they both seem consistent with $A_{22} \leq 0$.

TABLE I. ^{22}Na experimental ϵ/β^+ decay branching ratios.

ϵ/β^+	∀, skew ratio	Ref.
0.1048 ± 0.0007	0.910 ± 0.008	Leutz and Wenninger (1967), Ref. 4
0.1042 ± 0.0010	0.905 ± 0.011	Vatai, Varga, and Uchrin (1968), Ref. 5
0.1041 ± 0.0010	0.904 ± 0.011	Williams (1964), Ref. 3
0.1077 ± 0.0006	0.935 ± 0.008	MacMahon and Baerg (1976), Ref. 6
0.1128 ± 0.0018	0.979 ± 0.018	Bosch *et al.*, Ref. 7.
0.1152 ± 0.0003	···	Theory—this paper

TABLE II. ^{22}Na shape-factor experimental slopes.

Slope (%/MeV)	Experiment
1.0 ± 1.2	Daniel (1958), Ref. 9
−0.05 ± 0.12	Leutz (1961), Ref. 10
0.3 ± 0.9	Gils *et al.* (1972), Ref. 11
0.05 ± 0.17	Wenninger *et al.* (1968), Ref. 8—original result
5.9 ± 3.0	Wenninger *et al.* (1968), Ref. 8—recalculated result[a]

[a]Corrected for photon emission, annihilation in flight, and escape from the crystal.

By the time the ^{22}Na ϵ/β^+ ratio anomaly had been firmly established experimentally, Fierz interference was assumed to be nonexistent, so that alternative causes were suggested. One argument[5] employed an extrapolation of Bahcall's papers on orbital electron exchange and overlap effects.[15] Such arguments were rejected by Williams[16] because an exact calculation of such effects should reveal a change only in the relative subshell capture rates, not in the total rate. Capture of a K-orbital electron of the parent nucleus, for example, can result in an L-shell vacancy in the final system due to imperfect orbital overlap. The effect of the electron configuration on the total nuclear capture rate is presumably quite small except for the case of extremely low-energy transitions. (Note, however, that this conclusion is based on existing Hartree-Fock calculations, which do not include correlations.) Later arguments were put forth by Firestone *et al.*[17,18] to the effect that the anomaly is most likely the result of the exclusion, in the simple allowed calculation, of higher-order forbidden terms, which can make significant contributions to hindered allowed decays. A limited qualitative discussion of these effects has been given by several authors.[7,19,20] The implications of such higher-order forbidden terms are discussed quantitatively and in greater detail in this paper.

II. THEORETICAL CALCULATIONS

The two principal formalisms used to calculate β decay are thoroughly discussed by Holstein[21]

TABLE III. ^{22}Na β-γ experimental directional correlation A_{22}.

A_{22}	E_β(keV)	Experiment
$-1.8(3)\times10^{-3}$	350	Steffen (1959), Ref. 13
$-0.4(7)\times10^{-3}$	140–250	Müller (1965), Ref. 14
$-0.5(6)\times10^{-3}$	250–480	Müller (1965), Ref. 14

(based on "elementary-particle" amplitudes) and by Behrens and Jänecke[22] (an extension of "standard" nuclear β-decay multipole matrix elements). The two formalisms are completely equivalent. The actual calculated values presented in this paper were generated using the Behrens and Jänecke approach; however, in the main text the discussion is given in terms of the somewhat more transparent elementary particle approach. A translation dictionary between these terminologies is given in the Appendix.

We assume the canonical V-A form for the weak interaction. Thus, for β^+ decay,

$$T_{\rm wk}=\frac{G}{\sqrt{2}}\cos\theta_c\langle\beta_{p_2}|V_\lambda+A_\lambda|\alpha_{p_1}\rangle\bar{u}_\nu(k)\gamma^\lambda(1+\gamma_5)v_e(p)\,, \tag{1}$$

where p_1, p_2, p, and k represent the respective four-momenta of the parent nucleus α, daughter nucleus β, positron, and neutrino; $G(\approx10^{-5}m_p^{-2})$ is the weak decay constant; and $\theta_c(\approx15°)$ is the Cabibbo angle. Letting M_1 and M_2 be the respective parent and daughter masses, we also define

$$\begin{aligned}P&=p_1+p_2\,,\\ q&=p_1-p_2=p+k\,,\\ M&=\tfrac{1}{2}(M_1+M_2)\,,\end{aligned} \tag{2}$$

and

$$\Delta=M_1-M_2\,.$$

Then, to first order in recoil, the decay spectrum becomes

$$\begin{aligned}d\lambda_{\beta^+}=\frac{|T|^2}{(2\pi)^5}&\left(1+\frac{3E-E_0-3\vec{p}\cdot\hat{k}}{M}\right)\\ &\times(E_0-E)^2pEd\Omega_e d\Omega_\nu dE\,,\end{aligned} \tag{3}$$

where $E(\vec{p})$ is the positron energy (momentum), $\hat{k}$ is a unit vector in the direction of neutrino momentum, and E_0 is the maximum positron energy,

$$E_0=\Delta\left(\frac{1+m_e^2/2M\Delta}{1+\Delta/2M}\right). \tag{4}$$

We define for an arbitrary Gamow-Teller transition,[21]

$$\begin{aligned}\langle\beta_{p_2}|V_\lambda+A_\lambda|\alpha_{p_1}\rangle l^\lambda &=-\frac{i}{4M}C^{M'\,k;M}_{J'\,1;\,J}\epsilon_{ijk}\Big(2bl_iq_j+i\epsilon_{ij\lambda\eta}l^\lambda(cP^\eta-dq^\eta)\\ &\qquad+i\epsilon_{ij\lambda\eta}q^\lambda p^\eta q\cdot l\,\frac{h}{(2M)^2}\Big)+\cdots\,,\end{aligned} \tag{5}$$

where J and J' are the spins of the parent and daughter nucleus, respectively, and M and M' represent the initial and final components of nuclear spin along some axis of quantization. Here c represents the usual Gamow-Teller matrix element, b is the so-called weak magnetism contribution, h is the induced pseudoscalar, while d, often called the induced tensor, is uniquely correlated with the existence of a second-class axial current if α and β are isobaric analog states.[23,24] Note that although in Eq. 5 we do not list the Coulomb terms required for gauge invariance, these are included in our calculations using the definitions in Eq. (A1).

Each form factor (b, c, d, and h) is a function of the four-momentum transfer q^2. However, for present purposes it is sufficient to include this feature only for the Gamow-Teller term,

$$c(q^2) \equiv c_1 + c_2 q^2 + \cdots . \tag{6}$$

In terms of this notation, ϵ/β^+ has been calculated in a previous communication.[25] If $(\epsilon/\beta^+)_0$ is the theoretical electron capture to positron ratio for a strictly allowed decay, we define,

$$\forall = (\epsilon/\beta^+)_{\exp}/(\epsilon/\beta^+)_0 , \tag{7}$$

where $\forall$ is hereafter referred to as the skew ratio. Then, neglecting the electron binding energy with respect to m_e, we have[23]

$$\begin{aligned}\forall \approx 1 &+ \frac{1}{2M}\left(m_e - \tfrac{20}{3}\langle E\rangle + \tfrac{4}{3}m_e^2\left\langle\frac{1}{E}\right\rangle + \tfrac{7}{8}\Delta_0\right)\\ &- \frac{c_2}{c_1}\left[\tfrac{40}{9}\Delta(m_e+\langle E\rangle) + \tfrac{40}{9}(m_e^2 - \langle E^2\rangle) - \tfrac{4}{9}m_e^2\Delta\left(\frac{1}{m_e}+\left\langle\frac{1}{E}\right\rangle\right) - \tfrac{2}{3}\alpha E_0 R - \tfrac{9}{2}\alpha^2(2Z'-1) + \tfrac{20}{3}\frac{\alpha Z'}{R}(m_e+\eta\langle E\rangle)\right]\\ &- \frac{\alpha}{2MR}\frac{c_1-2d-2b}{c_1} + \frac{4}{3M}\frac{b}{c_1}(m_e+\langle E\rangle) - \frac{m_e^2}{3M}\left(\frac{-d+2b+h[(E_0+3\alpha Z'/2R)/2M]}{c_1}\right)\left(\frac{1}{m_e}+\left\langle\frac{1}{E}\right\rangle\right) ,\end{aligned} \tag{8}$$

where $Z'=11$ is the charge of the parent ^{22}Na nucleus,

$$\eta = \frac{Z'-1}{Z} , \tag{9}$$

and

$$\langle E^n\rangle = \frac{\int_{m_e}^{E_0} dE\, pE^{n+1}(E_0-E)^2 F_+(Z,E)}{\int_{m_e}^{E_0} dE\, pE(E_0-E)^2 F_+(Z,E)} \tag{10}$$

is the nth moment of the positron energy for the β^+ transition. There does exist an important omission in Eq. (8)—the radiative correction, which accounts for real photon emission and other non-Coulombic electromagnetic effects. This has been calculated for the β^+ decay and reduces $(\epsilon/\beta^+)_0$ by about 1.6%.[26] There exists in addition a radiative correction to account for similar effects in the ϵ process. This should tend to reduce the 1.6% number somewhat. However, a calculation of the ϵ radiative correction has not yet been made, so that in the following discussion we discard the β^+ radiative correction.

We find, then, for ^{22}Na, using $E_0 = 2.068 m_e$,[27] that

$$(\epsilon/\beta^+)_0 = 0.1152 \pm 0.0003 \tag{11}$$

and

$$\forall - 1 \approx -\left(18.0\frac{c_2}{c_1 R^2} - 1.56\frac{b}{Ac_1} - 0.70\frac{d}{Ac_1} + 0.0013\frac{h}{A^2 c_1}\right)\times 10^{-3} . \tag{12}$$

In Fig. 1 we show the dependence of $\forall$ on b, c_2, d, and h separately for reasonable values of these parameters. The slight deviation of the graph from the approximate expression given in Eq. (12) results from the fact that the figures were generated using a more complete form, including higher-order quadratic and Coulomb effects.

The shape factor $f_1(E)$ is defined for this β^+ decay by

$$d\lambda_{\beta^+} = F_+(Z,E)\frac{G^2\cos^2\theta_c}{2\pi^3}(E_0-E)^2 pEf_1(E)dE , \tag{13}$$

where $F_+(Z,E)$ is the Behrens-Jänecke Fermi function[22] with $Z=10$ (for the daughter ^{22}Ne nuclear charge) and

$$\begin{aligned}f_1(E) = c_1^2 &- \frac{2}{3}\frac{E_0}{M}c_1(c_1+d-b) + \frac{2}{3}\frac{E}{M}c_1(5c_1-2b) - \frac{m_e^2}{3ME}c_1(2c_1+d-2b)\\ &+ 2c_1c_2\left[\tfrac{11}{9}m_e^2 + \tfrac{20}{9}EE_0 - \tfrac{20}{9}E^2 - \tfrac{2}{9}m_e^2\frac{E_0}{E} - \frac{1}{3}\frac{\alpha Z E_0}{R} + \frac{10}{3}\left(\frac{\alpha Z E}{R}\right) - \frac{9}{4}\left(\frac{\alpha Z}{R}\right)^2\right]\\ &- \frac{\alpha Z}{2MR}c_1(c_1-2b-2d) + \frac{c_1 h}{(2M)^2}\left(-\tfrac{2}{3}m_e^2 + \tfrac{2}{3}m_e^2\frac{(E_0+3\alpha Z/2R)}{E}\right) .\end{aligned} \tag{14}$$

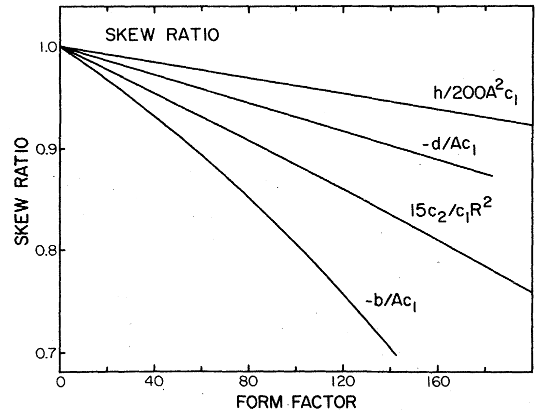

FIG. 1. Dependence of the ^{22}Na ϵ/β^+ skew ratio $\forall$ on the "elementary-particle" amplitudes b (weak magnetism), c_2 (Gamow-Teller four-momentum dependent term), d (induced tensor), and h (induced pseudoscalar).

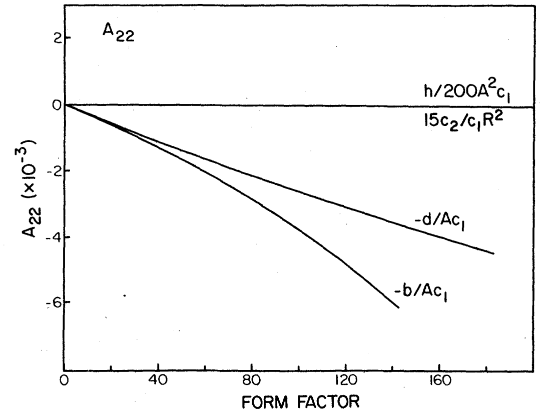

FIG. 3. Variation of the ^{22}Na β-γ directional correlation coefficient A_{22} with the "elementary-particle" amplitudes b, c_2, d, and h.

An important feature of the shape factor is its energy dependence. Most of the experimental data points are taken between $E_1 = 600$ keV and $E_2 = 900$ keV. Thus, we define the average slope parameter S by

$$S = \frac{1}{E_2 - E_1}\frac{f_1(E_2) - f_1(E_1)}{f_1(E_1)}$$
$$\approx -\left(1.79\frac{b}{Ac_1} - 7.78\frac{c_2}{c_1R^2} - 0.178\frac{d}{Ac_1} + 0.00074\frac{h}{A^2c_1}\right)\times 10^{-3}\ \mathrm{MeV}^{-1}. \quad (15)$$

Shown in Fig. 2 is the variation of S with respect to b, c_2, d, and h.

Finally, for the β-γ directional correlation, we define

$$d\lambda_{\beta^+,\gamma} \approx \tfrac{1}{2}F_+(Z,E)\frac{G^2\cos^2\theta_c}{(2\pi)^5}(E_0 - E)^2 pE\,dE\,d\Omega_e\,d\Omega_\gamma \times\left\{f_1(E) + f_3(E)\left[\left(\frac{\vec{p}\cdot\hat{s}}{E}\right)^2 - \frac{1}{3}\frac{p^2}{E^2}\right]\right\}, \quad (16)$$

where $\hat{s}$ is a unit vector in the photon direction. Here $f_1(E)$ is the shape factor as given in Eq. (14), while the β-γ directional correlation parameter A_{22} is given by

$$A_{22} \equiv \frac{2f_3(E)}{3f_1(E)} = -\frac{E}{21M}\frac{c_1 - b - d + \frac{8}{3}c_2M(E_0 - E)}{c_1}$$
$$\approx (\text{at } E = 850\ \text{keV})\left(4.4\frac{b}{Ac_1} + 4.4\frac{d}{Ac_1} - 0.60\frac{c_2}{c_1R^2}\right)\times 10^{-5}. \quad (17)$$

In Fig. 3 the variation of A_{22} with respect to b, c_2, d, and h is shown.

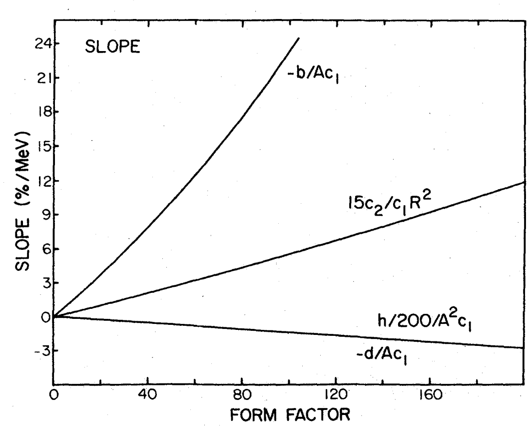

FIG. 2. Variation of the ^{22}Na β^+-decay shape factor S on the "elementary-particle" amplitudes b, c_2, d, and h.

III. DISCUSSION OF EXPERIMENTAL DATA

According to the error bars associated with the measured ϵ/β^+ ratios (cf. Table I), all of these results are quite precise. The deviation of the McMahon and Baerg[6] experiment is therefore disturbing; however, even this result differs by 6.0% from the strictly allowed theoretical value. The errors in the allowed theory are quite small because the decay energy is precisely known. Radiative corrections can account for at most ≈1.6% of this deviation, so at least a 4.4% effect remains unaccounted for. (The Bosch *et al.* experiment[7] removes the anomaly entirely; however, the great difference between this result and all of the others must be adequately resolved. All further discussion herein assumes that an anomaly indeed exists.)

From Fig. 1 we see that somewhat large values of weak magnetism, $b - b/Ac_1 \approx -35$; the Gamow-Teller q dependence, $c_2 - c_2/c_2R^2 \approx +3.5$; and the induced pseudoscalar, $h - h/A^2c_1 \approx 3.5\times 10^4$, or the induced tensor $d/Ac_1 \approx -80$ can result in a 6% deviation. We discuss the implications of such values of these parameters in detail below with regard to the remaining body of ^{22}Na data.

The value of c_1 can be determined from the experimental $\log ft = 7.42$:

$$|c_1| \approx [2ft^{\text{Fermi}}/ft(^{22}\text{Na})]^{1/2} \approx 0.016\,, \tag{18}$$

where $ft^{\text{Fermi}} = 3085$ sec is the ft value for pure Fermi transitions.[28] This value for c_1 is strictly correct only in the purely allowed sense; however, the introduction of second-forbidden corrections changes c_1 by no more than a few percent.

The weak magnetism term b can be obtained by measurement of the analog $M1$ γ-ray transition in ^{22}Na from the $T=1$ analog of the $J^\pi = 2^+$, 1.274-MeV state in ^{22}Ne. The $M1$ component of this transition is directly related to the weak magnetism β decay form factor by the conserved vector current (CVC) theory.[29] The $T = 1$ analog in ^{22}Na is well known to occur at 1.952 MeV and is observed to decay only to the $T=1$, 0.583-MeV state, as indicated in Fig. 4. The half-life of the $T=1$ state was measured to be 9 ± 5 fsec,[30] and an upper bound of 0.25% was recently set for the branching ratio to the ground state.[31] This yields, using CVC theory, an upper limit of $|b/Ac_1| < 14$ and a deviation of the ϵ/β^+ ratio of less than 2.1%. It is therefore not possible for weak magnetism *alone* to account for the skew ratio. It should be emphasized that this upper limit assumes a pure $M1$ transition. Any $E2$ competition lowers the weak magnetism prediction even further.

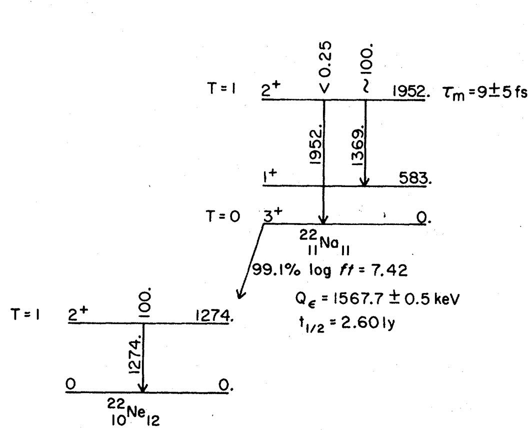

FIG. 4. Level schemes for $A = 22$ showing the decays of the $T=1$ analog states in ^{22}Na and ^{22}Ne.

There exist no simple analog experiments to determine c_2, d, or h, as these derive from the nonconserved axial current. We noted above, however, that the experimental skew ratio can be understood if $c_2/c_1R^2 \approx +3.5$ or $h/A^2c_1 \approx 3.5 \times 10^4$. In order to decide if such values are "reasonable" we have calculated all the relevant form factors in the impulse approximation using the extensive s-d shell-model wave functions generated by Chung and Wildenthal.[32] The results[33] are

$$\begin{aligned}
c_1 &\approx g_A M_{GT} = +0.00266\,,\\
\frac{b}{Ac_1} &\approx \frac{g_M}{g_A} + \frac{g_V}{g_A}\frac{M_L}{M_{GT}} = -117\,,\\
\frac{d}{Ac_1} &\approx \frac{M_{\sigma L} + m_A E_0 (M_{\sigma r^2} + 2M_{\sigma\cdot rr})}{5M_{GT}} \approx -19\,,\\
\frac{c_2}{c_1R^2} &\approx \frac{1}{10R^2}\,\frac{2M_{\sigma r^2} - M_{\sigma\cdot rr}}{M_{GT}} \approx -2.25\,,
\end{aligned} \tag{19}$$

and

$$\frac{h}{A^2c_1} \approx \tfrac{3}{10}(2m_A)^2\,\frac{M_{\sigma\cdot rr} - \frac{1}{3}M_{\sigma r^2}}{M_{GT}} \approx +7.3\times 10^3\,,$$

where m_A is the atomic mass unit, and the M's represent the reduced matrix elements

$$\begin{aligned}
M_{GT} &= \Big\langle \Big\| \sum_j \tau_j \vec{\sigma}_j \Big\| \Big\rangle,\\
M_{\sigma L} &= \Big\langle \Big\| i \sum_j \tau_j \vec{\sigma}_j \times \vec{\mathrm{L}}_j \Big\| \Big\rangle,\\
M_L &= \Big\langle \Big\| \sum_j \tau_j \vec{\mathrm{L}}_j \Big\| \Big\rangle,\\
M_{\sigma r^2} &= \Big\langle \Big\| \sum_j \tau_j \vec{\sigma}_j r_j^2 \Big\| \Big\rangle\,,
\end{aligned} \tag{20}$$

and

$$M_{\sigma\cdot rr} = \Big\langle \Big\| \sum_j \tau_j \sigma_j \cdot \vec{\mathrm{r}}_j \vec{\mathrm{r}}_j \Big\| \Big\rangle.$$

Comparison of the calculated value of c_1 with its experimental value [from Eq. (18)] indicates that the shell-model prediction is nearly an order of magnitude too small. However, the wave function calculation of M_{GT} involves considerable cancellation among several terms of comparable magnitude and thus could well be unreliable even as to the overall sign. Such cancellations are not such important features in the remaining reduced matrix elements, so that perhaps estimates of b, c_2, d, and h may be more reliable.

Thus, using the shell-model calculation and the experimental $c_1 = 0.016$, we find

$$\frac{b}{Ac_1} \approx -19\,,$$
$$\frac{d}{Ac_1} \approx -3.2\,,$$
$$\frac{c_2}{c_1 R^2} \approx -0.37\,, \tag{21}$$

and

$$\frac{h}{A^2 c_1} \approx 1200\,.$$

Together, these results predict a ≈3% effect on the skew ratio, primarily due to the large calculated weak-magnetism contribution. The experimental limit on b is already below this calculated value, making the effect even smaller. Both h and c_2 would need to be an order of magnitude larger to explain the skew ratio; and, there are indications to suspect that the impulse approximation may be unreliable for hindered transitions in calculating c_2.[34] Similar considerations may also apply to d and h.[35] In the best possible case, assuming $b \approx -13$, we would need values like $c_2/c_1R^2 = 1.2$ and $h/A^2c_1 = 1.2 \times 10^4$ simultaneously to explain the ϵ/β^+ anomaly.

Now finally, we apply these considerations to the interpretation of the remaining pieces of ^{22}Na-decay data. The best available shape-factor measurements are given in Table II. Wenninger, Stiewe, and Leutz[8] made two independent measurements of the spectral shape factor with a ^{22}Na doped NaI(Tl) crystal and with a magnetic spectrometer. We have reanalyzed their published NaI(Tl) data in the range $100 < E_\beta < 400$ keV, including corrections for photon emission,[36,37] annihilation in flight,[38] and escape of the positron from the detector. The corrected data are displayed in Fig. 5.[39] The best linear slope to these corrected data is presented in Table II. These corrections are quite large and may be indicative of possible systematic errors inherent to the other measurements. The published data all indicate essentially zero slope; however, corrections such as are discussed here all shift higher-energy β particles to lower energies in the measured spectrum. This leads to a slope that may be greater than zero; therefore, the data must be considered as inconclusive at this time.

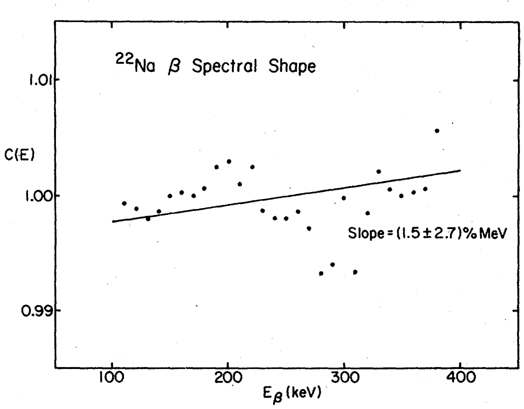

FIG. 5. ^{22}Na β^+-decay shape-factor data obtained by Wenninger, Stiewe, and Leutz,[8] using a NaI(Tl) crystal. See text for an explanation of the analysis and corrections.

In Fig. 2 we showed that only an extraordinarily large value of d can give a large negative slope, h gives a small negative slope, and b or c_2 give positive slopes for values consistent with the measured skew ratio. A negative slope is therefore very unlikely because d cannot explain the skew ratio; thus, a positive slope from the measured data is probably correct. Nevertheless, the measured slope discussed here is too uncertain to be trusted completely.

The best measurements of the β-γ directional correlation A_{22}, are given in Table III. Here, unfortunately, we cannot draw any firm conclusions because there exist two additional axial form factors of rank 2 which can in principle also affect the resulting A_{22}, although not the skew ratio or slope.[40] In Fig. 3 we showed the calculated A_{22} values for b, c_2, d, and h. Only b and d can give a significant negative A_{22}, and values of h and c_2 can give virtually zero A_{22}.

IV. CONCLUSIONS

The explanation of the ^{22}Na ϵ/β^+ anomalous ratio offers several avenues. The most likely possibility seems to us to be that the second-forbidden terms are larger than expected. Thus for example if $b/Ac_1 = h/A^2c_1 = 0$, $c_2/c_1R^2 = +2$, and $d/Ac_1 = -30$, we can obtain $\forall = 0.94$, $S = +1\%$/MeV, and $A_{22} = -1.3 \times 10^{-3}$, which are consistent with all the existing data for ^{22}Na. Given the experimental uncertainties this solution is not unique, but it must be emphasized that *any solution consistent with the skew ratio requires large values for the second-forbidden terms.* Additionally, we mention for completeness that a Fierz term of size $b_F \approx -0.05$ will also explain the skew ratio; however, it will require a significant negative slope for the shape factor. The ^{22}Na slope data do not support this conclusion and in view of the present limits on the absence of Fierz interference[34] we consider this possibility a remote one.

In order to clear up this situation further we suggest three courses. First, of course, is a careful analysis of the discrepancy between the various

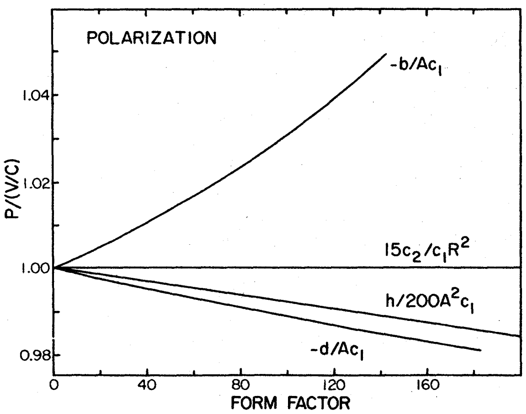

FIG. 6. Predicted variation of the ^{22}Na β^+ longitudinal polarization P_L [actually $(E/p)P_L$] with the "elementary-particle" amplitudes b, c_2, d, and h.

ϵ/β^+ measurements. Second, is a remeasurement of the shape factor in order to confirm the size and sign of the slope S. Third, it is important to have an additional *independent* experiment on this system. One possibility is a precise measurement of the positron longitudinal polarization P_L to a precision of about 1 part in 10^3. This may be quite feasible using a newly designed polarimeter.[41] We find

$$\frac{E}{p}P_L \approx 1 - 10^{-4}\left(2.46\frac{b}{Ac_1} + 1.23\frac{d}{Ac_1} + 0.0039\frac{h}{A^2c_1} - 0.13\frac{c_2}{c_1R^2}\right) \tag{22}$$

at $E=760$ keV. This is displayed graphically in Fig. 6. Large values of b or d, as discussed previously, will yield larger deviations from unity in opposite directions, h yields only a moderate deviation; and c_2 can yield none. Thus, both a new spectral shape remeasurement and a measurement of P_L would be most welcome in order to resolve the ^{22}Na question entirely.

Note added in proof. The experimental limit for $|b/Ac_1|$ relied on a poorly measured half-life of the $T=1$ state in ^{22}Na of $(9\pm5)\times10^{-15}$ sec. In order to obtain an upper limit on b, the lower value of $t_{1/2}=4\times10^{-15}$ sec was used. It has since come to our attention that the single-particle estimate for this $M1$ transition is $t_{1/2}=9.6\times10^{-15}$ sec. Using this value as our lower limit in $t_{1/2}$ we obtain a more stringent value of $|b/Ac_1|\leq 10$, which limits the weak magnetism contribution to the skew ratio as $\leq 1.6\%$.

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APPENDIX

We give here the relationships between the "elementary-particle" amplitudes $c, d, \ldots$, used in the multipole matrix elements ${}^AF_{101}^{(0)}$, ${}^VF_{111}^{(0)}, \ldots$, employed by Behrens and Jänecke.[22] The symbols are defined in the text for the "elementary-particle" amplitudes and in Ref. 22 for the multipole matrix elements. (These relations have been corrected so as to be gauge invariant.)

$$\begin{aligned}
{}^AF_{101}^{(0)} &= c_1 - d\frac{(E_0 - 3\alpha Z/2R)}{2M} + E_0^2 c_2 ,\\
{}^AF_{101}^{(1)} &= \frac{1}{R^2}\left(6c_2 + 2\frac{h}{4M^2}\right) ,\\
{}^AF_{110}^{(0)} &= \frac{\sqrt{3}}{2MR}\left(c_1 + d + h\frac{(E_0 - 3\alpha Z/2R)}{2M}\right),\\
{}^AF_{121}^{(0)} &= \frac{5\sqrt{2}}{(2MR)^2}h ,\\
{}^VF_{111}^{(0)} &= -\left(\frac{3}{2}\right)^{1/2}\frac{1}{MR}b .
\end{aligned} \tag{A1}$$

Equivalently,

$$\begin{aligned}
c_1 &= \frac{1}{[1 + (E_0 - 3\alpha Z/2R)/2M]}\\
&\quad\times\left[{}^AF_{101}^{(0)} - \tfrac{1}{6}(E_0R)^2\,{}^AF_{101}^{(1)} + \frac{(E_0 - 3\alpha Z/2R)R}{\sqrt{3}}\,{}^AF_{110}^{(0)}\right.\\
&\qquad\left. + \frac{R^2}{15\sqrt{2}}\,[E_0^2 - 3(E_0 - 3\alpha Z/2R)^2]\,{}^AF_{121}^{(0)}\right],\\
\frac{1}{R^2}c_2 &= \tfrac{1}{6}\,{}^AF_{101}^{(1)} - \frac{1}{15\sqrt{2}}\,{}^AF_{121}^{(0)} ,\\
d &= \frac{-1}{(1 + (E_0 - 3\alpha Z/2R)/2M)}\\
&\quad\times\left[{}^AF_{101}^{(0)} - \tfrac{1}{6}(E_0R)^2\,{}^AF_{101}^{(1)} - \frac{2MR}{\sqrt{3}}\,{}^AF_{110}^{(0)}\right.\\
&\qquad\left. + \frac{2MR^2}{15\sqrt{2}}\left(3(E_0 - 3\alpha Z/2R) + \frac{E_0^2}{2M}\right){}^AF_{121}^{(0)}\right],\\
h &= \frac{(2MR)^2}{5\sqrt{2}}\,{}^AF_{121}^{(0)} ,\\
b &= -\left(\frac{2}{3}\right)^{1/2} MR\,{}^VF_{111}^{(0)} .
\end{aligned} \tag{A2}$$

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