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Elastic scattering of positive pions on ^{12}C at 49.9 MeV

M. A. Moinester,* R. L. Burman, R. P. Redwine, and M. A. Yates-Williams
Los Alamos Scientific Laboratory†, Los Alamos, New Mexico 87545

D. J. Malbrough,§ C. W. Darden, R. D. Edge, T. Marks,§§ S. H. Dam, and B. M. Preedom
University of South Carolina,‖ Columbia, South Carolina 29208

F. E. Bertrand, T. P. Cleary, E. E. Gross, and C. A. Ludemann
Oak Ridge National Laboratory,¶ Oak Ridge, Tennessee 37830

M. Blecher, K. Gotow, D. Jenkins, and F. Milder
Virginia Polytechnic Institute and State University,†† Blacksburg, Virginia 24061
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Differential cross sections for the elastic scattering of positive pions on ^{12}C at 49.9 MeV were measured. The results differ from those deteermined by other investigators. The data are analyzed in terms of π^+ nucleus partial waves and are compared to various model calculations.

[NUCLEAR REACTIONS $^{12}C(\pi^+,\pi^+)^{12}C$. $E=49.9$ MeV, $\theta=25°-160°$ measured $d\sigma/d\Omega(\theta)$. Partial wave and optical model analyses.]

I. INTRODUCTION

There has been much discussion concerning the importance of low-energy pion-nucleus elastic scattering.[1-16] The discussion is concerned mainly with the observation that a first-order optical model whose strengths are determined from free πN amplitudes cannot describe the measured angular distributions and therefore higher-order effects such as two-nucleon correlations and true pion absorption must be included in the model. While data are now becoming available for nuclei other than ^{12}C (e.g., ^{16}O in Ref. 6), there has been considerable effort[8-16] spent trying to describe existing data for $\pi^+ + {}^{12}C$ elastic scattering.[7,8] This paper presents a new measurement of the angular distribution for the elastic scattering of π^+ from ^{12}C at 50 MeV. The methods used in this measurement are substantially different from those used in other experiments. For example, instead of using a one-detector or two-detector system, we used a ten-counter array. We find disagreement with previously published data.

II. EXPERIMENTAL TECHNIQUES AND APPARATUS

The measurements were made using the LAMPF low-energy pion channel. The experimental setup has been described in detail elsewhere[6]; we will emphasize here only those points particular to the ^{12}C measurements. Two targets (10 cm × 10 cm) mounted on plastic frames were used, one of natural carbon, 223(± 2) mg/cm thick, and one 251(± 2) mg/cm thick CH_2. The targets were mounted at 45° to the pion beam direction. An identical target frame, with no target, was used for background measurements. The pion energy at the center of the targets was 49.9 MeV. Ten plastic scintillator detector telescopes distributed in reflection or transmission geometry viewed scattering in the vertical plane. These detectors are described in detail elsewhere.[6,17] Systematic normalization corrections, common to all detectors, are the same as those described previously.[6] The π^+ beam flux was measured at full intensity (~ 10^7 sec^{-1}) using three different pion monitor systems; two detector telescopes fixed at 90° and 105° and two small scintillators used in coincidence to detect muons resulting from in-flight pion decay. Using low π^+ intensity (~ 10^5 sec^{-1}), these monitors were calibrated against the number of incident π^+'s detected by a pair of in-beam plastic scintillators whose two-dimensional spectra provided pulse height separation between π's, μ's, and e's. The agreement between the various monitors was within counting statistics. Corrections for the finite geometry of the beam, for the finite 4° acceptance of the detectors, and for multiple scattering in the target were carried out as described[6,18] and found to be less than 1% for the worst case at small angles.

III. EXPERIMENTAL RESULTS

The 49.9 MeV differential cross sections are listed in Table I. The cross-section values ob-

TABLE I. Differential cross sections for 49.9 MeV π^+-^{12}C elastic scattering.

$\theta_{c.m.}$ (deg)	$(d\sigma/d\Omega)_{c.m.}$ (mb/sr)[a]
30.5	8.98 ± 0.43
35.6	8.69 ± 0.43
40.6	7.13 ± 0.33
45.7	5.80 ± 0.30
55.8	3.43 ± 0.17
60.8	2.80 ± 0.14
70.9	2.63 ± 0.12
81.0	3.86 ± 0.07
86.0	4.66 ± 0.11
91.0	5.11 ± 0.05
101.0	6.37 ± 0.11
105.9	6.45 ± 0.13
110.9	6.64 ± 0.13
120.8	6.59 ± 0.13
130.7	6.07 ± 0.12
140.6	5.47 ± 0.25
150.5	4.94 ± 0.11
160.3	4.39 ± 0.08

[a]Uncertainties indicate only relative uncertainties between data points. There is in addition a normalization uncertainty of ±7%.

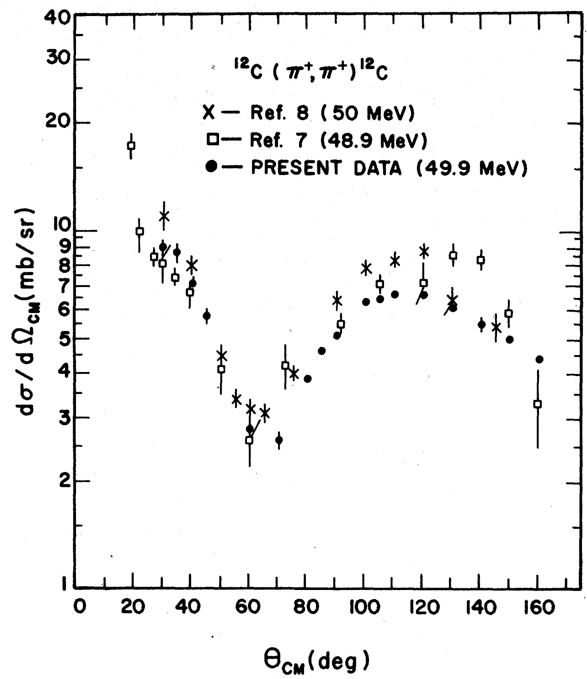

FIG. 1. Angular distribution for the elastic scattering of 49.9 MeV positive pions on ^{12}C (present data). Data from Ref. 8 at 48.5 MeV and from Ref. 7 at 48.9 MeV are also shown.

tained with the ^{12}C and CH_2 targets agreed to within counting statistics. The errors indicate only relative uncertainties between the data points. There is an additional uncertainty of ± 7% in the absolute normalization. The differential cross sections are plotted in Fig. 1, together with the results of the previous π^+-^{12}C experiments of Johnson *et al.*[7] (normalization uncertainty = ± 15%) and of Dytman *et al.*[8] (normalization uncertainty = ± 15%). There are differences in the shapes of the angular distributions of the different experiments, even allowing for normalization shifts of 15%.

To discuss the significance of the differences in the measured differential cross sections, it is necessary to summarize briefly the experimental techniques used by the three groups. In the work of Dytman *et al.*,[8] one detector system was used at different angles to detect the elastically scattered pions. In order to use the large, dispersed beam spot of the LAMPF EPICS channel, the detector system consisted of three multiwire chambers to determine the trajectory of the π^+ and two intrinsic germanium detectors to stop the π^+ and provide total energy information. They used a CH_2 target and normalized their data to measured $\pi^+ + p$ cross sections.[19] In the work of Johnson *et al.*,[7] two-detector systems were used simultaneously. Each system consisted of three plastic scintillators, the first two defining the solid angle subtended by the telescopes and the third providing total energy information. To determine the pion flux, they used an in-beam counter.

In order to minimize the relative error between differential cross sections measured at various angles, we used the ten-detector array discussed above. Keeping the monitor detectors fixed, we measured a complete angular distribution (18 angles) in only two runs. The 10 angles taken in each run have the same absolute normalization and we observed no significant differences in normalizations between the two runs. An indication of the correctness of our absolute normalization is given by the comparison shown in Fig. 2 between our measured values of the absolute differential cross-section for $\pi^+ + p$ scattering with that measured by Bertin *et al.*[19] As is seen in Fig. 2 the two data sets agreed within the uncertainty on our data (~± 10%).

IV. ANALYSIS

We present a partial-wave fit to the angular distribution, a fit using the form of a first-order Kisslinger optical potential, and a comparison of our data with various contemporary optical

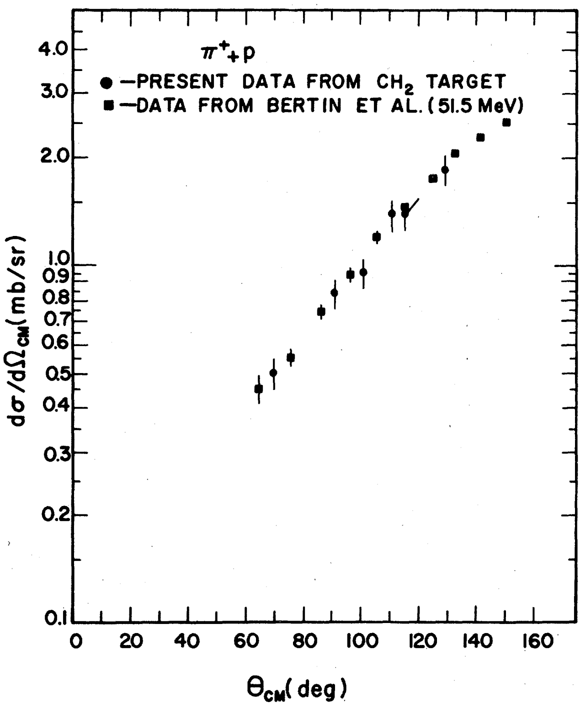

FIG. 2. Angular distribituons for $\pi^+ + p$.

model calculations which include second-order corrections.

A. Partial wave representation of the data

We have analyzed the data using a partial-wave analysis code written by Gibbs, Gibson, and Stephenson.[20] Using the standard notation[6] of η_l (inelasticity parameter) and δ_l (phase shift) for the l^{th} partial wave, the results of two equivalent fits to the data are presented in Table II. Considering the uncertainties in determining η and δ by a statistical fit, both solutions are consistent with unitarity. Also shown in Table II are the total cross sections as determined by the imaginary part of the forward scattering amplitude. We consider the good fits obtained with reasonably small values of l_{max} to be an indication of the general consistency of the data. The χ^2 value of the phase-shift analysis sets the reasonable lower limit for fits based on optical model calculations.

B. Phenomenological optical model fits

The elastic scattering data were compared to optical model calculations carried out with the FITPI code of Cooper and Eisenstein.[21] We used this code to solve a modified Klein-Gordon equation with a Kisslinger-model potential, incorporating a modified Gaussian density function,[22] as described in Table III.

With the charge density fixed, we varied the potential parameters b_0 and b_1, and the nuclear potential radius R. The optical model fit (solid curve) is compared with the data in Fig. 3. The best-fit potential parameters are given in Table III. The χ^2 per degree of freedom is 1.4. Table III also shows the values for σ (reaction) and σ (elastic) calculated from the best-fit optical potential parameters. We found the best-fit potential radius to be 2.50 fm but we also attempted to fit the data keeping the potential radius value at $R = 2.31$ fm, corresponding to subtracting the nucleon matter radius of 0.80 fm from the electron scattering rms value of 2.44 fm. This procedure

TABLE II. 49.9 MeV π^+-^{12}C elastic scattering phase shift parameters deduced from a partial-wave analysis and from the best-fit Kisslinger potential.

Analysis	Solution	l	η_l	δ_l (deg)	χ^2 per degree of freedom	σ(total)[a] (mb)
Partial wave	*A*	0	0.93	−8.9	0.66	229
		1	0.76	14.7		
		2	0.98	6.1		
		3	1.01	0.7		
Partial wave	*B*	0	0.53	−16.1	0.66	235
		1	0.97	12.2		
		2	0.96	5.7		
		3	0.99	0.6		
Optical model		0	0.79	−14.5	1.4	204
		1	0.85	14.1		
		2	0.97	5.8		
		3	1.00	0.9		

[a] Uncertainties were not determined in the analysis.

TABLE III. Best-fit values (uncertainties for the best-fit values were not determined in the analysis) for the Kisslinger potential. The Kisslinger potential form is $U(r) = -b_0k^2\rho(r) + b_1\vec{\nabla}\cdot\rho(r)\vec{\nabla}$, where $\rho(r)$ is the nuclear density, k is the incident pion wave number, and b_0 and b_1 are complex parameters related to the pion-nucleon amplitudes. We use the notation b_{0R} and b_{0I} for the real and imaginary parts of b_0, and similarly for b_1. A modified Gaussion density function

$$\rho(r) = \frac{\kappa^3}{3(\sqrt{\pi}R)^3}\left[1+\frac{4}{3}\left(\frac{\kappa r}{R}\right)^2\right]\exp\left(\frac{-\kappa^2r^2}{R^2}\right)$$

was used for the nuclear potential and charge densities, where R is the appropriate rms radius and $\kappa = 1.472$ for ^{12}C. The charge distribution in ^{12}C as determined by electron scattering has an rms radius of 2.44 fm. The value R which was used for the charge density was 2.54 fm, corresponding to adding the pion charge radius of 0.71 fm in quadrature to the rms radius of the charge distribution. The value of R which should, in principle, be used for the nuclear potential is 2.31 fm, corresponding to subtracting the proton charge radius of 0.80 fm. Parameters b_0 and b_1, and for σ (reaction) and σ (elastic) for 49.9 MeV π^+-^{12}C elastic scattering.

b_{0R} fm^3	b_{0I} fm^3	b_{1R} fm^3	b_{1I} fm^3	χ^2 Per degree of freedom	σ (elastic) mb	σ (reaction) mb	Nuclear potential R (fm)
−3.28	0.31	6.75	0.27	1.4	87.	117.	2.50

resulted in a fit giving a χ^2 per degree of freedom of 19.3. A similar analysis was carried out for $\pi^+ + {}^{16}$O scattering.[6] There also, the χ^2 fits are improved by changing the potential radius. The increased value of the radius in our simple phenomenological analysis may mock up some other effects such as the finite range of the πN interaction, the Lorentz-Lorenz effect, and true pion absorption as discussed recently by Gibbs et al.[23] and by Miller.[24] The best-fit parameters in Table III differ from those of Dytman *et al.*[8] primarily in the absorptive strengths. While their S-wave potential was slightly pion producing [$\mathrm{Im}b_0 = -(0.60\pm 68)\%$] and their potential was slightly nonunitary ($|\eta_0| = 1.06$), our best-fit potential is better behaved and is unitary.

We can calculate the scattering amplitude parameters η_l and δ_l and the total cross section from the best-fit optical model solution. Table II shows these values for comparison with those obtained from the phase-shift fits described in Sec. IV A.

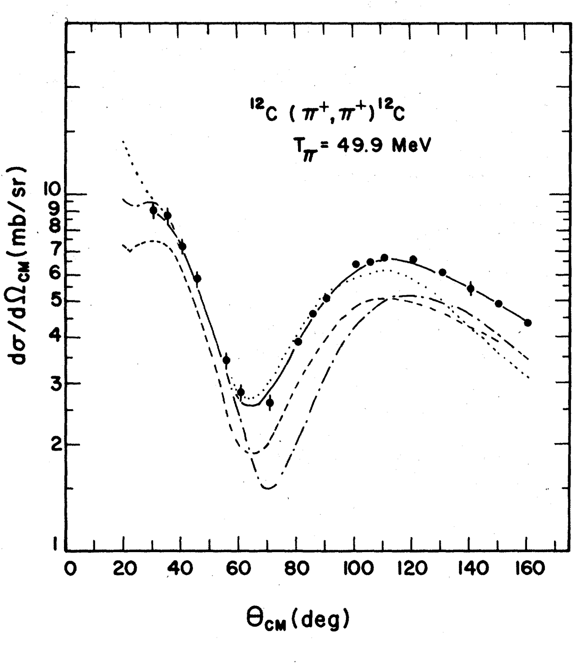

FIG. 3. Angular distribution at 49.9 MeV for the elastic scattering of positive pions on ^{12}C (present data). The solid curve is a first-order optical model fit to the data, as descibed in the text. The other curves are calculations using first-order optical models with second-order corrections as published by Liu and Shakin, Ref. 12 (---), Landau and Thomas, Ref. 13 (-·-), and Stricker *et al.*, Ref. 14 (···).

C. Higher-order optical model calculations

Several recent calculations can be compared with the data presented here. In Fig. 3 we show calculations due to Landau and Thomas,[13] Liu and Shakin[12] and by Stricker *et al.*[14] The first two calculations use a finite range πN interaction and include second-order effects accounting for "true pion absorption" and nuclear-binding effects. While not in quantitative agreement with the data, the shape of the angular distribution is clearly reproduced. The calculation by Stricker *et al.* uses a modified pionic-atom potential and thus while not fitted to the $\pi^+ + {}^{12}$C data, it is nevertheless a phenomenological potential.

V. CONCLUSIONS

We have presented angular distributions for elastic scattering of π^+ on ^{12}C at 49.9 MeV. The present data differ from results published by other investigators. Fits to the data were made using both a π^+ – nucleus partial-wave analysis and a potential with the form of a first-order Kisslinger optical model. Good fits were obtained with scattering amplitudes which preserve unitarity. The data were also compared with contemporary optical model calculations which include some second-order effects. These calculations show general agreement with the data.

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*On leave from Tel Aviv University, Ramat Aviv, Israel.

§Present address: Louisiana State University, Baton Rouge, Louisiana.

§§Present address: TRIUMF, University of British Columbia, Vancouver, BC8, Canada.

**Present address: Boston University, Boston, Massachusetts.

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