Effect of N^* resonances and the effective channel approach to nucleon-nucleus scattering

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Recent experimental results for p-^{4}He elastic scattering at 1 GeV are in fair agreement with the previous Saclay data, and also with the effective channel approach calculation, while a recent multiple diffraction analysis has found the N^* effect important near the first diffraction minimum. In view of these, we comment on the role of the N^* and other rearrangement channels within the effective channel approach, and show that the effective channel approach contains these effects collectively. The nonorthogonality and double-counting problems are discussed.

[NUCLEAR REACTIONS Effective channels for proton-^{4}He scattering, rearrangement channels, N^* effects, nonorthogonality.]

The multiple scattering, optical potential approach[1-4] has been used previously in the analysis of nucleon-nucleus scattering at high energies. In particular, the effective channel approach[4] (ECA) was applied with some success[5,6] to proton-^{4}He scattering at 1 GeV; it was demonstrated that the Saclay original 1974 data[7] had the angular distribution which was more consistent with the theory and the available input data than that of an earlier Brookhaven experiment.[8] Although the input parameters used for the nucleon-nucleon scattering and for the He nucleus were not very accurate, and the spin and isospin effects were averaged, the analysis was still sufficiently reliable to distinguish between the two available data at that time.

More recently, two additional sets of experimental data[9,10] have been reported for p-He elastic scattering, and a quite different theoretical study[11] based on the multiple diffraction theory[12] with various refinements[13-15] has also been reported, which also favors the original Saclay and the more recent data. However, the effect of the N^* resonance propagation[16] was found essential in reproducing the correct diffraction minimum, while the spin and isospin effects are minimal at this energy.

In view of these developments, both in theory and in experiment, it is important to clarify the role of N^* in these theoretical analyses and to point out some of the difficulties in explicitly incorporatint the resonance propagation and other rearrangement channels into the optical potential analysis, and into the diffraction formulation. The ECA employed in Refs. 5 and 6 will be shown to include these effects collectively through the parameters which were introduced for the NN interaction and the total p-^{4}He cross section.

1. In the effective channel approach, the scattering function Ψ is written as

$$\Psi(\vec{r}, \vec{R}) = P\Psi + Q\Psi \approx \psi_0(\vec{r})u_0(\vec{R}) + \varphi(\vec{r}, \vec{R})u_\varphi(\vec{R}), \quad (1)$$

where $\psi_0(\vec{r})$ is the initial target function and $\varphi(\vec{r}, \vec{R})$ is an effective inelastic excitation function which depends in general on the projectile position $\vec{R}$ and is chosen to be orthogonal to ψ_0; in (1), $P = |\psi_0\rangle\langle\psi_0|$ and $Q = 1 - P$, so that

$$(\psi_0, \varphi)_{\vec{r}} = 0 \quad (PQ = 0). \quad (2)$$

The orthogonality property (2) is required to avoid possible double counting of the contributions from P and Q. Furthermore, if one particular subset Q_s of the Q space is strongly coupled to the P channel, then the Q space may more appropriately be divided up as $Q = Q_r + Q_s$, again with

$$PQ_r = PQ_s = 0 \quad (\text{and } Q_rQ_s = 0). \quad (3)$$

When the channels contained in Q_s involve particle rearrangements, explicit construction of this operator is difficult, and an alternative procedure was developed earlier.[17]

It is important to recognize, however, that all these inelastic channels *can* be treated together collectively because of the closure property $Q = 1 - P$. Thus, the ECA calculation *contains* in principle *all* the rearrangement channels through the function φ; the propagation of the composite projectile-target system in the intermediate states is described by the operator

$$[E - (\varphi, H\varphi)_{\vec{r}} + i\epsilon]^{-1},$$

and the overall magnitude of the coupling between the P and Q spaces is also fixed by the use of the experimental total cross section for the p-^{4}He system.

2. The effect of various resonance propagations on the proton-nucleus scattering has been studied previously,[16] and found to be significant near the first diffraction minimum and for the polarization.[16,11] It is generally expected that the addition of such a channel fills in the diffraction minima, because two different parts of an amplitude will seldom have vanishing magnitudes at the same momentum transfer values.[15] It is a much more difficult task to ensure that such an addition of a new channel is mathematically consistent and thus avoids the problem of double counting. We first examine the procedure involved in the ECA, with particular attention directed to the effect of N^* and the double counting problem.

To clarify the contents of the effective NN potential v used in the ECA, we consider a model three-body system involving p, π, and N_c, with $N=(\pi+N_c)_0$ and $\Delta=(\pi+p)_0$. We also let the Hamiltonian and the energy of this system be h and e, respectively, and define the projection operators p_s and p_Δ for the pN and $N_c\Delta$ channels. Obviously, $p_s p_\Delta \neq 0$, unless the isospin components are included explicitly, where the N has $I=\frac{1}{2}$ and Δ has $I=\frac{3}{2}$. In the simple two-channel approximation describing the coupling $p+N \leftrightarrow \Delta+N_c$, we have the pN channel scattering equation[17] of the form

$$p_s\left[(h-e)+(h-e)p_\Delta\frac{1}{p_\Delta(e-H+i\epsilon)p_\Delta}p_\Delta(h-e)\right]p_s\psi_s = 0 \equiv p_s D_s p_s \psi_s . \quad (4)$$

In terms of the pN relative kinetic energy operator k_s, the first term in (4) is written as

$$p_s(h-e)p_s = p_s(k_s-e_s+v^s)p_s \equiv -g_s^{-1} . \quad (5)$$

Therefore, the effective potential in the pN channel is given by

$$w_s \equiv p_s\left[v^s+(h-e)p_\Delta\frac{1}{p_\Delta(e-h+i\epsilon)p_\Delta}p_\Delta(h-e)\right]p_s \equiv p_s w p_s , \quad (6)$$

which in turn gives the pN amplitude

$$t_s = w_s + w_s g_s t_s . \quad (7)$$

The important property of the operator D_s is that, although $p_s p_\Delta \neq 0$,

$$D_s p_\Delta = p_\Delta D_s = 0 ; \quad (8)$$

that is, D_s is an operator in the p_s space and orthogonal to the p_Δ channel. It is crucial, however, to retain the exact form (4) for D_s in order to satisfy (8); any modification of (4) immediately destroys (8).

Evidently, w_s contains the effect of the p_Δ channel, but, because of the relation (8), *cannot* directly couple the p_Δ channel to the p_s. This is also the case with t_s, whose on-shell behavior is used in the parametrization of the effective w_s. So long as a Hamiltonian whose structure is identical to (4) is used in the proton-nucleus problem, there will be no ambiguity of double counting. To completely include the effect of the Δ propagation in the nucleon-nucleus scattering, it is necessary to construct, in addition to w_s and w_Δ, the coupling potential $v_{s\Delta}=p_s(h-e)p_\Delta$; this is possible only if the NN problem is studied together with the NN^* and other channels in the parametrization of the elementary potential. A similar procedure was advocated for quite different reasons by Londergan *et al.*[18] in connection with the π-nucleus scattering.

In the ECA, the NN potential v is parametrized, with its form quite different from that of (6), so that the orthogonality properties (8) are no longer applicable. Therefore, it would in general be impossible to prevent v from coupling to other two-body channels, such as the NN^* and πd, and the effect of the N^* resonances is implicitly included in the ECA.

3. Next, let us examine the N^* problem in the original optical potential approach[3] and in the multiple diffraction theory.[16] In both cases, the elementary input is the projectile-target nucleon amplitude τ in the presence of other nucleons as spectators, with the antisymmetrization correctly taken into account. As τ is very difficult to evaluate, it is usually replaced by an on-shell two-nucleon amplitude $t_s^{\rm on}$, which is in turn parametrized in a form similar to v. The orthogonality of the type (8) is again lost and some effect of Δ may have crept in.

We next examine the double counting problem in the multiple diffraction formalism.[16] The nucleon-nucleon elastic profile function Γ_j, the N^* production profile Γ_j^Φ, and the nucleon-N^* elastic profile Γ_j^* are used to expand the proton-nucleus Γ, as

$$\Gamma = \Gamma_s + \Gamma_\Delta , \quad (9)$$

where

$$\Gamma_s = \sum_{j=1}^{A}\Gamma_j - \sum_{j<m}\Gamma_j\Gamma_m + \cdots ,$$
$$\Gamma_\Delta = -\sum_{j<m}\Gamma_j^\Phi\bar{\Gamma}_m^\Phi + \sum_{j<m<k}(\Gamma_j^\Phi\Gamma_m^\Phi\Gamma_k + \Gamma_j^\Phi\Gamma_m^*\bar{\Gamma}_k^\Phi + \cdots) + \cdots . \quad (10)$$

The Γ_j is related to the elastic nucleon-nucleon amplitude t_{sj} through a two-dimensional Fourier transform and similarly for Γ_j^Φ. A linearized propagator in the closure approximation is assumed in (10), between the t_{sj} and t_{sm} interactions. While the effect of $NN \leftrightarrow NN^*$ on t_{sj} is already included in Γ_j, as is clear from (6) and (7), its contribution for $j \neq m$ is to be included via Γ_Δ, thus avoiding a double counting of the Δ effect. However, the parametrization of Γ_j, Γ_j^*, and Γ_j^Φ is done in practice in exactly the same way as in the ECA and in the optical potential approach. Therefore, the orthogonality property is again lost, and the intermediate states between (Γ_j and Γ_m) and also between (Γ_j^Φ and $\overline{\Gamma}_m^\Phi$) may overlap, especially through the target excitation, rearrangement, and exchange effects, thus again giving rise to a double counting problem. This difficulty should be much less severe when explicit isospin states are used,[11] with special care in parametrizing the necessary inputs and extending them off shell.

4. Summary. We have pointed out that the ECA calculation of Ref. 6 includes collectively the effects of N^* and other rearrangement channels through (a) the parameter β' in the effective excitation function φ which was determined by the p-^{4}He total cross section, (b) the propagation of the system in the intermediate states described by the operator $(\varphi, H\varphi)_{\bar{r}}$, and (c) the parameter $\rho_v = \mathrm{Re}v/\mathrm{Im}v$ which is not well determined by the existing NN data. The shape of the diffraction minimum near $\theta \approx 24°$ was found to be sensitive to both β' and ρ_v. On the other hand, an explicit inclusion of the N^* channel into the ECA requires (i) a careful parametrization of the $NN \rightarrow N'N^*$ coupling and (ii) the use of the spin and isospin state functions to enforce the orthogonality property between the proton and the N^* channels.

In view of the theoretical difficulties discussed above and also of the poor input information available at present, any finer details in the agreement between the experiments[7-10] and theoretical calculations[3,5,6,11,19-21] should be taken with caution; the latest experiment[22] favors the Saclay 1976 data, while theoretical uncertainties[6] are probably larger than the differences among the experimental data.

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