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Isospin nonconservation in the N-N interaction

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A relativistic formalism of the N-N scattering including the electromagnetic contributions is presented. Relativistic electro- magnetic phase shifts are defined and evaluated. The difference between the nuclear phase shifts of the p-p and n-p interactions is evaluated taking into account the interference between strong and Coulomb forces, electromagnetic n-p contributions and the mass differences between the charged and neutral pions. The n-p interaction is described with the aid of six amplitudes. Neutron- proton observables are calculated with these six amplitudes. The mixing angle of the singlet-triplet transitions is evaluated. Experiments which should show these transitions are discussed.

[NUCLEAR REACTIONS N-N interaction, electromagnetic effects, isospin nonconservation.]

I. INTRODUCTION

The study of the amount of isospin nonconservation in the N-N interaction is interesting for two reasons. First, one would like to have an exact evaluation and experimental verification of it. The second reason is of practical nature. At present there are not enough experimental data to perform the n-p phase shift analysis and one is obliged to use the isospin $T=1$ phase shifts obtained from the p-p phase shift analysis. If one would like to perform a precise n-p phase shift analysis the differences between the p-p and n-p $T=1$ phase shifts should be taken into account as well as the electromagnetic contributions of the n-p interaction. The isospin nonconservation comes from two sources: from the isospin noninvariant electromagnetic interaction, from the mass differences between the proton and neutron and the mass differences of exchanged charged and neutral particles (especially the pions). In order to cope with all these problems we had to develop a six amplitude formalism of the n-p interaction and to use a sixth phase shift parameter, the mixing angle of the singlet-triplet transitions.

The presentation of our work follows this plan: In Sec. II we present the six amplitude formalism of the n-p interaction including the electromagnetic effects. In Appendix A we give formulas for c.m. observables in terms of these six amplitudes and in Appendix B we give formulas for lab system observables. In Sec. III we evaluate the electromagnetic contributions to the n-p interaction, while in Sec. IV we evaluate the differences between the p-p and n-p phase shifts. In relation to this we give in Appendix C formulas for the isospin nonconserving one-pion-exchange (OPE) contribution. In Sec. V we show results of some calculated observables and suggest possible experiments for detecting the isospin nonconservation. In Sec. VI we evaluate our results.

II. SIX AMPLITUDE FORMALISM OF THE N-N INTERACTION

There are many ways of choosing the six independent amplitudes of the isospin nonconserving N-N interaction. As the electromagnetic contributions are in a simple way evaluated in the framework of the helicity formalism[1] we shall start with the helicity amplitudes $\langle\lambda_3\lambda_4|\phi|\lambda_1\lambda_2\rangle$ which have the partial wave expansion

$$\langle\lambda_3\lambda_4|\phi|\lambda_1\lambda_2\rangle=\frac{1}{p}\sum_{J=0}^{\infty}(2J+1)\phi^J(\lambda_1,\lambda_2,\lambda_3,\lambda_4)\times d^J_{\lambda_1-\lambda_2,\lambda_3-\lambda_4}(\theta)\,, \tag{2.1}$$

where p is the c.m. momentum.

In order to include properly the electromagnetic contributions we split the right-hand side of Eq. (2.1) into

$$\langle\lambda_3\lambda_4|\phi|\lambda_1\lambda_2\rangle=\frac{1}{p}\sum_{J=0}^{\infty}(2J+1)[\phi^J(\lambda_1,\lambda_2,\lambda_3,\lambda_4)-\phi^J_{\rm EM}(\lambda_1,\lambda_2,\lambda_3,\lambda_4)]\times d^J_{\alpha\beta}(\theta)+\langle\lambda_3\lambda_4|\phi_{\rm EM}|\lambda_1\lambda_2\rangle, \tag{2.2}$$

where $\alpha=\lambda_1-\lambda_2$ and $\beta=\lambda_3-\lambda_4$,

$$\langle\lambda_3\lambda_4|\phi_{\rm EM}|\lambda_1\lambda_2\rangle=\frac{1}{p}\sum_J(2J+1)\phi^J_{\rm EM}(\lambda_1,\lambda_2,\lambda_3,\lambda_4)d^J_{\alpha\beta}(\theta) \tag{2.3}$$

is the electromagnetic contribution.

It should be noted that due to the infinite range of the electromagnetic interaction the expansions (2.1) or (2.3) might diverge. It was shown by Taylor[2] and Semon and Taylor[3] that Eqs. (2.1)–(2.3) are correct in terms of distributions.

The six independent helicity amplitudes of the n-p interaction are

$$\phi_1(\theta)=\langle ++\,|\phi|++\rangle=\frac{1}{p}\sum_J(2J+1)\phi_1^J d^J_{00}(\theta)\,,$$

$$\phi_2(\theta)=\langle ++\,|\phi|--\rangle=\frac{1}{p}\sum_J(2J+1)\phi_2^J d^J_{00}(\theta)\,,$$

$$\phi_3(\theta)=\langle +-\,|\phi|+-\rangle=\frac{1}{p}\sum_J(2J+1)\phi_3^J d^J_{11}(\theta)\,,\tag{2.4}$$

$$\phi_4(\theta)=\langle +-\,|\phi|-+\rangle=\frac{1}{p}\sum_J(2J+1)\phi_4^J d^J_{-1,1}(\theta)\,,$$

$$\phi_5(\theta)=\langle ++\,|\phi|+-\rangle=\frac{1}{p}\sum_J(2J+1)\phi_5^J d^J_{10}(\theta)\,,$$

$$\phi_6(\theta)=\langle ++\,|\phi|-+\rangle=-\frac{1}{p}\sum_J(2J+1)\phi_6^J d^J_{10}(\theta)\,,$$

where + or − denotes $\lambda=+\frac{1}{2}$ or $\lambda=-\frac{1}{2}$. For the p-p interaction only five amplitudes are independent (due to the fact that the protons are identical) with

$$\phi_6(\theta)=-\phi_5(\theta).$$

Another set of six independent amplitudes can be chosen which have the advantage that with them the N-N observables are relatively easily calculated and the observables are presented in a rather concise form. The N-N elastic scattering matrix is represented by[4,5]

$$\begin{aligned}M(\vec{k}_f,\vec{k}_i)=\tfrac{1}{2}\{&(a+b)+(a-b)(\vec{\sigma}_1\cdot\vec{n})(\vec{\sigma}_2\cdot\vec{n})\\&+(c+d)(\vec{\sigma}_1\cdot\vec{K})(\vec{\sigma}_2\cdot\vec{K})\\&+(c-d)(\vec{\sigma}_1\cdot\vec{P})(\vec{\sigma}_2\cdot\vec{P})+e[(\vec{\sigma}_1+\vec{\sigma}_2)\cdot\vec{n}]\\&+f[(\vec{\sigma}_1-\vec{\sigma}_2)\cdot\vec{n}]\},\end{aligned}\tag{2.5}$$

where $\vec{k}_i$ and $\vec{k}_f$ are c.m. initial and final momenta, respectively, a, b, c, d, e, and f are the six amplitudes and

$$\vec{P}=\frac{\vec{k}_f+\vec{k}_i}{|\vec{k}_f+\vec{k}_i|}\,,\quad \vec{K}=\frac{\vec{k}_f-\vec{k}_i}{|\vec{k}_f-\vec{k}_i|}\,,\quad \vec{n}=\frac{\vec{k}_i\times\vec{k}_f}{|\vec{k}_f\times\vec{k}_i|}\,.\tag{2.6}$$

The amplitude f is the isospin violating one. In Appendix A we give a list of c.m. observables calculated with these six amplitudes. The notation and the procedure follows that presented in the review of Hoshizaki.[6] The exception is P_{n0} which denotes the neutron polarization in neutron-proton scattering, while P_{0n} denotes the polarization of the proton. As we shall see the difference between these two observables is a sensitive measure of isospin nonconservation. In Appendix B we give a list of observables defined in the laboratory system. We again follow the notation and procedure used in Hoshizaki's review with the x,y,z directions determined by the unit vectors

$$\vec{z}=\vec{p}_i/|\vec{p}_i|\,,\quad \vec{y}=\vec{n}\,,\quad \vec{x}=\vec{n}\times\vec{z}\,,\tag{2.7}$$

where $\vec{p}_i$ and $\vec{p}_f$ are laboratory system initial and final momenta respectively.

The transition between the a, b, c, d, e, f amplitudes and the helicity amplitudes can be obtained using a procedure outlined in Hoshizaki's review. We obtain

$$\begin{aligned}\phi_1&=\tfrac{1}{2}(a\cos\theta+b-c+d+ie\sin\theta),\\\phi_2&=\tfrac{1}{2}(a\cos\theta-b+c+d+ie\sin\theta),\\\phi_3&=\tfrac{1}{2}(a\cos\theta+b+c-d+ie\sin\theta),\\\phi_4&=\tfrac{1}{2}(-a\cos\theta+b+c+d-ie\sin\theta),\\\phi_5&=\tfrac{1}{2}(-a\sin\theta+ie\cos\theta-if),\\\phi_6&=\tfrac{1}{2}(a\sin\theta-ie\cos\theta-if),\end{aligned}\tag{2.8}$$

where θ is the c.m. scattering angle.

The inverse relations are

$$\begin{aligned}a&=\tfrac{1}{2}[(\phi_1+\phi_2+\phi_3-\phi_4)\cos\theta-2(\phi_5-\phi_6)\sin\theta],\\b&=\tfrac{1}{2}(\phi_1-\phi_2+\phi_3+\phi_4),\\c&=\tfrac{1}{2}(-\phi_1+\phi_2+\phi_3+\phi_4),\\d&=\tfrac{1}{2}(\phi_1+\phi_2-\phi_3+\phi_4),\\e&=-\tfrac{1}{2}i[(\phi_1+\phi_2+\phi_3-\phi_4)\sin\theta+2(\phi_5-\phi_6)\cos\theta],\\f&=i(\phi_5+\phi_6).\end{aligned}\tag{2.9}$$

In order to complete our discussion of the six amplitudes we still have to give a convenient representation of the partial wave amplitudes. For this purpose we shall generalize the bar phase shift L-S representation, where L is the total orbital angular momentum and S is the total spin. The T-matrix elements in this representation, $\langle LS|T(J)|L'S'\rangle$, are given by

$$\begin{aligned}2i\langle J0|T(J)|J0\rangle&=\cos2\bar{\gamma}_J\exp(2i\bar{\delta}_J)-1,\\2i\langle J1|T(J)|J1\rangle&=\cos2\bar{\gamma}_J\exp(2i\bar{\delta}_{JJ})-1,\\2i\langle J1|T(J)|J0\rangle&=-i\sin2\bar{\gamma}_J\exp(i\bar{\delta}_{J+i}\bar{\delta}_{JJ}),\\2i\langle J\pm1,1|T(J)|J\pm1,1\rangle&=\cos2\bar{\epsilon}_J\exp(2i\bar{\delta}_{J\pm1,J})-1,\\2i\langle J\pm1,1|T(J)|J\mp1\rangle&=-i\sin2\bar{\epsilon}_J\exp(i\bar{\delta}_{J-1,J+i}\bar{\delta}_{J+1,J}).\end{aligned}\tag{2.10}$$

The new parameter $\bar{\gamma}_J$ is the mixing angle of the singlet-triplet transition.

The relation between the T-matrix elements (2.10) and the T-matrix elements ϕ_i^J of the helicity representation (2.4) are[7]

$$\begin{aligned}
&\langle J,0\,|\,T(J)\,|\,J,0\rangle=\phi_1^J-\phi_2^J,\\
&\langle J,1\,|\,T(J)\,|\,J,1\rangle=\phi_3^J-\phi_4^J,\\
&\langle J,0\,|\,T(J)\,|\,J,1\rangle=\phi_6^J-\phi_5^J,\\
&\langle J\mp1,1\,|\,T(J)\,|\,J\mp1,1\rangle=\{(J+\tfrac12\mp\tfrac12)(\phi_1^J+\phi_2^J)+(J+\tfrac12\pm\tfrac12)(\phi_3^J+\phi_4^J)\pm2[J(J+1)]^{1/2}(\phi_5^J+\phi_6^J)\}/(2J+1),\\
&\langle J-1,1\,|\,T(J)\,|\,J+1,J\rangle=\{-[J(J+1)]^{1/2}(\phi_1^J+\phi_2^J-\phi_3^J-\phi_4^J)-\phi_5^J-\phi_6^J\}/(2J+1).
\end{aligned}\tag{2.11}$$

The inverse relations are

$$\begin{aligned}
&\phi_1^J=\tfrac12[\langle J,0\,|\,T(J)\,|\,J,0\rangle+\phi_{12}^J], \quad \phi_2^J=\tfrac12[-\langle J,0\,|\,T(J)\,|\,J,0\rangle+\phi_{12}^J],\\
&\phi_3^J=\tfrac12[\langle J,1\,|\,T(J)\,|\,J,1\rangle+\phi_{34}^J], \quad \phi_4^J=\tfrac12[-\langle J,1\,|\,T(J)\,|\,J,1\rangle+\phi_{34}^J],\\
&\phi_5^J=\tfrac12[-\langle J,0\,|\,T(J)\,|\,J,1\rangle+\phi_{56}^J], \quad \phi_6^J=\tfrac12[\langle J,0\,|\,T(J)\,|\,J,1\rangle+\phi_{56}^J],
\end{aligned}\tag{2.12}$$

where

$$\begin{aligned}
&\phi_{12}^J=\{(J+1)\langle J+1,1\,|\,T(J)\,|\,J+1,1\rangle+J\langle J-1,1\,|\,T(J)\,|\,J-1,1\rangle-2[J(J+1)]^{1/2}\langle J+1,1\,|\,T(J)\,|\,J-1,1\rangle\}/(2J+1),\\
&\phi_{34}^J=\{J\langle J+1,1\,|\,T(J)\,|\,J+1,1\rangle+(J+1)\langle J-1,1\,|\,T(J)\,|\,J-1,1\rangle+2[J(J+1)]^{1/2}\langle J+1,1\,|\,T(J)\,|\,J-1,1\rangle\}/(2J+1),\\
&\phi_{56}^J=\{[J(J+1)]^{1/2}(\langle J-1,1\,|\,T(J)\,|\,J-1,1\rangle-\langle J+1,1\,|\,T(J)\,|\,J+1,1\rangle)-(J+1,1\,|\,T(J)\,|\,J-1,1\rangle\}/(2J+1).
\end{aligned}$$

The formulas presented so far in this section allow a complete description of the isospin nonconserving N-N scattering. For the p-p case we have $\phi_5^J=\phi_6^J$. It is interesting to have a formula for the isospin nonconserving amplitude f in terms of phase shifts. From Eqs. (2.9), (2.4), (2.12), and (2.10) we obtain

$$\begin{aligned}
f(\theta)&=i[\phi_5(\theta)+\phi_6(\theta)]\\
&=\frac{1}{p}\sum_{J=1}^{\infty}(2J+1)(\phi_5^J-\phi_6^J)d_{10}^J(\theta)\\
&=-\frac{1}{p}\sum_{J=1}^{\infty}(2J+1)\langle J,0\,|\,T(J)\,|\,J,1\rangle d_{10}^J(\theta)\\
&=\frac{1}{2p}\sum_{J=1}^{\infty}(2J+1)\sin2\bar\gamma_J\exp(i\bar\delta_J+i\bar\delta_{JJ})d_{10}^J(\theta)\,.
\end{aligned}\tag{2.13}$$

The last equality gives a direct relation between the singlet-triplet mixing angles $\bar\gamma_J$ and the isospin nonconserving amplitude f.

The nuclear phase shifts can be defined in the following way. Let δ denote one of the six phase shifts of Eq. (2.10). We define the "quasinuclear" phase shift δ^N as

$$\delta^N\equiv\delta-\delta^{EM},\tag{2.14}$$

where δ^{EM} is the corresponding electromagnetic phase shift. In practice the amplitudes are calculated according to Eqs. (2.2) and (2.3).

III. ELECTROMAGNETIC CONTRIBUTIONS

The details of the treatment of the electromagnetic contributions are described in our earlier papers.[8,9] Before reviewing the results it seems to us necessary to discuss some of the peculiarities of the electromagnetic interactions.

For the spinless case the Coulomb amplitude is determined up to an overall phase factor[2] and up to a distribution with a support at zero scattering angle.[8] The Coulomb phase shifts are determined up to an overall additive constant.[8] For particles with spin the nature of ambiguities is more complicated. For the p-p interaction all helicity amplitudes are singular at zero scattering angle. This implies that no information on the phase of the amplitudes at zero angle is available. Therefore one can multiply all the helicity amplitudes by a common factor. By doing this all the p-p observables are unchanged. This also implies that one can add an overall common constant to the electromagnetic phase shifts of $\bar\delta_J$, $\bar\delta_{JJ}$, $\bar\delta_{J\pm1,J}$. The result of this change is that all amplitudes are multiplied by a common phase factor and distributions with support at zero scattering angle (δ functions) are added to the amplitudes. This can be verified by inspecting Eqs. (2.4), (2.12), and (2.10) and by noting that the sums

$$\sum_J(2J+1)d_{\lambda\mu}^J(\theta)$$

are distributions with a support at zero scattering angle only.

Other peculiarities arise from the large anomalous magnetic moments of the nucleons. The inclusion of magnetic moments requires a derivative coupling interaction which makes the interaction nonrenormalizable. Therefore if Feynman diagrams are used for calculating electromagnetic amplitudes or electromagnetic phase shifts, at present time one can not go beyond the Born approximation. Therefore in deriving relativistic formulas for the electromagnetic phase shifts we have to limit ourselves to the Born approximation.

Our partial wave expansions of the Born approximation are based on the formula which we derived earlier,[8]

$$1/(1-\cos\theta) = -\sum_{l=0}^{\infty}(2l+1)[\psi(l+1)+\epsilon]P_l(\cos\theta), \quad (3.1)$$

where $\psi(l+1)$ is the digamma function, ϵ is an arbitrary constant and γ is Euler's constant $=0.577\ldots$,

$$\psi(l+1) = -\gamma + 1 + \tfrac{1}{2} + \cdots + 1/l. \quad (3.1a)$$

For the n-p interaction the helicity amplitudes $\phi_1(\theta)$, $\phi_3(\theta)$, and $\phi_5(\theta)$ are singular at zero scattering angle. However, the amplitude $\phi_2(\theta)$ is regular at $\theta=0$ and the phase of $\phi_2(\theta)$ is well determined. Therefore for the n-p scattering there is no common phase factor ambiguity and the electromagnetic phase shifts are well defined. On the other hand due to the above singularities, the total cross section is infinite and the differential cross section is singular at $\theta=0$.

From the Born approximation one can obtain the electromagnetic phase shifts. If in Eq. (2.10) one expands the T-matrix elements and phase shifts in power series of the Coulomb parameter, one finds for the first order (and also up to the second order)

$$\begin{aligned}
\overline{\delta}_J &\simeq \langle J0|T(J)|J0\rangle,\\
\overline{\delta}_{JJ} &\simeq \langle J1|T(J)|J1\rangle,\\
\overline{\gamma}_J &\simeq -\langle J1|T(J)|J0\rangle, \qquad (3.2)\\
\overline{\delta}_{J\pm1,J} &\simeq \langle J\pm1,1|T(J)|J\pm1,1\rangle,\\
\overline{\epsilon}_J &\simeq -\langle J+1,1|T(J)|J-1,1\rangle.
\end{aligned}$$

We shall calculate the electromagnetic phase shifts up to the first order in the Coulomb parameter η (taking also into account the anomalous magnetic moments) from the one-photon-exchange contribution, which is most easily calculable in the helicity representation. We shall give expressions for the matrix elements from which, using Eqs. (2.11) and (3.2), the phase shifts could be directly calculated.

The matrix elements ϕ_i^J can be obtained by using the technique and results of Ref. (8). For the p-p interaction we obtain

$$\begin{aligned}
\phi_1^J &= f_1\psi(J+1) + f_{10}\delta_{J0} - f_{11}\delta_{J1},\\
\phi_2^J &= f_{20}\delta_{J0} + f_{21}\delta_{J1},\\
\phi_3^J &= f_3\{\psi(J+1) - [2J(J+1)]^{-1}\} + f_{31}\delta_{J1}, \qquad (3.3)\\
\phi_4^J &= -f_4/[J(J+1)] + f_{41}\delta_{J1},\\
\phi_5^J &= f_5/[J(J+1)]^{1/2} + f_{51}\delta_{J1} = \phi_6^J,
\end{aligned}$$

where the $\psi(J+1)$ are obtained from Eq. (3.1) and one can add to them an arbitrary constant common to ϕ_1^J and ϕ_3^J (one can check that this will lead to adding a common constant to the phase shifts $\overline{\delta}_J$, $\overline{\delta}_{JJ}$, $\overline{\delta}_{J\pm1,J}$), the δ_{Ji} are Kronecker δ's, and

$$\begin{aligned}
\eta &= \tfrac{1}{2}e^2M/k,\\
f_1 &= f_3 = \eta(M^2+2k^2)/(ME),\\
f_{10} &= \eta(2M^2 - 8\nu_p k^2 - \nu_p^2k^2)/(4ME),\\
f_{11} &= f_{21} = \tfrac{1}{2}f_{31} = \tfrac{1}{2}f_{41} = \eta\nu_p^2k^2/(12ME),\\
f_{20} &= \eta(2M^2 - 4\nu_p k^2 + \nu_p^2k^2 + 2\nu_p^2k^4/M^2)/(4ME),\\
f_4 &= f_{20} + 3f_{11},\\
f_5 &= \eta(\nu_p k^2/M^2 - \tfrac{1}{2}),\\
f_{51} &= \sqrt{2}\,\eta\nu_p^2k^2,
\end{aligned}$$

where η is the Coulomb parameter and ν_p is the anomalous magnetic moment of the proton, $\nu_p = 1.792\,8456$.

The electromagnetic helicity amplitudes can be calculated from the electromagnetic phase shifts using Eqs. (2.10), (2.12), and (2.4). One should note that the partial wave series (2.4) are divergent and can not be summed directly. The resulting sums up to the first and second order in the Coulomb parameter are presented elsewhere[9] and will not be repeated here. The numerical techniques of summation of divergent partial waves by a generalization of Padé approximants will be treated in a separate paper.

In practice the first two orders in the Coulomb parameter are sufficient. We will quote here only the singular contributions which are sufficient for practical calculations,

$$\begin{aligned}
&\Delta \equiv \tfrac{1}{2}(1-\cos\theta), \quad y \equiv \sin\tfrac{1}{2}\theta,\\
&k\phi_1(\theta) = -f_1[1 - if_1(2\gamma+\ln\Delta)]/(1-\cos\theta) - 2if_5^2\ln\Delta,\\
&k\phi_2(\theta) = -2if_5^2\ln\Delta,\\
&k\phi_3(\theta) = -\tfrac{1}{2}f_1(1+\cos\theta)[1-if_1(2\gamma+\ln\Delta)]/(1-\cos\theta)\\
&\qquad\qquad + \tfrac{1}{2}i(1+\cos\theta)(f_1^2 - 2f_5^2)\ln\Delta, \qquad (3.4)\\
&k\phi_4(\theta) = \tfrac{1}{2}if_1f_4\ln\Delta,\\
&k\phi_5(\theta) = -f_5\sin\theta[1-if_1(2\gamma+\ln\Delta)]/(1-\cos\theta)\\
&\qquad\qquad - if_5(\tfrac{3}{2}f_1 - f_4)[y^{-1} - \ln(1+y^{-1})]\sin\theta.
\end{aligned}$$

The appearance of the Euler's constant γ in the above formulas is the result of a particular choice of the arbitrary constant, it is consistent with the usual definition of Coulomb phase shifts. The observables should not depend on the particular choice. However, for Eq. (3.4) this is not the case and further improvement of Eq. (3.4) is needed. We can observe from Eqs. (3.1a), (3.3), (3.2), and (2.11), that with the constant γ of Eq. (3.1a) an overall constant $-f_1\gamma$ is common to the phase shifts. From this fact and from Eqs. (2.12) we can infer that up to some δ functions at zero scattering angle the helicity amplitudes have an overall phase factor $\exp(-2if_1\gamma)$. Therefore if we multiply the helicity amplitudes by the factor $\exp(2if_1\gamma)$, the results should not depend on γ, and we can replace Eq. (3.4) by equations equivalent to it up to the second order in η:

$$
\begin{aligned}
&A \equiv \exp(2if_1\gamma),\\
&kA\,\phi_1(\theta) = -f_1(1 - if_1\ln\Delta)/(1-\cos\theta) - 2if_5^{\,2}\ln\Delta,\\
&kA\,\phi_2(\theta) = -2if_5^{\,2}\ln\Delta,\\
&kA\,\phi_3(\theta) = -\tfrac{1}{2}f_1(1+\cos\theta)(1-if_1\ln\Delta)/(1-\cos\theta)\\
&\qquad\qquad + \tfrac{1}{2}i(1+\cos\theta)(f_1^{\,2}-2f_5^{\,2})\ln\Delta, \qquad (3.5)\\
&kA\,\phi_4(\theta) = \tfrac{1}{2}if_1 f_4\ln\Delta,\\
&kA\,\phi_5(\theta) = -f_5\sin\theta(1-if_1\ln\Delta)/(1-\cos\theta)\\
&\qquad\qquad - if_5(\tfrac{3}{2}f_1 - f_4)[y^{-1} - \ln(1+y^{-1})]\sin\theta.
\end{aligned}
$$

An equivalent method of calculation is to replace γ by zero in Eq. (3.1a), and in all equations following it. For practical calculations we recommend the following procedure:

1. Define the electromagnetic phase shifts from Eqs. (3.2) and (3.3) only with terms containing f_1, $f_3(=f_1)$, f_4, and f_5. The rest can be absorbed in the definition of the nuclear phase shifts.

2. Define the electromagnetic and nuclear amplitudes according to Eq. (2.2).

3. For the electromagnetic part use Eq. (3.5). A very small regular part is absent from Eq. (3.5) which can be omitted. (For details see Ref. 9.)

For the neutron-proton interaction we obtain

$$
\begin{aligned}
&\phi_1^J = h_{10}\delta_{J0} + h_{11}\delta_{J1},\\
&\phi_{20}^J = h_{20}\delta_{J0} + h_{21}\delta_{J1},\\
&\phi_3^J = h_{31}\delta_{31},\\
&\phi_4^J = h_4/[J(J+1)] + h_{41}\delta_{J1}, \qquad (3.6)\\
&\phi_5^J = h_{51}\delta_{J1},\\
&\phi_6^J = h_6/[J(J+1)] + h_{61}\delta_{J1},
\end{aligned}
$$

where

$$
\begin{aligned}
&h_{10} = -\eta\nu_n k^2(1+\tfrac{1}{4}\nu_p)/(EM),\\
&h_{11} = -h_{21} = -\tfrac{1}{2}h_{31} = -\tfrac{1}{2}h_{41} = -\eta\nu_n\nu_p k^2/(12ME),\\
&h_{20} = -\eta\nu_n k^2[1-\nu_p(E^2+M^2)/M^2]/(2EM),\\
&h_4 = \tfrac{1}{2}\eta\nu_n k^2(1-\nu_p E^2/M^2)/(EM),\\
&h_{51} = h_{61} = \sqrt{2}\,\eta\nu_n\nu_p k^2/(12M^2),\\
&h_6 = -\eta\nu_n k^2/M^2,
\end{aligned}
$$

and ν_n is the anomalous magnetic moment of the neutron, $\nu_n = -1.913\,148$. The singlet triplet mixing parameter can be calculated from Eqs. (2.11), (3.6), and (3.2). The result is

$$\bar{\gamma}_J = \eta\nu_n k^2/\{M^2[J(J+1)]^{1/2}\}. \qquad (3.7)$$

The helicity amplitudes, up to the first and second order in the Coulomb parameter, obtained from the phase shifts, are calculated elsewhere.[9] Here we will list only the singular parts which are sufficient for practical calculations (the regular part is small and can be treated in the same level as the short range interaction),

$$
\begin{aligned}
&k\phi_6(\theta) = h_6\sin\theta/(1-\cos\theta),\\
&k\phi_1(\theta) = -ih_6^{\,2}\ln[\tfrac{1}{2}(1-\cos\theta)], \qquad (3.8)\\
&k\phi_3(\theta) = -\tfrac{1}{2}ih_6^{\,2}(1+\cos\theta)\ln[\tfrac{1}{2}(1-\cos\theta)],
\end{aligned}
$$

and $\phi_2(\theta)$, $\phi_4(\theta)$, $\phi_5(\theta)$ are regular.

From Eq. (3.8) we see that σ_{tot} for the n-p interaction is infinite. Also the differential cross section at very small angles

$$
\begin{aligned}
\frac{d\sigma}{d\Omega} &= \tfrac{1}{2}(|\phi_1|^2 + |\phi_2|^2 + |\phi_3|^2\\
&\quad + |\phi_4|^2 + 2|\phi_5|^2 + 2|\phi_6|^2)\\
&\simeq |\phi_6(\theta)|^2\\
&\simeq h_6^{\,2}\sin^2\theta/[k(1-\cos\theta)]^2 \qquad (3.9)
\end{aligned}
$$

has a pole at zero momentum transfer squared $-t = 2k^2(1-\cos\theta)$.

Some of the indirect electromagnetic contributions resulting from the mass differences of charged and neutral particles are discussed in Sec. IV.

IV. DIFFERENCES BETWEEN THE p-p AND n-p PHASE SHIFTS

The nuclear p-p and n-p phase shifts may differ as a result of the following effects:

A. mixed electromagnetic and strong interaction contributions,

B. differences in the field-theoretic electromagnetic contributions,

C. differences in masses of the exchanged neutral and charged mesons as well as the neutron-proton mass difference,

D. isospin symmetry breaking of the coupling constants.

The knowledge of these effects is important for the phase shift analysis of the n-p system. The isospin $T=1$ phase shifts are taken from the p-p phase shift analysis. In recent years significant progress has been achieved in fixing the mixed Coulomb and strong interaction contributions arising in potential scattering. Several formulas give contributions which are model independent.[10] Plessas, Streit, and Zingl[11] found the following simple relation:

$$\delta_l(p\text{-}p) \approx \left(1 - \frac{Me^2}{2l+1}\frac{\partial}{\partial k}\right)\delta_l(p\text{-}n)\,, \tag{4.1}$$

where M is the nucleon mass, e is the electric charge ($e^2 \simeq 1/13\sigma$), k is the c.m. momentum, $\delta_l(p\text{-}p)$ and $\delta_l(p\text{-}n)$ are nuclear p-p and n-p phase shifts, respectively. Formula (4.1) is in good agreement with $l \geq 1$ phase shifts which are calculated in potential models. A somewhat improved formula for $l=0$ was recently given by Streit, Fröhlich, Zankel, and Zingl,[12]

$$|\delta_l(p\text{-}p) - \delta_l(p\text{-}n)| = \left|\alpha_l\left(\frac{\partial}{\partial k}\delta_l(p-n) + \frac{i}{2k}\left(e^{-2i\delta_l'(p-n)} - 1\right)\right)\right|\,, \tag{4.2}$$

where

$\alpha_0 = -5.37746$ MeV, $\alpha_1 = -2.17940$ MeV,

$\alpha_2 = -1.34436$ MeV, $\alpha 3 = -0.96861$ MeV,

and for higher l, α_l approaches values consistent with Eq. (4.1), namely,

$$\alpha_l \simeq -\frac{Me^2}{2l+1}\,. \tag{4.3}$$

For practical use Eq. (4.2) can be transformed to

$$\delta_l(p-p) - \delta_l(p-n) = \alpha_l\left(\frac{\partial}{\partial k}\delta_l(n-p) + \frac{\sin 2\delta_l(p-n)}{2k}\right)\left[1 + \left(\frac{\cos 2\delta(p-n) - 1}{2k(\partial/\partial k)\delta(p-n) + \sin\delta(pn)}\right)^2\right]^{1/2}, \tag{4.4}$$

where the sign of α_l was chosen to agree with that of Eq. (4.1). For the n-p phase shift analysis the phase shifts $\delta_l(p\text{-}n)$ in the right-hand side of Eq. (4.4) can be replaced by $\delta_l(p\text{-}p)$ if their difference is small.

The effect of neutral-charged pion mass differences and differences in pion-nucleon coupling constants for the case of one and two pion exchanges was discussed by El-Ghabaty, Gupta, and Kaskas.[13] They found that for the $T=1$ np states one should use one pion exchange with an effective mass of $2m_c - m_0 = 144.173$ MeV, where m_c is the mass of the charged pions and m_0 the mass of neutral pions. This should be confronted with the trend to use an average pion mass (for example 136.5 MeV in a recent N-N phase shift analysis[14]). These differences in the effective pion masses might lead to small but significant differences in the phase shift analysis.

The problem of the isospin symmetry breaking of the pion-nucleon coupling constants was discussed by Morrison[15] but without any definitive conclusions. However, it seems that if symmetry breaking in the coupling constants exists, it is rather negligible. In Fig. 1 the one-pion-exchange (OPE) diagrams are given, allowing for isospin symmetry breaking. In Appendix C the corresponding partial wave projections are given. The corresponding OPE potential should be written now in the following form:

$$V(r) = \frac{g_{np\pi^+}{}^2}{4\pi} V_{\mathrm{OPE}}(r, m_C)(1 + \vec{\tau}_n \cdot \vec{\tau}_p) - \frac{|g_{nn\pi^0}|g_{pp\pi^0}}{4\pi} V_{\mathrm{OPE}}(r, m_0)\,, \tag{4.5}$$

where $V_{\mathrm{OPE}}(r, m)$ is the usual OPE potential with pion mass m. In the case of charge independence this potential reduces to. $(g^2/4\pi)V_{\mathrm{OPE}}(r,m)\vec{\tau}_n \cdot \vec{\tau}_p$. Above, m_c is the mass of charged pions and m_0 is the mass of neutral pions, the proton-neutron mass difference (being relatively very small) is neglected; $\vec{\tau}$ is the isospin vector composed of Pauli matrices.

In Fig. 2 we present results of calculations for the nuclear n-p and p-p phase shift differences $\delta_{np} - \delta_{pp}$. The curve C is obtained by switching off the Coulomb potential in a potential model of the N-N interaction (Paris potential[16]). The curves OPE and TPE (two-pion-exchange) were obtained by calculating phase shifts of one and two pion exchanges with the unitarization procedure as in Eq. (3.2). The pure electromagnetic np phase shift (denoted by E-M) is only shown for the $^1S_0$ state, for higher angular momenta it is rather negligible. From the above results we learn that the contributions due to pion mass differences are (for lab kinetic energies bigger than 10 MeV) of the same

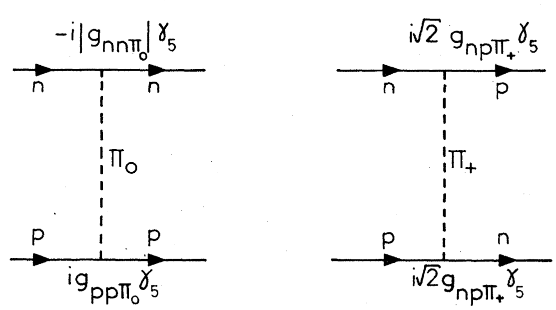

FIG. 1. One pion exchange diagrams. The condition for isospin conservation in the $\pi N\bar{N}$ Born vertices is $g_{nn\pi^0}{}^2 = g_{pp\pi^0}{}^2 = g_{np\pi^+}{}^2$.

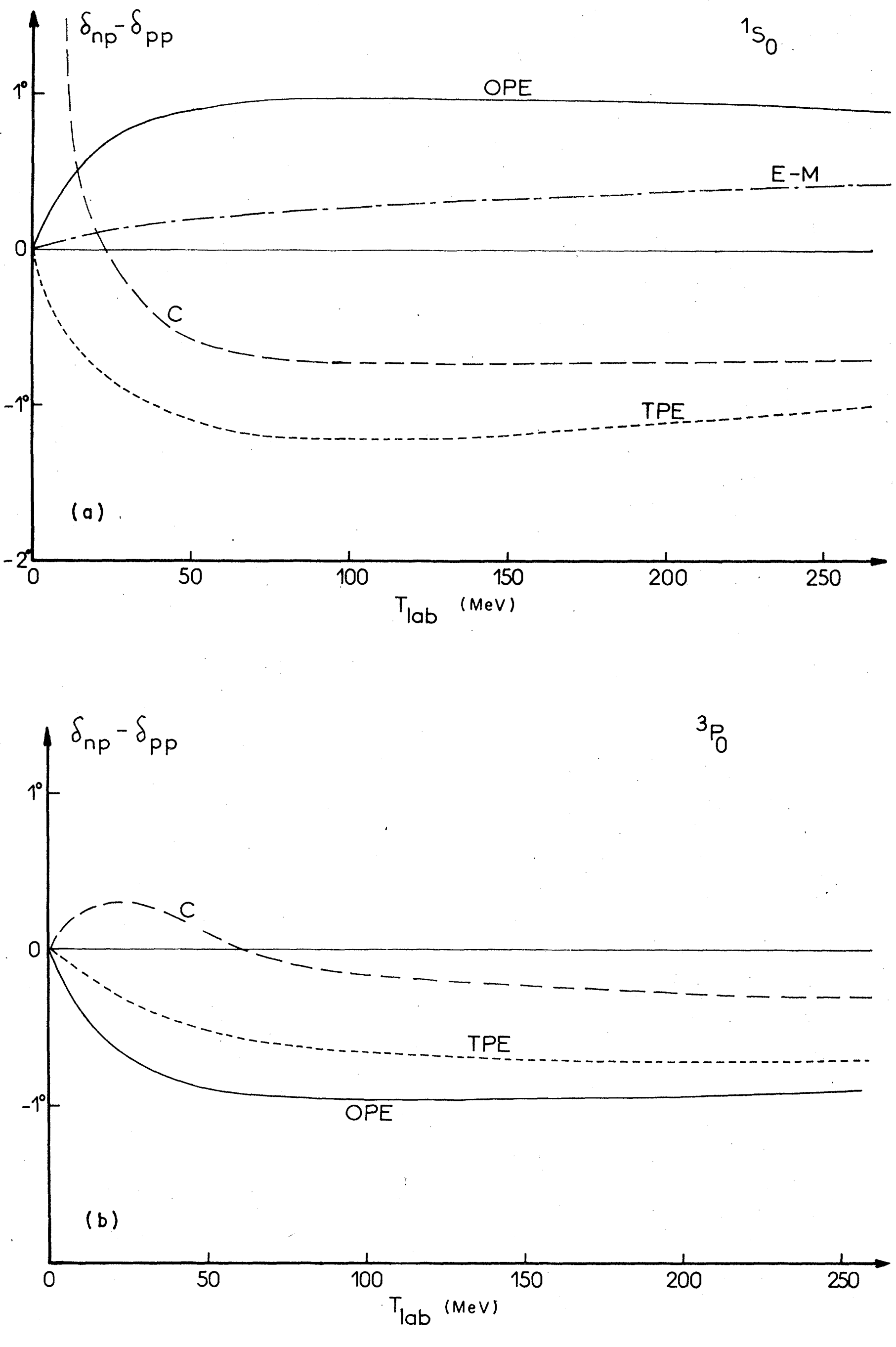

FIG. 2. The differences between various np and pp phase shifts, $\delta_{np}-\delta_{pp}$, in degrees. The solid line labeled OPE represents the differences in OPE due to the pion mass differences. The dashed line labeled TPE, represents the differences in two pion exchanges due to the pion mass differences. The dashed line C is the phase shift difference obtained by switching off the Coulomb potential in a potential model. The dashed point line labeled E-M gives the electromagnetic np phase shifts.

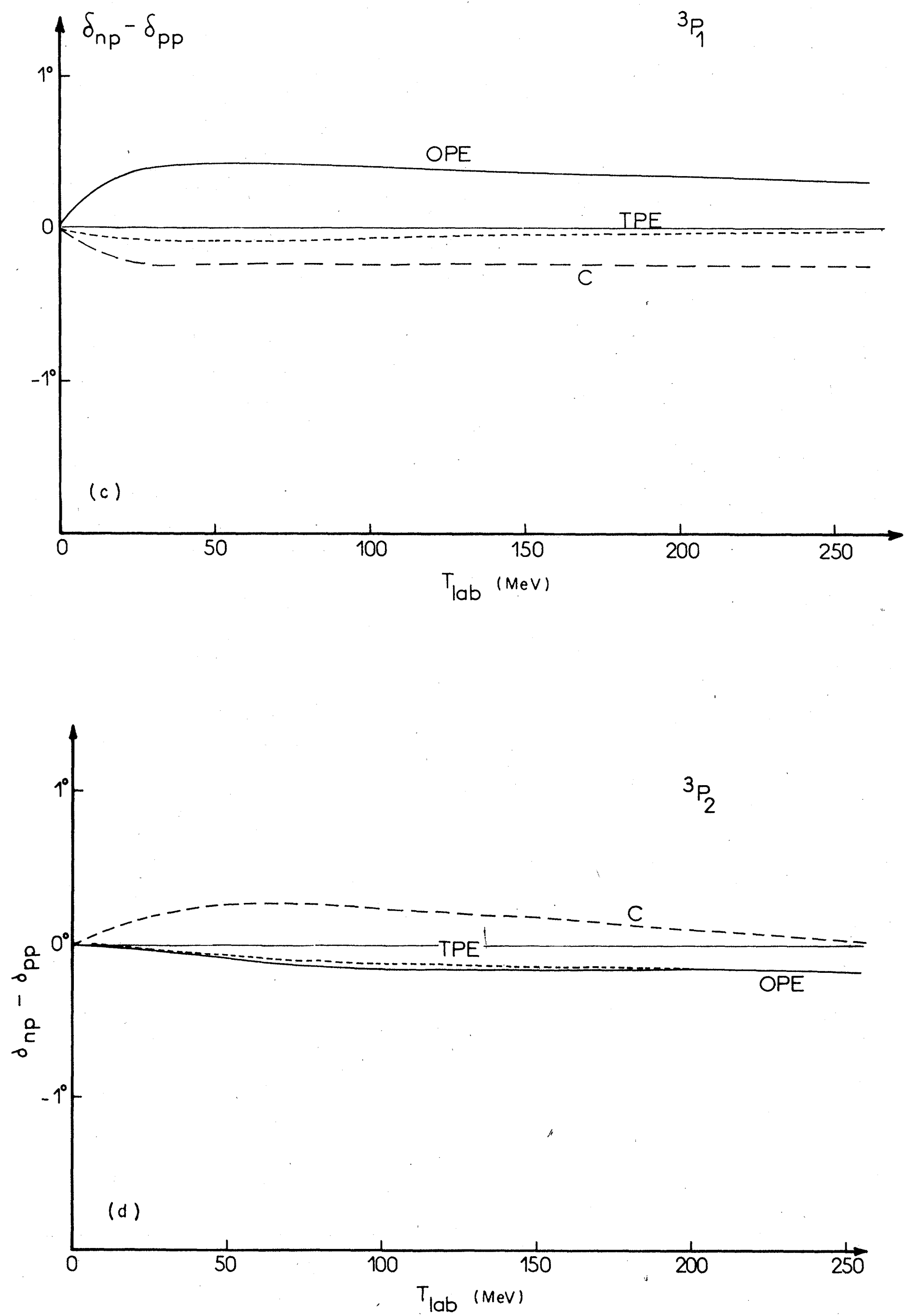

FIG. 2. (*Continued*).

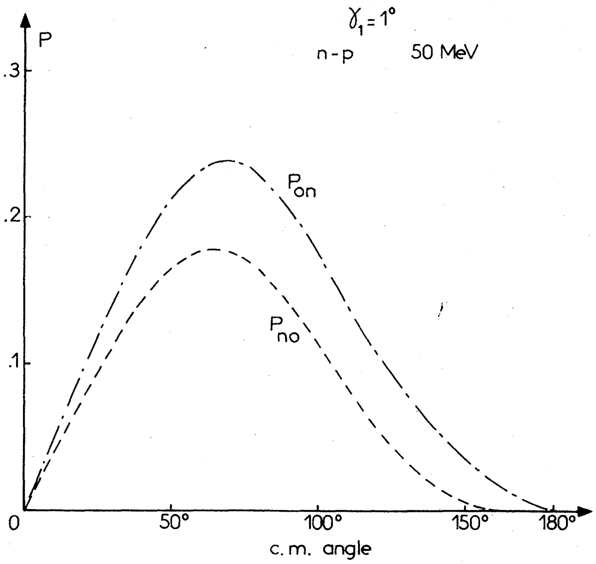

FIG. 3. The polarizations P_{n0} and P_{0n} obtained by assuming $\gamma_1 = 1°$ calculated with the phase shifts of Ref. 14 for lab kinetic energy 50 MeV.

order of magnitude as the contributions resulting from switching off the Coulomb potential. Unfortunately, for lower angular momenta the TPE contributions are of the same order as OPE contributions which indicates that summing only one and two pion contributions is rather insufficient for partial waves with low angular momentum.

V. DETECTION OF THE ISOSPIN NONCONSERVATION

In Appendixes A and B are accumulated formulas for n-p observables in terms of our six amplitudes. We have made calculations using the formalism developed in Secs. II and III including electromagnetic effects. Among the observables we have tested we found that the polarization might be a very sensitive observable to test isospin nonconservation. We used nuclear phase shifts from the recent phase shift analysis of Arndt, Hackman, and Roper.[14] In Fig. 3 we show an example of possible differences in the polarizations P_{n_0} (the polarization of the neutron is measured) and P_{0n} (the polarization of the proton is measured) taking the mixing angle of the ${}^1P_1 \longleftrightarrow {}^3P_1$ transition of 1°. [This is only an example (with a small angle) to indicate possible effects. There exists no theory to evaluate this angle.] As one can see, the difference $P_{0n} - P_{n_0}$ might be a sensitive way to detect isospin noninvariance. In Fig. 4 we show how the inclusion of the electromagnetic effects, discussed in Sec. III, changes the values of the polarizations. In this figure P is the polarization calculated without the electromagnetic effects. In experiments performed till now the P_{n_0} was measured. The differences are quite significant below 50° and become quite dramatic at very small angles [Fig. 4(b)]. The electromag-

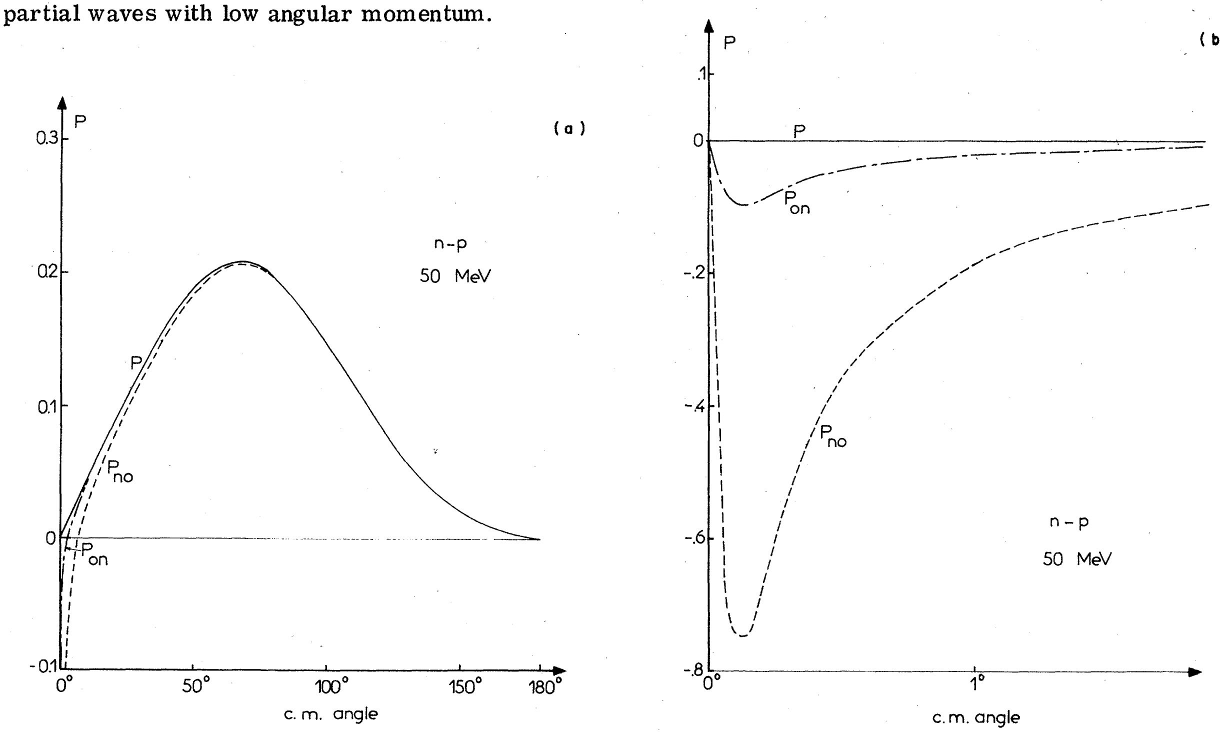

FIG. 4. The polarizations P_{n0}, P_{0n}, and P for lab kinetic energy 50 MeV. P is obtained without the electromagnetic contributions. The small angle's details of Fig. 4(a) are shown in Fig. 4(b).

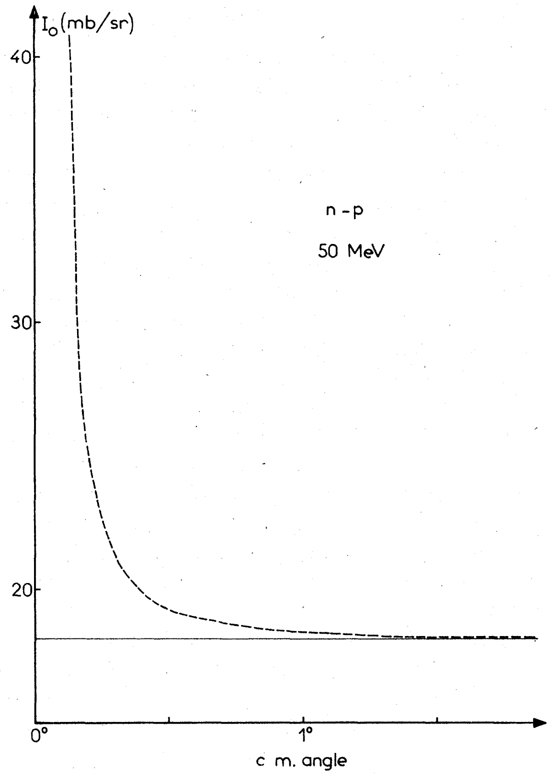

FIG. 5. The small angle differential cross section I_0 for lab kinetic energy 50 MeV.

netic contributions have an effect only on very small angles of the differential cross section, which should be singular at the forward direction according to Eq. (3.9). This effect of the electromagnetic contribution is shown in Fig. 5. We see that only at very small angles (at present time unmeasurable) one can observe deviations from the calculated differential cross section without taking into account the electromagnetic contributions.

Till now there are no reliable theoretical predictions for the mixing angle of the singlet-triplet transitions resulting from strong interactions. These transitions may be present due to the proton-neutron mass difference which is relatively small. If one meson exchanges are concerned, the singlet-triplet transitions might appear only in diagrams which have vertices with different nucleon masses, i.e., only diagrams with charge exchange. The mixing angles of the singlet-triplet transitions $\overline{\gamma}_J$ resulting from the OPE charge exchange diagram of Fig. 1 are evaluated in Appendix C. As the singlets and the uncoupled triplets (with $l \geq 1$) are qualitatively well described at low energies by the OPE, we conjecture that the mixing angles of the singlet-triplet transitions $\overline{\gamma}_J$, might be also qualitatively well described by the OPE. In Fig. 6 are plotted $\overline{\gamma}_1$ and $\overline{\gamma}_2$. In Fig. 7 the polarizations P_{0n} and P_{n0} are presented. The deviations at small angles are obtained mainly due to the electromagnetic effects. The deviations at the intermediate angles come from the singlet-triplet transitions evaluated with the OPE.

VI. SUMMARY AND CONCLUSIONS

In Sec. II we have developed the six amplitude formalism of the N-N interaction allowing for isospin nonconservation. Three different sets of amplitudes were given and also transition formulas linking the different sets. Using these amplitudes we obtained formulas for the observables of single, double, and triple scattering (Appendix-

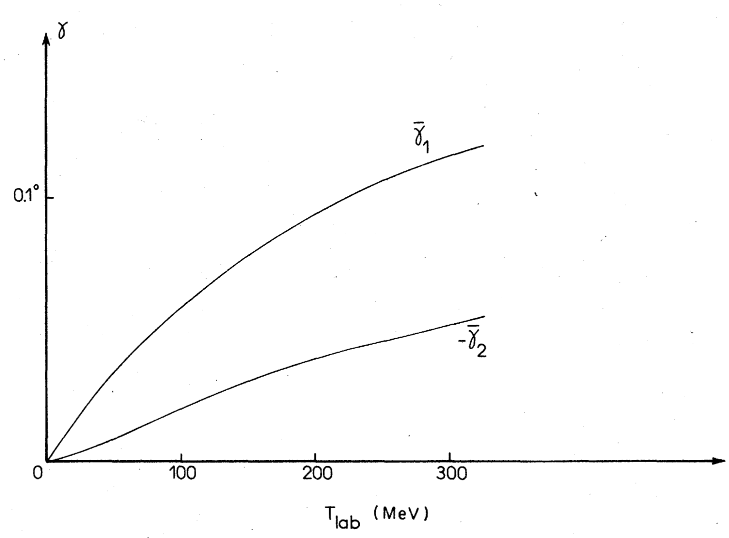

FIG. 6. The mixing angles γ_1 and γ_2 of the singlet-triplet transitions resulting from the n-p mass difference in OPE.

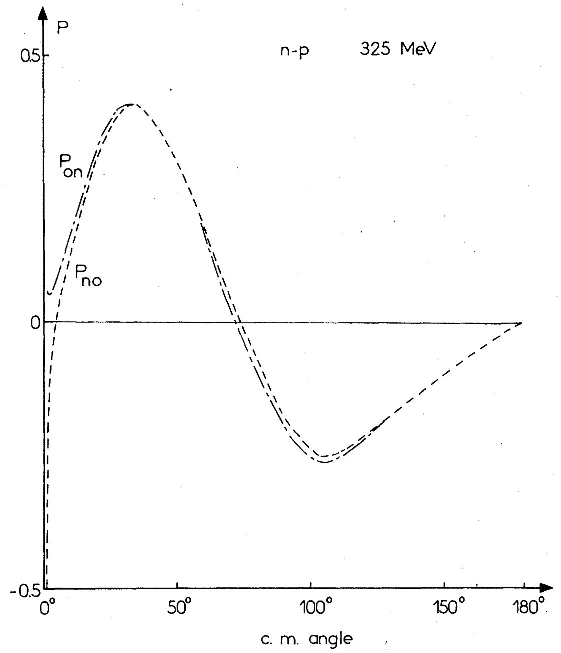

FIG. 7. The polarizations with the electromagnetic contributions and pion and nuclear mass differences in OPE (P_{0n} and P_{n0}) and without these contributions (P) for lab kinetic energy 325 MeV.

es A and B). The electromagnetic contributions are discussed in Sec. III. The treatment is based on the lowest order Feynman diagrams and includes the anomalous magnetic moments of the nucleons. This formalism can not be applied for too low energies, below about 10 MeV, as in this case more photon exchanges are needed. Due to the nonsymmetric spin-orbit interaction of the neutron's magnetic moment with the proton's current one obtains singlet-triplet transitions contributing to the isospin violating amplitude [Eqs. (2.13) and (3.7)]. Small contributions to the singlet-triplet transitions, proportional to the relative proton-neutron mass difference are obtained in the charge exchange OPE (Sec. V and Appendix C). The new features of the n-p interaction are the forward singularity of the differential cross section at very small angles and sensitivity of the polarization to the isospin violating effects. As we have demonstrated the difference between the polarization P_{n0}-P_{0n} might be sensitive to the singlet-triplet transitions. Experimental evaluation of this difference can lead to a qualitative evaluation of the mixing angles of the singlet-triplet transitions. We also found that the inclusion of the electromagnetic effects is important in the description of small angle n-p polarization. The obtained deviations are of the same order as the experimental errors for existing 50 MeV experiments for angles above 20°. The inclusion of the electromagnetic effects will become necessary if n-p polarization measurements will be performed with greater precision or for smaller (than 20°) angles, where P_{n0}-P_{0n} becomes quite large.

ACKNOWLEDGMENTS

I would like to thank Professor R. Vinh Mau for the hospitality extended to me during my stay at the Laboratoire de Physique Théorique. Discussions with J. Bystricky and B. Loiseau are highly acknowledged. Most of the formulas of Appendix A were supplied to me by J. Bystricky. I am grateful to J. Bystricky and F. Lehar for giving me their reports before their publication.

APPENDIX A: OBSERVABLES IN THE c.m. SYSTEM

$$I_0=\tfrac{1}{2}(|a|^2+|b|^2+|c|^2+|d|^2+|e|^2+|f|^2)\,,$$
$$I_0P_{n0}=I_0A_{n0}=\mathrm{Re}(a^*e+b^*f)\,,$$
$$I_0P_{0n}=I_0A_{0n}=\mathrm{Re}(a^*e-fb^*)\,,$$
$$I_0C_{nn}=I_0A_{nn}$$
$$=\tfrac{1}{2}(|a|^2-|b|^2-|c|^2+|d|^2+|e|^2-|f|^2)\,,$$
$$I_0D_{nn}=\tfrac{1}{2}(|a|^2+|b|^2-|c|^2-|d|^2+|e|^2+|f|^2)\,,$$
$$I_0K_{nn}=\tfrac{1}{2}(|a|^2-|b|^2+|c|^2-|d|^2+|e|^2-|f|^2)\,,$$
$$I_0D_{KP}=-I_0D_{PK}=\mathrm{Im}(b^*e+a^*f)\,,$$
$$I_0K_{KP}=\mathrm{Im}(c^*e-d^*f)\,,$$
$$I_0C_{KP}=-I_0A_{KP}=\mathrm{Im}(d^*e-c^*f)\,,$$
$$I_0C_{PK}=-I_0A_{PK}=\mathrm{Im}(d^*e+c^*f)\,,$$
$$I_0D_{KK}=\mathrm{Re}(a^*b+c^*d-e^*f)\,,$$
$$I_0D_{PP}=\mathrm{Re}(a^*b-c^*d-e^*f),$$
$$I_0K_{PP}=\mathrm{Re}(a^*c-b^*d)\,,$$
$$I_0C_{PP}=I_0A_{PP}=\mathrm{Re}(a^*d-b^*c)\,,$$
$$I_0K_{PK}=-\mathrm{Im}(c^*e+d^*f)\,,$$
$$I_0C_{KK}=\mathrm{Re}(a^*d+b^*c)\,,$$
$$I_0K_{KK}=\mathrm{Re}(a^*c+b^*d)\,,$$

APPENDIX B: OBSERVABLES IN THE LABORATORY SYSTEM

Let us denote the following:

θ = the scattering angle in the c.m. system,
θ_R = the angle of the recoiled particle in the c.m. system,
θ_L = the scattering angle in the lab system,
θ_{LR} = the scattering angle of the recoiled particle in the lab system,
$\alpha=\frac{1}{2}\theta-\theta_L$,
$\alpha_R=\frac{1}{2}\theta_R-\theta_{LR}$.

Using the notation of Ref. 6 for the observables and the directions as defined by Eqs. (2.6) and (2.7), we obtain

$$R = (D_{KK}\cos\tfrac{1}{2}\theta + D_{PK}\sin\tfrac{1}{2}\theta)\cos\alpha - (D_{KP}\cos\tfrac{1}{2}\theta + D_{PP}\sin\tfrac{1}{2}\theta)\sin\alpha ,$$

$$A = (-D_{KK}\sin\tfrac{1}{2}\theta + D_{PK}\cos\tfrac{1}{2}\theta)\cos\alpha - (-D_{KP}\sin\tfrac{1}{2}\theta + D_{PP}\cos\tfrac{1}{2}\theta)\sin\alpha ,$$

$$R' = (D_{KK}\cos\tfrac{1}{2}\theta + D_{PK}\sin\tfrac{1}{2}\theta)\sin\alpha + (D_{KP}\cos\tfrac{1}{2}\theta + D_{PP}\sin\tfrac{1}{2}\theta)\cos\alpha ,$$

$$A' = (-D_{KK}\sin\tfrac{1}{2}\theta + D_{PK}\cos\tfrac{1}{2}\theta)\sin\alpha + (-D_{KP}\sin\tfrac{1}{2}\theta + D_{PP}\cos\tfrac{1}{2}\theta)\cos\alpha,$$

$$R_t = (K_{KP}\cos\tfrac{1}{2}\theta + K_{PP}\sin\tfrac{1}{2}\theta)\cos\alpha_R - (K_{KK}\cos\tfrac{1}{2}\theta + K_{PK}\sin\tfrac{1}{2}\theta)\sin\alpha_R,$$

$$A_t = (-K_{KP}\sin\tfrac{1}{2}\theta + K_{PP}\cos\tfrac{1}{2}\theta)\cos\alpha_R - (-K_{KK}\sin\tfrac{1}{2}\theta + K_{PK}\cos\tfrac{1}{2}\theta)\sin\alpha_R,$$

$$R_t' = (K_{KP}\cos\tfrac{1}{2}\theta + K'_{PP}\sin\tfrac{1}{2}\theta)\sin\alpha_R + (K_{KK}\cos\tfrac{1}{2}\theta + K_{PK}\sin\tfrac{1}{2}\theta)\cos\alpha_R,$$

$$A_t' = (-K_{KP}\sin\tfrac{1}{2}\theta + K_{PP}\cos\tfrac{1}{2}\theta)\sin\alpha_R + (-K_{KK}\sin\tfrac{1}{2}\theta + K_{PK}\cos\tfrac{1}{2}\theta)\cos\alpha_R ,$$

$$C_{xz} = C_{KP}\cos\alpha\cos\alpha_R - C_{PP}\sin\alpha\cos\alpha_R - C_{KK}\cos\alpha\sin\alpha_R + C_{PK}\sin\alpha\sin\alpha_R,$$

$$C_{zz} = C_{KP}\sin\alpha\cos\alpha_R + C_{PP}\cos\alpha\cos\alpha_R - C_{KK}\sin\alpha\sin\alpha_R - C_{PK}\cos\alpha\sin\alpha_R ,$$

$$C_{xx} = C_{KP}\cos\alpha\sin\alpha_R - C_{PP}\sin\alpha\sin\alpha_R + C_{KK}\cos\alpha\cos\alpha_R - C_{PK}\sin\alpha\cos\alpha_R ,$$

$$C_{zx} = C_{KP}\sin\alpha\sin\alpha_R + C_{PP}\cos\alpha\sin\alpha_R + C_{KK}\sin\alpha\cos\alpha_R + C_{PK}\cos\alpha\cos\alpha_R ,$$

$$A_{xx} = C_{KK}\cos^2\tfrac{1}{2}\theta - \tfrac{1}{2}(C_{PK} + C_{KP})\sin\theta + C_{PP}\sin^2\tfrac{1}{2}\theta ,$$

$$A_{zz} = C_{KK}\sin^2\tfrac{1}{2}\theta + \tfrac{1}{2}(C_{PK} + C_{KP})\sin\theta + C_{PP}\cos^2\tfrac{1}{2}\theta ,$$

$$A_{zx} = -C_{PK}\cos^2\tfrac{1}{2}\theta + \tfrac{1}{2}(C_{PP} - C_{KK})\sin\theta + C_{KP}\sin^2\tfrac{1}{2}\theta ,$$

$$A_{xz} = -C_{KP}\cos^2\tfrac{1}{2}\theta + \tfrac{1}{2}(C_{PP} - C_{KK})\sin\theta + C_{PK}\sin^2\tfrac{1}{2}\theta .$$

APPENDIX C. ONE-PION-EXCHANGE CONTRIBUTION FOR THE n-p INTERACTION

Up to the lowest order in the coupling constants we have for the OPE T-matrix elements and phase shifts the following contributions. Let us introduce the notation

k = c.m. momentum,

$E = (k^2 + M^2)^{1/2}$,

M=nucleon mass,

$\beta_0 = k|\, g_{pp\pi_0} g_{nn\pi_0}| \,/(16\pi E)$,

$\beta_+ = k\, g_{pn\pi^+}{}^2/(16\pi E)$,

$g_{pp\pi_0}$, $g_{nn\pi_0}$, $g_{pn\pi^+}$ are the coupling constants for the $pp\pi_0$, $nn\pi_0$, and $pn\pi^+$ vertices, correspondingly,

$x_0 = 1 + \tfrac{1}{2}\mu_{\pi_0}{}^2/k^2$,

$x_+ = 1 + \tfrac{1}{2}\mu_{\pi^+}{}^2/k^2$,

μ_{π_0}, μ_{π^+} are neutral and charged pion masses, respectively,

$$\gamma_J(x) = \delta_{J0} - (x-1)Q_J(x) ,$$

$$\eta_J(x) = \frac{(J+1)Q_{J-1}(x) - (2J+1)(2J+1)Q_J(x) + JQ_{J+1}(x)}{(2J+1)} ,$$

we obtain

$$\bar{\delta}_J \simeq \langle J\,0|\, T(J)|\, J\,0\rangle \simeq \beta_0\gamma_J(x_0) - 2(-1)^J\beta_+\gamma_J(x_+) ,$$

$$\bar{\delta}_{JJ} \simeq \langle J1|\, T(J)|\, J1\rangle \simeq \beta_0\eta_J(x_0) + 2(-1)^J\beta_+\eta_J(x_+) ,$$

$$\bar{\delta}_{J,J-1} \simeq \langle J-1,1|\, T(J)|\, J-1,1\rangle \simeq \{-\beta_0[J\gamma_J(x_0) + (J+1)\eta_J(x_0)] + 2(-1)^J\beta_+[J\gamma_J(x_+) + (J+1)\eta_J(x_+)]\}/(2J+1) ,$$

$$\bar{\delta}_{J,J+1} \simeq \langle J+1,1|\, T(J)|\, J+1,1\rangle \simeq \frac{1}{2J+1}\{-\beta_0[(J+1)\gamma_J(x_0) + \eta_J(x_0)] + 2(-1)^J\beta_+[(J+1)\gamma_J(x_+) + J\eta_J(x_+)]\},$$

$$\bar{\epsilon}_J \simeq -\langle J+1,1|\, T(J)|\, J-1,1\rangle \simeq \frac{[J(J+1)]^{1/2}}{2J+1}\{-\beta_0[\gamma_J(x_0) - \eta_J(x_0)] + 2\beta_+(-1)^J[\gamma_J(x_+) - \eta_J(x_+)]\},$$

$$\phi_1^J \simeq \phi_5^J \simeq \phi_6^J \simeq 0, \quad \phi_2^J = -\beta_0\gamma_J(x_0) + 2(-1)^J\beta_+\gamma_J(x_+) ,$$

$$\phi_3^J = 2(-1)^J\beta_+\eta_J(x_+) ,$$

$$\phi_4^J = -\beta_0 \gamma_J (x_0).$$

The above expressions are accurate up to the first order in the mass difference $(M_n - M_p)$, where M_n and M_p are the neutron and proton masses, respectively. If one takes into account the first order corrections of $(M_n - M_p)$ a very small correction to the above formulas is added. The new feature is the appearance of singlet triplet transitions according to

$$\bar{\gamma}_J \simeq \langle J\,0 | \,T(J)| \,J1\rangle \simeq 4\beta_+ (-1)^J \left(\frac{J}{J+1}\right)^{1/2} \frac{W_n - W_p}{W_n + W_p} \left[x_+ Q_J(x_+) - Q_{J-1}(x_+)\right],$$

where $W = E + M$.

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$$\delta_l \equiv \delta_l(p-n),$$

$$\delta_l(p-p) - \delta_l = \tfrac{1}{2}\arctan\frac{\alpha_l \operatorname{Im} Z}{1+\alpha_l \operatorname{Re} Z},$$

$$Z = 2i\,\frac{2(\partial\delta_l/\partial k) + \sin 2\delta_l}{1+e^{2i\delta_l}},$$

which will be published in the Proceedings of the 8th international conference on few body systems and nuclear forces, Graz, 24–30 August 1978.

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