Emission of ^{24}Na fragments in the interaction of ^{197}Au with intermediate-energy pions and protons*

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Excitation functions for the formation of ^{24}Na from ^{197}Au have been measured for incident π^+ between 176 and 368 MeV, π^- between 239 and 368 MeV, and protons between 197 and 800 MeV. The cross sections increase sharply with energy and at any given energy obey the relation $\sigma_{\pi+} > \sigma_{\pi-} > \sigma_p$. The results are analyzed in terms of the contribution of hot-spot and coherent-cascade mechanisms to the proton yields. The role of statistical phase-space factors, as embodied in cascade-evaporation calculations, is invoked to explain the ratio of π^+ and π^- cross sections.

[NUCLEAR REACTIONS ^{197}Au(p,x), E_p=197–800 MeV; ^{197}Au(π^+,x), E_{π^+}=176–368 MeV; ^{197}Au(π^-,x), E_{π^-}=239–368 MeV; ^{24}Na excitation functions.]

I. INTRODUCTION

The interaction of high-energy particles with complex nuclei resulting in the emission of light fragments, commonly referred to as fragmentation, is a process that is not as yet completely understood. Wolfgang *et al.*[1] were the first to observe that the yields of light fragments in reactions of medium and heavy elements with protons showed a rapid increase with bombarding energy from an apparent threshold of a few hundred MeV to several GeV, at which point they began to level off. Since the excitation functions for the formation of light nuclides such as ^{24}Na were similar in shape and threshold to that for pion production in nucleon-nucleon interactions, these authors proposed that fragmentation resulted from localized heating of the struck nucleus by pion production and subsequent reabsorption. One of the key features of the proposed mechanism was the time scale for fragment emission; this process was thought to occur concurrently with the intranuclear cascade and prior to energy equilibration. This assumption has been confirmed by measurements of the double differential cross sections for fragment emission.[2,3] These experiments showed that the angular distributions of the fragments in the moving system defined by the energy spectra obtained at forward and backward angles were forward-peaked rather than symmetric, an indication of a fast, one-step process. A similar conclusion had been previously reached by Faissner and Schneider[4] on the basis of emulsion studies.

The evidence for the role of pion absorption in fragmentation is less conclusive than that bearing on the time scale. In their review article on fragmentation, Perfilov, Lozhkin, and Shamov[5] suggested that high energy transfer to a restricted region of the struck nucleus, whether accompanied by pion absorption or not, is responsible for fragmentation. On the other hand, Faissner and Schneider[4] qualitatively explained the forward emission of light fragments in high momentum-transfer processes induced by 660-MeV protons in terms of a hydrodynamic model. These workers pictured the intranuclear cascade as a stream of nuclear matter moving forward through the nucleus with a sufficiently low angular spread to permit a longitudinal deformation of the nucleus, the ultimate result of which might be the emission of a fragment. A more quantitative model, which has a similar physical basis, has been recently advanced to explain a different set of data. This is the coalescence model,[6] in which fragment emission is depicted as resulting from the coalescence of cascade nucleons moving through the nucleus with small relative momentum. This model is quantitatively able to account for the differential cross sections for the emission of very light ($A \leq 4$) fragments in relativistic heavy ion reactions and can apparently also account for the emission of energetic boron to oxygen fragments in relativistic heavy ion reactions[7] as well as that of energetic helium to beryllium fragments in high-energy proton reactions.[3]

Evidence on the role of pions in fragmentation has also been obtained in a study of fragment emission in reactions induced by ^{4}He ions. Crespo, Alexander, and Hyde[8] measured the cross sections and recoil properties of several light fragments in reactions induced by 320–880-MeV ^{4}He ions and compared them with those obtained for protons of comparable energy. The cross sections were a factor of 2 higher for the ^{4}He-induced reactions while in every other respect the results were quite similar. In view of the fact that the ^{4}He energy was at most 220 MeV per nucleon, these workers concluded that pion production and reabsorption was of minor importance in fragmentation induced by ^{4}He. A recent controversy[9,10] on the importance of pion production in reactions induced by 100–300-MeV/nucleon heavy ions does not appear to have affected the validity of this conclusion.

The various mechanisms discussed above fall into two broad categories: hot-spot formation (HS), and the development of a coherent cascade (CC). The HS process involves the transfer of high excitation energy to a moderately localized region of the nucleus, presumably, but not necessarily due to pion production and reabsorption. On the other hand, the CC process involves the development of a coherent cascade in which a number of nucleons find themselves traveling with very low relative momentum towards the nuclear surface. In either case the interplay between surface tension and Coulomb repulsion forces presumably determines the eventual outcome of such a process. It is clear, however, that the nature of the initial state is quite different in the two cases.

In order to determine the relative importance of these two mechanisms additional experiments are needed. The study of fragmentation in reactions induced by pions presents an interesting possibility in this respect. Because of the likelihood of absorption, pions are known to transfer more energy to a target nucleus than protons of the same kinetic energy up to incident energies of at least several hundred MeV. For instance, Garrett and Turkevich[11] report that the spallation cross sections for 65-MeV $\pi^{\pm}$ on copper are comparable to those obtained for protons of the same total energy, i.e., 205 MeV. Moreover, for products that are 20–30 mass units removed from the target, and thus require the highest excitation energies, pions appear to be even more effective than protons of the same total energy. On the other hand, Monte Carlo cascade simulations[12] suggest that pions should be less effective than protons of comparable kinetic energy in initiating a coherent cascade. This result must be related to the fact that for energies of several hundred MeV the momentum of a pion is only about half that of a proton with the same kinetic energy. These considerations suggest that a comparison of fragmentation yields in reactions induced by pions and protons may be of value in determining the relative importance of the HS and CC processes at intermediate energies. To be sure, the probability at these energies of cascades involving the participation of enough nucleons to permit fragment emission by the CC process must be exceedingly small. However, the observed fragmentation yields are also very small so that the CC process should not be ruled out just on the basis of low probability.

The recent availability of intense pion beams makes such measurements possible and the present study addresses itself to this question. We have measured the excitation functions for the production of a typical light fragment, ^{24}Na, from gold in reactions induced by 180–370-MeV $\pi^{\pm}$ and 200–800-MeV protons. The results will be analyzed in terms of the proposed HS and CC mechanisms. A preliminary account of our experiment has been published elsewhere.[13]

II. EXPERIMENTAL

The irradiations were performed with proton and pion beams at LAMPF. The 800-MeV proton irradiations were carried out in the nuclear chemistry irradiation facility in area B while those at 200–600 MeV were performed in the switchyard area. The duration of the bombardments ranged from 10 to 30 min and the beam intensity was in the vicinity of 1 μA. The pion irradiations were carried out in the P^3 channel and lasted from 6 to 12 h. The number of π^+ striking the target ranged from $1-5\times 10^8$/sec while that of π^- was lower by about a factor of 4. These values were obtained with a 6% (FWHM) momentum bite. The presence of protons in the π^+ beam was virtually eliminated by differential degradation and momentum separation. The maximum contribution of protons at the highest π^+ energy was estimated as 1%.

The targets consisted of 250-μm thick gold foils of the highest available purity. Spectrochemical analysis indicated that the only significant impurities were 30 ppm Si, 10 ppm Fe, and 100 ppm Ag. The target stack consisted of one or two of these foils, surrounded by 25-μm thick gold guard foils and preceded on the upstream side by beam monitor foils. In the case of the proton runs the beam intensity was monitored by means of the ^{27}Al(p, $3p3n$) reaction leading to the formation of ^{22}Na, and three 25-μm aluminum foils were incorporated in the target stack for this purpose. The cross sections of this reaction were taken from the compilation by Tobailem, de Lassus St. Genies, and Leveque.[14] The more commonly used ^{27}Al(p, $3pn$) monitor reaction was not chosen because of the possibility of secondary production of ^{24}Na via the (n, α) reaction induced by neutrons originating in the rather thick gold targets needed

TABLE I. Adopted values of monitor reaction cross sections.

Bombarding energy[a] (MeV)	$^{27}Al(p,3p3n)^{22}Na$ σ (mb)	$Si(\pi^+,x)^{24}Na$ σ (mb)	$Si(\pi^-,x)^{24}Na$ σ (mb)
177		7.7 ± 0.5	
198	16.1 ± 1.0		
237		7.5 ± 0.6	
240			9.4 ± 0.5
295		5.9 ± 0.6	
300			8.0 ± 0.5
369		4.7 ± 0.5	6.4 ± 0.5
398	17.7 ± 1.0		
597	17.3 ± 1.0		
800	16.3 ± 1.0		

[a] Energy at the midpoint of the monitor foil.

to make the experiment feasible. Comparison of $^{24}Na/^{22}Na$ ratios in thin aluminum stacks with those obtained in normal target stacks showed, in fact, that secondary production of ^{24}Na accounted for some 30% of the observed yield. On the other hand, the $^{22}Na/^{7}Be$ ratio was found to be independent of target thickness indicating that secondary production of ^{22}Na could be neglected.

The pion beam intensity was monitored by means of the $Si(\pi^{\pm},x)^{24}Na$ reaction, whose cross sections have recently been measured.[15] A 500-μm thick silicon disk was incorporated in the target stack for this purpose. In view of the fact that the $^{27}Al(p,3p3n)$ reaction was found to be free of secondary effects it seems reasonable to assume that the same is true for the roughly comparable $Si(\pi^{\pm},x)^{24}Na$ reaction for the same target configuration. Table I summarizes the various monitor cross sections used in this experiment and indicates the energies at which measurements were made.

After the various irradiations, the target foils were dissolved and sodium was separated by an adaptation of a standard procedure[16] whose key step is the selective absorption of sodium by hydrated antimony pentoxide.[17] The samples were assayed with a calibrated Ge(Li) detector connected to a multichannel analyzer. The monitor foils were assayed with the same detection system. The activity of the samples obtained in the proton runs was sufficiently high to permit the samples to be assayed in a low geometry configuration. The samples from the pion runs were assayed in the highest possible geometry but, since the radiations from the same nuclide, ^{24}Na, were measured for both target and monitor, no corrections for coincidence summing were necessary.

The possibility of secondary reactions contributing to ^{24}Na formation was investigated by measurement of the cross section as a function of target thickness. These experiments were performed with 368-MeV π^+ for targets ranging from 250 to 500 μm in thickness. The cross sections were found to be independent of target thickness indicating that secondary production is negligible.

III. RESULTS

The measured cross sections are summarized in Table II and the excitation functions plotted in Fig. 1. The results represent a weighted average of two or three determinations at each energy. The tabulated errors are the larger of the standard deviation and the estimated uncertainty of the individual determinations. The contribution to the measured proton cross sections from light element impurities was estimated on the basis of reported cross sections for the production of ^{24}Na from light elements by 100–400-MeV protons.[18,19] The effect of these impurities varies inversely with bombarding energy and decreases from approximately 8% at 200 MeV to 0.6% at 400

TABLE II. Summary of experimental cross sections for ^{24}Na production from gold.

Bombarding energy[a] (MeV)	σ_{π^+} (μb)	σ_{π^-} (μb)	σ_p (μb)
176	11.0 ± 0.9		
197			1.54 ± 0.15
236	18.8 ± 1.5		
239		8.5 ± 1.4	
294	21.5 ± 2.1		
299		15.2 ± 2.2	
368	39.3 ± 3.2	30.2 ± 3.1	
398			27.0 ± 1.1
597			92.5 ± 6.8
800			248 ± 10

[a] At midpoint of target.

MeV. The exponential decrease of the ^{24}Na cross section with target Z renders silicon the only contributing impurity of any consequence. The pion results were also corrected for the contribution from silicon since the excitation function has been measured for both π^+ and π^-.[15] The correction also decreases with increasing energy and ranges from 2% to 0.4% for π^+ and from 3% to 0.6% for π^-. The tabulated results have been corrected for this effect. An uncertainty of 50% in the magnitude of the correction was incorporated in the errors.

While our results on ^{24}Na formation from gold in reactions induced by pions constitute the first measurement of this type, a number of determinations have been reported for proton reactions. Lavrukhina *et al.*[20] thus measured the excitation function for the formation of ^{24}Na from gold in reactions induced by 220–660-MeV protons. Except for the lowest energy, their cross sections are substantially lower than the present values. Their reported value at 660 MeV thus is 81 μb, which may be compared with the 120-μb value read off our excitation function. Crespo, Alexander, and Hyde[8] measured the cross section at 700 MeV. Their value of 135 μb agrees within the limits of error with our interpolated value of 150 μb. Korteling and Caretto[18] performed a similar determination at 400 MeV and report a value of 21.5 μb compared to the present result of 27 μb.

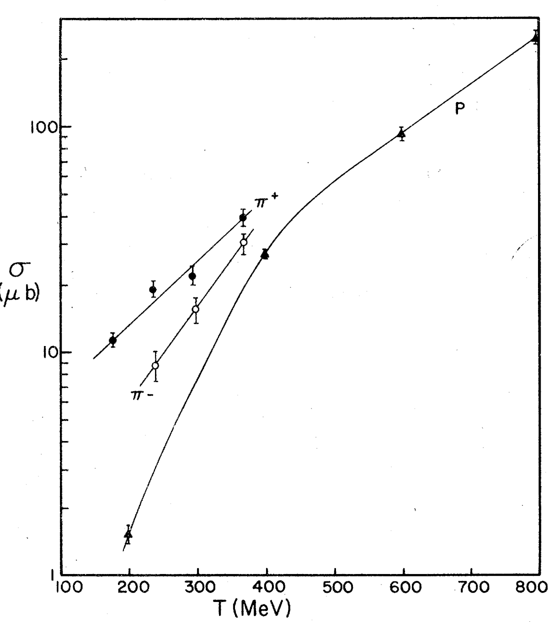

FIG. 1. Excitation functions for the formation of ^{24}Na from gold by protons, π^+, and π^-.

IV. DISCUSSION

The qualitative features of the data are apparent in Fig. 1. The cross sections of all three reactions increase in a pronounced way with energy indicating that ^{24}Na formation requires the transfer of high excitation energy or momentum to the struck nucleus. At a given bombarding energy the cross section is highest for π^+ and lowest for protons, the differences becoming largest at the lowest energies. The situation differs if the pion energies are shifted upward by 140 MeV in order to permit a comparison with the proton data at the same total available energy. The π^- cross sections are now uniformly lower than the proton values while the π^+ cross sections are lower at all but the lowest proton energies.

A. Comparison between π^+ and π^- cross sections

The difference between the π^+ and π^- cross sections, which on the average amounts to some 60%, is somewhat surprising. There are several factors that can lead to such differences. Since pion absorption is of importance in pion-induced reactions the relative probability of π^+ and π^- absorption in gold has to be considered. If we make the reasonable assumption that the pion is absorbed on a pair of nucleons, there are two absorption processes which cannot occur for both π^+ and π^-. The $\pi^+ + 2n \rightarrow p + n$ reaction cannot occur for π^- while the $\pi^- + 2p \rightarrow p + n$ cannot occur for π^+. Since gold has approximately 50% more neutrons than protons the contribution of the $\pi^+ + 2n$ reaction should be larger than that of the $\pi^- + 2p$ and so the absorption cross section should be correspondingly larger for π^+. Monte Carlo cascade calculations[21] confirm these conclusions but indicate that the difference in absorption cross sections amounts to no more than 2% and so cannot account for the observed effect.

Another possible explanation of the observed difference lies in the dependence on target Z of the ^{24}Na production cross sections in proton reactions. It has been found that for heavy elements the ^{24}Na cross sections increase with target Z. For instance, at 400 MeV the cross section increases from 19 μb for Ta to 45 μb for U.[18] Between tantalum and gold the increase is approximately 2% per Z unit, although the fluctuations in the heavy element cross sections are such as to make this value rather uncertain. The difference in Z between the composite systems formed in π^+ and π^- bombardment is 2 while that between the residual nuclei resulting from cascades[21] involving pion absorption is 1.3. If we assume that the cross sections in the vicinity of gold do, in fact, increase by 2% per Z in pion as in proton reactions, we expect the maximum difference between π^+ and π^- yields to be about 4%, again much smaller than the observed difference.

Yet another possible reason for the difference lies in the effect of target composition on the relative yield of neutron-excessive and neutron-deficient products. This "memory" effect was observed by Porile and Church[22] in proton spallation reactions and has more recently also been observed in pion spallation.[11,23] In the case of the latter, it was found that the yield of neutron-deficient products formed in the spallation of copper was much higher for π^+ than for π^- while the opposite held true for neutron-excessive products. However, the relevance of these results to the present situation is not clear since ^{24}Na formation from gold does not involve spallation. The results of Juliano and Porile[24] on the formation of ^{24}Na in reactions induced by 11.5-GeV protons are of interest in this connection. In an attempt to distinguish between the effects of target N/Z and target Z in determining the magnitude of the ^{24}Na production cross section from heavy elements, these workers measured the ^{24}Na yield from separated isotopes of uranium as well as from ^{208}Pb. Since ^{208}Pb and ^{233}U have practically the same N/Z a finding of comparable ^{24}Na cross sections from these targets would have constituted evidence that target composition rather than target Z played a major role in determining fragment yields. It was instead found that the ^{24}Na yield from ^{233}U was some 50% higher than that from ^{208}Pb and was, in fact, about the same as that from ^{235}U and ^{238}U. It was thus concluded that the observed variation in the ^{24}Na yield from heavy elements could be primarily ascribed to a dependence on target Z (or A). To be sure, the present situation is somewhat different since the comparison involves a given product made from isobaric target-projectile composite systems so that these results are not conclusive.

In a study of the formation of delayed neutron-emitting light fragments from various targets bombarded by high-energy protons, Dostrovsky *et al.*[25] calculated their relative formation cross sections on the assumption of an evaporation mechanism. The calculation involved a modified version of the DFF evaporation code[26] performed on the distribution of residual nuclei obtained from Monte Carlo cascade calculations. The experimental trends were well reproduced and the authors concluded that even if fragment emission occurred prior to equilibration, the role of statistical phase space factors was of importance in determining the yields of specific fragments. A similar calculation of ^{24}Na yields, in which careful attention was paid to the emission of unbound nuclides which could decay to ^{24}Na as well as to that of isobaric progenitors, was performed by Porile.[27] This calculation was also able to predict the experimental trends and indicated the importance of such evaporation theory factors as level densities and binding energies in accounting for the data. It is of interest to see whether evaporation theory can account for the effect of present interest.

We start with the results of Monte Carlo cascade calculations for 180- and 300-MeV π^+ and π^- incident on gold.[21] Since fragment emission occurs on a rapid time scale we make the simplifying assumption that fragment evaporation only competes with that of nucleons and light particles during the initial step. Since the HS process is bound to be of importance in pion reactions we restrict the calculation to those cascades in which pion absorption occurs. This assumption is actually not very restrictive since the fragment evaporation probability exhibits a strong dependence on excitation energy and cascades resulting in absorption lead to much higher energy transfers than those in which pion inelastic or charge-exchange scattering occurs. Finally, we simplify the calculation by grouping all absorptive cascades leading to a given nuclide together and characterizing this intermediate by an appropriate average excitation energy $\bar{E}^*$ and production cross section.

The above procedure yielded some 20–30 residual nuclei. The relative evaporation probability of a neutron, proton, α particle, and ^{24}Na, and its progenitors was evaluated for each of these nuclides by means of the DFF formalism.[26] The evaluation of the fragment evaporation probabilities required some important modifications of this formalism as described in detail by Porile.[27] This study indicated that in evaluating the total evaporation probability of ^{24}Na it was necessary to consider as well the emission of the ^{24}Ne isobaric progenitor and that of the following progenitors in unbound states: ^{25}Ne, ^{25}Na, ^{25}Mg, and ^{28}Al. Each of these nuclides can be emitted in any one of a myriad of excited states. The number of these states was estimated by means of the Fermi gas model level density integrated over an appropriate energy interval. Each fragment was then assumed to be emitted at a unique excitation energy such that the number of levels below this energy was equal to the number above. The resulting emission probability was multiplied by a weighting factor representing the combined effect of the number of levels in the energy interval of interest, their statistical weight, and their probability of deexciting to ^{24}Na or one of its isobaric progenitors. For instance, the evaporation width of ^{28}Al was evaluated at an energy of 28.2 MeV and was weighted by a factor of 148 relative to the ^{24}Na width evaluated at 5.0 MeV. The resulting fragment evaporation probabilities for each residual nucleus were weighted by the respective formation cross sections in order to obtain the ^{24}Na evaporation cross section.

It was found that the ^{24}Na cross sections are extremely sensitive to the assumed value of the level density parameter a of the residual nuclei formed by evaporation. For instance, the cross sections for 300-MeV π^+ increased from 0.0070 to 144 μb as a decreased from $A/10$ to $A/30$. Even if this had not been the case the calculation of *absolute* cross sections by this procedure would not have been particularly meaningful because of the severe approximation introduced by the level density representation of the fragment excited states. On the other hand, our approach has considerably more merit for an estimation of *relative* fragment yields and it is found, in fact, that the π^+/π^- cross section ratios are almost independent of a.

The calculated ratios are compared with the experimental values in Fig. 2. The former are averages of the $a = A/20$ and $a = A/30$ results. These were chosen since the experimental cross sections in all cases lie between these calculated values. The flags represent the difference between these two calculations. The curve drawn through the experimental points actually represents the ratio of the smooth curves drawn through the experimental cross sections in Fig. 1. Since the π^- measurements were not performed below 240 MeV, this curve permits an extrapolation of the experimental ratio down to 180 MeV on the assumption that the slope of the π^- excitation function does not change over this interval. It is seen that the calculated ratios agree with the experimental values indicating that, regardless of the details of the mechanism, phase space factors play an important role in determining the cross section ratios.

While the enhanced calculated yield of ^{24}Na in π^+ reactions arises from the combined effects of the distribution of residual nuclei in Z, A, and $\bar{E}^*$ on the evaporation widths, there is one factor that is dominant in a comparison between π^+ and π^-. This is the hitherto referred to difference in isobaric distribution of the residual nuclei resulting from π^+ and π^- reactions. This difference is graphically illustrated in Fig. 3, which shows the fractional isobaric yield of the residual nuclei resulting from bombardment by 180-MeV pions, integrated over all mass numbers. A shift of approximately one Z unit between the π^+ and π^- distributions is apparent. Also shown in the figure is the fractional emission probability of ^{24}Na and its progenitors as a function of Z. This probability was evaluated at a constant excitation energy for

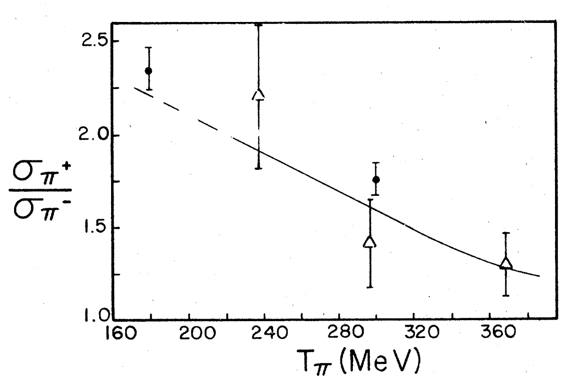

FIG. 2. Energy dependence of the ratio of π^+ to π^- ^{24}Na production cross sections. Open points, experimental values; closed points, Monte Carlo cascade-evaporation calculations. The curve represents the ratio of the smooth excitation functions drawn through the data in Fig. 1 and is based on an extrapolation of the π^- cross section below 240 MeV.

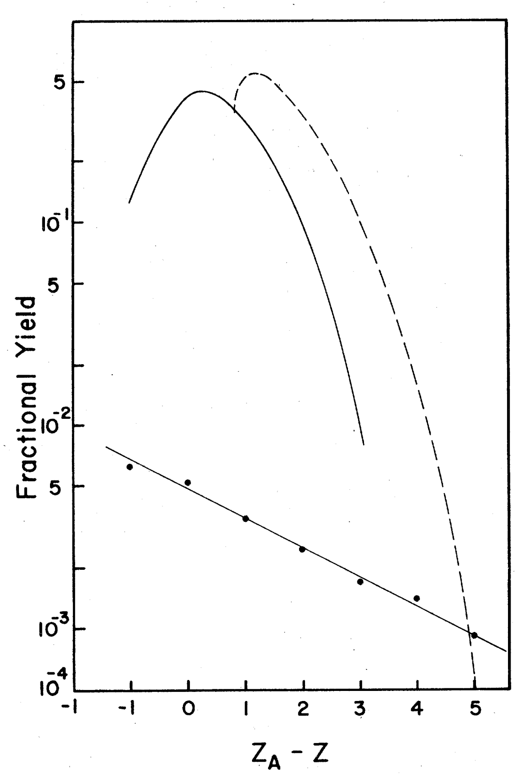

FIG. 3. Isobaric yield distribution of the residual nuclei resulting from cascades initiated by 180-MeV π^+ (solid curve) and π^- (dashed curve). The solid line through the points represents the isobaric dependence of the fragment evaporation probability averaged over mass number and evaluated at a constant excitation energy.

the most important mass chains and then averaged for a given $(Z_A - Z)$ over A. The fragment emission probability decreases exponentially with decreasing Z and this effect, coupled with the shift in the isobaric distribution of the emitting nuclei, is responsible for the observed effect. The variation of the fragment emission probability with $(Z_A - Z)$ appears, at first sight, to be somewhat surprising. One might thus have expected a neutron-rich fragment to be more readily emitted from a neutron-excessive nuclide, thereby leading to the opposite of the observed trend. It turns out, however, that neutron emission, which is the dominant decay channel, also exhibits a strong isobaric dependence. The neutron emission width thus increases with $(Z_A - Z)$ more sharply than the fragment width. Since the fragment evaporation probability is essentially given by the ratio of these widths the observed trend ensues. The ratio of π^+ to π^- cross sections thus ultimately hinges on the shape of the mass-energy surface. It should also be noted that at 300 MeV the difference in the isobaric yield distributions of the cascade products is less pronounced than at 180 MeV and so the ratio of π^+ to π^- cross sections is somewhat closer to unity.

B. Comparison between pion and proton cross sections

The comparison between the π^+ and proton cross sections may be used to obtain an estimate of the relative importance of the HS and CC processes in the formation of ^{24}Na in intermediate-energy proton reactions. We can at the outset state that the CC process must be of negligible importance in reactions induced by intermediate-energy pions. This follows from the previously mentioned fact[12] that the probability of coherent cascades in reactions induced by hadrons depends on their momentum. The momenta of 180- and 370-MeV pions are about equal to those of 40- and 120-MeV protons, respectively. The proton excitation function in Fig. 1 shows that the ^{24}Na production cross sections at these energies should be negligibly small compared to the 180–370-MeV pion cross sections. We can thus conclude that, even in the unlikely eventuality that the CC process were dominant in proton reactions, the expected yields for the same pion momentum must be lower than the observed values for 180–370-MeV pions by perhaps two or more orders of magnitude.

We will thus use the observed pion cross sections as a measure of the HS yield and attribute any excess yield in proton reactions to the CC process. In comparing the pion and proton cross sections we must choose the proton energy E_p at which the ^{24}Na yield is to be compared with that obtained from pions having energy E_π. It has been customary[11,12] when comparing proton and pion spallation yields to make the comparison at the same total energy, i.e., $E_p = E_\pi + 140$ MeV, on the assumption that pions transfer about the same energy to the struck nuclei as protons of the same total energy. Instead of making this assumption we chose to inspect in detail the results of Monte Carlo cascade simulations[21] in order to obtain the desired energy correspondence for the process of interest. Our procedure is as follows. We first examine the excitation energy distribution of π^+ cascades resulting in pion absorption and determine the interval associated with the most energetic interactions. Next, we use proton cascade simulations to determine the proton bombarding energy for which the excitation energy interval in question constitutes the same fraction of the total cross section as it does for pions. Finally, we use the cascade-evaporation formalism described in the preceding section to correct the results for any differences between the two distributions of residual nuclei. In addition, we must also normalize the results to take into account the difference between the total reaction cross section for protons, $\sigma_R(E_p)$, and the π^+ absorption cross section, $\sigma_A(E_{\pi^+})$. If the corrected proton cross section determined in this fashion is higher than the comparable pion value the excess may be attributed to the CC process. The cross section for the CC process at a proton energy E_p is thus given by the relation

$$\sigma_{\rm CC}(E_p) = \sigma_{\rm exp}(E_p) - [\sigma_{\rm exp}(E_{\pi^+} = E_p - \Delta E)(\sigma_R(E_p)/\sigma_A(E_{\pi^+}))f(Z,A,\bar{E}^*)], \quad (1)$$

where $\sigma_{\rm exp}(E_p)$ and $\sigma_{\rm exp}(E_{\pi^+})$ are the experimental p and π^+ cross sections for ^{24}Na formation at the indicated energies, ΔE is the difference between E_p and E_{π^+} obtained in the manner described above, and $f(Z,A,\bar{E}^*)$ is the ratio of fragment evaporation yields from the proton and π^+ cascade distributions for the common excitation interval of interest.

We have performed the above calculation for proton energies corresponding to 180- and 300-MeV π^+. We now present the details of the calculation performed at the lower of these energies.

The distribution of excitation energies resulting from the absorption of 180-MeV π^+ by ^{197}Au is shown as a probability plot in Fig. 4. We shall characterize this spectrum by the lower energy limit above which a certain fraction of the interactions lie, e.g., $E^*_{50\%} \geq 160$ MeV and $E^*_{25\%} \geq 208$ MeV. These designations mean that the top quarter

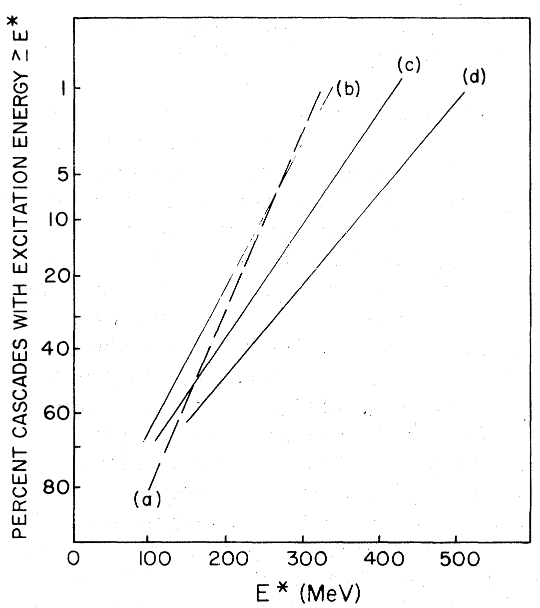

FIG. 4. Probability plot of the cumulative excitation energy spectrum of residual nuclei resulting from the interaction of ^{197}Au with 180-MeV π^+ (line a) and 300- (b), 440- (c), and 600- (d) MeV protons. Data are based on Monte Carlo cascade simulations (Ref. 21).

and the top half of the absorptive interactions result in energy transfers in excess of 208 and 160 MeV, respectively. We perform separate calculations for each of these intervals on the assumption that ^{24}Na emission is associated with either the most energetic 25% or 50% of the interactions. While these choices are, of course, arbitrary, the steepness of the excitation functions indicates that only the more inelastic interactions can lead to fragment emission. If our calculation is to be meaningful the results must be reasonably insensitive to the choice of cutoff energy. Figure 4 includes results of cascade calculations performed for protons of several energies. By interpolation between these data it is found that the proton energies for which $E^*_{50\%} \geq 160$ MeV and $E^*_{25\%} \geq 208$ MeV are 430 and 345 MeV, respectively. We thus compare the ^{24}Na cross section measured for 180-MeV π^+ with the corresponding value obtained for either 430- or 345-MeV protons. Note that these energies exceed the π^+ energy by substantially more than the π^+ rest mass. The correction for differences in the $(Z, A, \bar{E}^*)$ distribution of the residual nuclei was based on calculations performed for 300- and 440-MeV proton interactions and subsequent interpolation to the desired energies. The evaporation calculations were performed for each residual nucleus and the fragment emission probability was evaluated at the average excitation energy of those events lying in the top 50% or 25% E^* interval. It was found that in spite of the fact that these intervals had the same lower cutoff energy for the various cascade calculations, the $\bar{E}^*$ values for a given nuclide varied due to the differences in the shape of the E^* spectrum. For instance, $\bar{E}^*$ of the interactions leading to the ^{196}Au residual nucleus with $E^* \geq 160$ MeV was 217 MeV for 180-MeV π^+, 209 MeV for 300-MeV p, and 246 MeV for 440-MeV p. The difference in $\bar{E}^*$ values in fact turned out to be the major contribution to $f(Z, A, \bar{E}^*)$. Finally, the values of $\sigma_R(E_p)$ and $\sigma_A(E_{\pi^+})$ were obtained directly from the cascade calculations.

The results obtained in this fashion are summarized in Table III. The comparison of the 180-MeV π^+ cross section with the 345–430-MeV proton data shows that at these proton energies there is no significant contribution from the CC process; the HS mechanism can account for the entire cross section. On the other hand, a significantly different result is obtained in the comparison of the 300-MeV π^+ cross section with the 560–700-MeV proton values. At these higher proton energies the CC process accounts for about half the fragment yield. This trend is physically reasonable since the probability of the CC process must increase with the mean number of cascading nucleons and this number in turn increases with bombarding energy.

How reliable are the quantitative estimates obtained in our analysis? The most sensitive feature of the calculation is the choice of proton energy at which to compare the pion and proton data. For instance, if the 300-MeV π^+ cross section had been compared with the proton value obtained at 500 MeV rather than that at 700 MeV the CC contribution would have been reduced to zero. The choice

TABLE III. Evaluation of the contribution of the coherent cascade (CC) mechanism to the formation of ^{24}Na in proton reactions.

E_{π^+} (MeV)	$\sigma_{\rm exp}(E_{\pi^+})$ (μb)	Excitation energy interval	E^* (MeV)	E_p (MeV)	$\sigma_{\rm exp}(E_p)$ (μb)	$\sigma_R(E_p)/\sigma_A(E_{\pi^+})$	$f(Z,A,E^*)$	$\sigma_{CC}(E_p)$ (μb)	$\sigma_{CC}(E_p)/\sigma_{\rm exp}(E_p)$
180	11.6	Top 50%	≥160	430	36.0	1.92	1.62	0	0
		Top 25%	≥208	345	14.3	1.86	1.01	0	0
300	25.2	Top 50%	≥222	700	153	1.86	1.22	96	0.63
		Top 25%	≥280	560	76.5	1.79	1.04	30	0.39

of proton energy is critically dependent on the validity of the shape of the excitation energy spectrum obtained from the Monte Carlo cascade calculation. If the pion calculation were to substantially overestimate the probability of large energy transfers or if the proton calculation were to underestimate it, the necessity of invoking the CC mechanism could be eliminated. The validity of the calculated E^* spectrum can be checked by comparison of calculated and experimental spallation yields. Such comparisons[23,28,29] indicate that the cascade code of present interest[21] does a reasonably good job of reproducing the shape of the mass yield curve in the mass region resulting from the deposition of high energies. Even if the E^* spectrum should not be correctly predicted in the case of present interest, it is reasonable to assume that the deviations for incident pions and protons are comparable since the calculation is based on the same model in both cases. However, as noted above, the CC contribution can only be reduced if the deviations in the calculated E^* spectrum for protons and pions lie in opposite directions, and this seems unlikely.

The calculated CC cross sections also depend, though in a less sensitive way, on the other quantities listed in Table III. It has been pointed out[28] that the cascade calculation underestimates the pion absorption cross section by as much as 35%, whereas it closely matches the experimental values of the proton reaction cross section. This effect would increase the estimated contribution of the CC process. The uncertainty in the $f(Z,A,\bar{E}^*)$ values is difficult to evaluate but is probably not large. The discussion in the preceding section indicated, in fact, that this evaporation theory factor provides the only quantitative explanation of the observed ratio of π^+ to π^- cross sections. The fact that the f values are comfortably close to unity further indicates that they affect the results in only a minor way. Finally, the experimental cross sections, which were read off the excitation function curves in Fig. 1, have 10–30% uncertainties, depending on the extent of the interpolation between measured values.

In summary, we feel that while the calculated values of the CC cross sections are subject to sizable uncertainties they are sufficiently accurate to provide at least a qualitative picture of the changes in mechanism with increasing bombarding energy. The 50% difference between the two values derived from the comparison with the 300-MeV π^+ data appears to be consistent with the overall uncertainty in the estimate. In spite of the quantitative aspects of this estimate we must, however, add a cautionary note concerning the speculative nature of any mechanistic analysis band solely on integral cross section data.

Confirmatory evidence for the results of the above calculation come from a somewhat different analysis. We have followed the procedure described in the preceding section and used the cascade-evaporation formalism to evaluate the ratio of ^{24}Na cross sections formed by protons and π^+ having the same kinetic energy. This ratio may then be compared with that of the corresponding experimental values. We find that at 180 MeV the values of $(\sigma_p/\sigma_{\pi^+})_{\rm calc}$ and $(\sigma_p/\sigma_{\pi^+})_{\rm exp}$ are approximately equal whereas at 300 MeV the latter is 3 times larger than the former, suggesting the presence of an additional mechanism for fragment emission in proton reactions. While it is encouraging that this calculation yields qualitatively similar results it probably should not be given too much weight because the large difference in the shapes of the excitation energy spectra in proton and pion reactions undoubtedly strains it beyond the limit of its validity.

V. CONCLUSIONS

A detailed analysis of the cross sections for ^{24}Na production in reactions induced in ^{197}Au by intermediate energy protons and π^+ performed with the aid of Monte Carlo cascade simulations yields some new information about the fragmentation process. While it may be concluded on quite general grounds that only a hot-spot (HS) mechanism can lead to fragmentation in intermediate-energy pion reactions, there is an additional mechanism involving the occurrence of coherent cascades (CC), which may be of importance in proton reactions. Our analysis shows that while this mechanism does not contribute at 300–400 MeV, it may become of comparable importance to the HS process at 600–700 MeV.

The comparison of the ^{24}Na cross section in π^+ and π^- reactions shows a higher yield for π^+. The magnitude of this ratio as well as its energy dependence can only be accounted for by invoking phase space factors which are more commonly associated with an equilibrium process. While all the accepted mechanisms of fragmentation involve pre-equilibrium emission, we conclude that the role of statistical phase space remains of importance even in these fast processes.

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