Effect of the spin-dependent electromagnetic forces on low-energy proton-proton scattering

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The effects of electromagnetically induced spin-dependent potentials on the analyzing power for low-energy proton-proton scattering are calculated using the Coulomb-distorted-wave Born approximation. The amplitudes which arise from these potentials are nearly in phase with the usual Coulomb amplitude and as a result the potentials have virtually no effect on the analyzing power.

[NUCLEAR REACTIONS polarized p-p scattering, electromagnetic effects.]

In the scattering of particles with spin, polarization effects can arise from purely electromagnetic interactions. For example, the interaction of the magnetic moment of a projectile with the Coulomb field of the target produces a spin-dependent force which can result in a nonzero polarization or analyzing power. In proton-proton scattering at energies of a few hundred MeV these effects are significant. Thus if one wishes to isolate the nuclear effects, it is necessary to correct for the electromagnetic processes. At lower energies ($E_p < 50$ MeV) corrections have been unnecessary since the errors in the measured analyzing powers were much larger than the predicted electromagnetic effects. However, recent polarized beam experiments[1,2] have produced highly accurate analyzing power data at 10 and 16 MeV. These new measurements make it necessary to reconsider the importance of the spin-dependent electromagnetic potentials. In this paper we calculate the effects of these potentials using approximations which are valid at low energies.

From relativistic theories of the proton-proton interaction (see Ref. 3) one finds that, to first order in the fine-structure constant, the proton-proton potential contains two spin-dependent terms of electromagnetic origin: a spin-orbit potential,

$$V_{ls} = -8\mu_0(\mu_T - \tfrac{1}{4}\mu_0) r^{-3}\vec{l}\cdot\vec{S}\,, \qquad (1)$$

and a tensor potential,

$$V_T = -\mu_T{}^2 r^{-3} S_{12}\,, \qquad (2)$$

where

$$\mu_0 = e\hbar/2mc\,, \qquad (3)$$

$$\mu_T = 2.792\,76\mu_0\,, \qquad (4)$$

$$\vec{S} = \tfrac{1}{2}(\vec{\sigma}_1 + \vec{\sigma}_2)\,, \qquad (5)$$

$$S_{12} = 3(\vec{\sigma}_1\cdot\hat{r})(\vec{\sigma}_2\cdot\hat{r}) - \vec{\sigma}_1\cdot\vec{\sigma}_2\,, \qquad (6)$$

and where m is the proton mass. The spin-orbit potential describes the interaction of the magnetic moment of each proton with the Coulomb field of the other and includes a correction for the Thomas precession, while the tensor potential corresponds to the moment-moment interaction.

In order to calculate the effect of these spin-dependent potentials we make use of the distorted-wave Born approximation for the scattering from two potentials. For a potential of the form

$$U = V + V' \qquad (7)$$

the elastic scattering wave function, $\psi^{(+)}$, satisfies an integral equation

$$\psi^{(+)}(\vec{k},\vec{r}) = \phi^{(+)}(\vec{k},\vec{r}) - \int G^{(+)}(\vec{r},\vec{r}\,')V'(\vec{r}\,')\psi^{(+)}(\vec{k},\vec{r}\,')d^3r'\,. \qquad (8)$$

Here $\phi^{(+)}$ is a wave function for scattering from V alone, and G is a Green's function corresponding to V. The distorted-wave Born approximation is obtained by replacing ψ on the right-hand side of Eq. (8) by ϕ.

We take V to be the sum of the Coulomb and nuclear potentials and treat the potentials of Eqs. (1) and (2) as perturbations. Now for low energies the nuclear potential has a large effect only in the $l=0$ partial wave. Since the Pauli principle restricts the $l=0$ scattering to the singlet spin state (where the spin-dependent forces vanish), it is reasonable to neglect the effect of the nuclear forces when evaluating the integral in Eq. (8). We therefore replace $\psi^{(+)}$ in the integral by a pure Coulomb wave function, $\phi_C^{(+)}$, and take $G^{(+)}$ to be the usual Coulomb Green's function. For large r, $G^{(+)}$ becomes

$$G^{(+)}(\vec{r},\vec{r}\,') \xrightarrow[r\to\infty]{} \frac{m}{4\pi\hbar^2}\frac{1}{r}\exp\{i[k'r - \eta\ln(2k'r)]\} \times \phi_C^{(-)*}(\vec{k}\,',\vec{r}\,')\,, \qquad (9)$$

where η is the Coulomb parameter

$$\eta = \tfrac{1}{2}me^2/\hbar^2 k\,. \qquad (10)$$

In order to describe the scattering of identical particles, we must ensure that $\psi^{(+)}$ is properly antisymmetrized. This is done by simply choosing $\phi_C^{(+)}$ to be an antisymmetric wave function with the correct boundary conditions. On the other hand, the Green's function [i.e., the wave function $\phi_C^{(-)}$ in Eq. (9)] should not be antisymmetrized. Thus we find that the scattering amplitude is given by

$$f(\vec{k},\vec{k}')=f_C(\vec{k},\vec{k}')+f_N(\vec{k},\vec{k}') -\frac{m}{4\pi\hbar^2}\int \phi_C^{(-)*}(\vec{k}',\vec{r}')V'(\vec{r}')\phi_C^{(+)}(\vec{k},\vec{r}')d^3r' , \tag{11}$$

where f_C and f_N are the Coulomb and nuclear scattering amplitudes, respectively, and where it is understood that $\phi_C^{(+)}$ is to be antisymmetrized but that $\phi_C^{(-)}$ is not.

Since the matrix elements of $\vec{l}\cdot\vec{s}$ and S_{12} between singlet spin states are zero we consider only the triplet states. In this case the appropriate Coulomb amplitude is

$$f_C(k,k')=-\frac{\eta}{k}\left\{\frac{1}{1-\mu}\exp[-i\eta\ln\tfrac{1}{2}(1-\mu)] -\frac{1}{1+\mu}\exp[-i\eta\ln\tfrac{1}{2}(1+\mu)]\right\}e^{2i\alpha_0}, \tag{12}$$

where

$$\mu=\hat{k}\cdot\hat{k}' . \tag{13}$$

For the Coulomb wave functions we write

$$\phi_C^{(+)}(\vec{k},\vec{r})=(8\pi/kr)\sum_{\text{odd } l}\sum_m i^l e^{i\alpha_l}F_l(kr)Y_l^m(\hat{r})Y_l^{m*}(\hat{k}) , \tag{14}$$

$$\phi_C^{(-)*}(\vec{k},\vec{r})=(4\pi/kr)\sum_{\text{all } l}\sum_m i^{-l}e^{i\alpha_l}F_l(kr)Y_l^{m*}(\hat{r})Y_l^m(\hat{k}) . \tag{15}$$

Here α_l is the usual Coulomb phase shift, and F_l is the regular solution to the Coulomb radial wave equation.

We now consider the spin-orbit potential in Eq. (1). We must evaluate the matrix element

$$\langle\sigma'|f_{ls}|\sigma\rangle=C_{ls}\int\phi_C^{(-)*}(\vec{k}',\vec{r})r^{-3} \times\langle\chi_1^{\sigma'}|\vec{l}\cdot\vec{s}|\chi_1^{\sigma}\rangle\phi_C^{(+)}(\vec{k},\vec{r})d^3r , \tag{16}$$

where $|\chi_1^\sigma\rangle$ is a triplet spin function and where

$$C_{ls}=2m\mu_0(\mu_T-\tfrac{1}{4}\mu_0)/\pi\hbar^2 . \tag{17}$$

We choose a coordinate system which has its z axis along $\vec{k}$ and its y axis along $\vec{k}\times\vec{k}'$. It is then straightforward to show that

$$\langle\sigma'|f_{ls}|\sigma\rangle=8\pi C_{ls}\langle\chi_1^{\sigma'}|iS_y|\chi_1^{\sigma}\rangle \times\sum_{\text{odd } l}e^{i\alpha_l}(2l+1)M_{ll}^{-3}P_l^1(\mu) , \tag{18}$$

where

$$M_{ll'}^{-3}=k^{-2}\int_0^\infty r^{-3}F_l(kr)F_{l'}(kr)dr \tag{19}$$

and

$$P_l^\lambda=(1-\mu^2)^{\lambda/2}\frac{d^\lambda}{d\mu^\lambda}P_l(\mu) . \tag{20}$$

From Ref. 4 we find

$$M_{ll}^{-3}=[2l(l+1)(2l+1)]^{-1} \times\left(2l+1-\eta\pi+1+\eta\pi\coth\eta\pi-2\eta^2\sum_{p=0}^{l}\frac{1}{p^2+\eta^2}\right) . \tag{21}$$

In principle, one could evaluate f_{ls} by summing Eq. (18) term by term with a computer. However, the sum converges too slowly for this to be a practical method. Instead we split f_{ls} into two pieces,

$$\langle\sigma'|f_{ls}|\sigma\rangle=[f_1(\mu)+f_2(\mu)] \times\langle\chi_1^{\sigma'}|iS_y|\chi_1^{\sigma}\rangle , \tag{22}$$

where f_1 is defined as

$$f_1(\mu)=4\pi C_{ls}\sum_{\text{odd } l}e^{2i\alpha_l}(2l+1)[l(l+1)]^{-1}P_l^1(\mu) . \tag{23}$$

This separation is convenient because f_2 converges rapidly enough to be summed term by term, while f_1 can be summed analytically:

$$f_1=4\pi C_{ls}e^{2i\alpha_0}(1-\mu^2)^{-1/2} \times\{\exp[-i\eta\ln\tfrac{1}{2}(1-\mu)]+\exp[-i\eta\ln\tfrac{1}{2}(1+\mu)]-1\} . \tag{24}$$

If one were to use the plane-wave Born approximation to calculate the scattering amplitude these results would be somewhat modified. The plane-wave result is obtained by setting $f_2=0$ and setting $\alpha_0=\eta=0$ in Eq. (24). This has little effect on the magnitude of f_{ls}, but does change the phase (see below). Ebel and Hull[5] have used the plane-wave Born approximation to calculate the phase shifts resulting from the spin-orbit potential, but have included the effect of the Coulomb potential by including factors $\exp(2i\alpha_l)$ in the partial wave expansion for f_{ls}. Using this approach they obtain precisely the result in Eq. (24); however, the term f_2 is missing. At energies near 10 MeV, this is not serious since $|f_2|$ is about an order of magnitude smaller than $|f_1|$.

We now consider the tensor potential defined in Eq. (2). Using an approach similar to that described above we find

$$\langle\sigma'|f_T|\sigma\rangle=(4\sqrt{10}m\mu_T{}^2/\hbar^2)\sum_{\text{odd } l}\sum_{l',\lambda}(2l+1)i^{l-l'}[(l'-\lambda)!/(l'+\lambda)!]^{1/2}e^{i(\alpha_l+\alpha_{l'})}M_{ll'}{}^{-3}\langle l0,2\lambda|l'\lambda\rangle\langle l0,20|l'0\rangle P_{l'}^{\lambda}(\mu)\,, \tag{25}$$

where $M_{ll}{}^{-3}$ is given in Eq. (19) and[4]

$$M_{l,l+2}{}^{-3}=M_{l+2,l}{}^{-3} =(6|l+1+i\eta||l+2+i\eta|)^{-1}\,. \tag{26}$$

The quality f_T is easily evaluated by summing Eq. (25) term by term. Actual calculations show that the effect of the tensor force on the analyzing power is extremely small at low energies (since f_T can influence the analyzing power only through interference with other spin-dependent amplitudes) and therefore we consider this potential no further.

Numerical calculations show that the effect of the spin-orbit potential on the analyzing power for low-energy p-p scattering is also extremely small. This point is illustrated in Fig. 1, which shows the analyzing power for p-p scattering at $E_p=10$ MeV (Ref. 1). The solid curve was obtained from a phase shift calculation[1] in which the electromagnetic effects were neglected. When the amplitude f_{ls} is included one obtains a curve which is indistinguishable from the solid curve (the values of A change by less than 10^{-5}).

This result is somewhat surprising since $|f_1|$ is only about two orders of magnitude smaller than the Coulomb amplitude $|f_C|$. In order to understand why the effect on the analyzing power is so small, we set equal to zero the nuclear phase shifts for all partial waves with $l\geq 1$. In this case the analyzing power resulting from f_1 is given by[6]

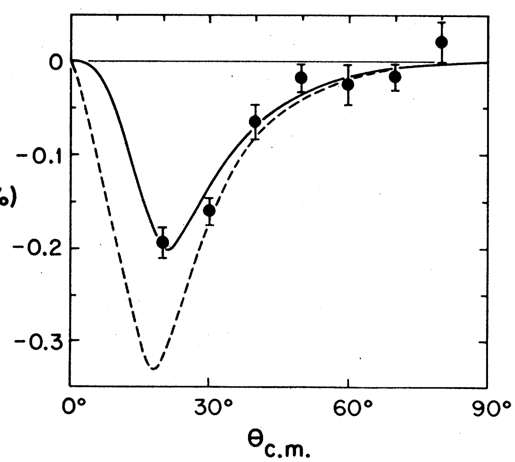

FIG. 1. Analyzing powers for p-p scattering at 10 MeV. The solid curve was obtained from a standard phase shift calculation in which the spin-dependent electromagnetic forces are neglected. When the amplitude f_{ls} [Eq. (18)] is included in the calculation the analyzing power is virtually unchanged. The dashed curve is obtained when the plane-wave expression for f_{ls} is used.

$$A=\text{Im}(f_1^*f_C)/(\tfrac{3}{4}|f_C|^2+\tfrac{1}{4}|f_{ss}|^2)\,. \tag{27}$$

Here f_{ss} is the scattering amplitude for the singlet state. At $\theta_{\text{c.m.}}=30°$, for example, $|f_1|\simeq 0.02\,|f_C|$ and $|f_{ss}|\simeq 4|f_C|$. Thus one might expect A to change by as much as 4×10^{-3} when f_1 is included. However, the change is much smaller than that because f_1 and f_C are almost exactly in phase. The phases of the amplitudes f_1, f_2, and f_C are shown in Fig. 2. Except for very small angles the phase differences are less than 1°.

Thus we see that it is important to get the phase of f_{ls} right. For example, if we use plane waves instead of Coulomb waves in calculating f_{ls} (as is commonly done for higher energies) the phase of the amplitude changes and the analyzing power is modified. The dashed curve in Fig. 1 shows the result which is obtained in this case.

Our results disagree with previously published calculations by Imai *et al.*[7] and by McKee and Osborn.[8] In Ref. 7 the information given is insufficient to determine the source of the error (although the use of plane waves could easily produce the results shown), while in Ref. 8, the error may be attributed to the use of wave functions which are not antisymmetric.

In conclusion, it has been shown that electromagnetically induced spin-dependent forces have virtually no effect on the analyzing power for low-energy p-p scattering, and that the phase change induced by the Coulomb distortion is partly responsible for this null effect.

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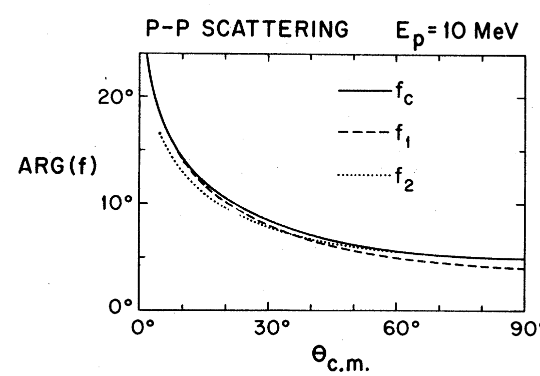

FIG. 2. Phase of the amplitudes f_C, f_1, and f_2 as a function of scattering angle.

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