Exponential model with pairing attenuation and the backbending phenomenon

P. C. Sood and A. K. Jain
Department of Physics, Banaras Hindu University, Varanasi 221005, India
(Received 31 October 1978)

Rotational energy calculations with an exponential dependence of the nuclear moment of inertia on pairing correlations coupled with an explicit spin dependence of the effective pairing gap analogous to that used in superconductivity give an excellent fit to the experimental data on excitation energies of yrast band levels in well-deformed even-even nuclei and also reproduce the backbending feature satisfactorily. The variation of the pairing gap parameter with spin is found to be gradual and smooth as compared to the abrupt variation evidenced in the pairing collapse treatment. Variations of the limiting moments of inertia parameter with mass number derived from our model exactly match with those coming out of the cranking model calculations for neutrons and are in no way related to the proton results for lighter rare earth nuclei; also the fluctuations in the values of the moment of inertia are found to correspond to those of the neutron single particle level spacings in this region.

[NUCLEAR STRUCTURE Even-even well-deformed rare earth nuclei, calculated yrast band level energies, exponential dependence of moment of inertia on pairing, explicit variation of pairing with spin, compared with experimental data and other calculations.]

I. INTRODUCTION

Investigations of the nuclear moments of inertia $\mathcal{I}$ and their variation with angular momentum I have been topics of immense interest, particularly since the observation in 1971 of the backbending feature depicting an abrupt rise in the values of the moments of inertia of yrast levels in certain deformed nuclei at angular momentum in the range 12–16. With development of the heavy-ion accelerators, the experimental data have flowed in rapidly while several models and theories have been proposed to explain this feature; the subject has been treated in several recent review articles[1-7] wherein individual references are available. Basically, two mechanisms have been invoked to describe the phenomenon. One approach considers as its basis the Mottelson-Valatin effect,[8] i.e., a collective superfluid to normal phase transition as the pairing correlations disappear at a certain critical angular momentum. Alternatively, in the alignment scheme of Stephens and Simon,[9] backbending corresponds to the shift to a configuration leaving two nucleons unpaired and in a high j orbital with the maximum projection of their angular momentum along the rotation axis. Alignment need not be perfect for this process to be competitive and several investigators have considered the situations wherein both these competing processes are operative. Approaches based on band mixing[10-14] and other phenomenological or empirical considerations[15-18] have also been pursued; such approaches, while providing more or less satisfactory fits to the data, do not generally shed much light on the physical processes, and the parameters of the models are just effective parameters with no straightforward physical interpretation.

However, the basic fact arising from all the studies so far is that the phenomenon involves attenuation of the pairing correlations, maybe to the extreme of the pairing collapse and phase transition or simply the decoupling of a pair with subsequent alignment and appearance of a decoupled band. In fact, as discussed below, even smaller-than-the-rigid-body value of the moment of inertia in the ground state and also its slow monotonic rise at low spins is mainly accounted for by the inclusion of the pairing correlations. Thus it seems logical to work with a model which incorporates an explicit relationship between the moment of inertia and the pairing correlation parameter. Such a relationship was indicated in the phenomenological studies of Draper[19] and was later established by Ma and Rasmussen[20] through cranking model calculations. These studies established that the dependence of the nuclear moments of inertia on pairing correlations is nearly exponential over most of the region of physical interest. We have also arrived[21,22] at a similar conclusion based on the single particle level density considerations in the shell-correction approach.[23] We have further included an explicit spin dependence of the pairing gap parameter similar to that adopted in the statistical model caluclations of Moretto[24] and analogous to the temperature dependence observed in superconductivity.[25] In the following we briefly discuss the exponential dependence of the moment of inertia on pairing correlations and suggest an explicit spin dependence of the latter (Sec. II). This formulation is employed for numerical calculation of the yrast band excitation energies for specific nuclei and the results compared with the experiment in Sec. III. A discussion of the physical significance of the model parameters derived from calculations *vis-à-vis* results from earlier investigations is presented in Sec. IV, and, finally, in Sec. V we summarize the conclusions arising from our studies.

II. PAIRING CORRELATIONS AND THE EXPONENTIAL DEPENDENCE

The three main features of the moment of inertia that are directly affected by the pairing correlations are (a) the ground state value of the moment of inertia that lies in between the rigid body and the irrotational flow limits, (b) the slow rise of the moment of inertia at moderate and low spins, and (c) the sudden rise of the moment of inertia observed at higher spins (the backbending feature) in some cases.

The effect of the pairing correlations in bringing down the rigid body estimate to the observed value is now well known.[26] The slow rise in the moment of inertia at lower spins may in part be the result of the shape changes due to centrifugal stretching; but recent empirical as well as theoretical findings indicate that this does not bring in significant contribution at least for the well-deformed nuclei. Measurement of the $B(E2)$ values in the ground state rotational bands of the well-deformed rare earth[27-29] and the actinide nuclei[7] reveals no rise up to the highest measured spins. For example, the $B(E2)$ values for Yb isotopes up to spin 18 are consistent with the rigid rotor values,[27] while the moment of inertia rises rapidly (a backbending in two cases, namely, ^{164}Yb and ^{166}Yb). Similar results have also been reported for Hf isotopes[28] and Er isotopes[29]; rather a drop in $B(E2)$ values near the backbend is observed in several cases. A number of theoretical calculations also report similar results.[30,31]

Coming to the studies involving an explicit relationship between the moments of inertia and the pairing correlations, it is noted that an exponential dependence was first indicated in the phenomenological studies of Draper,[19] who credited the idea to Rasmussen's report at the Alushta conference in April 1972. Draper's formulation may be viewed as a modification of the variable moment of inertia (VMI) model[32] in that the rotational energy expression can be written[33] as

$$E(I)=\frac{I(I+1)}{A\exp(x)}+\tfrac{1}{2}Cx^2;\quad \frac{dE}{dx}=0. \tag{1}$$

This expression provided good fits to rotational band energies at low and moderate spins, but is not very satisfactory at high spins.

Ma and Rasmussen[20] carried out cranking model calculations of the moments of inertia for a range of values of the pairing parameter Δ in the rare earth region. Results obtained show excellent straight line behavior in the $\ln\mathcal{I}$ versus Δ plots over a wide range of Δ values above a lower limit Δ_L which is about 20–30% of the ground state value of the energy gap Δ_0. Thus,

$$2\mathcal{I}(\Delta)/\hbar^2=(2\mathcal{I}_0/\hbar^2)e^{-\gamma\Delta},\quad \text{for } \Delta_L\leqslant\Delta\leqslant\Delta_0, \tag{2}$$

$$\Delta_L/\Delta_0\sim 0.3, \tag{3}$$

where $2\mathcal{I}_0/\hbar^2$ is the extrapolated value in the limit of zero pairing from the $\ln\mathcal{I}$ versus Δ plots. The region $\Delta_L\leqslant\Delta\leqslant\Delta_0$ usually covers the region of physical interest for most problems. They also concluded that the pairing does not completely disappear even at spins as high as 18. The application of the exponential relation through a pairing stretch model to calculation of rotational energies by Ma and Rasmussen led to "quite good energy fits" for the five cases studied, none of which, however, had known levels beyond $I=12$, and the predicted spectrum in none of these cases showed backbending.

We have been led to the exponential model through our consideration of the single particle level densites. It was empirically found[34] that a correlation exists between the presence of the gaps near the Fermi energy in the neutron single particle level scheme and the backbending feature for deformed nuclei. It may be intuitively argued that a large single particle level density around the Fermi energy implies a strong pairing and therefore an abrupt transition (backbending) to depairing, whereas a low level density implies a weak pairing and hence a smooth transition. In an attempt to provide a theoretical basis for this empirical interrelationship we found the philosophy of the shell-correction approach useful. In this approach we find[21,22] that the underlying physical process here is that of a transition from quasiparticles in the presence of pairing, to independent particles as the pairing disappears. Thus the variations in the quasiparticle density $g_{qp}(\epsilon_F)$ are passed on to the moment of inertia and this may or may not lead to backbending depending on the strength of the pairing correlations; this arises from the relations[21,22]

$$\begin{aligned}&\mathcal{I}\propto g_{qp}(\epsilon_F),\\&2\mathcal{I}/\hbar^2=Kg_{sp}(\epsilon_F)\exp(-\Delta),\end{aligned} \tag{4}$$

where $g_{sp}(\epsilon_F)$ is the single particle density around the Fermi energy and Δ is an effective pairing gap parameter containing an implicit scale to make it dimensionless. The first two factors are nearly constant for a given nucleus but may vary from nucleus to nucleus.

In the present study we do not wish to stress the underlying theoretical or physical basis of the exponential dependence of the moment of inertia on the effective pairing gap, but proceed to use such a dependence for calculating the rotational energies. For this purpose we have to bring in the explicit spin dependence of the pairing gap. We recall[26] that since $\mathcal{I}$ increases with decreasing Δ, it must be expected that Δ will be a decreasing function of I; for sufficiently large I the coupling to the rotation is expected to destroy the pair correlation (at least for the region of importance in the neighborhood of the Fermi energy), an effect analogous to the destruction of superconductivity by a magnetic field. In the latter case, the pairing gap is known[25] to have a temperature dependence of the type

$$\Delta(T) \propto \Delta(0)(1 - T/T_c)^{0.5}, \tag{5}$$

such that the pairing vanishes at the critical temperature T_c. A similar dependence of pairing gap on the angular momentum

$$\Delta(I) = \Delta(0)(1 - I/I_c)^{0.5} \tag{6}$$

for nuclear problems was obtained by Moretto[24] in his statistical model calculations. We adopt Eq. (6) along with the exponential dependence of Eq. (4) and apply these relations for calculation of the rotational energies.

III. RESULTS AND COMPARISON WITH EXPERIMENT

In accordance with the above discussion, we write the rotational energy expression as

$$E(I) = \frac{\hbar^2}{2\mathcal{I}_0} I(I+1) \exp[\Delta_0(1 - I/I_c)^{0.5}], \tag{7}$$

and use it to calculate the energies of the yrast levels in deformed nuclei. It is to be noted that, for even-even nuclei under consideration, I and I_c assume only even integral values and I_c is expected to lie in the neighborhood of 18 (higher for nonbackbending cases) for good rotors; it is to be remembered that our I_c is not the value corresponding to the onset of anomalous variation of moment of inertia $\mathcal{I}$ as usually adopted, but is the value where $\mathcal{I}$ has reached a maximum value. The other two parameters are left as free parameters to be determined for each individual nucleus by a least squares fit to the experimental data. For the the backbending nuclei, the maximum spin I_{max} used as the input cutoff corresponds to the point where the rotational frequency ω in $\mathcal{I}-\omega^2$ plots reaches a minimum value, i.e., the forward or down-bending region is not included, since in our description the moment of inertia becomes constant after the critical spin; alternatively, the maximum spin corresponds to that value where the γ transition energies start rising again after a flattening or a dip.[35] For better predictive power we scan for an optimum I_c by making a least squares fit for a range of I_c values keeping $\mathcal{I}_0$ and Δ_0 as free parameters. Illustration of the results of such calculations is provided in Fig. 1 which is a plot of the total percentage rms deviation of the calculated energies from the experiment as a function of the critical spin I_c. The rms deviation is seen to either reach a minimum or acquire an almost constant value at some stage. I_c is chosen corresponding to the minimum or to the point where flattening starts.

We have carried out calculations for the yrast level energies of even-even nuclei following the procedure outlined above. It is found that our expression (7) gives an excellent fit to the data for all the nuclei for which the ratio of the energy of the 4^+ state to that of the 2^+ state exceeds 3.0, i.e., the expression is found to be valid for good rotors

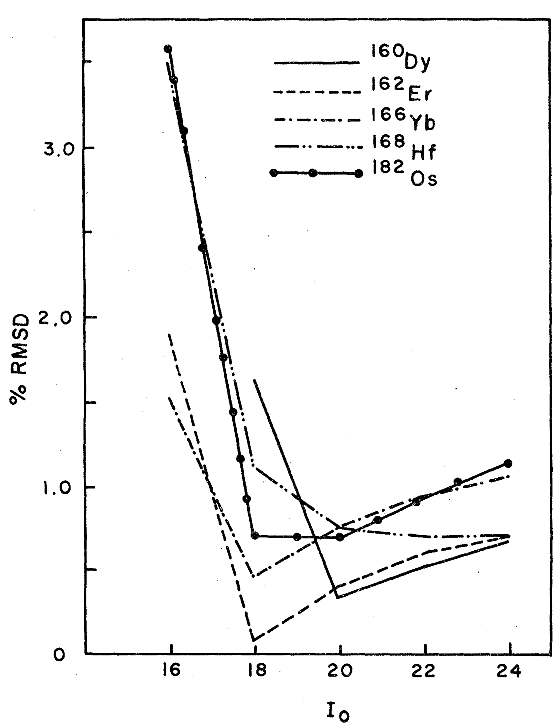

FIG. 1. Percentage rms deviations of the calculated from the experimental level energies as a function of the critical spin parameter I_c for five typical cases.

only. This confirms the theoretical conclusion of Ma and Rasmussen[20] that the exponential dependence of the moment of inertia on pairing holds only for well-deformed nuclei.

The results of the least squares fitting for as many as 29 well-deformed nuclei are summarized in Table I. The table lists the experimental I_{max} as defined above and the percentage rms deviation of the calculated from the experimental level energies up to I_{max} followed by the model parameter values. The last column lists the ground state moment of inertia given by the relation

$$2\mathcal{I}_{g.s.}/\hbar^2 = (2\mathcal{I}_0/\hbar^2)\exp(-\Delta_0)\,, \qquad (8)$$

i.e., the moment of inertia corresponding to maximum pairing.

As is evident from Table I for every case included therein, the rms deviation of the calculated from the experimental energies is within a fraction of 1%. Such an impressive agreement for such a wide range of nuclei, particularly with the inclusion of the backbending region levels, has not been achieved so far to our knowledge, except perhaps to a limited extent with the empirical cubic polynomial formula[16,36] or, semiempirical formula[37,38] reported by us earlier. To illustrate the quantitative agreement obtained for the excitation energies of yrast states we present in Fig. 2

TABLE I. The results of the least square fitting to the well-deformed nuclei ($E_4/E_2 > 3.0$). Here we list the total percent rmsd, the spin I_{max} denoting the maximum spin included in the fitting, which for backbending nuclei corresponds to the spin having minimum angular velocity, the critical spin I_c which is a scanned parameter, and the two free parameters, namely, the moment of inertia $2\mathcal{I}_0/\hbar^2$ and the effective pairing gap Δ_0. The last column lists the ground state value of the moment of inertia $2\mathcal{I}_{g.s.}/\hbar^2$ defined in Eq. (8). The asterisk over spin I_{max} indicates that, for that particular nucleus, higher spin states are known which lie in the forward or down-bending region and have not been included in our fitting.

Nucleus	I_{max}	% rmsd	I_c	$2\mathcal{I}_0/\hbar^2$ (MeV^{-1})	Δ_0	$2\mathcal{I}_{g.s.}/\hbar^2$ (MeV^{-1})
Dy 158	18*	0.57	22	127.14	0.77	58.87
160	18	0.31	20	108.69	0.48	67.25
162	12	0.16	18	95.77	0.27	73.11
164	12	0.23	18	102.95	0.25	80.17
Er 160	16*	0.87	22	142.16	1.12	46.38
162	16*	0.07	18	99.61	0.56	56.89
164	18	0.82	18	92.02	0.36	64.20
166	16	0.44	20	110.70	0.43	72.01
Yb 164	16*	0.60	20	124.52	0.98	46.73
166	16	0.43	18	99.42	0.56	56.79
168	20	0.67	24	128.91	0.67	65.96
170	18	0.58	20	100.20	0.37	69.21
172	12	0.15	20	94.29	0.23	74.91
174	20	0.26	24	102.69	0.28	77.61
176	18	0.42	20	93.39	0.27	71.29
Hf 168	16*	0.71	20	129.98	1.02	46.87
170	18*	0.87	26	166.60	1.06	57.67
172	18	0.42	22	115.38	0.63	61.45
174	14	0.25	16	92.12	0.36	64.21
176	16	0.26	20	98.23	0.39	66.20
W 174	18	0.89	26	167.84	1.18	51.32
176	18	0.37	24	139.42	0.97	52.85
178	16	0.26	20	104.62	0.65	54.53
180	16*	0.67	18	94.85	0.53	55.74
182	14	0.08	18	75.98	0.25	59.05
Os 180	14	0.82	22	169.39	1.37	43.04
182	16*	0.65	20	134.49	1.11	44.32
184	16	0.82	18	85.40	0.56	48.78
186	16*	0.90	18	86.51	0.72	42.12

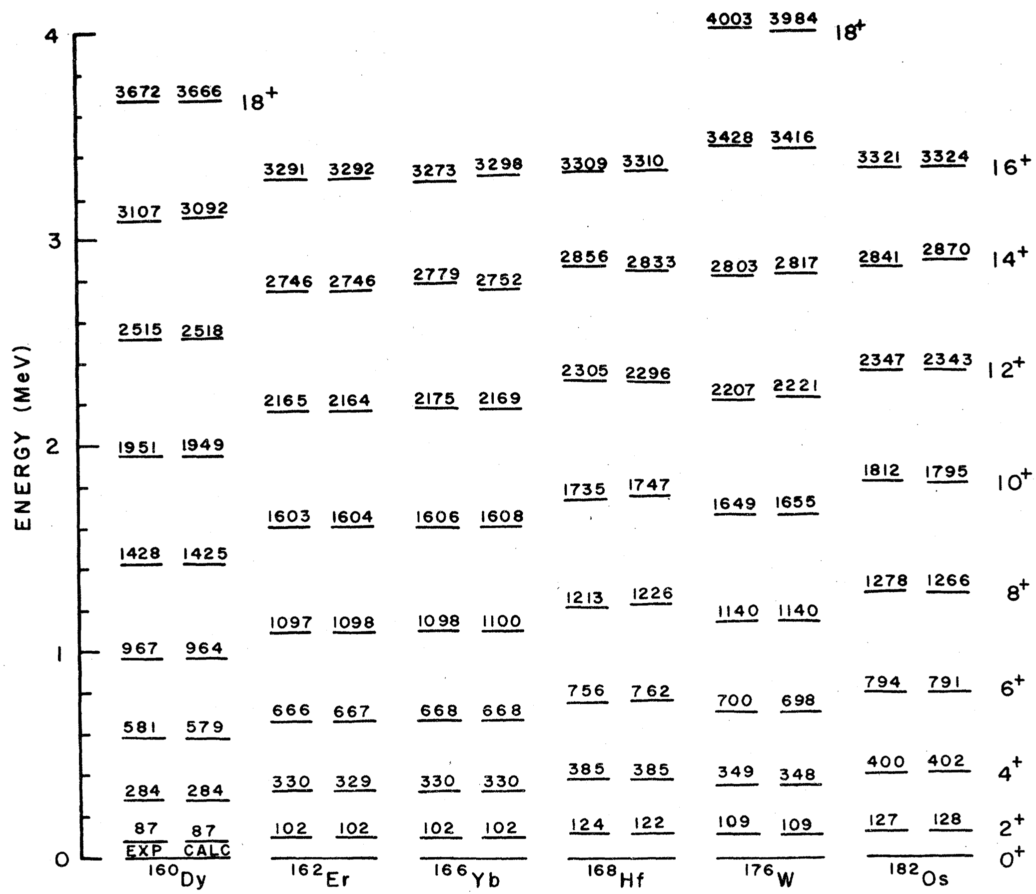

FIG. 2. Comparison of the experimental and the calculated level energies from our model for six typical cases, one each from the six sets of isotopes investigated in our calculations.

a comparison of the experimental and the calculated energies for six typical backbending nuclei, one each from the six sets of isotopes for which calculations have been carried out. The degree of agreement obtained is quite evident; it is to be noted that these are representative, and not necessarily the best-fitted, cases.

In line with the commonly adopted procedure for displaying the high spin data for yrast bands, we now take up the comparison of the experimental data with the calculations through $\mathscr{I}-\omega^2$ plots. The moment of inertia $\mathscr{I}$ and the squared rotational frequency ω^2 are related to the spin derivatives of the energy through the relations

$$2\mathscr{I}/\hbar^2 = (4I-2)/E_\gamma(I \to I-2)\,, \tag{9}$$

$$(\hbar\omega)^2 = \{dE/d[I(I+1)]^{1/2}\}^2\,,$$
$$= [E_\gamma(I \to I-2)/(2I-1)]^2(I^2-I+1)\,, \tag{10}$$

where we have adopted the "Stockholm" choice.[2] Results for four typical backbending nuclei are displayed in Fig. 3 and the excellent agreement between the theory and the experiment is clearly noticed.

IV. DISCUSSION OF MODEL PARAMETERS

Having established the adequacy of our expression (7) for satisfactorily predicting the excitation energies of the yrast band levels and reproducing the backbending feature for good rotors, we look into the physical content and characteristics of the computed model parameters.

As already mentioned in the previous section, we have confirmed the theoretical conclusion of Ma and Rasmussen that for good rotors the nuclear moment of inertia has an exponential dependence on the pairing gap parameter. Going a step further, we have introduced an explicit spin dependence in Eq. (6) for the pairing gap parameter; this dependence has been adopted in analogy with superconductivity and is in agreement with the statistical model studies for nuclei. The quantitative success of this choice has been demon-

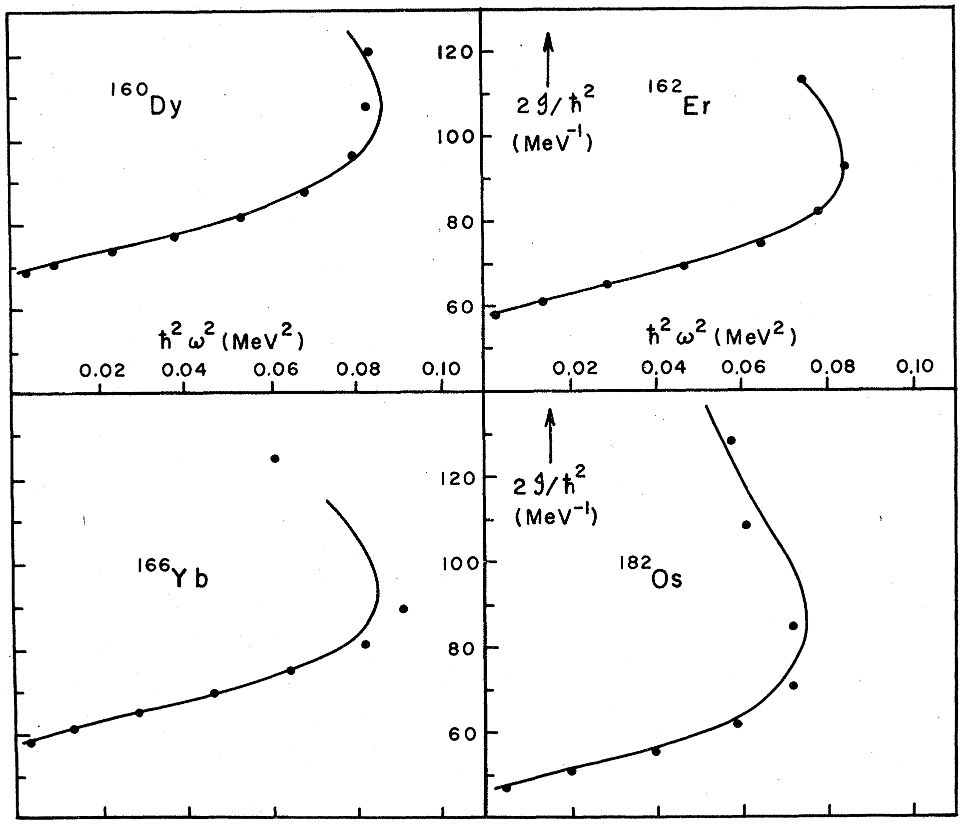

FIG. 3. Nuclear moments of inertia $2\mathfrak{I}/\hbar^2$ plotted as a function of the rotational frequency squared $(\hbar\omega)^2$ for four selected even-even nuclei. The solid line represents our calculations and the full circles are the experimental points.

strated above in our numerical calculations. A comparison of our results for $\Delta(I)$ with those from the pairing stretch model of Ma and Rasmussen is presented in Fig. 4; it is seen that the spin dependence resulting from their calculations is consistent with our choice for an appropriate I_c.

It is more interesting to compare our results with those of Kumar[1] who investigated the phase transition phenomenon employing the constrained-Hartree-Fock-Bogoliubov treatment of the pairing-plus-quadrupole model. The results for the spin variation of the energy gap in the nucleus ^{160}Dy from the calculations of Kumar are compared with those from our formulation in Fig. 5; it is seen that his results can be fitted with an expression similar to our Eq. (6) if the exponent is changed from 0.50 to 0.25. What is interesting is the fact that in his phase transition picture the almost sudden pairing collapse is very evident, e.g., the moment of inertia value suddenly drops by over 50% as we move from spin 16 to spin 18, this decrease being more than the total decrease seen in moving from 0 to 16. In contrast, the variation is much more gradual in our case. In view of the

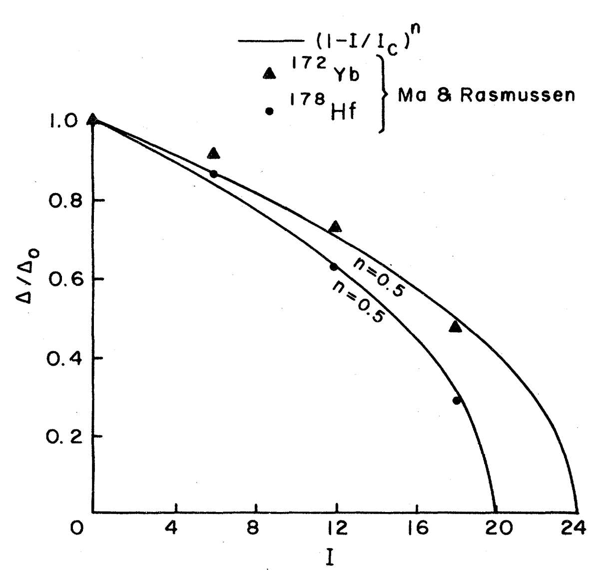

FIG. 4. Variation of the quantity (Δ/Δ_0) from our model which has $n = 0.5$ dependence (full lines) in comparison with the values from the calculations of Ma and Rasmussen (Ref. 20) for two cases, namely, ^{172}Yb and ^{178}Hf.

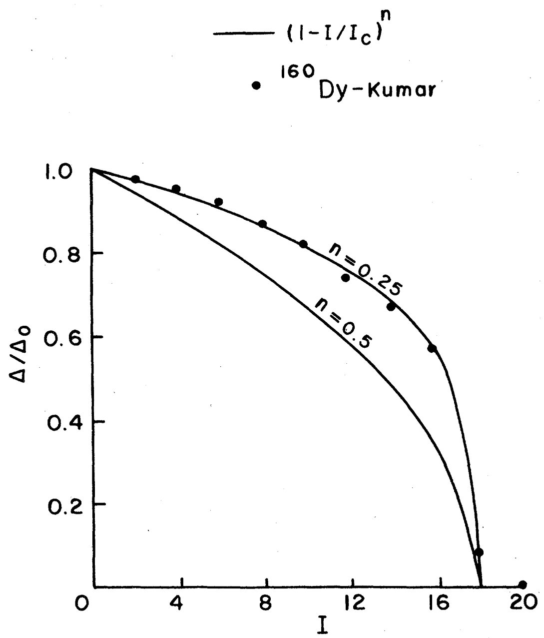

FIG. 5. Variation of the quantity (Δ/Δ_0) from our model ($n=0.5$) in comparison with the values from the calculations of Kumar[1] for ^{160}Dy. Kumar's values show a very rapid variation and match with a $n=0.25$ dependence.

established wide-range applicability of our formulation, it is logical to conclude that the exponential model does not favor a sudden pairing collapse picture for the backbending nuclei.

It may be added that we do not attempt to identify our parameter Δ_0 numerically with the energy gap for two reasons, although it is tempting to do so on observing the apparent agreement between the two. Firstly, we have kept Δ_0 as a dimensionless parameter leaving the scale undefined; alternatively, in terms of the formulation of Ma and Rasmussen[20] [vide Eq. (2) above] we are working with a constant value $\gamma = 1.0$ MeV^{-1} for all nuclei, whereas actually in their[20] calculations it is found to vary appreciably from nucleus to nucleus. Secondly, and more significantly, we believe, particularly in view of the results of recent calculations by Ma and Rasmussen,[31] that this parameter should be more appropriately considered as an effective or a "generalized pairing parameter" (in the spirit of similar earlier approaches[39,40]), which, in view of the excellent yrast band energy predictions, may effectively include contributions from Coriolis antipairing effects as well as from higher order cranking and decoupled band crossing.

Next we examine the second free parameter of our model, i.e., the moment of inertia parameter. In Fig. 6 we plot the parameter $2\mathscr{I}_0/\hbar^2$ as

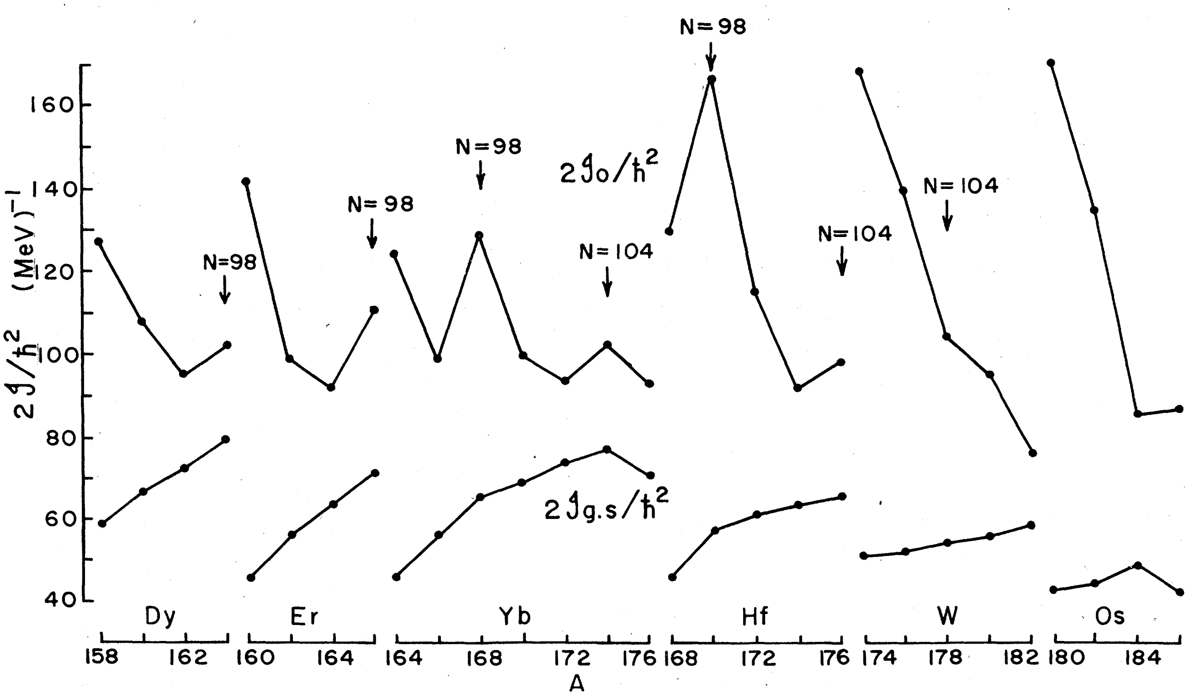

FIG. 6. The moment of inertia parameter $(2\mathscr{I}_0/\hbar^2)$ from our model for six sets of isotopes investigated in our calculations (upper part of the figure). In the lower part of the figure we show the ground state values of the moment of inertia from our model.

a function of the mass number A for different sets of isotopes; this parameter represents the moment of inertia at zero pairing, or the maximum value of the moment of inertia attainable in our model. Also plotted in the lower part of the same figure is the ground state value of the moment of inertia $2\mathcal{I}_{g.s.}/\hbar^2$ defined in Eq. (8). Two interesting features are evident from the figure. One is the strong fluctuations in the values of $2\mathcal{I}_0/\hbar^2$, which, as we shall see, may be correlated with the shell fluctuations in the single particle structure. The second is an almost complete washing out of these fluctuations in the ground state moments of inertial.

The fluctuations in the values of $2\mathcal{I}_0/\hbar^2$ can be correlated with the shell fluctuations, or the existence of gaps in the single particle structure[34]; the appearance of a gap, or a sudden change in the neutron level density, gets reflected in a sudden change in the value of $\mathcal{I}_0$ at the corresponding neutron number. The strongest evidence for this appears at $N=98$. All the $N=98$ isotones show a sudden break in the value of $\mathcal{I}_0$ and this coindices with the existence of a considerable gap in the neutron single particle scheme at $N=98$. Also, our earlier study[34] indicated the occurrence of a gap in neutron level scheme at $N=104$, which was said to diminish as we move from ^{174}Yb to ^{178}W. A similar situation is seen to exist from the variations of $\mathcal{I}_0$ in Fig. 6.

Looking at the values of the parameters as listed in Table I, one observes that the variations in $\mathcal{I}_0$ and Δ_0 are generally in the same pattern and their contribution through Eq. (8) results in effective cancellation of the fluctuation effects with the consequence that $\mathcal{I}_{g.s.}$ on the whole shows a smooth variation with A. This can be taken as the manifestation of the antishell nature of the pairing correlations. The latter tend to oppose the shell effects by smearing out the single particle density around the Fermi energy.[23] This is most pronounced in the ground state which corresponds to maximum pairing strength. As the pairing decreases with increasing angular momentum, the masking of shell effects diminishes and, ultimately, as the pairing vanishes, shell fluctuations become most pronounced.

The values of the ground state moment of inertia

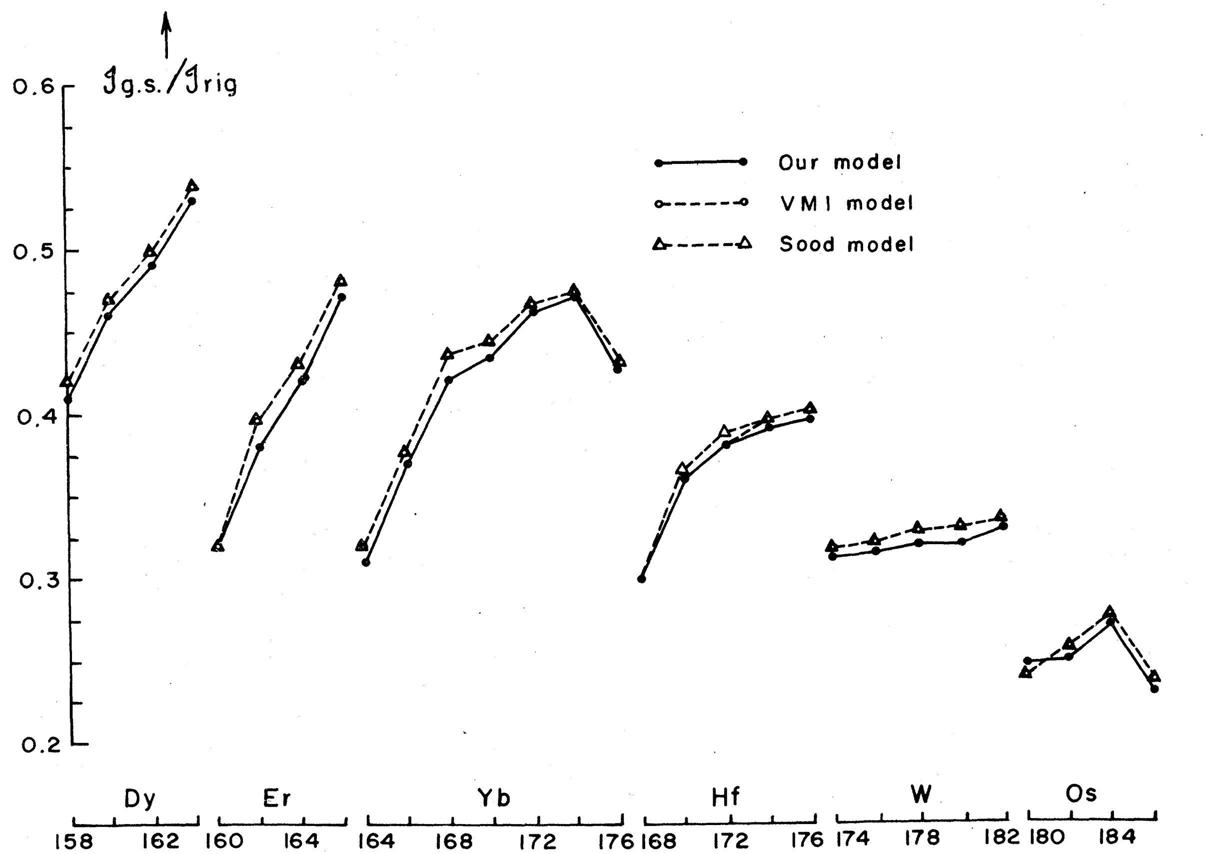

FIG. 7. Comparison of the values of the ratio $(\mathcal{I}_{g.s.}/\mathcal{I}_{rig})$ from our model with the same from the VMI model (Ref. 32) and the semiempirical formula of Sood (Ref. 40).

as a fraction of the rigid body value

$$\mathscr{I}_{\text{rig}} = \tfrac{2}{5}\,\text{MAR}^2(1+0.3\beta)\,, \qquad (11)$$

with $R = 1.2 \times A^{1/3}$ fm, from our calculations are compared with those from two earlier widely employed models[32,37] in Fig. 7. Our values are marginally smaller than earlier calculations, but all of them follow precisely the same trends.

The variation of moment of inertia with angular momentum arising from our present calculations is compared in Fig. 8 with that from the semiempirical formula of Sood.[37,38] It is observed that the semiempirical formula, which is possibly the only formulation giving the spin variation of $\mathscr{I}$ in an analytical form, shows qualitatively the same behavior but somewhat smaller rate of rise of moments of inertia around the backbending limit.

It is instructive to compare the values of our parameter $\mathscr{I}_0$ with the corresponding quantity obtained from the cranking model calculations of Ma and Rasmussen,[20] who have listed the extrapolated values of cranking moment of inertia at zero pairing for neutrons $\mathscr{I}_0^n$ and for protons $\mathscr{I}_0^p$ separately. This comparison is presented in Fig. 9. It is seen that for the lighter rare earth nuclides the variations as well as the fluctuations of the model parameter $\mathscr{I}_0$ are remarkably matched by the cranking model results for the neutrons, whereas the theoretical values for protons remain structureless and practically constant for each set of isotopes. On the other hand, we see from Fig. 9 that for Os isotopes with $A = 184$–186 the variations trend from our calculations is similar to that from the cranking model results for protons. Such an observation is in accordance with the evidence[41,42] suggesting the importance of protons in the Os region, although Refs. 41 and 42 have somewhat conflicting opinion on the role of protons for backbending. It is certainly of interest to extend the cranking model calculations of Ma and Rasmussen[20] to the lighter Os, W, and Hf isotopes to determine how far the close similarity observed for their neutron values with our model results from Dy, Er, and Yb isotopes persists for higher Z nuclides.

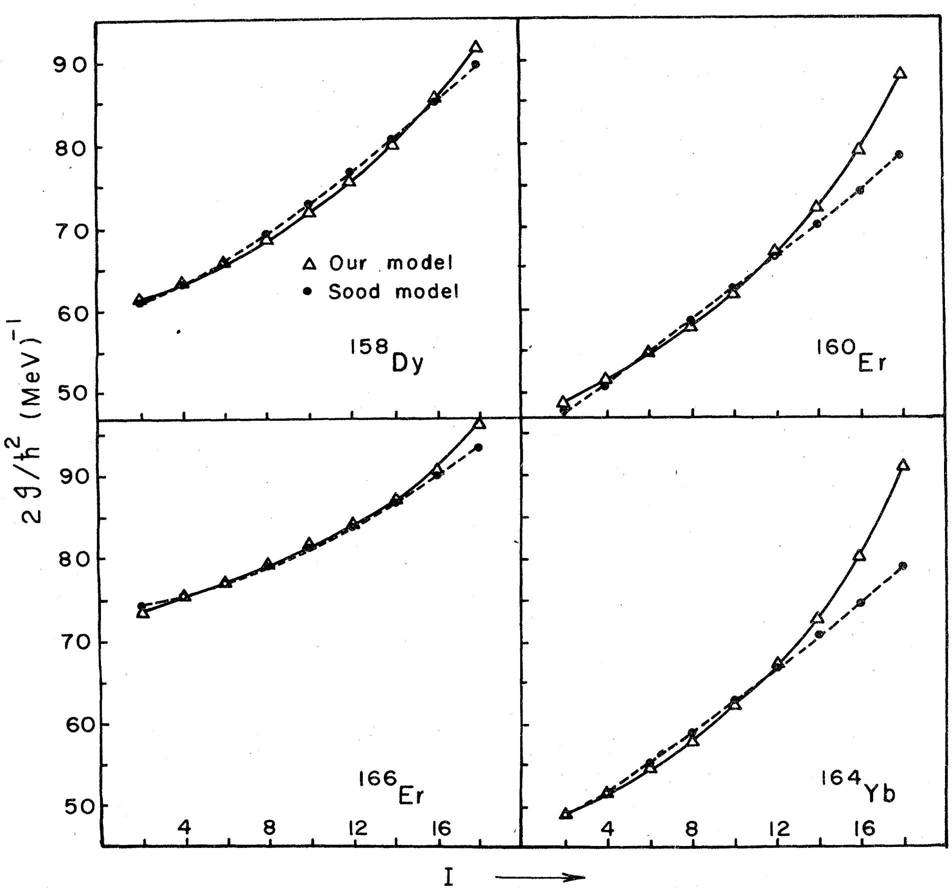

FIG. 8. Variation of the moment of inertia with spin from our model (triangles) compared with the same from the semiempirical formula of Sood (Ref. 40) (full circles).

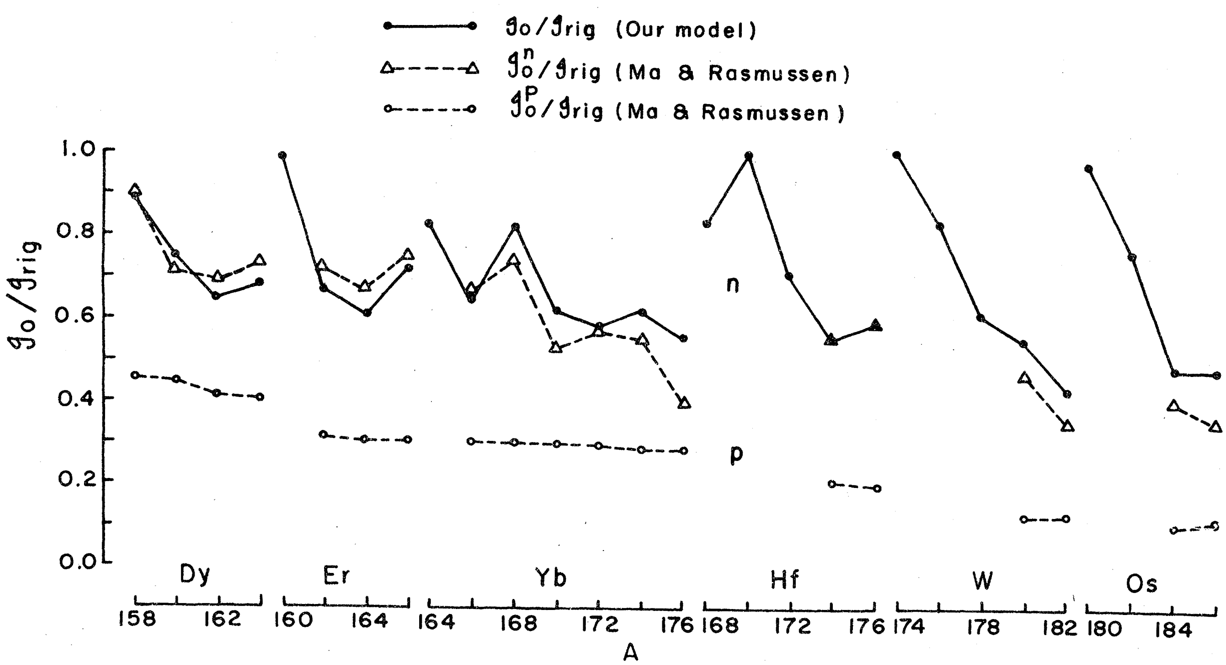

FIG. 9. Comparison of the ratio ($\mathscr{I}_0/\mathscr{I}_{rig}$) from our model (full lines) with the same for neutrons (triangles) and protons (open circles) from the calculations of Ma and Rasmussen (Ref. 20).

V. CONCLUSIONS

In summary, we present our conclusions as follows:

(a) The rotational energy calculations assuming an exponential dependence of the nuclear moment of inertia on the pairing correlations and a spin dependence of the pairing gap analogous to that used in superconductivity give an excellent fit to the experimental data on excitation energies of yrast band levels in well-deformed even-even nuclei.

(b) In contrast with the earlier observations of Ma and Rasmussen,[20] the exponential model in our formulation does reproduce the backbending feature satisfactorily.

(c) In agreement with the theoretical cranking model calculations of Ma and Rasmussen,[20] we find that the exponential dependence of the moment of inertia on pairing holds only for well-deformed nuclei. Our formulation provides a satisfactory fit to the data only for the nuclei for which the energy ratio $E(4^+)/E(2^+)$ exceeds 3.0.

(d) Comparison of the spin variation of the energy gap parameter as deduced from our computations with the results of the constrained-Hartree-Fock-Bogoliubov calculations for phase transitions by Kumar[1] indicates that pairing attenuation is not so sudden as expected from pairing collapse approach.

(e) The moments of inertia values in the limit of no pairing exhibit strong fluctuations matching those of the neutron single particle level spacings[34] and in good agreement with the cranking model calculations[20] for the neutrons for lighter rare earth nuclides. Theoretical calculations for heavier rare earths, i.e., isotopes of Hf, W, and Os, are not available for extended sets. From the available results, we find a similarity between the variations for our results and for the microscopic calculations for protons in the case of the two Os isotopes; however, no definitive conclusions can be drawn until extensive microscopic calculations for this region become available.

(f) Comparison with the experiment indicates that the maximum value of the moment of inertia is reached, i.e., backbending is terminated, at a spin value one step lower than I_c; this may be taken to signify, as suggested by Ma and Rasmussen,[20] that the very low pairing region does not come in the picture for actual cases, which remain within the region of validity of the exponential model defined in our Eq. (3).

ACKNOWLEDGMENTS

One of us (P. C. S.) gratefully acknowledges the joint hospitality of the Theoretical Physics Institute and the Nuclear Research Centre of the

University of Alberta, Edmonton (Canada), during the summer of 1977, where this work was continued. Valuable discussions with Professor J. O. Rasmussen (Berkeley) and Professor J. B. French (Rochester) are gratefully acknowledged. One of us (A. K. J.) acknowledges the award of a research fellowship by Council of Scientific and Industrial Research (India). These investigations are supported in part by the Department of Atomic Energy, Government of India.

[1]K. Kumar, Phys. Scr. 6, 270 (1972).
[2]R. A. Sorensen, Rev. Mod. Phys. 45, 353 (1973).
[3]A. Johnson and Z. Szymanski, Phys. Rep. 7C, 181 (1973).
[4]F. S. Stephens, Rev. Mod. Phys. 47, 43 (1975).
[5]A. Faessler, *Heavy Ion High-Spin States and Nuclear Structure* (IAEA, Vienna, 1975), Vol. I, p. 405.
[6]G. S. Goldhaber, C. B. Dover, and A. L. Goodman, Annu. Rev. Nucl. Sci. 26, 239 (1976).
[7]F. S. Stephens, in Proceedings of the International School of Physics, Varenna, Italy, 1976 (unpublished).
[8]B. R. Mottelson and J. G. Valatin, Phys. Rev. Lett. 5, 511 (1960).
[9]F. S. Stephens and R. S. Simon, Nucl. Phys. A183, 257 (1972).
[10]A. Molinari and T. Regge, Phys. Lett. 41B, 93 (1972).
[11]R. A. Broglia, A. Molinari, G. Pollarolo, and T. Regge, Phys. Lett. 50B, 295 (1974); 57B, 113 (1975).
[12]C. Flaum and D. Cline, Phys. Rev. C 14, 1224 (1976).
[13]M. Reinecke and H. Ruder, Z. Phys. A279, 93 (1976).
[14]Y. El Masri, J. Vervier, and A. Faessler, Nucl. Phys. A279, 223 (1977).
[15]R. K. Gupta, S. Wahlborn, and L. Hjertman, Phys. Scr. 6, 261 (1972).
[16]A. N. Mantri and P. C. Sood, Phys. Rev. C 9, 2076 (1974).
[17]A. L. Goodman and A. Goswami, Phys. Rev. C 9, 1948 (1974).
[18]C. A. Heras and S. M. Abecasis, Can. J. Phys. 53, 1651 (1975).
[19]J. E. Draper, Phys. Lett. 41B, 105 (1972).
[20]C. W. Ma and J. O. Rasmussen, Phys. Rev. C 9, 1083 (1974).
[21]P. C. Sood and A. K. Jain, in Proceedings of the International Conference on Selected Topics in Nuclear Structure, Dubna, 1975 (unpublished), Vol. 1, p. 35.
[22]A. K. Jain, Ph.D. thesis, Banaras Hindu Univeristy, India, 1977 (unpublished).
[23]M. Brack, J. Damgaard, A. S. Jensen, H. C. Pauli, V. M. Strutinsky, and C. Y. Wong, Rev. Mod. Phys. 44, 320 (1972).
[24]L. G. Moretto, Nucl. Phys. A185, 145 (1972).
[25]G. Rickayzen, in *Superconductivity*, edited by R. D. Parks (Dekker, New York, 1969), Vol. 1, p. 51.
[26]A. Bohr and B. R. Mottelson, *Nuclear Structure* (Benjamin, Massachusetts, 1975), Vol. 2, p. 82.
[27]B. Bochev, S. A. Karamian, T. Kutsarova, E. Nadjakov, and Y. T. Oganessian, Nucl. Phys. A267, 344 (1976).
[28]B. Bochev, S. Iliev, R. Kalpakchieva, S. A. Karamian, T. Kutsarova, E. Nadjakov, and Ts. Venkova, Nucl. Phys. A282, 159 (1977).
[29]F. Kearns, G. Varley, G. D. Dracoulis, T. Inamura, J. C. Lisle, and J. C. Willmott, Nucl. Phys. A278, 109 (1977).
[30]A. Faessler, K. R. Sandhyadevi, F. Grummer, K. W. Schmid, and R. R. Hilton, Nucl. Phys. A256, 106 (1976).
[31]C. W. Ma and J. O. Rasmussen, Bull. Am. Phys. Soc. 529 (1977).
[32]M. A. J. Mariscotti, G. S. Goldhaber, and B. Buck, Phys. Rev. 178, 1864 (1969).
[33]J. O. Rasmussen, in *Nuclear Spectroscopy and Reactions*, edited by J. Cerny (Academic, New York, 1975), Pt. D, p. 97.
[34]A. K. Jain and P. C. Sood, J. Phys. G 4, 81 (1978).
[35]P. C. Sood and A. N. Mantri, J. Phys. Soc. Jpn. Suppl. 34, 453 (1973).
[36]P. C. Sood and A. K. Jain, Phys. Rev. C 12, 1064 (1975).
[37]P. C. Sood, Phys. Rev. 161, 1063 (1967).
[38]P. C. Sood, Nucl. Data A4, 281 (1968).
[39]C. W. Ma and J. O. Rasmussen, Phys. Rev. C 2, 198 (1970).
[40]A. N. Mantri and P. C. Sood, Phys. Rev. C 7, 1294 (1973).
[41]A. Neskakis, R. M. Lieder, M. Muller-Veggian, H. Beuscher, and W. F. Davidson, Nucl. Phys. A261, 189 (1977).
[42]S. Andre, J. Genevey-Rivier, J. Treherne, J. Jastrzebski, R. Kaczarowski, and J. Lukasiak, Phys. Rev. Lett. 38, 327 (1977).