

Gamma-neutrino angular correlation in muon capture by ^{28}Si

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The theory of γ -neutrino angular correlation in nuclear muon capture is developed using the density matrix technique. A closed expression for the correlation coefficient, for unpolarized muon capture and including relativistic terms, is obtained in the particle-hole model for the nucleus. An interesting relation between the correlation coefficient and the longitudinal polarization of the recoil nucleus in muon capture is established. This relation is independent of nuclear models and the muon capture coupling constants, and its importance in connection with time-reversal invariance is pointed out. Utilizing the close analogy between muon capture and inelastic electron scattering, the numerical results for the process $\mu^- + {}^{28}\text{Si}(0^+) \rightarrow {}^{28}\text{Al}^*(1^+) + \nu_\mu$, ${}^{28}\text{Al}^*(1^+) \rightarrow {}^{28}\text{Al}(0^+) + \gamma$ are presented. It is found that the correlation coefficient is extremely sensitive to the nuclear model, contrary to common belief, and also sensitive to the induced pseudoscalar coupling constant in muon capture. The results are compared with the available experimental data and a range for g_p/g_A is obtained as $3 < g_p/g_A < 20$, in agreement with other predictions, indicating a remote possibility of the quenching of g_p due to virtual pion effects in the $A = 28$ system. With the "canonical" value for g_p , our results give $g_T/g_A = +4.5 \pm 7.5$ due to the large uncertainty in the experimental data.

NUCLEAR REACTIONS ${}^{28}\text{Si}(\mu^-, \gamma\nu) {}^{28}\text{Al}$; γ -neutrino angular correlation coefficient; particle hole model; induced pseudoscalar and tensor form factors.

I. INTRODUCTION

The study of muon capture by complex nuclei, leading both to specific bound final nuclear states and to continuum neutrons, has been established as a powerful tool to examine the nuclear structure and to study the capture mechanism in general and to obtain a value for the induced pseudoscalar coupling (g_p) in particular. Such a study of g_p is also expected to throw light upon the induced tensor coupling which is being searched for with some success in recent experiments.¹ In addition to the capture rate, the recoil nuclear polarization in $\mu^- + {}^{12}\text{C}(0^+) \rightarrow {}^{12}\text{B}(1^+) + \nu_\mu$ was looked for by Wolfenstein² and Devanathan, Parthasarathy, and Subramanian.³ In this case, the recoil nuclear polarization is completely insensitive to the nuclear model (the contrary holds for capture rates), and hence can provide a better estimate for g_p . The recent measurements by the Louvain group⁴ indicate $4.8 < g_p/g_A < 9.8$. It has been pointed out by Grenacs *et al.*⁵ that one can measure the γ -neutrino angular correlation in muon capture in certain nuclei, after muon capture decay by γ emission, by observing the Doppler broadening of the γ rays due to the recoil of the nucleus. This has been successfully carried out by Miller *et al.*⁶ Although the theory of γ -neutrino angular correlation has been developed in a series of papers by Popov *et al.*⁷⁻¹¹ in terms of the multipole expansion of the weak hadronic operators (in close analogy

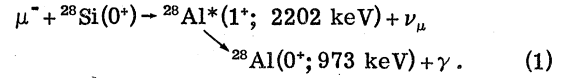
with orbital electron capture theory), the formalism seems to be complicated and not easily applied by experimentalists. Recently, Ciechanowicz¹² has applied the "multipole" theory of Popov to muon capture by ${}^{28}\text{Si}$; he obtained $-4.9 < g_p/g_A < 1.2$ and claimed that this result indicates a downward renormalization of the Goldberger-Treiman relation for the $A = 28$ system. This conclusion is certainly surprising for two reasons. First, the downward renormalization of g_p has recently been established theoretically only for infinite nuclear matter¹³ and it is not clear what happens in the case of a finite nucleus. Even if one assumes such a renormalization for g_p in a heavy nucleus, it is improbable to have such a small value (as predicted by Ciechanowicz¹²) for a medium nucleus like ${}^{28}\text{Si}$. Second, while the treatment of the muon capture by Ciechanowicz¹² is essentially an impulse approximation approach which treats the nucleons in the nucleus as free, the reason for the renormalization of g_p is the many-body effect¹³ (possible scattering of the virtual pions by other nucleons and the introduction of the pion optical potential).

It is the purpose of this paper to develop the theory of γ -neutrino angular correlation in a simple way in terms of nuclear matrix elements in the particle-hole model and evaluate the correlation coefficient for muon capture by ${}^{28}\text{Si}$. In the course of the development, we have obtained an interesting relation between the γ -neutrino angular

correlation coefficient and the longitudinal polarization of the intermediate nucleus in a nuclear-model-independent manner. This relation does not depend upon the muon capture coupling constants and this provides a method for verifying the time-reversal invariance in muon capture by means of accurate measurements of the correlation coefficient, since the longitudinal polarization is difficult to measure. It seems that Bernabeu¹⁴ has obtained the same relation using general invariance arguments. Our derivation, however, is independent in the sense that we start from the Hamiltonian for muon capture, derive an explicit expression for the correlation coefficient, and compare it with the longitudinal polarization.

The close analogy between muon capture and inelastic electron scattering has been studied in detail in a unified approach by Donnelly and Walecka,¹⁵ who exploited the fact that, on account of the conserved vector current (CVC) theory, half the matrix elements in the semileptonic weak processes (those coming from the vector current part) are identical to those measured in electron scattering. It has been pointed out by Überall¹⁶ that the 1^+ states of the final nucleus in muon capture are analogous to the $M1$ excitation in inelastic electron scattering. If the inelastic electron scattering process leading to 1^+ states is studied, that knowledge can thus be applied to the muon capture process. Inelastic electron scattering to 1^+ final nuclear levels has been extensively studied by Donnelly and Walker¹⁷ using the Serber-Yukawa residual interaction. The experimental data on the yield of muon capture and inelastic electron scattering in ^{28}Si by Miller¹⁸ at 100 MeV/c momentum transfer (in muon capture the momentum transfer is of the same magnitude) indicate that the transition to the 2202 keV 1^+ level of ^{28}Al is dominant. Comparing this with the theoretical studies of Donnelly and Walker,¹⁷ where the excitation of 1^+ at 13.67 MeV in ^{28}Si is dominant at precisely the same momentum transfer, we have used the wave functions of Donnelly and Walker¹⁷

to evaluate the correlation coefficient in



In Sec. II, the expression for the γ -neutrino angular correlation coefficient is derived, and in Sec. III the relation between the correlation coefficient and the longitudinal polarization of the recoil nucleus in muon capture is first conjectured and then established. Its relevance to time-reversal invariance is pointed out. In Sec. IV numerical results are presented for various values of g_P/g_A and, comparing with the experimental results of Miller *et al.*,⁶ ranges for g_P/g_A and g_T/g_A are given.

II. THEORY

The theory of γ -neutrino angular correlations can be considerably simplified if one uses the density matrix formalism.^{18,19} In this section we give the density matrix of the intermediate nucleus in muon capture and then evaluate the density matrix of the final nucleus after γ emission. Consider muon capture by a nucleus $|J_i M_i\rangle$ to $|J_f M_f\rangle$ which can be described by the Fujii-Primakoff Hamiltonian. For simplicity and comparison with the experimental data, we confine ourselves to unpolarized muon capture by an initial-spin-zero nucleus. The formalism can, however, be extended naively to polarized muon capture. Construction of the density matrix for the final nucleus in muon capture has been described in detail by Devanathan, Parthasarathy, and Subramanian.²⁰ In our present problem, as we are interested in the angular correlation, the integration over neutrino directions should not be carried out. This will change the form of the density matrix given in Ref. 20. Since the details of evaluation by Racah algebra are rather straightforward and the principles are outlined in Ref. 20, we only give the final expression for the density matrix element of the final nucleus after muon capture, preserving the angular identity of the neutrino:

$$\begin{aligned} \rho_{M_f M_f'}^{\mu c} = \sum_J \left\{ G_V^2 g(J_f 0 J_f; J_f 0 J_f) C(J_f J_f J; 000) + G_A^2 \sum_{l, l'} (i)^{l'-1} (-1)^{l'-J_f} g(l 1 J_f; l' 1 J_f) [l][l'] \right. \\ \left. C(l l' J; 000) W(J_f 1 l l'; l J_f) \right. \\ + (G_P^2 - 2G_P G_A) \sum_{l, l'} (i)^{l'-1} \frac{[l][l']}{[J_f]^2} C(l 1 J_f; 000) C(l' 1 J_f; 000) g(l 1 J_f; l' 1 J_f) C(J_f J_f J; 000) \\ + \frac{2}{M} (G_P - G_A) g_A \sum_{l, \lambda} (i)^{-J_f+l-1} (-1)^{\lambda-J_f} \frac{[\lambda][l]}{[J_f]^2} C(l 1 J_f; 000) g(l 1 J_f; J_f 1 \lambda 1 J_f) C(J_f J_f J; 000) \\ + \frac{2}{M} G_A g_V \sum_{l, l'} \sum_{\lambda} \sqrt{2} (i)^{l'-1+l} C(l 1 \lambda; 000) (-1)^{l'-\lambda} \\ \left. \times [J_f][1][\lambda][l'] W(J_f 1 \lambda 1; l 1) C(\lambda l' J; 000) g(l 1 J_f; l' 1 J_f 0 J_f) W(J_f \lambda J_f l'; 1 J) \right\} \\ \times (-1)^{M_f} \frac{[J_f]^2}{\sqrt{4\pi} [J]} C(J_f J_f J; -M_f M_f' M_f) Y_J^M(\hat{\nu}), \quad (2) \end{aligned}$$

where M is the nucleon mass, G_V , G_A , G_P , g_V , and g_A are muon capture coupling constants,¹⁹ and we follow the notation of Rose²¹ for angular momentum coefficients. Further, we have

$$\mathcal{G}(l n J_f; l' n J_f) = 16\pi^2 \sum_{p,h} \sum_{p',h'} \frac{[j_p][j_{p'}]}{[J_f]^2} X_{p,h}^{J_f} X_{p',h'}^{J_f} \langle j_p \| \{Y_i(\hat{p}) \times \sigma_n\}_{J_f} \| j_h \rangle \langle j_{p'} \| \{Y_i(\hat{p}) \times \sigma_n\}_{J_f} \| j_{h'} \rangle^* \\ \times \langle j_i(\nu r) \rangle_{ph} \langle j_i(\nu r) \rangle_{p'h'} | \phi_\mu |^2, \quad (3)$$

$$\mathcal{G}(1 J_f; J_f 1 \lambda n J_f) = 16\pi^2 \sum_{p,h} \sum_{p',h'} \frac{[j_p][j_{p'}]}{[J_f]^2} X_{p,h}^{J_f} X_{p',h'}^{J_f} \langle j_p \| \{Y_i(\hat{p}) \times \sigma_1\}_{J_f} \| j_h \rangle \langle j_i(\nu r) \rangle_{ph} | \phi_\mu |^2 \\ \times \langle j_{p'} \| [\{Y_{J_f}(\hat{p}) \times \nabla_1\}_\lambda \times \sigma_n]_{J_f} j_{J_f}(\nu r) \| j_{h'} \rangle^*, \quad (4)$$

where n can be 0 or 1 such that $\sigma_0=1$ and $\sigma_1=\sigma_\lambda$, the $X_{ph}^{J_f}$'s are the particle-hole mixing coefficients, and

$$\langle j_i(\nu r) \rangle_{ph} = \int_0^\infty R_{n_p i_p}(\nu r) j_i(\nu r) R_{n_h i_h}(\nu r) r^2 dr,$$

with the R_{ni} being the harmonic-oscillator radial wave functions. $|\phi_\mu|^2$ is the square of the muon wave function in the K orbit, averaged over the nuclear volume. The above reduced nuclear matrix elements and the radial integrals (and those involving derivatives of the hole radial wave functions) are analytically evaluated using the method of de Forest and Walecka.²² It is to be noted that when integration over neutrino angles is carried out (giving $\delta_{J,0}$), Eq. (2) readily gives the partial muon capture rate to the nuclear level $|J_f M_f\rangle$.

The operator responsible for the γ emission from from $|J_f M_f\rangle$ to $|J_F M_F\rangle$ is taken to be $\vec{j}_N \cdot \vec{A}_p$ following Rose,²³ where \vec{j}_N is the nucleon current and \vec{A}_p is the vector potential of the emitted γ ray with circular polarization $p(\pm 1)$. The density matrix element $(\rho_F)_{M_F M_F}$ of the final nuclear state after γ emission from $|J_f M_f\rangle$ (from muon capture by $|J_i M_i\rangle$) then is

$$(\rho_F)_{M_F M_F} = \sum_{M_f, M_f'} \langle J_F M_F | H_\gamma | J_f M_f \rangle (\rho^{i,c})_{M_f, M_f'} \\ \times \langle J_F M_F | H_\gamma | J_f M_f' \rangle^*, \quad (5)$$

with $\rho_{M_f, M_f'}^{i,c}$ given by Eq. (2). Denoting the multipolarity of the γ radiation by L and expanding \vec{A}_p in multipoles, we obtain

$$\sum_{M_f} (\rho_F)_{M_F, M_F} = |a_\tau|^2 |\langle J_F \| L(\tau) \| J_f \rangle|^2 \\ \times \sum_{M_f, M_f'} \rho_{M_f, M_f'}^{i,c} \sum_{\gamma=0}^{2L} (-1)^p (-1)^{J_f - J} F C(LL\gamma; p - p0) \sqrt{4\pi} [J_F]^2 / [J_f]^2 \\ \times W(J_f L J_f L; J_F \gamma) C(J_f \gamma J_f; M_f M_f M_f) [Y_\gamma^M(\hat{p})]^*, \quad (6)$$

where $|a_\tau|^2$ is a constant factor depending on the nature of the multipolarity, $|\langle J_F \| L(\tau) \| J_f \rangle|^2$ is the square of the γ decay matrix element, and τ stands for either an electric or a magnetic transition. Substituting for $\rho_{M_f, M_f'}^{i,c}$ from Eq. (2) and summing over M_F , we find $\delta_{\nu, J} \delta_{M_f, M_f'}$ by using the orthogonality of Clebsch-Gordon coefficients. The two spherical harmonics $Y_J^M(\hat{p})$ and $Y_J^M(\hat{p})^*$ combine to give $P_J(\cos\theta_{\nu})$, where θ_{ν} is the angle between γ and neutrino directions. The result is

$$\mathcal{I}(\theta_{\nu}) = \frac{1}{6\pi} |a(M1)|^2 |\langle 0^+ \| M1 \| 1^+ \rangle|^2 \\ \times \sum_J \left\{ G_V^2 [1]^2 C(11J; 000) \mathcal{G}(101; 101) + G_A^2 \sum_{i, i'} (i)^{i'-i} (-1)^{i'-1} [1]^2 \mathcal{G}(111; i'11) [i][i'] C(i'J; 000) W(11J i'; 11) \right. \\ + (G_P^2 - 2G_P G_A) \sum_{i, i'} (i)^{i'-i} [i][i'] C(111; 000) C(1'1; 000) \mathcal{G}(111; i'11) C(11J; 000) \\ + \frac{2}{M} (G_P - G_A) g_A \sum_{i, \lambda} (i)^{i-2} (-1)^{\lambda-1} [\lambda][i] C(i'11; 000) \mathcal{G}(111; 11\lambda 11) C(11J; 000) \\ + \frac{2}{M} G_A g_V \sum_{i, i', \lambda} \sqrt{2} (i)^{i'-i+3} C(11\lambda; 000) (-1)^{i'+\lambda} [1]^4 [\lambda][i'] W(11\lambda 1; 11) \mathcal{G}(111; i'1101) \\ \left. \times W(1\lambda 1 i'; 1J) C(\lambda i' J; 000) \right\} C(11J; 1-10) P_J(\cos\theta_{\nu}). \quad (7)$$

When the summation over J is carried out, the term $J=0$ is angle independent and $J=1$ does not contribute. By dividing the term $J=2$ by the angle-independent part ($J=0$), we obtain the angular correlation as

$$I(\theta_{\gamma\nu}) = I(0)[1 + \alpha P_2(\cos\theta_{\gamma\nu})], \quad (8)$$

where the angular correlation coefficient α is given by

$$\alpha = A/B, \quad (9)$$

with A and B obtained from Eq. (7) by putting $J=0$ and $J=2$, respectively. Equation (7) is written for the process (1). The numerical evaluation of α is carried out in Sec. IV.

III. RELATION BETWEEN CORRELATION COEFFICIENT AND LONGITUDINAL POLARIZATION

Equation (9) gives the γ -neutrino angular correlation coefficient in terms of reduced nuclear matrix elements in the particle-hole model including the relativistic terms in the Fujii-Primakoff Hamiltonian. We now neglect the relativistic terms and confine ourselves to the S-wave neutrino. This is known as the Fujii-Primakoff approximation (FPA). Then the reduced nuclear matrix elements in A and B cancel out and a simple expression for the correlation coefficient is obtained:

$$\alpha = \frac{2G_P G_A - G_P^2}{3G_A^2 + G_P^2 - 2G_P G_A}. \quad (10)$$

Under the same approximation (FPA) Wolfenstein²⁴ gives the longitudinal polarization of the final nucleus in muon capture [in our case $^{28}\text{Al}(1^+; 2202 \text{ keV})$] as

$$P_L = -\frac{2G_A^2}{3G_A^2 + G_P^2 - 2G_P G_A}. \quad (11)$$

By comparing Eqs. (10) and (11), we find

$$-\alpha = 1 + \frac{3}{2}P_L. \quad (12)$$

Equation (12) can be shown to be true when the relativistic terms are included and higher partial wave neutrinos are taken into account. The method of derivation is straightforward when we compare the complete expression for α [given in Eq. (9)] with that for P_L given by Devanathan and Subramanian,²⁵ and relate the appropriate nuclear matrix elements. Thus relation (12) is rigorous and independent of nuclear structure or muon capture coupling constants. This result complements the relation between P_L and asymmetry in the angular distribution of the recoil nucleus in muon capture,

derived by Devanathan and Subramanian²⁵ from the Fujii-Primakoff Hamiltonian and by Bernabeu²⁶ from rotational invariance.

We now discuss the significance of Eq. (12). We find

$$\alpha = G_P(2G_A - G_P)/I_0,$$

where

$$I_0 = 3G_A^2 - 2G_P G_A + G_P^2.$$

We have $G_P \ll 2G_A$, where $\alpha \approx \frac{2}{3}G_P G_A$, showing that α directly involves G_P and can be expected to be more sensitive to induced pseudoscalar couplings than other observables in muon capture. This fact is reflected in Fig. 1, where we present the complete value of α for various values of g_P/g_A . Secondly, relation (12) can be used to test time-reversal invariance. Bernabeu²⁶ has shown that the limits for P_L admitted by time-reversal invariance are 0 and -1 and any deviation from the above limits is claimed to be an indication of the violation of time-reversal invariance in muon capture. Experimental measurement of P_L is complicated, however, and high accuracy is improbable. Nevertheless, the above time-reversal limits for P_L imply through relation (12) that

$$-1 \leq \alpha \leq 0.5, \quad (13)$$

and experimental measurements of α can be carried out using highly efficient γ -ray detectors (so as to observe the Doppler broadening). Any deviation of α from the above limits is then an indication of time-reversal violation in muon capture. There is only one measurement of α , by Miller *et al.*,⁶ whose results agree with Eq. (13). It would be worthwhile to obtain additional measurements to confirm time-reversal invariance in muon capture.

IV. NUMERICAL RESULTS AND DISCUSSION

Equation (9) gives the α -neutrino angular correlation coefficient in terms of nuclear matrix elements and muon capture coupling constants. The nuclear matrix elements are evaluated in the particle-hole model of Donnelly and Walker,¹⁷ which describes the residual interaction by the Serber-Yukawa potential.

We have also evaluated the correlation coefficient in the pure shell model (PSM) and in the configuration mixing model of Donnelly and Walker.¹⁷ The results are given in Fig. 1 along with the experimental results of Miller *et al.*⁶ From Fig. 1 we find that the correlation coefficient is extremely sensitive to the nuclear wave functions. This conclusion contradicts that of Popov *et al.*⁷⁻¹¹ who have claimed that, within 10% deviation, the correlation coefficient should be

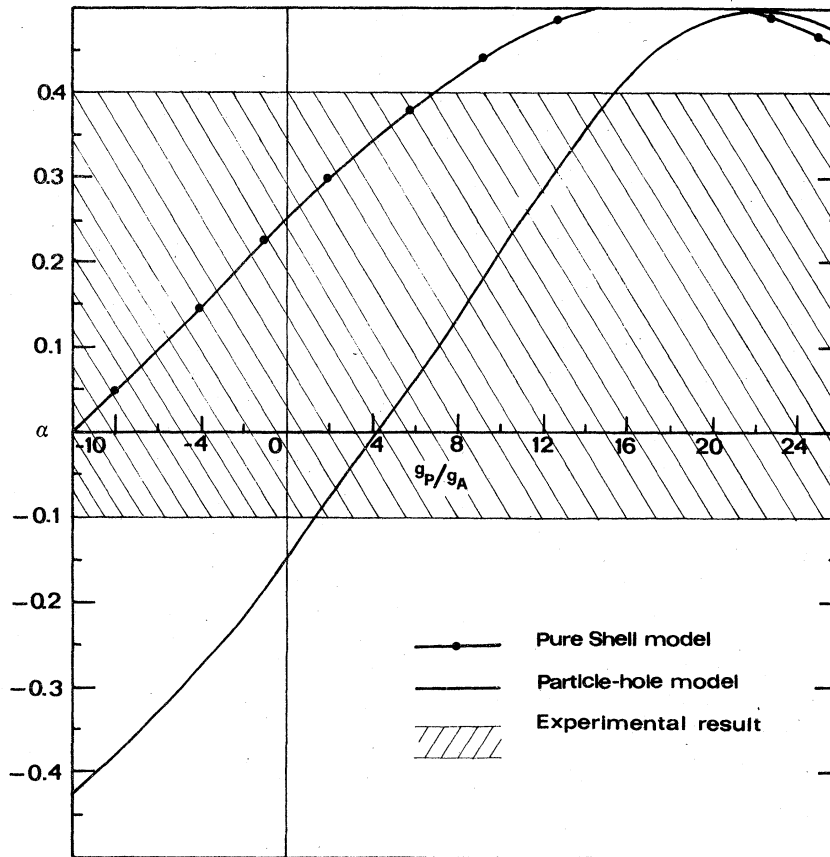


FIG. 1. Variation of α , the γ -neutrino angular correlation coefficient in muon capture in ^{28}Si as a function of g_P/g_A . The "dashed line" region corresponds to the experimental data (Ref. 6). Though PSM results are given for comparison, only the particle-hole model results are used in the analysis.

independent of the nuclear wave functions involved. It is a fairly well-known fact that the PSM is a crude model and cannot describe many of the nuclear properties. Also, Donnelly and Walker,¹⁷ using the configuration mixing scheme and describing the residual interaction by the Serber-Yukawa potential, have obtained an extremely good fit to inelastic electron scattering by ^{12}C , ^{16}O , ^{28}Si , ^{32}S , and ^{40}Ca . We confine ourselves to ^{28}Si ; Donnelly and Walker have found that the 1^+ level of ^{28}Si at 13.67 MeV is predominantly excited in inelastic electron scattering at 100 MeV/c momentum transfer. Miller *et al.*⁶ have also measured the muon capture probability of ^{28}Si to ^{28}Al and inelastic electron scattering by ^{28}Si at the same momentum transfer; they confirm the conclusion of Donnelly and Walker.¹⁷ Thus, ^{28}Si is fairly well described by the model of Donnelly and Walker¹⁷ and hence we have used their model to analyze the γ -neutrino angular correlation in muon capture by $^{28}\text{Si}(0^+)$ leading to $^{28}\text{Al}(1^+; 2.202 \text{ MeV})$.

In their measurement of α , Miller *et al.*⁶ have

considered two silicon targets. For natural silicon they report $\alpha = 0.15 \pm 0.25$, and for SiO_2 they give $\alpha = 0.29 \pm 0.3$. As the experimental uncertainties are large, more precise measurements are needed. However, we present the range for g_P/g_A for the above two sets of experimental data, respectively, as

$$2g_A < g_P < 15g_A \quad \text{for set I,} \quad (14)$$

and

$$4g_A < g_P < 22g_A \quad \text{for set II} \quad (15)$$

in the configuration mixing model. These ranges are different from those of Ciechanowicz¹² and do not indicate any quenching of the induced pseudo-scalar coupling constant. Our range for g_P along with other results are listed in Table I, which shows our agreement with Holstein²⁷ and Possoz *et al.*⁴ Recently, Castro and Dominguez²⁸ have shown that the upper bound for g_P in nuclear muon capture is the Goldberger-Treiman value. Since in muon capture one always finds the combination $g_P + g_T$, this allows us to draw a range for g_T , the

TABLE I. Ranges for g_p/g_A and g_T/g_A in various muon capture processes in light nuclei. a and b of present work correspond to the two sets I and II of the experimental data (Ref. 6) defined in the text.

No.	Observable used in muon capture	Nuclei	Ref.	Range for g_p/g_A	Range for g_T/g_A
1	Capture rate	^{12}C , ^{16}O	32	9.0 ± 4.5	1.8 ± 4.5
2	Capture rate	^{12}C	27	8.5 ± 2.5	1.3 ± 2.5
3	Recoil polarization	^{12}B	27	15.0 ± 4.0	7.8 ± 4.0
4	Recoil polarization	^{12}B	4	7.1 ± 2.7	1.0 ± 2.7
5	γ -neutrino angular correlation	^{28}Si	12	-1.9 ± 3.1	-9.0 ± 3.1
6	γ -neutrino angular correlation	^{28}Si	Present work	8.5 ± 6.5^a 13.0 ± 9.0^b	1.4 ± 6.5 5.9 ± 9.0

induced tensor coupling constant. This range is also indicated in Table I.

From Fig. 1 and Table I, we draw the following conclusions:

(i) Our results for the range of g_p/g_A (assuming $g_T=0$) agree with those of others, except for those of Ciechanowicz.¹² Our results therefore do not indicate the possible quenching of g_p in a finite nuclear medium. As mentioned in the Introduction, it is ambitious to expect an indication of the quenching (a many-body effect) from a naive impulse approximation theory, especially in a medium-sized nucleus like ^{28}Si .

(ii) Our results indicate a range for g_T of $-3 < g_T/g_A < 12$ due to the large uncertainty in the experimental results of Miller *et al.*⁶ However, the most probable value²⁹ for g_T from our analysis is $g_T = +3.5 \pm 1.3$, which agrees with that of Possoz *et al.*⁴ ($g_T = 1.0 \pm 2.7$), but which stands in strong contradiction with the experimental results of

Sugimoto and Calaprice¹ and with a value of $g_T = 6$, recently advocated by Kubodera, Delorme, and Rho.³⁰

We wish to point out that a more precise measurement of the γ -neutrino angular correlation in ^{28}Si is needed, as is a measurement of the variation with energy of the asymmetry coefficients in ^{12}B and ^{12}N (as suggested by Hwang and Primakoff³¹) to obtain a clear view of the situation regarding the induced-tensor form factor.

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