Elastic scattering of deuterons off carbon nuclei

Jayati Ghosh and V. S. Varma
Department of Physics and Astrophysics, University of Delhi, Delhi-110007, India
(Received 7 March 1978)

Elastic scattering of deuterons off carbon nuclei is studied in the Glauber model. Fits to the 650 MeV elastic scattering data are attempted using, firstly, nucleon-nucleon data as input and the α-particle model of the carbon nucleus, and, secondly, deuteron-proton scattering data as input with both an independent particle and an α-particle description of the carbon nucleus. In all these cases the best fits are qualitatively the same as those obtained in a recent calculation using nucleon-nucleon data as input with an independent particle model of the nucleus.

[NUCLEAR REACTIONS $^{12}C(d,d)$, $E=650$ MeV; calculation of $\sigma(E,\theta)$, $\sigma_T(E)$, and comparison with experiment.]

I. INTRODUCTION

There have been various attempts at analyzing the elastic scattering of 650 MeV deuterons off ^{12}C nuclei. The first analysis[1] used optical potentials for neutron and proton scattering from carbon within the framework of the Glauber model. A subsequent analysis[2] using the Glauber model but with an independent-particle model of the ^{12}C nucleus and two-particle scattering data as input seemed to give better agreement with the experimental measurements. However, a more recent calculation by Varma and Franco[3] indicates that the better agreement reported in Ref. 2 was a result of the truncation of the multiple-scattering series and the use of too simple a deuteron wave function. Use of the full multiple-scattering series and more realistic deuteron wave functions destroys this agreement, leading Varma and Franco to conjecture that better agreement with the data within the framework of the Glauber model may only be possible provided nuclear correlations and spin-dependent effects are taken into account.

In the present study, we have once again examined the data of Ref. 1 attempting to take account of nuclear correlations by use of an α-particle model for the ^{12}C nucleus. Let us emphasize at the outset that this model is not completely realistic in that it does not take into account short-range nucleon-nucleon correlations. In the first part of the study we use two-particle scattering data as input, while in the second part, we use deuteron-proton elastic scattering data as input in conjunction with both an independent particle as well as an α-particle description of the ^{12}C nucleus. In no case do we get qualitatively better fits than have been obtained in Ref. 3.

II. α-PARTICLE MODEL AND TWO-PARTICLE INPUT

The ^{12}C nucleus is known to be deformed, a feature which is not reflected in the independent-particle model on which both the analyses[2,3] using two-nucleon data were based. A simple method of taking into account such deformation as well as correlations within the ^{12}C nucleus is to use the α-particle model which assumes that this nucleus consists of three α-particles in their ground state located at the vertices of an equilateral triangle with sides of fixed length d. This model has already been applied with considerable success to the study of the scattering of electrons, protons, and pions from carbon nuclei.[4-9] We therefore use suitably parametrized nucleon-nucleon amplitudes to construct the d-α scattering amplitude in the Glauber model, and use this in conjunction with the α-particle model to obtain the d-^{12}C scattering amplitude.

If we take the deuteron wave function as a sum of Gaussians

$$|\psi_d(\vec{r})|^2=\sum_{j=1}^{3}\alpha_j(4\pi\beta_j)^{-3/2}e^{-r^2/4\beta_j}\,,$$

the nucleon-nucleon amplitude is given by

$$f(q)=\frac{k\sigma(i+\rho)}{4\pi}\,e^{-\alpha q^2/2}$$

and describes the ground state of the α particle by an independent-particle model with a form factor given by

$$S_\alpha(q)=e^{-c^2q^2/4}\,,$$

where c is the size parameter of the Gaussian single-particle density distribution, then the elastic d-α scattering amplitude inclusive of the center-of-mass correction in the Glauber model is given by[3]

$$F_{d\alpha}(q) = G \sum_{j=1}^{4} \sum_{k=0}^{j} \sum_{l=0}^{k} \sum_{m=1}^{3} \Gamma_{jklm} e^{-\tilde{\alpha}_{jklm} q^2}, \quad (1)$$

where

$$G = -ik,$$

$$\Gamma_{jklm} = \binom{4}{j}\binom{j}{k}\binom{k}{l}\left(\frac{-\sigma(1-i\rho)}{2\pi(c^2+2a)}\right)^{j-l} \times \left(\frac{\sigma^2(1-i\rho)^2}{16\pi^2 a(c^2+a)}\right)^{l} \frac{\alpha_m}{A(j,l)E(j,k,l,m)},$$

$$\tilde{\alpha}_{jklm} = F(j,k,l,m) - \frac{c^2}{16},$$

and the quantities $E(j,k,l,m)$ and $F(j,k,l,m)$ are identical to those defined in Ref. 3 (with the replacement $R \rightarrow c$). A typographical error in Eq. (10) of this reference necessitates the correction

$$A(j,l) = \frac{2(j-l)}{(c^2+2a)} + \frac{2l}{(c^2+a)}.$$

The d-α scattering amplitude given by Eq. (1) can now be used in the Glauber model with the α-particle picture of the ^{12}C nucleus to obtain an expression for the d-^{12}C scattering amplitude operator, in terms of first-, second-, and third-order d-α scattering amplitudes. For this purpose we choose the origin of our coordinate system to coincide with the center of the equilateral triangle at whose vertices the α particles are located, integrate over the intermediate momentum transfers, and finally take the expectation value over the ground state of the ^{12}C nucleus, i.e., average all operators over all possible orientations of the nucleus. This averaging involves multiple integrations over the angles describing the orientations of the α-particle triangle in space and any attempt at carrying out these integrations analytically destroys the essential simplicity of the model. We therefore adopt the procedure followed by Ahmed[7] and replace the squared lengths of the sides of the α-particle triangle when projected on the impact-parameter plane by their average value $\frac{2}{3}d^2$ whenever they occur in the second- and third-order scattering terms. Khan and Ahmed[8] have shown by explicit calculation that the simplification referred to above does not introduce any significant errors, while reducing considerably the computational work, so that it may be used reliably for the quantitative study of fits to experimental cross sections. Using this simplification one can write the d-^{12}C scattering amplitude as

$$F_{dc}(\Delta) = F^{(1)}(\Delta) + F^{(2)}(\Delta) + F^{(3)}(\Delta), \quad (2)$$

with

$$F^{(1)}(\Delta) = 3F_{d\alpha}(\Delta) j_0(\Delta d/\sqrt{3}),$$

$$F^{(2)}(\Delta) = 3\left(\frac{i}{2\pi k}\right) G^2 \sum_{(j)} \sum_{(l)} \Gamma_{(j)}\Gamma_{(l)} \frac{\pi}{(\tilde{\alpha}_{(j)} + \tilde{\alpha}_{(l)})} \exp\left[-\left(\frac{\tilde{\alpha}_{(j)}\tilde{\alpha}_{(l)}}{(\tilde{\alpha}_{(j)} + \tilde{\alpha}_{(l)})}\Delta^2 + \frac{d^2}{6(\tilde{\alpha}_{(j)} + \tilde{\alpha}_{(l)})}\right)\right] j_0\left(\frac{\Delta d'}{2}\right),$$

where

$$d' = \frac{d}{\sqrt{3}}\left[1 + 3\left(\frac{\tilde{\alpha}_{(j)} - \tilde{\alpha}_{(l)}}{\tilde{\alpha}_{(j)} + \tilde{\alpha}_{(l)}}\right)^2\right]^{1/2}$$

and

$$F^{(3)}(\Delta) = \left(\frac{1}{2\pi i k}\right)^2 G^3 \sum_{(j)} \sum_{(l)} \sum_{(m)} \Gamma_{(j)}\Gamma_{(l)}\Gamma_{(m)} \frac{4\pi^2}{S_{(j)(l)(m)}(\tilde{\alpha}_{(l)} + \tilde{\alpha}_{(m)})} \exp\left[-\left(\tilde{\alpha}_{(j)} - \frac{4\tilde{\alpha}_{(j)}^2}{S_{(j)(l)(m)}}\right)\Delta^2\right] \times \exp\left\{-\frac{d^2}{6(\tilde{\alpha}_{(l)} + \tilde{\alpha}_{(m)})}\left[1 + \frac{3(\tilde{\alpha}_{(l)} + \tilde{\alpha}_{(m)})^2 + (\tilde{\alpha}_{(l)} - \tilde{\alpha}_{(m)})^2}{(\tilde{\alpha}_{(l)} + \tilde{\alpha}_{(m)})S_{(j)(l)(m)}}\right]\right\} \times j_0(\Delta d''),$$

where

$$d'' = \frac{d}{\sqrt{3}}\left[\left(1 - \frac{6\tilde{\alpha}_{(j)}}{S_{(j)(l)(m)}}\right)^2 + \frac{12\tilde{\alpha}_{(j)}^2(\tilde{\alpha}_{(l)} - \tilde{\alpha}_{(m)})^2}{(\tilde{\alpha}_{(l)} + \tilde{\alpha}_{(m)})^2 S_{(j)(l)(m)}^2}\right]^{1/2}$$

and

$$S_{(j)(l)(m)} = 4\,\frac{\tilde{\alpha}_{(j)}\tilde{\alpha}_{(l)} + \tilde{\alpha}_{(j)}\tilde{\alpha}_{(m)} + \tilde{\alpha}_{(l)}\tilde{\alpha}_{(m)}}{(\tilde{\alpha}_{(l)} + \tilde{\alpha}_{(m)})}.$$

In the above expressions, we have represented the fourfold summation indices j,k,l,m occurring in Eq. (1) by the collective symbol (j), so that each summation indicated in the expressions for

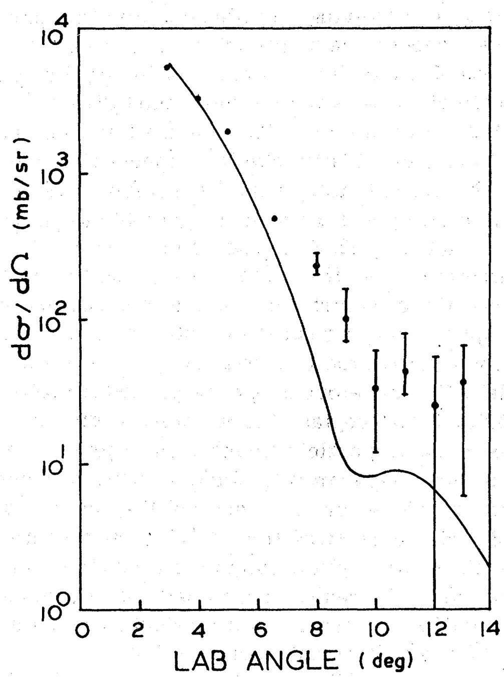

FIG. 1. $d-{}^{12}C$ elastic differential cross section at 650 MeV as a function of the laboratory angle in the α-particle model of ^{12}C nucleus with two-particle scattering data as input. Experimental data are from Ref. 1.

$F^{(2)}(\Delta)$ and $F^{(3)}(\Delta)$ is, in fact, a fourfold sum with limits as specified in Eq. (1).

Equation (2) is now used to obtain the elastic differential cross section. The values of the parameters for the nucleon-nucleon scattering amplitude are the same as in Ref. 3. The α-particle parameters used are $d=3.0$ fm (Ref. 5) and $c=1.366$ fm (Ref. 10) which correspond to a rms radius of ^{12}C of 2.40 fm.

In Fig. 1 we display the results of our calculation using mean values of nucleon-nucleon parameters. The calculated cross section lies systematically lower than the experimental data. The value of the deuteron-carbon total cross section is $\sigma_{tot} = 482$ mb, compared with the experimental value of 456 ± 18 mb.[1]

We have tested the sensitivity of the fit to variations in the α-particle parameters keeping the rms radius of the α cluster within the nucleus either greater than or equal to the rms radius of a free α particle in view of the possibility that interactions between the α clusters within the nucleus may cause them to be smeared out.[5,6] We find that for no reasonable values[7] of these parameters is the fit to experimental data any better than what is shown in Fig. 1.

The effect of variation of ρ, the ratio of the real to imaginary part of the nucleon-nucleon amplitude, was also studied and we found that no good overall fit to the data could be obtained for any reasonable values of this parameter either.

We are therefore forced to conclude that no satisfactory explanation of $d-{}^{12}C$ data at the present energy in terms of nucleon-nucleon scattering measurements exists as of the moment.

III. DEUTERON-PROTON INPUT

The analysis of Sec. II, as well as the studies in Refs. 2 and 3, are all based on the use of two-particle data. In the present section therefore we wish to investigate whether or not better agreement with $d-{}^{12}C$ measurements can be obtained by using deuteron-nucleon rather than two-particle scattering data as input.

If we parametrize the strong interaction deuteron-nucleon scattering amplitude as

$$f_d(q) = \frac{k\sigma_d}{4\pi}(i+\rho_d)e^{-b_d q^2/2} \tag{3}$$

and use an independent-particle model of the ^{12}C nucleus with a harmonic-oscillator form factor

$$S_c(q) = (1 - q^2R^2/9)e^{-R^2q^2/4},$$

where $R = 1.59$ fm (which corresponds to an rms radius of 2.41 fm), the $d-{}^{12}C$ scattering amplitude in the Glauber model is given by

$$F_{dc}(\Delta) = -\frac{ikQ}{2}\sum_{j=1}^{12}\binom{12}{j}\left(\frac{-\sigma_d(1-i\rho_d)}{2\pi Q}\right)^j \times \sum_{l=0}^{j}\binom{j}{l}\left(1-\frac{4R^2}{9Q}\right)^{j-l}\left(\frac{4R^2}{9Q}\right)^l \frac{l!}{j^{l+1}} \times \sum_{m=0}^{l}\binom{l}{m}\frac{(-1)^m}{m!}\left(\frac{\Delta^2 Q}{4j}\right)^m \times \exp\left[-\frac{\Delta^2}{4}\left(\frac{Q}{j}-\frac{R^2}{12}\right)\right], \tag{4}$$

with $Q = R^2 + 2b_d$.

Alternatively one can use the deuteron-nucleon amplitude to first construct the $d-\alpha$ scattering amplitude in the Glauber model, viz.,

$$F_{d\alpha}(q) = -\frac{ik}{2}(c^2+2b_d) \times \sum_{j=1}^{4}\binom{4}{j}\frac{1}{j}\left(-\frac{\sigma_d(1-i\rho_d)}{2\pi(c^2+2b_d)}\right)^j e^{-X_j q^2/4},$$
$$X_j = \frac{(c^2+2b_d)}{j} - \tfrac{1}{4}c^2, \tag{5}$$

and then use the α-particle model to yield yet another expression for the $d-{}^{12}C$ scattering amplitude which is obtained by using Eq. (5) above for the $d-\alpha$ scattering amplitude in Eq. (2). The resulting expression as well as Eq. (4) can be used to give estimates of the differential and total cross section for d-^{12}C scattering.

The parameters appearing in the deuteron nucleon scattering amplitude [Eq. (3)] were determined by analyzing the d-p data of Dutton *et al.*[11] at 1.69 GeV/c by assuming that the total dp phase is the sum of pure nuclear and Coulomb phases:

$$\chi^{dp}_{\text{total}} = \chi^{dp}_{\text{strong}} + \chi^{dp}_{\text{coulomb}} ,$$

which leads to

$$F_{dp}(\Delta) = f_c(\Delta) + \frac{ik}{2\pi}\int e^{i\vec{\Delta}\cdot\vec{b}} e^{i\chi^{dp}_{\text{coulomb}}} \Gamma^{dp}_{\text{strong}}\, d^2b ,$$

where the Coulomb scattering amplitude is taken as

$$f_c = -\frac{2\eta k}{\Delta^2}\exp\{-2i[\eta\ln(\Delta/2k) - \arg\Gamma(1+i\eta) + \eta\ln(2kR)]\},$$

the Coulomb phase shift is $\chi_c(b) = (b/2R)^{2i\eta}$, ($\eta = e^2/\hbar v$), and the profile function $\Gamma^{dp}_{\text{strong}}$ is just the Fourier transform of the dp scattering amplitude in Eq. (3). We are, of course, assuming the deuteron to be a point charge, which it is not.

The total cross section σ_d was taken to be 57.1 mb,[12-14] with $\rho_d = -0.105$ and $b_d = 27.8$ (GeV/c)$^{-2}$. These values of ρ_d and b_d correspond to the minimum value of χ^2, where χ^2 is defined as

$$\chi^2 = \sum_{i=1}^{n}\left(\frac{\sigma_{\exp}(\theta_i) - \sigma_{\text{th}}(\theta_i)}{\Delta\sigma(\theta_i)}\right)^2 ,$$

where n is the total number of data points, $\sigma_{\exp}(\theta_i)$ the experimental differential cross sections, $\Delta\sigma(\theta_i)$ the experimental errors, and $\sigma_{\text{th}}(\theta_i)$ the predicted cross sections. The best fit parameters correspond to the value of $\chi^2 = 36$, and the experimental d-p data together with the best fit curve is shown in Fig. 2.

The deuteron-nucleon parameter as determined is used in conjunction with Eqs. (4) and (5) to calculate fits to the d-^{12}C differential cross section with the ^{12}C nucleus described by an independent-particle model and the α-particle model, respectively. These fits are displayed in Fig. 3. Not only are these fits no better than those obtained by using two-particle scattering data as input, they also overestimate the total cross section whose values in the two cases are given by 474 and 484 mb, respectively.

We have even tried obtaining best fits to the d-^{12}C data by treating the deuteron nucleon parameters ρ_d and b_d as free variables in Eqs. (4) and (5) and in neither case do we get a better fit than was obtained by taking these parameters to be fixed by best fits to the d-p data of Ref. 11. This shows that the present d-^{12}C data cannot be successfully explained by taking d-p data as input either in the independent particle or the α-particle model of the ^{12}C nucleus.

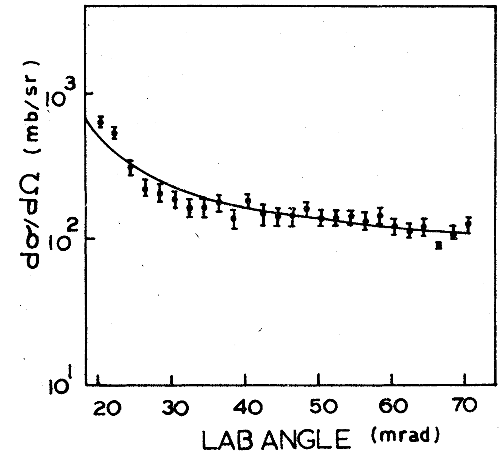

FIG. 2. Differential cross section for elastic dp scattering at 1.69 GeV/c (Ref. 11), showing the best fit curve.

IV. DISCUSSION

The effects of Coulomb interactions have not been included in our analysis, and instead of using separate p-p and n-p parameters in Sec. II we

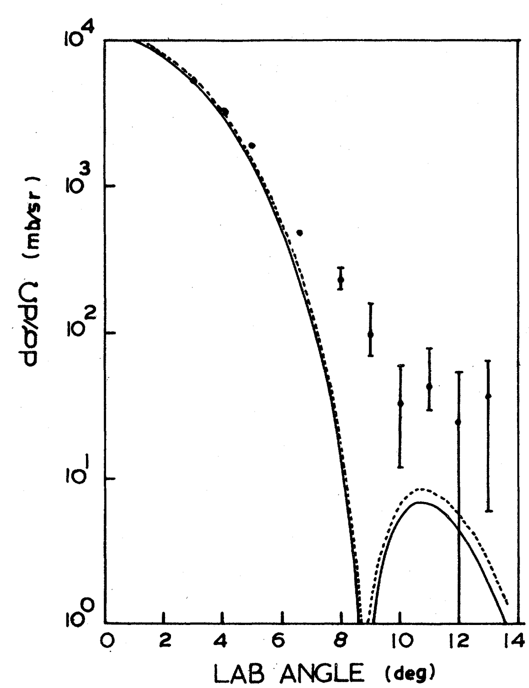

FIG. 3. d-^{12}C elastic differential cross section at 650 MeV using dp input for both independent-particle model and α-particle model (dotted line) of the ^{12}C nucleus.

have used average values of these parameters. Apart from the resulting simplicity, the justification for such a procedure is that the analysis of Varma and Franco[3] has shown that both these factors cause insignificant changes in the differential cross section other than either in the extreme forward direction or in the region of the interference minimum. Neither of these factors can therefore cause changes of the magnitude necessary to bring about agreement between the present calculation and experiment. Had we obtained better fits to start with, the inclusion of these factors would have been necessary before pronouncing on the ability of the theory to explain the experimental data.

One is therefore forced to conclude that no satisfactory explanation of d-^{12}C data at the present energy exists as of the moment. To get better fits to the data one must take into account nuclear correlations in a fashion which is more realistic than is done by the α-cluster model and/or spin and off-shell effects. Only further analysis can reveal which of these effects plays a relatively more important role and whether or not their inclusion leads to better agreement with experimental results within the framework of the Glauber model.

[1] L. M. C. Dutton, J. D. Jafar, H. B. Van der Raay, D. G. Ryan, J. A. Steigelmair, R. K. Tandon, and J. F. Reading, Phys. Lett. 16, 331 (1965).

[2] K. S. Chadha and V. S. Varma, Phys. Rev. C 13, 715 (1976).

[3] G. K. Varma and V. Franco, Phys. Rev. C 15, 813 (1977).

[4] E. V. Inopin and B. I. Tishenko, Zh. Eksp. Teor. Fiz. 38, 1160 (1960) [Sov. Phys.—JETP 11, 840 (1960)].

[5] E. V. Inopin, A. A. Kresin, and B. I. Tishenko, Yad. Fiz. 2, 802 (1965) [Sov. J. Nucl. Phys. 2, 573 (1966)].

[6] A. N. Antonov and E. V. Inopin, Yad. Fiz. 16, 74 (1972) [Sov. J. Nucl. Phys. 16, 38 (1973)].

[7] I. Ahmad, Phys. Lett. 36, 301 (1971).

[8] Z. A. Khan and I. Ahmad, Pramana 8, 149 (1977).

[9] J. F. Germond and C. Wilkin, Nucl. Phys. A237, 477 (1975); *ibid.* 249, 457 (1975).

[10] R. H. Bessel and C. Wilkin, Phys. Rev. 174, 1189 (1968).

[11] L. M. C. Dutton, R. J. W. Howells, J. D. Jafar, and H. B. Van der Raay, Nucl. Phys. B9, 594 (1969).

[12] W. N. Hess, Rev. Mod. Phys. 30, 368 (1958).

[13] F. F. Chen, C. P. Levitt, and A. M. Shapiro, Phys. Rev. 103, 211 (1956).

[14] H. G. De-Carvalho, Phys. Rev. 96, 398 (1954).