PHYSICAL REVIEW C VOLUME 18, NUMBER 4 OCTOBER 1978

Microscopic analysis of the three-particle transfer reaction $^{92}Zr(p,\alpha)^{89}Y$

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(Received 27 April 1978)

The reaction $^{92}Zr(p,\alpha)^{89}Y$ is analyzed by applying a microscopic theory to the three-particle transfer reaction. Good fits to the angular distributions are obtained for the transitions to the four low-lying states of ^{89}Y. Relative cross sections of all four transitions, calculated from wave functions given in the literature, are strongly influenced by core-excited configurations. With their inclusion the calculations are in excellent agreement with the experiment. It is thus shown that in the microscopic description the (p,α) reaction is a sensitive tool for testing wave functions.

[NUCLEAR REACTIONS $^{92}Zr(p,\alpha)$; $E=35$ MeV, measured $\sigma(\theta)$; microscopic DWBA analysis; ^{92}Zr, ^{89}Y test of shell model wave functions.]

I. INTRODUCTION

At energies between 20 and 50 MeV the (p,α) reaction predominantly consists of a direct three-nucleon pickup.[1-4] The reaction has a number of attractive features for nuclear structure studies, e.g., the possibility to populate proton hole states two neutrons away from the stability line, i.e., states which are not reached by single proton pick-up, the coherence[5] in the single-particle states of the three transferred nucleons, the ability to form high spin final states and the well-known strong j dependence of the angular distributions for $l=1$ transitions. In spite of these potentially useful properties the (p,α) reaction has scarcely been used for spectroscopic studies. The reason may be that a microscopic description of the reaction is needed if quantitative results are to be obtained. As explained below, the microscopic approach in general does not allow one to deduce wave functions of the target and the final nuclei from experimental (p,α) results in a straightforward manner. However, it provides a very sensitive test of wave functions obtained from other sources.

An account of recent experimental and theoretical work on the (p,α) reaction has been given by Falk[6] and Smits.[7] In many cases the reaction mechanism was assumed to be the pickup of a triton cluster. The form factor is then computed by binding a triton with proper cluster quantum numbers at the experimental separation energy in a Woods-Saxon well. While this simple model is adequate for the extraction of transferred l values or even j values in some cases,[8,9] it does not permit quantitative analyses of data such as extraction of spectroscopic strengths.

More quantitative analyses of the (p,α) reaction have been performed by using semimicroscopic models. Smits and Siemssen[10] successfully combined a semimicroscopic model with a weak-coupling model in the Sn region. The proton and neutron structure amplitudes are factored out from the DWBA matrix element and the dineutron structure amplitude is taken from a (p,t) experiment. In this treatment the microscopic form factor is approximated by a cluster form factor utilizing to some extent the merits of the microscopic description. Suck and Coker[11] suggested a semimicroscopic model for the (p,α) reaction in the Zr region which was able to fit the angular distributions, but it was not successful in the case of $^{96}Mo(p,\alpha)^{93}Nb$ investigated by Park *et al.*[12] In this model a form factor is used consisting of a product of a single-particle proton state with a microscopic two-neutron form factor. However, in this approach it is assumed that the transferred proton is always localized at the center of mass of the two transferred neutrons, thus the motion of the proton relative to the dineutron system is neglected.

For a fully microscopic treatment of the (p,α)

reaction a theory analogous to the microscopic two-nucleon transfer theory has been developed and applied to the $^{42}Ca(p, \alpha)^{39}K$ and $^{12}C(\alpha, p)^{15}N$ reactions by Falk.[6,13] With these investigations some progress has been made towards establishing the (p, α) reaction as a quantitative spectroscopic tool. Nevertheless, more studies of this kind seem necessary. We therefore have undertaken a series of (p, α) experiments in the mass 90 region.[14]

In this paper we present an analysis of the $^{92}Zr(p, \alpha)^{89}Y$ reaction which lends itself well to a microscopic description since wave functions of the initial and final nuclei have been given in the literature.

Former work on the Zr isotopes includes investigations by Peterson and Rudolph[15] who have studied the (p, α) reaction at 22.8 MeV with the simple cluster pickup model. For the $^{90}Zr(p, \alpha)^{87}Y$ reaction Fulmer and Ball[3] assumed a simple radial form factor consisting of a product of three single-particle harmonic oscillator wave functions. The observed strong energy dependence of the angular distributions at 20.2 and 22.9 MeV indicates that compound nucleus formation may not be negligible. In order to minimize such contributions we have chosen $E_p = 35$ MeV for our measurements. A preliminary account of the present work has been given in Ref. 16.

II. EXPERIMENTAL METHOD

The injector cyclotron of the Swiss Institute of Nuclear Research (SIN) provided a 35 MeV proton beam of 12 keV resolution. With beam currents of 80–300 nA dead time corrections in the detecting system were below 10%. Self-supporting targets with a thickness of 500 $\mu g/cm^2$, enriched to 96% of ^{92}Zr, were used.

Three 1 mm Ortec silicon surface-barrier detectors registered the α particles. Two of the detectors at different angles but fixed relative to one another were located on one side of the beam and the third on the other side. The angle relative to the beam of the two systems could be varied independently. Due to the low energy loss in the detector or large negative Q values, peaks from other reaction products appeared well below the α peaks in the spectra. No particle identification was therefore necessary. In order to reduce pulse pileup from the high flux of elastically scattered protons, veto detectors positioned behind the α detectors rejected the signals from penetrating particles.

A spectrum taken at $\theta_{lab} = 30°$ is shown in Fig. 1. The overall energy resolution, determined mainly by the target thickness, was about 100 keV. The angular resolution of the detecting system was 1.5°.

III. ANALYSIS

A. DWBA calculation

The distorted wave Born approximation (DWBA) analysis was performed with the code DWUCK.[17] The details of the microscopic calculation are given in the Appendix. Equation (A15) shows that several amplitudes may contribute coherently to any given matrix element. From (p, α) reactions alone it is therefore in general not possible to deduce unambiguous wave functions of the target and residual nuclei. Taking them from other sources (Sec. III E), we calculate the spectroscopic ampli-

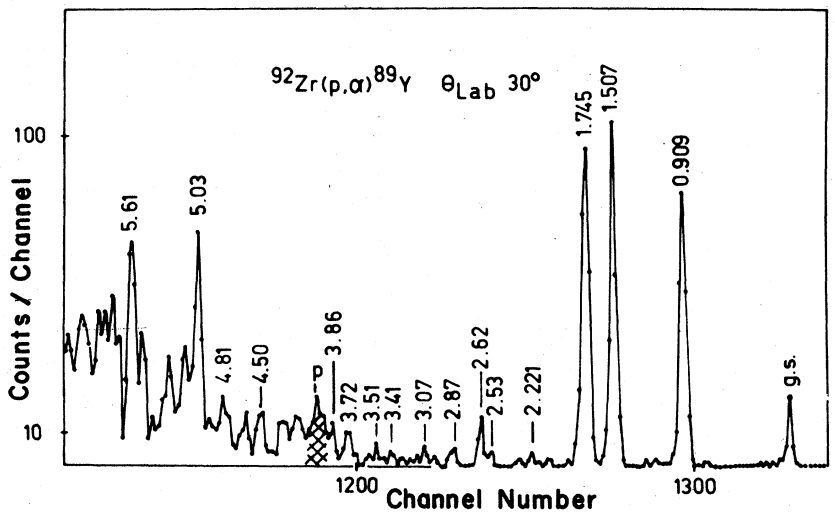

FIG. 1. α-particle spectrum of the reaction $^{92}Zr(p, \alpha)^{89}Y$ at $E_p = 35$ MeV and for $\theta_{lab} = 30°$.

TABLE I. Optical model and bound-state potential parameters.

Reaction	V_0 (MeV)	r_0 (fm)	a_0 (fm)	W (MeV)	r_i (fm)	a_i (fm)	V_{so} (MeV)	r_{so} (fm)	a_{so} (fm)
(p,p)	44.26	1.232	0.627	10.92	1.275	0.536	7.0	1.232	0.627
(α,α)	180.0	1.373	0.582	33.18	1.373	0.582			
Boundstate									
p	59.0	1.25	0.65				6.35	1.25	0.65
n	52.0	1.25	0.65				6.35	1.25	0.65

tudes $S_{AB}^{1/2}$ defined in Eq. (A2) and (A3) with methods which Glendenning[18] and Towner and Hardy[19] applied to two-nucleon transfer processes. Our results will show that in such an approach the (p, α) reaction provides a sensitive test of proposed wave functions.

We relate the experimental to the theoretical cross section as follows:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{exp}} = \epsilon\left(\frac{d\sigma}{d\Omega}\right)_{\text{DWBA}} .$$

The zero range approximation which is used in our calculation introduces a normalizing factor. It is chosen in such a way that the enhancement factor ϵ is unity for the transition to the ground state of ^{89}Y. The enhancement factors ϵ for the excited states will then be a measure of how well the experimental data are described by the theory. Within the limits of the DWBA description, a value $\epsilon = 1$ would indicate an ideal choice of wave functions.

B. Optical-model parameters

The optical-model and bound-state parameters used in our calculation are listed in Table I. The proton parameters reproduce the elastic scattering cross section data of ^{90}Zr(p,p) at $E_p = 40$ MeV.[20]

The ambiguities in the α optical potentials deduced from elastic scattering are well known. The shape of the angular distribution for (p, α) reactions, on the other hand, is very sensitive to the α potential used. We selected the parameters from a family of 16 potentials, having well depths between 40 and 210 MeV which all reproduce the α elastic scattering data on ^{92}Zr at 34.4 MeV.[21] Only the deep potential well with parameters as listed in Table I gives the correct slope of the (p, α) angular distribution. The same preference for a well depth of 170 MeV, which is 15% lower than the simple assumption $V_\alpha \approx 4V_N = 200$ MeV, is also found by microscopic analyses in the folding model.[22]

The bound nucleon wave functions are calculated for Woods-Saxon potentials with parameters reproducing the neutron and proton separation energy of ^{92}Zr and the level splittings of the $d_{5/2}$ and $d_{3/2}$ single-particle states.

C. Three-nucleon form factor

The three-nucleon form factor $F_M^{\gamma L}$ of Eq. (A16) was calculated with a code written by Falk.[6] Due to the difference between the sum of the separation energies of the three nucleons in their respective single-particle states and the separation energy for a triton, the microscopic form factor does not have the correct asymptotic behavior. As usual we replace it in the external region by the tail of the triton cluster form factor which is calculated for the required separation energy, L transfer, and number of nodes. Assuming an s state for the internal motion of the triton cluster one obtains the number of nodes[23]

$$N = (n_1 + n_2 + n_3 - 2) + \tfrac{1}{2}(l_1 + l_2 + l_3 - L) .$$

The geometrical parameters of the triton bound-state potential were chosen in such a way that the triton form factor could be joined smoothly to the microscopic form factor at the nuclear surface.

D. Importance of nuclear interior and finite range

It has been argued[10] that the (p, α) reaction is a surface reaction as the α particle is a strongly absorbed particle. Nevertheless, due to the large angular momentum mismatch in (p, α) reactions, we expect the nuclear interior to play an important role in the evaluation of the DWBA integral.[24] In order to investigate its effect in our case we performed DWBA calculations with lower cutoff radii up to 4.7 fm. The results given in Fig. 2 show that the angular distributions are in fact very sensitive to the cutoff radius. The angle-integrated cross section varies by more than a factor of 2 for different cutoff radii. An equally strong influence of the nuclear interior was found by Falk[13] in the ^{12}C(α, p) reaction. The use of a simple triton cluster form factor is thus necessarily a crude approximation.

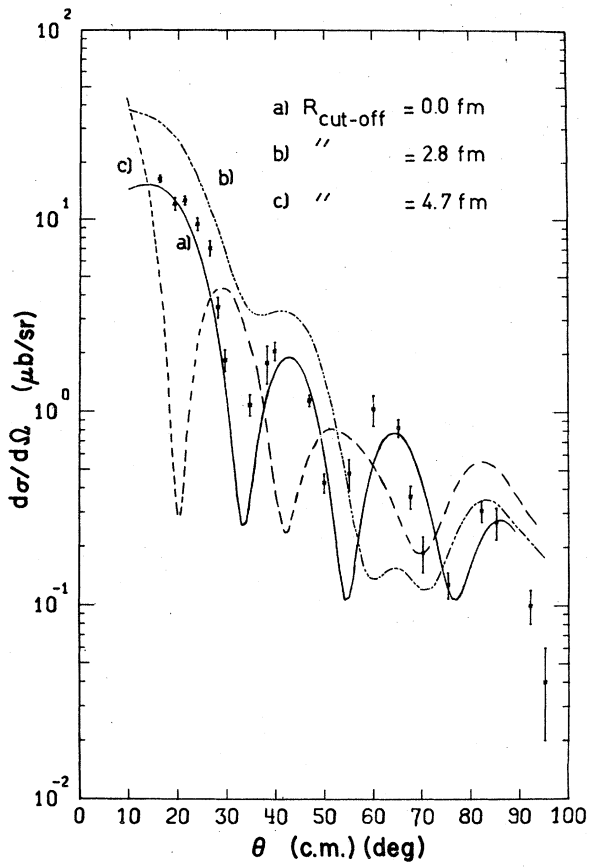

FIG. 2. Calculated angular distributions showing the dependence of the cross section on the inner cutoff radius used in the evaluation of the DWBA radial integral. Curve (a) has been normalized to the data points by introducing a normalization factor into the DWBA calculation. The same normalization factor has then been used for the calculation of the curves (b) and (c).

The success of cluster calculations in predicting the shapes of the experimental angular distributions may well be due to a rather arbitrary choice of Woods-Saxon well geometry for the calculations of the triton form factor.[12]

Our calculations were done in the zero range approximation because an exact finite range formalism for the microscopic description of the three-nucleon transfer process is at present not available. This may seem unsatisfactory since finite range corrections can be appreciable for reactions with a large momentum transfer. However, in the two-particle transfer (t, p) process a careful investigation[25] showed that finite range corrections have little effect on the relative cross sections and the angular distributions. To what extent this conclusion holds for (p, α) reactions can only be decided when calculations have been performed.

E. Wave functions

In many shell-model treatments of the Zr nuclei, $^{88}_{38}Sr_{50}$ is considered to be an inert closed-shell core. With this assumption we write for the ground state of $^{92}_{40}Zr_{52}$

$$|^{92}\mathrm{Zr}\rangle_{\text{g.s.}} = \pi[a_0(2p_{1/2})^2 + b_0(1g_{9/2})^2]_{0^+} \times \nu[d(2d_{5/2})^2 + e(2d_{3/2})^2 + f(3s_{1/2})^2]_{0^+} .$$

The magnitude of $a_0 = -0.66$ and $b_0 = 0.75$ is taken from the (^{3}He, d)-stripping results (Table XIII of Cates *et al.*[26]), the relative sign from Bayman[27] and Vervier.[28]

The neutron pair is coupled to $L' = 0$ with the amplitudes $d = 0.92$, $e = 0.19$, and $f = 0.21$ as obtained by Flynn *et al.*[29] from ^{90}Zr$(t, p)^{92}$Zr data. In the reverse (p, t) reaction, 2^+ states of ^{90}Zr are only weakly excited.[30] We therefore neglect components of the kind

$$[\pi(1g_{9/2})^2_{2^+,4^+}, \nu(2d_{5/2})^2_{2^+,4^+}]_{0^+}$$

in the ^{92}Zr ground state. As the neutrons in the low-lying levels of ^{89}Y which we study in our experiment form a closed shell, it is only the $L' = 0$ coupled pair which is picked up in the transfer process. The proton wave functions of ^{89}Y have been calculated by Vergados and Kuo.[31] The $\frac{1}{2}^-$ ground state and the $\frac{9}{2}^+$ first excited state are nearly pure single-particle $2p_{1/2}$ and $1g_{9/2}$ states, respectively. The $\frac{3}{2}^-$ second and $\frac{5}{2}^-$ third excited state are 2p-1h states given as

$$|^{89}\mathrm{Y}\rangle_{3/2^-} = 0.7774[(2p_{1/2})^2_0; (2p_{3/2})^{-1}] - 0.5962[(1g_{9/2})^2_0; (2p_{3/2})^{-1}] ,$$

$$|^{89}\mathrm{Y}\rangle_{5/2^-} = 0.9136[(2p_{1/2})^2_0; (1f_{5/2})^{-1}] - 0.3168[(1g_{9/2})^2_0; (1f_{5/2})^{-1}] .$$

These wave functions are somewhat restricted as evidenced by the fact that their normalization is less than unity. The authors themselves suggest that the missing amplitudes contain 3p-2h and 4p-3h excitations. In fact several authors have found that admixtures of $(2p_{3/2})^{-2}$-core excitations improve theoretical fits to experimental results for the Zr nuclei, as, e.g., Cates *et al.*[26] in the (^{3}He, d) reaction and Preedom *et al.*[32] in the (d, ^{3}He) pickup. Courtney and Fortune[33] show that such configurations in ^{90}Zr and ^{89}Y remove the previous discrepancy between the measured and calculated lifetime of the excited 0^+ state in ^{90}Zr. The same degree of $2p_{3/2}$ core excitation has been found by Hughes[34] in the ^{88}Sr nucleus. The results of the analysis of the reaction ^{88}Sr(^{3}He, d)^{89}Y studied by Picard and Bassani[35] show that in the ^{88}Sr ground state the fullness of the $2p_{3/2}$ orbital is ≈90%. From the analysis of the reaction ^{88}Sr$(d, n)^{89}$Y Hor-

TABLE II. ^{89}Y proton wave functions relative to ^{88}Sr.

J^π	Proton wave functions
$\frac{1}{2}^-$	$0.94(2p_{1/2})-0.35[(2p_{1/2})(1g_{9/2})^2(2p_{3/2})^{-2}]$
$\frac{9}{2}^+$	$0.95(1g_{9/2})+0.32[(1g_{9/2})(2p_{1/2})^2(2p_{3/2})^{-2}]$
$\frac{3}{2}^-$	$0.78[(2p_{1/2})^2(2p_{3/2})^{-1}]-0.60[(1g_{9/2})^2(2p_{3/2})^{-1}]+0.20[(2p_{3/2})^{-3}(2p_{1/2})^2(1g_{9/2})^2]$
$\frac{5}{2}^-$	$0.91[(2p_{1/2})^2(1f_{5/2})^{-1}]-0.32[(1g_{9/2})^2(1f_{5/2})^{-1}]-0.25[(2p_{3/2})^{-2}(2p_{1/2})^2(1g_{9/2})^2(1f_{5/2})^{-1}]$

ton and Hollandsworth[36] have found a 14%, and Harrison and Hiebert[37] from their study of the reaction ^{88}Sr$(d,{}^3\mathrm{He})^{87}$Rb and ^{88}Sr$({}^3\mathrm{He},d)^{89}$Y a 20% $(2p_{3/2})^{-2}$ proton configuration in the ground state of ^{88}Sr. In order to investigate the sensitivity of the (p,α) reaction to such an admixture we include the $(2p_{3/2})^{-2}$ configurations in the proton wave functions as follows:

$$|^{92}\mathrm{Zr}\rangle_{\mathrm{g.s.}} = a_c(2p_{1/2})^2 + b_c(1g_{9/2})^2 + c_c[(2p_{3/2})^{-2}(2p_{1/2})^2(1g_{9/2})^2] .$$

The amplitudes are calculated from the $({}^3\mathrm{He},d)$-spectroscopic strengths (Table IX of Cates *et al.*[26]) to give

$$a_c=-0.39, \quad b_c=0.73, \quad c_c=0.56 .$$

The sign of the admixture c_c is treated as a free parameter. We add similar core-excited configu-

TABLE III. Three-nucleon spectroscopic amplitudes $S^{1/2}[\gamma, \mathrm{LSJ}]$. The numbers given in the table include the coefficients required for the transformation from *jj* to LS coupling.

^{89}Y	E_x (MeV)	Angular momentum transfer L	S	J	Configuration of the transferred nucleons	$S^{1/2}[\gamma, \mathrm{LSJ}]$
$\frac{1}{2}^-$	0.000	1	$\frac{1}{2}$	$\frac{1}{2}$	$\pi 2p_{1/2};\nu(2d_{5/2})^2{}_{0^+}$	−0.57
					$\pi 2p_{1/2};\nu(2d_{3/2})^2{}_{0^+}$	−0.10
					$\pi 2p_{1/2};\nu(3s_{1/2})^2{}_{0^+}$	−0.17
$\frac{9}{2}^+$	0.909	4	$\frac{1}{2}$	$\frac{9}{2}$	$\pi 1g_{9/2};\nu(2d_{5/2})^2{}_{0^+}$	−0.88
					$\pi 1g_{9/2};\nu(2d_{3/2})^2{}_{0^+}$	−0.15
					$\pi 1g_{9/2};\nu(3s_{1/2})^2{}_{0^+}$	−0.26
$\frac{3}{2}^-$	1.507	1	$\frac{1}{2}$	$\frac{3}{2}$	$\pi 2p_{3/2};\nu(2d_{5/2})^2{}_{0^+}$	−0.95
					$\pi 2p_{3/2};\nu(2d_{3/2})^2{}_{0^+}$	−0.16
					$\pi 2p_{3/2};\nu(3s_{1/2})^2{}_{0^+}$	−0.28
$\frac{5}{2}^-$	1.745	3	$\frac{1}{2}$	$\frac{5}{2}$	$\pi 1f_{5/2};\nu(2d_{5/2})^2{}_{0^+}$	−1.27
					$\pi 1f_{5/2};\nu(2d_{3/2})^2{}_{0^+}$	−0.21
					$\pi 1f_{5/2};\nu(3s_{1/2})^2{}_{0^+}$	−0.38

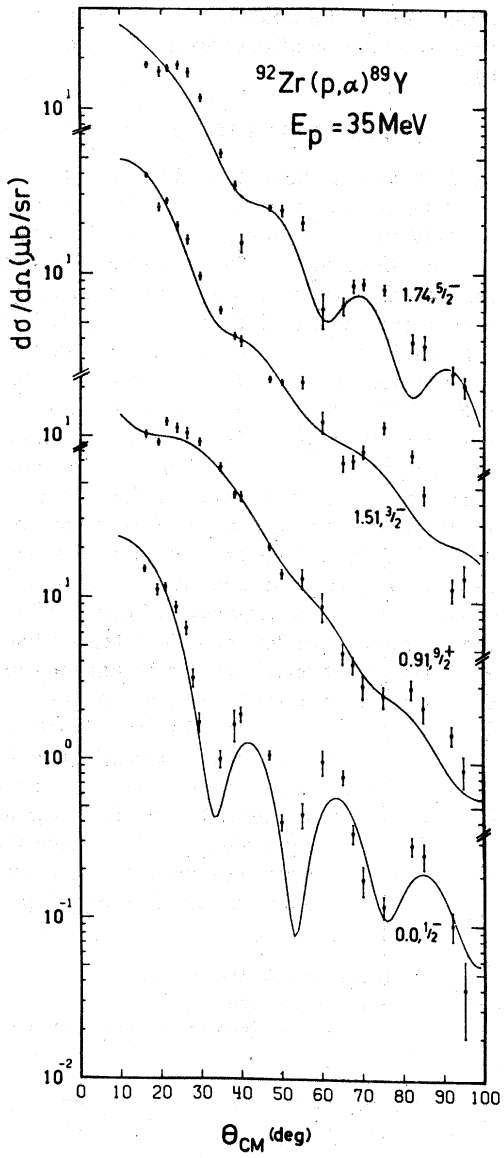

FIG. 3. Experimental and theoretical angular distributions for the reaction ^{92}Zr$(p,\alpha)^{89}$Y at E_p = 35 MeV leading to the ground and the first three excited states in ^{89}Y.

rations to the final states in ^{89}Y (Table II). Their amplitudes are determined by normalizing the wave functions of Vergados[31] to unity. The 12% admixture of the core-excited configuration to the ^{89}Y ground state which we obtain thus is in good agreement with the authors mentioned above.[33,35] Other excitations, as, e.g., $(f_{5/2})^{-2}$ configurations are not included here since there is no evidence for $l=3$ transitions in the (^{3}He, d) data on ^{92}Zr (Ref. 26) and ^{89}Y (Ref. 38). The same is not true for the case of ^{90}Zr considered below.

The spectroscopic amplitudes defined in the Appendix, Eq. (A3) calculated with wave functions including core-excited configurations are given in Table III.

IV. RESULTS AND DISCUSSION

Figure 3 shows the experimental and theoretical angular distributions for the reaction ^{92}Zr(p, α)^{89}Y at $E_p = 35$ MeV leading to the ground and the first three excited states in ^{89}Y. The microscopic model reproduces the data quite well. The characteristic j dependence observed as well at 22.5 MeV[15] and at 28 MeV[39] shows up very clearly in the $L=1$ transitions to the $J=\frac{1}{2}^-$ and $J=\frac{3}{2}^-$ states. The solid lines in Fig. 3 are obtained with the restricted wave functions discussed in Sec. III E. The inclusion of core excitations does not change the angular dependence, but the relative magnitudes of the cross sections are strongly affected as shown in Table IV. The first and second columns give the J^π values and the excitation energies of the final states. The ϵ_0 values of the third column are the enhancement factors as defined in Eq. (1) for the restricted wave functions. For the transitions to the first and second excited state they deviate significantly from unity. Using the wave functions which contain core-excitation components, we obtain enhancement factors ϵ_c in much better agreement with the expected value. The relative signs of the wave function amplitudes strongly affect the calculated cross sections. For the main components of the wave functions we obtain them in agreement with the literature. The signs of the core-excited components have, to our knowledge, not been given before.

TABLE IV. Enhancement factors ϵ without (ϵ_0) and with (ϵ_c) $(2p_{3/2})^{-2}$-core excitations.

J^π (^{89}Y)	E_x (MeV)	ϵ_0	ϵ_c
$\frac{1}{2}^-$	0.000	1 [a]	1 [a]
$\frac{9}{2}^+$	0.909	1.60	0.88
$\frac{3}{2}^-$	1.507	0.56	0.97
$\frac{5}{2}^-$	1.745	0.97	1.07

[a] Normalized to unity.

In order to test the sensitivity of the theory to the wave functions we calculated the cross sections with a different set of ^{92}Zr amplitudes which we calculated from the ^{92}Zr(d, ^{3}He) data of Preedom *et al.*[32] ($a=-0.66$, $b=0.55$, $c=0.51$). These calculations disagree with the experimental values and thus clearly favor the amplitudes of Cates *et al.*[26] The fact that Preedom *et al.*[32] had to extract spectroscopic strengths from unresolved doublets in their spectra may explain this discrepancy. A repetition of their experiment with higher resolution would certainly be desirable.

Figure 1 shows that up to $E_x \simeq 4$ MeV only the four lowest states of ^{89}Y are strongly populated in the ^{92}Zr(p, α) reaction. According to Vergados and Kuo[31] there is one additional $\frac{3}{2}^-$ state below 3 MeV with one hole in the ^{92}Zr-$J=0$-coupled proton pair configuration. With their wave functions we predict a cross section which is approximately 10 times smaller than that for the lowest $\frac{3}{2}^-$ state in good agreement with our experimental results.

All the other low-lying states given by Vergados and Kuo[31] require proton pairs in ^{92}Zr coupled to $J \geq 2$ or excitation of such pairs in the reaction. It is again consistent with these wave functions that such states are only weakly populated in the experiment.

The fact that only $L'=0$ coupled neutron pairs enter into our analysis is not due to the limitation of the theory but to the particular choice of nuclei participating in the reaction. Experimental data for cases with more complicated configurations are now being analyzed.

Both the ^{92}Zr(p, α) and the ^{90}Zr(d, ^{3}He) reactions essentially consist of a proton pickup leading to the same final states in ^{89}Y. A comparison between these two reactions has been made by Suck and Coker.[11] However, these authors have neglected the difference of the proton configurations of the two target nuclei ^{90}Zr and ^{92}Zr. Due to this difference the two reactions are connected only through the wave functions of the common final states in ^{89}Y.

In order to check the consistency of our results with those of the (d, ^{3}He) reaction we have to determine the ground-state wave function of ^{90}Zr which can be obtained from different sources. We might again use the (^{3}He, d) data of Cates *et al.*[26] However, one of their spin assignments in ^{91}Nb has been put to doubt by a later experiment of Knöpfle *et al.*[40] Their spectroscopic factor $S_{9/2}/S_{3/2}$ seems to be reliable and we obtain from it the value of b^2/c^2 (see Table V) as 1.89. The ratio a^2/c^2 can be calculated from the (d, ^{3}He) reaction leading to

TABLE V. Proton wave function amplitudes for ^{90}Zr and ^{92}Zr relative to ^{88}Sr.

	a	b	c
^{90}Zr	−0.75	0.54	0.39
^{92}Zr	−0.39	0.73	0.56

the two lowest $\frac{1}{2}^-$ and $\frac{9}{2}^+$ states in ^{89}Y for which the ratio $S_{1/2}/S_{9/2} = 1.74$ has been given by Preedom *et al.*[32] in reasonable agreement with Kavalovski *et al.*[41] With the ^{89}Y wave functions of Vergados and Kuo[31] we then obtain $a^2/c^2 = 3.61$. The resulting normalized amplitudes for ^{90}Zr (Table V), taken with the appropriate signs, are used, again with the wave functions of Table II, to calculate the spectroscopic factors $S_{3/2}$ and $S_{5/2}$ of the remaining two states in ^{89}Y. As Table VI shows they agree with the experimental results to within ±10% which is less than the uncertainties quoted by the authors. The considerations above show that the wave functions of Table II for ^{89}Y and of Table V for the Zr nuclei yield consistent results for the $(d, {}^3\mathrm{He})$ and the (p, α) reaction leading to the same four final states in ^{89}Y.

The ^{90}Zr amplitudes of Table V differ from the ones calculated from the same experimental data by Courtney and Fortune[33] and which yield a value of the EO-transition strength in ^{90}Zr in good agreement with the experiment. Nevertheless we prefer our values because these authors use absolute rather than relative spectroscopic factors from different experiments $[({}^3\mathrm{He}, d)$ and $(d, {}^3\mathrm{He})]$, a procedure which is certainly liable to larger errors. Following their procedure to calculate the EO strength, but using our wave functions, we obtain a value twice as high as the experimental one. However, shell model calculations[42] as well as experimental data[26,40] indicate that $(f_{5/2})^{-2}$ or $(f_{7/2})^{-2}$ excitations may not be neglected in the low-lying states of ^{90}Zr. Their amplitudes are at present not available from theory or experiment. It is therefore impossible to take this excitation into account.

V. SUMMARY

We have analyzed the ^{92}Zr$(p, \alpha)^{89}$Y reaction with a fully microscopic theory using shell model wave functions based on an inert ^{88}Sr core. The good agreement of the theoretical angular distributions with the experimental data shows that the microscopic theory is able to describe the (p, α) reaction satisfactorily. However, two of the resulting enhancement factors ϵ_0 (Table IV) deviate significantly from unity. Taking excitations of the ^{88}Sr core into account as described above we obtain enhancement factors ϵ_c close to unity for all four transitions. The ^{92}Zr-wave functions (Table V) used in this calculation are obtained from the ^{92}Zr$({}^3\mathrm{He}, d)$ experiment of Cates *et al.*[26] The ^{89}Y-wave functions calculated by Vergados and Kuo[31] had to be supplemented by 3p-2h configurations (Table II). These extended wave functions in conjunction with the ^{90}Zr functions of Table V are fully consistent with the ^{90}Zr$(d, {}^3\mathrm{He})$ and ^{90}Zr$({}^3\mathrm{He}, d)$ data of Refs. 26 and 32.

TABLE VI. $(d, {}^3\mathrm{He})$-spectroscopic factors.

^{89}Y	S_{exp} [a]	S_{th}
$\frac{1}{2}^-$	1.91	1.91 [b]
$\frac{9}{2}^+$	1.10	1.10 [b]
$\frac{3}{2}^-$	4.25	3.91
$\frac{5}{2}^-$	7.8	7.43

[a] From Preedom *et al.* (Ref. 32).
[b] $S_{1/2}/S_{9/2}$ normalized to experimental values.

In conclusion this work shows, we believe, that the (p, α) reaction, interpreted in a microscopic three-nucleon pickup theory, is a useful spectroscopic tool.

ACKNOWLEDGMENTS

We are grateful to Professor W. R. Falk (University of Manitoba, Winnipeg, Canada) for his assistance and for having placed at our disposal his computer code, and to Professor G. Eder (Atominstitut der Oesterreichischen Universitäten, Wien) for valuable suggestions and discussions. Further, we are obliged to H. Leeb (Atominstitut der Oesterreichischen Universitäten, Wien), P. Schober, and S. Steiner for their assistance. We also wish to express our thanks to the staff of the SIN cyclotron laboratory.

APPENDIX

In this Appendix the microscopic formalism developed by Falk[6,13] for the (p, α) reaction is presented. We rewrite the theory in an antisymmetrized form stating all normalization factors explicitly.

The transition matrix element for the reaction $p + A \rightarrow \alpha + B$

$$T_{p,\alpha} = \binom{2}{1}^{1/2} \binom{Z}{1}^{1/2} \binom{N}{2}^{1/2} \times \int\int d\vec{R} d\vec{r}\, \chi_{k'}^{(-)*}(\vec{r}_{\alpha B}) \langle \psi_\alpha \psi_B | V | \psi_p \psi_A \rangle \chi_k^{(+)}(\vec{r}_{pA}) . \quad \text{(A1)}$$

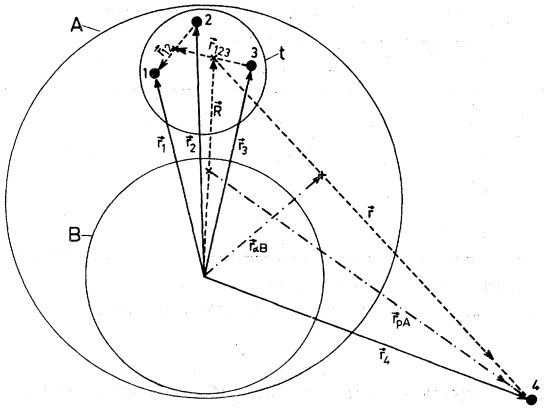

FIG. 4. Representation of the relevant coordinates in the description of the $A(p,\alpha)B$ reaction. Particles 1 and 2 refer to the two neutrons and particle 3 refers to the proton.

The relevant coordinates are given in Fig. 4. The integration has to be performed over the c.m. coordinate of the three transferred nucleons $\vec{R} = \frac{1}{3}(\vec{r}_1 + \vec{r}_2 + \vec{r}_3)$ and the relative coordinate of this three-nucleon system with respect to the incident proton $\vec{r} = \vec{r}_4 - \frac{1}{3}(\vec{r}_1 + \vec{r}_2 + \vec{r}_3)$. The optical model wave functions $\chi_k^{(-)}(\vec{r}_{\alpha B})$ and $\chi_k^{(+)}(\vec{r}_{pA})$ depend on the relative coordinates $\vec{r}_{pA} = (m_B/m_A)\cdot\vec{R} + \vec{r}$ and $\vec{r}_{\alpha B} = \vec{R} + (m_p/m_\alpha)\vec{r}$ in the entrance and exit channel, respectively. The factor

$$\binom{2}{1}^{1/2}\binom{Z}{1}^{1/2}\binom{N}{2}^{1/2}$$

stems from the antisymmetrization of the entrance and exit channel wave functions where Z and N are the number of protons and neutrons in the target nucleus. We thereby consider protons and neutrons as distinguishable particles and assume that the wave functions ψ_α, ψ_B and ψ_p, ψ_A are already antisymmetrized.

The wave function $\psi^{J_A}_{M_A}(\xi_A)$ of the target nucleus can be expanded using the wave functions $\psi^{J_B}_{M_B}(\xi_B)$ of the residual nucleus and the three-nucleon wave function $\psi^{\gamma LSJ}_{M_J}(\xi_1, \xi_2, \xi_3)$ written in the LS representation[19]

$$\psi^{J_A}_{M_A}(\xi_A) = \psi^{J_A}_{M_A}(\xi_B, \xi_1, \xi_2, \xi_3) = \sum_{\gamma,LSJ}\sum_{M_J}\sum_{J_B,M_B} \mathcal{J}_{AB}(\gamma, LSJ)(J_B M_B JM_J | J_A M_A) \times \psi^{J_B}_{M_B}(\xi_B)\psi^{\gamma,LSJ}_{M_J}(\xi_1, \xi_2, \xi_3). \quad \text{(A2)}$$

The internal coordinates of the target nucleus A and the residual nucleus B are ξ_A and ξ_B, respectively. The coordinates of the three nucleons bound in the target nucleus are ξ_1, ξ_2, ξ_3. The angular momentum quantum numbers of the three-nucleon system are L, S, J and M, M_S, M_J. The set $\gamma \equiv [n_1, n_2, n_3, l_1, l_2, l_3, j_1, j_2, j_3, L_{12}, S_{12}]$ characterizes the internal quantum numbers of the three-nucleon system. Here the number of nodes, the orbital angular momentum, and total angular momentum quantum numbers of the three nucleons are given by n_i, l_i, j_i, whereas L_{12} and S_{12} are the orbital angular momentum and spin quantum numbers of the subsystem consisting of the two neutrons.

The spectroscopic amplitude $S^{1/2}_{AB}(\gamma, L, S, J)$ is related to the expansion coefficient $\mathcal{J}_{AB}(\gamma, L, S, J)$ in Eq. (A2) by[19]

$$S^{1/2}_{AB}(\gamma, LSJ) = \binom{Z}{1}^{1/2}\binom{N}{2}^{1/2}\mathcal{J}_{AB}(\gamma, LSJ). \quad \text{(A3)}$$

In the L-S representation the three-nucleon wave function in Eq. (A2) is written in analogy to the two-nucleon case[43]

$$\psi^{\gamma,LSJ}_{M_J}(\xi_1, \xi_2, \xi_3) = \sum_{M,M_S}(LMSM_S|JM_J)\phi^{\gamma,L}_M(\vec{r}_1, \vec{r}_2, \vec{r}_3)\chi^{\gamma,S}_{M_S}(1,2,3), \quad \text{(A4)}$$

where

$$\chi^{\gamma,S}_{M_S}(1,2,3) = \sum_{m_{s_3},M_{S_{12}}}(\tfrac{1}{2}m_{s_3}S_{12}M_{S_{12}}|SM_S)\sum_{m_{s_1},m_{s_2}}(\tfrac{1}{2}m_{s_1}\tfrac{1}{2}m_{s_2}|S_{12}M_{S_{12}})\chi^{1/2}_{m_{s_1}}(1)\chi^{1/2}_{m_{s_2}}(2)\chi^{1/2}_{m_{s_3}}(3) \quad \text{(A5)}$$

and

$$\phi^{\gamma L}_M(\vec{r}_1, \vec{r}_2, \vec{r}_3) = N(\gamma)\Big(\sum_{m_3,M_{12}}(l_3 m_3 L_{12}M_{12}|LM)\sum_{m_1,m_2}(l_1 m_1 l_2 m_2|L_{12}M_{12})\phi^{n_3 l_3 j_3}_{m_3}(\vec{r}_3) \times [\phi^{n_1 l_1 j_1}_{m_1}(\vec{r}_1)\phi^{n_2 l_2 j_2}_{m_2}(\vec{r}_2) + (-1)^{l_1+l_2-L_{12}+S_{12}}\phi^{n_2 l_2 j_2}_{m_2}(\vec{r}_1)\phi^{n_1 l_1 j_1}_{m_1}(\vec{r}_2)]\Big). \quad \text{(A6)}$$

The antisymmetrization factor

$$N(\gamma) = \begin{cases} \frac{1}{2} \text{ for } (n_1, l_1, j_1) \equiv (n_2, l_2, j_2) \\ 1/\sqrt{2} \text{ otherwise} \end{cases}$$

and the single-particle wave functions are defined by

$$\phi^{n_i l_i j_i}_{m_i}(\vec{r}_i) = \frac{u_{n_i l_i j_i}(r_i)}{r_i} \mathcal{Y}^{m_i}_{l_i}(\vartheta_i, \varphi_i) . \tag{A7}$$

In order to calculate the DWBA integral a transformation from the coordinates $\vec{r}_1, \vec{r}_2, \vec{r}_3$ of the three nucleons to the c.m. coordinate $\vec{R}$ and the internal coordinates $\vec{r}_{12}$ and $\vec{r}_{123}$ of the three-nucleon wave function has to be performed. The coordinate $\vec{r}_{12}$ is the relative coordinate between particles 1 and 2. The coordinate $\vec{r}_{123}$ is the relative coordinate between the c.m. of particles 1 and 2 and the particle 3 (Fig. 4). To utilize the Brody-Moshinsky transformation we expand the radial part of the single-particle wave function (in our case the Saxon-Woods wave function) in spherical oscillator wave functions $H_{p_i l_i}(\nu r_i^2)$

$$u_{n_i l_i j_i}(r_i) = \sum_{p_i} a_{p_i} H_{p_i l_i}(\nu r_i^2) . \tag{A8}$$

Performing now two successive generalized Brody-Moshinsky transformations[44] from the coordinates $(\vec{r}_1, \vec{r}_2, \vec{r}_3)$ to the coordinates $(\vec{R}, \vec{r}_{12}, \vec{r}_{123})$ we get the following expression for the three-nucleon wave functions:

$$\begin{aligned}\phi^{\gamma,L}_M(\vec{R}, \vec{r}_{12}, \vec{r}_{123}) = \sum_{p_1,p_2,p_3} a_{p_1} a_{p_2} a_{p_3} N(\gamma) \sum_{\substack{m_1,m_2 \\ m_3,M_{12}}} (l_1 m_1 l_2 m_2 | L_{12} M_{12})(l_3 m_3 L_{12} M_{12} | LM) \\ \times \sum_{\substack{N'_{12} L'_{12} M'_{12} \\ n' l' m'}} \sum_{\substack{N' L' M' \\ n l m}} \left\langle \begin{matrix} p_1 & l_1 & m_1 \\ p_2 & l_2 & m_2 \end{matrix} \middle| \mu, \mu \middle| \begin{matrix} N'_{12} & L'_{12} & M'_{12} \\ n' & l' & m' \end{matrix} \right\rangle \\ \times \left\langle \begin{matrix} p_3 & l_3 & m_3 \\ N'_{12} & L'_{12} & M'_{12} \end{matrix} \middle| \mu, 2\mu \middle| \begin{matrix} N' & L' & M' \\ n & l & m \end{matrix} \right\rangle \\ \times \phi^{N'L'}_{M'}(\vec{R}) \phi^{nl}_m(\vec{r}_{123}) [\phi^{n'l'}_{m'}(\vec{r}_{12}) + (-1)^{l'-S_{12}} \phi^{n'l'}_{m'}(-\vec{r}_{12})] .\end{aligned} \tag{A9}$$

In this formula (p_i, l_i, m_i) are the quantum numbers of the single-particle harmonic oscillator wave functions, $(N'_{12}, L'_{12}, M'_{12})$ and (n', l', m') refer to the c.m. and relative motion of the subsystem consisting of particles 1 and 2, and (n, l, m) refers to the relative motion of the c.m. of particles 1 and 2 and the particle 3. Finally, (N', L', M') are the quantum numbers for the c.m. motion of the three-nucleon system.

The quantities $\langle \cdots | \mu, \mu | \cdots \rangle$ and $\langle \cdots | \mu, 2\mu | \cdots \rangle$ are the generalized Talmi-Moshinsky brackets defined in Ref. 44. The harmonic oscillator wave functions in the new coordinate system are

$$\phi^{N'L'}_{M'}(\vec{R}) = \frac{H_{N'L'}(3\nu R^2)}{R} \mathcal{Y}^{M'}_{L'}(\theta, \phi) ,$$

$$\phi^{n'l'}_{m'}(\vec{r}_{1,2}) = \frac{H_{n'l'}(\frac{1}{2}\nu r_{12}^2)}{r_{12}} \mathcal{Y}^{m'}_{l'}(\vartheta_{12}, \varphi_{12}) ,$$

and

$$\phi^{nl}_m(\vec{r}_{123}) = \frac{H_{nl}(\frac{2}{3}\nu r_{123}^2)}{r_{123}} \mathcal{Y}^{m}_{l}(\vartheta_{123}, \varphi_{123}) . \tag{A10}$$

For the α-particle wave functions we write

$$\Psi\alpha(\xi_1, \xi_2, \xi_3, \xi_4) = \phi_\alpha(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4) \chi^{S\,\alpha=0}_{M\,\alpha=0}(1, 2, 3, 4) , \tag{A11}$$

where

$$\begin{aligned}\chi^{S\,\alpha=0}_{M\,\alpha=0}(1, 2, 3, 4) = \tfrac{1}{2} [\chi^{1/2}_{1/2}(1) \chi^{1/2}_{-1/2}(2) - \chi^{1/2}_{1/2}(2) \chi^{1/2}_{-1/2}(1)] \\ \times [\chi^{1/2}_{1/2}(3) \chi^{1/2}_{-1/2}(4) - \chi^{1/2}_{1/2}(4) \chi^{1/2}_{-1/2}(3)]\end{aligned}$$

and

$$\begin{aligned}\phi_\alpha(\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4) &= N_\alpha \exp\left(-\eta^2 \sum_{i<j} r_{ij}^2\right) \\ &= N_\alpha \exp[-\eta^2 (3r^2 + 2r_{12}^2 + \tfrac{8}{3} r_{123}^2)] .\end{aligned} \tag{A12}$$

The size parameter[18] is $\eta = 0.233$ fm^{-1} and the normalization factor is

$$N_\alpha = \left(\frac{128}{\pi^3} \eta^6\right)^{3/4} .$$

It is convenient to separate the α-particle wave function into two parts, the first one describing the motion between the proton and the three-nucleon system and the second one describing the internal motion of the three-nucleon system

$$\phi_\alpha(\vec{r}, \vec{r}_{12}, \vec{r}_{123}) = \phi_\alpha^{(1)}(\vec{r})\phi_\alpha^{(2)}(\vec{r}_{12}, \vec{r}_{123}),$$

where

$$\phi_\alpha^{(1)}(\vec{r}) = \left(\frac{6\eta^2}{\pi}\right)^{3/4} \exp(-3\eta^2 r^2) \tag{A13}$$

and

$$\phi_\alpha^{(2)}(\vec{r}_{12}, \vec{r}_{123}) = \left(\frac{64}{3\pi^2}\eta^4\right)^{3/4} \exp[-\eta^2(2r_{12}^2 + \tfrac{8}{3}r_{123}^2)] .$$

In order to simplify the evaluation of the six-dimensional integral in the DWBA-matrix element in Eq. (1) the zero range approximation is used

$$V(\vec{r})\phi_\alpha^{(1)}(\vec{r}) = D_0\delta(\vec{r}) . \tag{A14}$$

Inserting now into the expression for the transition matrix Eq. (A1), the three-nucleon wave function (A9), and the α-particle wave function (A12), we obtain in the zero range approximation

$$T_{p\alpha}(M_B, M_\alpha = 0, M_A, M_p) = \sum_\gamma \sum_{\substack{LJ \\ MM_J}} (J_B M_B J M_J | J_A M_A)(LMS_p - M_p | JM_J) S_{AB}^{1/2}(\gamma, L, S_p, J)$$
$$\times \int d\vec{R}\, \chi_{k'}^{(-)*}(\vec{R}) F_M^{\gamma,L}(R) \mathcal{Y}_L^M(\theta, \phi) \chi_k^{(+)}\left(\frac{m_B}{m_A}\vec{R}\right), \tag{A15}$$

where the integration over the spin variables has been performed.

The radial form factor is

$$F_M^{\gamma,L}(R) = 2D_0 N(\gamma) \sum_{p_1 p_2 p_3} \sum_{\substack{m_1 m_2 m_2 \\ M_{12}}} \sum_{\substack{N_{12} N \\ n' n}} a_{p_1} a_{p_2} a_{p_3} (l_1 m_1 l_2 m_2 | L_{12} M_{12})(l_3 m_3 L_{12} M_{12} | LM) \left\langle \begin{matrix} p_1 & l_1 & m_1 \\ p_2 & l_2 & m_2 \end{matrix} \middle| \mu, \mu \middle| \begin{matrix} N_{12} & L_{12} & M_{12} \\ n' & 0 & 0 \end{matrix} \right\rangle$$
$$\times \left\langle \begin{matrix} p_3 & l_3 & m_3 \\ N_{12} & L_{12} & M_{12} \end{matrix} \middle| 2\mu, \mu \middle| \begin{matrix} N & L & M \\ n & 0 & 0 \end{matrix} \right\rangle I_{n'n} \frac{H_{NL}(3\nu R^2)}{R} . \tag{A16}$$

The integral over the internal coordinates of the three-nucleon system is

$$I_{n'n} = \int \phi_\alpha^{(2)}(\vec{r}_{12}, \vec{r}_{123}) \phi_0^{n'0}(\vec{r}_{12}) \phi_0^{n0}(\vec{r}_{123}) d\vec{r}_{12} d\vec{r}_{123}$$
$$= \left(\frac{64}{3}\eta^4\right)^{3/4} \left\{ \left[\frac{(2n'+1)!!}{n'!2^{n'}}\right]^{1/2} \frac{(\nu/2)^{3/4}}{\bar{\gamma}^{3/2}} \left[1 - \frac{\nu}{2\bar{\gamma}}\right]^{n'} \right\} \left\{ \left[\frac{(2n+1)!!}{n!2^n}\right]^{1/2} \frac{(\frac{2}{3}\nu)^{3/4}}{\Gamma^{3/2}} \left[1 - \frac{2\nu}{3\Gamma}\right]^n \right\}, \tag{A17}$$

where $\bar{\gamma} = 2\eta^2 + \frac{1}{4}\nu$ and $\Gamma = \frac{8}{3}\eta^2 + \frac{1}{3}\nu$.

Using the generalized transformation brackets

$$\langle n_1 l_1 n_2 l_2 : \lambda | \mu_1 \mu_2 | NLnl : \lambda \rangle = \sum_{\substack{m_1 m_2 \\ Mm}} (l_1 m_1 l_2 m_2 | \lambda M_0)(LMlm | \lambda M_0) \left\langle \begin{matrix} n_1 & l_1 & m_1 \\ n_2 & l_2 & m_2 \end{matrix} \middle| \mu_1 \mu_2 \middle| \begin{matrix} N & L & M \\ n & l & m \end{matrix} \right\rangle \tag{A18}$$

defined in Ref. 44, we can write the radial form factor as follows:

$$F_M^{\gamma L}(R) = F^{\gamma L}(R) = 2D_0 N(\gamma) \sum_{p_1 p_2 p_3} \sum_{\substack{N_{12} N \\ n' n}} a_{p_1} a_{p_2} a_{p_3} \langle p_1 l_1 p_2 l_2 : L_{12} | \mu\mu | N_{12} L_{12} n'0 : L_{12} \rangle$$
$$\times \langle p_3 l_3 N_{12} L_{12} : L | 2\mu, \mu | NLn0 : L \rangle I_{n'n} \frac{H_{NL}(3\nu R^2)}{R} . \tag{A19}$$

This means that the radial form factor does not depend on the angular momentum projection quantum numbers.

The differential DWBA cross section is finally

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{DWBA}} = \frac{\mu_{pA}\mu_{\alpha B}}{(2\pi\hbar^2)^2} \left(\frac{k'}{k}\right) \frac{1}{2(2J_A+1)} \sum_{\substack{M_A M_B \\ M_p}} |T_{p\alpha}(M_B, M_\alpha = 0, M_A, M_p)|^2 , \tag{A20}$$

where the transition matrix element $T_{p\alpha}$ is given by Eq. (A15).

*Work supported by the Swiss National Science Foundation.
†Work supported by Fonds zur Förderung der wissenschaftlichen Forschung.

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