Electromagnetic form factors of odd-A axially symmetric deformed nuclei*

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(Received 28 March 1978)

Expressions for electron scattering form factors for odd-A deformed nuclei are derived by projecting states with definite angular momentum from an axially symmetric intrinsic wave function. Through an expansion of the multipole matrix elements in terms of $1/\langle J_y^2\rangle$ both single-particle and collective contributions are taken into account in a systematic way. Selection rules are discussed and some schematic calculations of magnetic form factors are presented for ^{25}Mg and ^{181}Ta.

[NUCLEAR REACTIONS Electromagnetic form factors of odd-A deformed nuclei; projected Hartree-Fock approximation. Schematic calculations of transverse form factors for ^{181}Ta and ^{25}Mg.]

I. INTRODUCTION

Electron scattering is a powerful tool for the study of electromagnetic charge and current distributions in nuclei. Recent measurements at the new high resolution electron scattering facilities of form factors for the $0^+ \rightarrow I^+$ transitions within the ground state rotational band in even-even deformed nuclei have enabled the determination of the intrinsic nuclear charge distributions with high precision.[1] This information has successfully been interpreted in terms of Hartree-Fock theory.[2] Extension of these experiments to larger angles (or preferably 180°), where also electromagnetic current distributions can be probed, are being planned and can be expected to contribute to our understanding of nuclear rotational motion. Up to now little is known about currents in deformed nuclei. In the long wavelength limit the only easily accessible source of information on the collective part of the convection current is the value of the gyromagnetic ratio $g_R = (\mu_I/I)$ (for $K=0$). An analysis of magnetic dipole moments and transition rates indicates that its value is close to but somewhat less than Z/A, which corresponds to the value of a rigid-body rotation.[3] Experimental data at higher q values could in principle provide information about details of the radial distribution of the collective convection current density, which are expected to be sensitive to the characteristics of the rotational motion.

Electron scattering experiments also enable one to study higher multipole ($\lambda \geqslant 3$) moments which are very difficult to measure otherwise. An interesting question in this respect is whether there exist large collective contributions to the $M3$ matrix elements. These could be expected on the basis of the observation that in a collective model the $M3$ operator can be expressed, similarly to the $M1$ operator, in terms of a quadratic function $[\alpha_2 \times \alpha_2]^{\lambda=3}$ of the collective quadrupole variables α_2. For the same reason collective contributions to the $\lambda=5$ and higher multipolarities are expected to be smaller since they require at least cubic functions of α_2.

The aim of this paper is to discuss some aspects of transverse electron scattering on odd-even axially symmetric nuclei in terms of a microscopic theory. We focus our attention on transitions within the ground state rotational band. The most fundamental description of deformed nuclei that is currently available is the projected Hartree-Fock (PHF) approximation. This method is based upon the fact that the overlap integral $\langle \phi_K | e^{-i\beta J_y} | \phi_K \rangle \approx e^{-(1/2)\beta^2 \langle J_y^2 \rangle}$ for an axially symmetric intrinsic state ϕ_K has an angular width $1/\langle J_y^2 \rangle$, which serves as an expansion parameter.[4] This approach has been shown[2] to give a rather satisfactory description of elastic and inelastic Coulomb electron scattering on even-even and odd-even deformed nuclei already in zeroth order.

In Sec. II general expressions for the Coulomb and transverse electromagnetic form factors are presented and the resulting selection rules are discussed. Since in practice the numerical aspects of the full projected Hartree-Fock method are rather involved, it seems appropriate to see to what extent qualitative results can be obtained by simpler methods. To this end in Sec. III the Nilsson model is considered to obtain some qualitative predictions of the transverse scattering cross sections for ^{25}Mg and ^{181}Ta. The conclusions of this work are discussed in the final section.

II. PROJECTED HARTREE-FOCK APPROACH

At present the best approximation to the intrinsic ground state of an axially symmetric nucleus is the density dependent Hartree-Fock approximation (DDHF). Since the Hartree-Fock wave function ϕ_K is not an eigenstate of the angular momentum operator, it is necessary to project ϕ_K onto eigenstates of good angular momentum. In this section we derive a general expression for electromagnetic form factors following a projection method proposed by Villars.[4]

In general a projected HF wave function with good angular momentum I and parity π can be expressed as (for $K\neq 0$)

$$\psi^{I^\pi}_{M,K}(x)=\frac{\pi^{1/2}}{2N^I_K}\int d\Omega\,[D^{*I}_{KM}(\Omega)\mathcal{R}(\Omega)\phi^\pi_K(x)+(-1)^{I-K}D^{*I}_{-KM}(\Omega)\mathcal{R}(\Omega)\phi^\pi_{-K}(x)]\,, \tag{2.1}$$

where N^I_K is a normalization constant. Conventions for the Wigner D functions and the rotation operator $\mathcal{R}$ are the same as in Ref. 4. The wave function (2.1) has the normal behavior under time reversal

$$T\psi^{I^\pi}_{M,K}=(-1)^{I-M}\psi^{I^\pi}_{-M,K}\,. \tag{2.2}$$

The projection operator $P^I_{KM}=\int d\Omega\, D^{*I}_{KM}(\Omega)\mathcal{R}(\Omega)$ projects out from ϕ_K a component with definite $\vec{I}^2=I(I+1)$ and $I_z=M$. The reduced matrix elements of an arbitrary tensor operator $T^\lambda_\mu(x)$ that transforms as $T^\lambda_\mu(Rx)=\sum_\nu T^\lambda_\nu(x)D^\lambda_{\nu\mu}(\Omega)$ can be expressed in the form[4]

$$\langle I'K\|T^\lambda\|IK\rangle=\frac{(2I'+1)^{-1/2}}{N^I_KN^{I'}_K}\frac{1}{2}\int_0^{\pi/2}\sin\beta\,d\beta\sum_\nu\langle IK-\nu\,\lambda\,\nu|I'K\rangle \times\{d^I_{K-\nu,K}(\beta)\,\mathrm{Re}\langle\phi_K|T^\lambda_\nu e^{-i\beta J_y}|\phi_K\rangle+(-1)^{I-K}d^I_{K-\nu,-K}\,\mathrm{Re}\langle\phi_K|T^\lambda_\nu e^{-i\beta J_y}|\phi_{-K}\rangle\}\,. \tag{2.3}$$

With the use of the many-body theorem[5]

$$\langle\phi|AB|\phi\rangle=\langle\phi|AB|\phi\rangle_L\exp[\langle\phi|B|\phi\rangle_C]\,, \tag{2.4}$$

where B is a unitary operator, and $\langle\,\rangle_L$ and $\langle\,\rangle_C$ denote linked and connected diagrams, respectively; the matrix elements that occur in the right-hand side of Eq. (2.3) can be rewritten as

$$\langle\phi_K|T^\lambda_\nu e^{-i\beta J_y}|\phi_{\pm K}\rangle=\langle\phi_K|T^\lambda_\nu e^{-i\beta J_y}|\phi_{\pm K}\rangle_L\exp(\langle\phi_K|e^{-i\beta J_y}|\phi_K\rangle_C). \tag{2.5}$$

For strongly deformed nuclei the overlap integral $\langle\phi_K|e^{-i\beta J_y}|\phi_K\rangle_C$ is large only for the values of β around $\beta=0$ (see Refs. 4 to 6). One can use the approximation[4]

$$\exp(\langle\phi_K|e^{-i\beta J_y}|\phi_K\rangle_C)\approx\exp(-\tfrac{1}{2}\beta^2\langle J_y^2\rangle), \tag{2.6}$$

where $\langle J_y^2\rangle$ is the expectation value of the operator J_y^2 with respect to the HF wave function ϕ_K (upper (lower) case letters denote particles (holes))

$$\langle J_y^2\rangle=\sum_{Aa}|\langle a|j_y|A\rangle|^2. \tag{2.7}$$

An expansion of the integrand in Eq. (2.3) around $\beta=0$ with expansion parameter $1/\langle J_\perp^2\rangle$ ($\langle J_\perp^2\rangle=2\langle J_y^2\rangle\gtrsim 100$) is therefore expected to converge rapidly. Keeping only the more significant terms up to first order in $1/\langle J_\perp^2\rangle$ Eq. (2.3) can be rewritten as

$$\begin{aligned}\langle\psi^{I'}_K\|T^\lambda(q)\|\psi^I_K\rangle\cong{}&(2I+1)^{1/2}N^{(0)}_{I,I',K}\\&\times\{\langle IK\lambda 0|I'K\rangle\langle\phi_K|T^\lambda_0(q)|\phi_K\rangle\tfrac{1}{2}(1+(-1)^{\lambda+p})\\&+(-1)^{I-K}\langle I-K\lambda\,2K|I'K\rangle\langle\phi_K|T^\lambda_{2K}(q)|\phi_{-K}\rangle\tfrac{1}{2}(1+(-1)^{2K+p})\}\\&+\frac{(2I+1)^{1/2}}{\langle J_\perp^2\rangle}\{\mathrm{Re}\langle\phi_K|T^\lambda_1(q)J_-|\phi_K\rangle_L\langle IK-1\,\lambda\,1|I'K\rangle[(I+K)(I-K+1)]^{1/2}\\&+\mathrm{Re}\langle\phi_K|T^\lambda_{-1}(q)J_+|\phi_K\rangle_L\langle IK+1\,\lambda\,-1|I'K\rangle[(I-K)(I+K+1)]^{1/2}\\&-\mathrm{Re}\langle\phi_K|T^\lambda_0(q)2J_y^2|\phi_K\rangle_L\langle IK\lambda 0|I'K\rangle\}\,.\end{aligned} \tag{2.8}$$

Here $p=0(1)$ for Coulomb (transverse multipoles). The normalization factor $N^{(0)}_{I,I',K}$ is given by

$$N^{(0)}_{I,I',K}=(N^0_{IK}/N^0_{I'K})^{1/2}, \tag{2.9}$$

where

$$N^0_{IK}\cong\sum_{t\geq 0}\frac{(I+K)!(I-K)!}{(I+K-t)!(I-K-t)!(t!)^2}\sum_{n\geq 0}\frac{(I-t)!(-1)^{n+t}}{n!(I-t-n)!}\frac{(n+t)!}{(\langle J_\perp^2\rangle)^{n+t}}. \tag{2.10}$$

$N^{(0)}_{I,I',K}$ has the value 1 for elastic scattering and increases slowly for inelastic scattering with increasing I':

$$N^{(0)}_{I,I',K}\simeq 1-\frac{1}{2}\frac{I(I+1)-I'(I'+1)}{\langle J_\perp^2\rangle}.$$

The zeroth-order contributions that are obtained from the first term in Eq. (2.8) by approximating $N^{(0)}_{I,I',K}=1$ correspond to the result of the classical rotational model in which coupling between rotational and intrinsic degrees of freedom is neglected, i.e., the total wave function factorizes.[7] For odd-even nuclei the HF problem is usually solved in the "pair filling approximation" (i.e., the conjugate orbitals are pairwise occupied). In this case the wave function of the even-even core is time reversal invariant and only the odd nucleon contributes to the zeroth-order matrix elements of the transverse electric and magnetic multipole operators. There are several kinds of contributions that are first order in $1/\langle J_\perp^2\rangle$: the second set of terms in Eq. (2.8) which receives contributions from the core nucleons, and in addition a contribution arising from the expansion of $N^{(0)}_{I,I',K}$, which gives rise to a q-independent scaling of the zeroth-order terms for $I'\neq I$. It is worth discussing briefly the various selection rules for electron scattering on odd-A nuclei that follow from Eq. (2.8) for the different multipole transitions. For simplicity it will be assumed that the Hartree-Fock intrinsic state ϕ_K has been calculated in the pair filling approximation. We come back to this question at the end of this section.

Coulomb multipoles C^λ. Only even multipoles contribute that satisfy the condition $|I'-I|\leq\lambda\leq I+I'$. Of the two zeroth-order terms the $\Delta K=2K$ one does not contribute, whereas the $\Delta K=0$ one receives contributions from all protons. The effect of the first-order terms has been investigated in Ref. 8 for the case of $I=0\to I'$ transitions in even-even nuclei ($K=0$) in the rare-earth region. It was found that these terms yield negligible corrections to the cross section for small I' and an increase of up to 15% on $0^+\to 6^+$ transitions.

Transverse magnetic multipoles M^λ: $\lambda=$ *odd and* $|I'-I|\leq\lambda\leq I+I'$. Both zeroth-order terms contribute to M^λ, which receive only contributions from the odd nucleon. The first two first-order terms within the curly brackets represent important collective contributions from the core nucleons. For magnetic dipole moments this contribution (in the limit $q\to 0$) can be expressed as

$$\Delta\mu=g_R\frac{I(I+1)-K^2}{I+1}$$

with

$$g_R=\langle\phi_K|M^1_xJ_x+M^1_yJ_y|\phi_K\rangle/\langle J_\perp^2\rangle. \tag{2.11}$$

Noting that

$$\vec{\mathrm{M}}\approx\sum_i\frac{1+\tau_3^{(i)}}{2}\vec{\mathrm{l}}^{(i)}$$

one recovers the well-known result $g_R\sim Z_{\mathrm{eff}}/A_{\mathrm{eff}}$. We note that Eq. (2.11) represents an average over the excitation energy of the self-consistent cranking formula[9] for the collective g factor.

Transverse electric multipoles E^λ: $\lambda=$ *even*, $|I'-I|\leq\lambda\leq I'+I$ *and* $I'\neq I$. There is no zeroth-order contribution to the E^λ multipoles except for the single-particle contribution $\langle\phi_K|E^\lambda_{2K}|\phi_{-K}\rangle$ for inelastic transitions with $\lambda\geq 2K+1$. It is interesting to note that in the case $I=I'$ the three first-order terms in Eq. (2.8) cancel each other, and therefore one recovers the familiar result (which follows from time reversal invariance) that transverse electric multipoles do not contribute to elastic scattering. In first order the collective contribution to transverse electric multipole operators is given by

$$\langle\psi_K^{I'}\|E^\lambda(q)\|\psi_K^I\rangle=(-1)^{I+K}\frac{[(2I+1)(2I'+1)]^{1/2}}{\langle J_\perp^2\rangle}\mathrm{Re}\langle\phi_K|E^\lambda_1(q)J_-|\phi_K\rangle$$
$$\times\left\{[(I+K)(I-K+1)]^{1/2}\begin{pmatrix}I&\lambda&I'\\K-1&1&-K\end{pmatrix}-[(I'+K)(I'-K+1)]^{1/2}\begin{pmatrix}I'&\lambda&I\\K-1&1&-K\end{pmatrix}\right\}. \tag{2.12}$$

In the pair filling approximation the polarization of the core due to the spin of the odd nucleon is ignored. If a more strict HF calculation for spin-nonsaturated systems were to be performed this effect would be reflected in the fact that the even-even core would no longer be time reversal invariant. This implies that the zeroth-order terms in Eq. (2.8) for transverse multipoles would receive also a contribution from the core nucleons. In order to see that it is in these terms that this effect would show up more let us consider again the static magnetic dipole moment. To first order in $1/\langle J_\perp^2\rangle$ the magnetic moment for $K \neq \frac{1}{2}$ is given by

$$\mu \cong g_K \frac{K^2}{I+1} + \Delta\mu \tag{2.13}$$

with $\Delta\mu$ as given by Eqs. (2.11) and

$$g_K = \langle \phi_K \,|\, M_z^1 \,|\, \phi_K \rangle / K. \tag{2.14}$$

If the even-even core is time reversal invariant only the odd nucleon contributes to g_K. Therefore the difference between the "true" g_K value and the single particle value gives a "measure" of the core polarization. Finally we point out that although it is not apparent from Eq. (2.8), it can be shown that the reduced matrix elements obey the relation

$$\langle I'K \,\|\, T^\lambda(q) \,\|\, IK \rangle = (-1)^{I-I'+\lambda+p} \times \langle IK \,\|\, T^\lambda(q) \,\|\, I'K \rangle . \tag{2.15}$$

III. APPLICATION OF DEFORMED HARMONIC OSCILLATOR MODEL

Since full projected Hartree-Fock calculations are rather complex and in order to obtain a qualitative idea about the merits of the projection technique for the calculation of transverse form factors, we have performed a schematic calculation, approximating the intrinsic state ϕ_K by a Slater determinant consisting of deformed harmonic oscillator wave functions. The intrinsic matrix elements $\langle K \,|T_\mu^\lambda(q)\,|\, \pm K\rangle$ that occur in the zeroth-order terms of Eq. (2.8) can then be calculated directly by replacing ϕ_K with a Nilsson single-particle state $[Nn_z\Lambda]K^\pi$. By expanding the latter in terms of spherical orbitals $|Nlj\rangle$ one finds

$$\langle K' \,|\, T_\mu^\lambda(q) \,|K\rangle = \sum_{\substack{l,l' \\ j,j' \\ \Lambda,\Lambda'}} a_{l\Lambda}^K a_{l'\Lambda'}^K \langle l\Lambda\tfrac{1}{2}\Sigma \,|\, jK\rangle \times \langle l'\Lambda'\tfrac{1}{2}\Sigma' \,|\, j'K'\rangle (2j'+1)^{-1/2} \langle jK\lambda\mu \,|\, j'K'\rangle \langle l'j' \,\|\, T^\lambda(q) \,\|\, lj\rangle . \tag{3.1}$$

It is instructive to compare the resulting multipole distributions in this strong coupling scheme

$$\langle I'K \,\|\, T^\lambda(q) \,\|\, IK\rangle = (2I+1)^{1/2}\{\langle IK\lambda 0 \,|\, I'K\rangle\langle K \,|\, T_0^\lambda(q) \,|\, K\rangle \delta_{\lambda,\mathrm{odd}} + (-1)^{I-K}\langle I-K\lambda 2K \,|\, I'K\rangle\langle K \,|\, T_{2K}^\lambda(q) \,|{-K}\rangle\} , \tag{3.2}$$

with the result for a spherical nucleus. For the important case $I'=I=K$ and I not too small the geometrical factors $\langle II\lambda 0 \,|\, II\rangle$ that multiply the $\Delta K=0$ term fall off rapidly with increasing λ. Thus one predicts that in elastic magnetic scattering in the strong coupling limit the intermediate multipoles ($\lambda = 3, \ldots, 2I-2$) are strongly suppressed compared to the spherical single-particle values, whereas the highest multipole ($\lambda = 2I$) that receives a contribution from the $\Delta K = 2K$ term is comparable to the single-particle value.

As a specific example we first consider ^{181}Ta, motivated by a current experiment at Bates.[10] In the Nilsson model ^{181}Ta is described by a $[404]\frac{7}{2}^+$ proton intrinsic state; assuming a deformation $\delta = 0.3$ one has $[404]\frac{7}{2}^+ = 0.976\,|N=4, l=4, \Lambda=4, \Sigma=-\frac{1}{2}\rangle - 0.218\,|N=4, l=4, \Lambda=3, \Sigma=\frac{1}{2}\rangle$. All necessary matrix elements of the transverse multipole operators can be expressed as linear combinations of the matrix elements in the spherical basis $\langle \alpha \,\|\, T^\lambda(q) \,\|\, \beta\rangle$, where $\alpha, \beta = g_{9/2}$ and $g_{7/2}$. For simplicity the radial wave functions are approximated by harmonic oscillator wave functions.

The calculated transverse form factor for elastic scattering is shown in Fig. 1, together with the prediction of the extreme single-particle model. One sees that compared to the weak coupling case there is an appreciable reduction of the cross section for the intermediate q values, due to the large quenching of the M^3 and M^5 multipoles. We note that this effect enables one to experimentally determine the lowest and the highest multipole distributions over a larger q region than is possible in the weak coupling case. In the present case with $I=K$ large the collective M^1 contribution

$$\frac{\mu_{\mathrm{coll}}}{\mu^{\mathrm{sp}}} \sim \frac{g_R}{g_K}\frac{1}{I} ,$$

is relatively unimportant, and has therefore been neglected.

In the inelastic form factors within the $K=\frac{7}{2}^+$ band that are also being measured the various allowed multipole matrix elements $\langle K \,|\, T_\mu^\lambda(q) \,|\, \pm K\rangle$ occur with different weighting for each I'. A simultaneous measurement of a number of inelastic $I \to I'$ transverse form factors within the ground state rotational band would thus in principle enable one to map out each nuclear current multipole

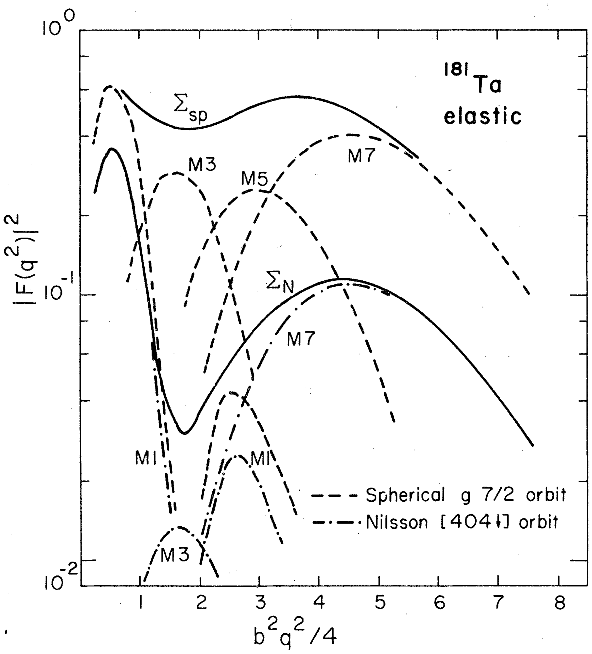

FIG. 1. Transverse form factor squared (arbitrarily normalized) calculated for elastic scattering on ^{181}Ta ($I=K=\frac{7}{2}^+$). The various multipole distributions for the spherical $g_{7/2}$ ([404↓] Nilsson) case are represented by dashed (dashed-dotted) curves.

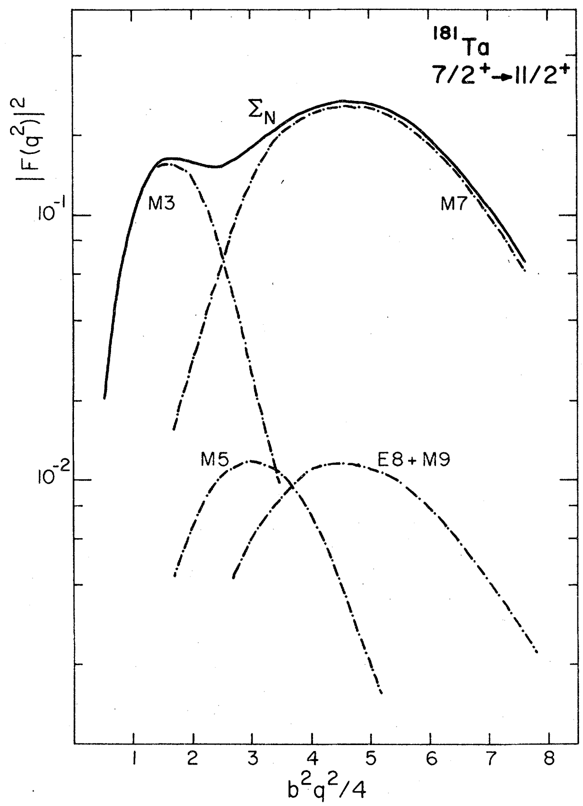

FIG. 2. Transverse form factor squared calculated in the Nilsson model for the $\frac{7}{2}^+ \rightarrow \frac{11}{2}^+$ ground state band transition in ^{181}Ta.

separately. One easily sees that with increasing I' the $\lambda=3$ and $\lambda=5$ multipoles become more important. As an example we have shown in Fig. 2 the predicted $\frac{7}{2}^+ \rightarrow \frac{11}{2}^+$ transverse form factor. In this case there are also nonvanishing E^8 and M^9 multipole contributions that come from the $j=\frac{9}{2}$ component in the Nilsson wave function. However, these contributions are rather small, and, moreover, have a q dependence very similar to that of the M^7 multipole.

As a second illustration we consider ^{25}Mg. Although in this case axial symmetry may be broken (as suggested by structure calculations[12] on ^{24}Mg) and the quantity $1/\langle J_y^2\rangle$ might not be a good expansion parameter, we have chosen this example because recently elastic electron scattering for this nucleus was measured[11] over a wide range of momentum transfer. With $I=K=\frac{5}{2}$ and $a^{K=5/2}_{l=2,\Lambda=2}=1$ the zeroth-order matrix elements are directly expressed in terms of $\langle d_{5/2}\,\|\,T^\lambda(q)\,\|\,d_{5/2}\rangle$. As a simple model to estimate the collective effect we have considered a spherical ^{16}O core with 8 nucleons in the Nilsson orbitals $[Nn_z\Lambda]\Omega^\pi=[220]\frac{1}{2}^+$ and $[211]\frac{3}{2}^+$. If $\Delta N=2$ mixing is neglected there is no collective contribution to the M^5 distribution since the matrix elements of the orbital part of the M^5 operator vanish in this model space. Therefore a q-independent scaling of the M^5 distribution is predicted by a factor 0.55. For the M^3 form factor there is a small first-order contribution generally opposite in sign to the already small zeroth-order contribution (which is 0.12 times the single neutron value). Compared to the zeroth-order M^1 multipole distribution the collective contribution has the opposite sign and the first zero occurs at higher q. (In coordinate space the convection current distribution occurs further inside than the spin magnetization distribution.) The effect of the inclusion of the collective contribution is to shift the zero in the M^1 form factor from $q=0.95$ fm^{-1} to 0.75 fm^{-1}. In Fig. 3 the cross section for elastic magnetic scattering on ^{25}Mg calculated in the present approach is compared to the experimental data of Ref. 11. To this end we have applied corrections for proton finite size and center-of-mass effects. The effective q was taken to be $q_{eff}=q(1+1.2Z\alpha\hbar c/ER_0)$ and the harmonic oscillator range parameter $b=1.83$ fm. Also shown is the result of a full size shell model calculation.[11] Note that there is rather good mutual agreement between both calculations and experiment. However, because of the large uncertainties in the low

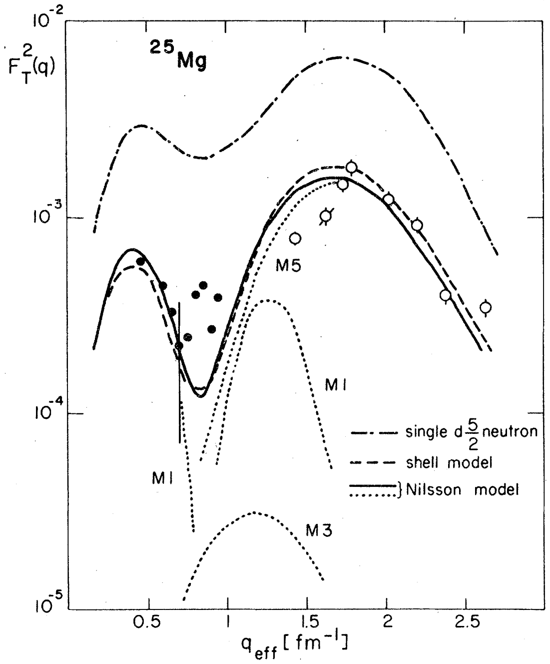

FIG. 3. Transverse form factor for the elastic electron scattering on ^{25}Mg. The dashed-dotted curve is the result for the extreme single-particle model. The full line is the sum of the three multipole distributions calculated in the Nilsson model including the core contribution. The experimental points are taken from Ref. 11. All the low q points (full circles) contain error bars comparable to the one shown in the figure.

q data, it is not possible to make a quantitative comparison between experiment and various models for the collective M^1 form factor.

IV. SUMMARY AND DISCUSSION

We have derived expressions for Coulomb and transverse electron scattering form factors for odd-A nuclei in the framework of the projected Hartree-Fock approach. Through a systematic expansion in terms of the quantity $1/\langle J_\perp^2\rangle$ both single nucleon and collective contributions are taken into account. We have shown that the various transverse multipole distributions for elastic scattering in the strong coupling scheme have a strength very different from the spherical case. The collective contribution to the M^1 form factor has a q dependence different from the zeroth-order odd nucleon form factor. Therefore from an accurate measurement of the transverse form factor in the intermediate q region information might be obtained about the collective convection current distribution.

Finally we want to point out that although in the present approach the charge and current densities are derived in the same framework, a problem arises due to the fact that the projected HF states do not minimize the energy. Since the projected Hartree-Fock states are not eigenstates of the Hamiltonian H the equation of continuity

$$\vec{\nabla}\langle\psi_f|\vec{j}|\psi_i\rangle \equiv -i\langle\psi_f|[H,\rho]|\psi_i\rangle = -i(E_f-E_i)\langle\psi_f|\rho|\psi_i\rangle \tag{4.1}$$

is not exactly satisfied. As a consequence one does not have a guarantee that Siegert's theorem for $q\to 0$,

$$\langle\psi_f\|E^\lambda(q)\|\psi_i\rangle \approx \frac{E_f-E_i}{q}\left(\frac{\lambda+1}{\lambda}\right)^{1/2} \times\langle\psi_f\|C^\lambda(q)\|\psi_i\rangle, \tag{4.2}$$

will be satisfied in this approach. Numerical computations must be performed to find out whether conditions (4.1) and (4.2) are approximately fulfilled or badly violated. To obtain a more rigorous determination of the collective contribution to the current density and to satisfy in a better approximation the continuity equation the variation after projection Hartree-Fock approximation should be used.

ACKNOWLEDGMENTS

We would like to thank Professor Villars and Professor Bertozzi for stimulating discussions. One of us (A.E.L.D.) would like to thank Dr. A. M. Bernstein for the hospitality offered to him at the Laboratory for Nuclear Science during the 1977 MIT Summer Study.

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*This work is supported in part through funds provided by the U. S. Department of Energy (DOE) under Contract No. EY-76-C-02-3069.

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