Search for Efimov states in ^{12}C

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When ^{12}C is treated as a bound system of three α particles interacting through the Ali-Bodmer α-α force, as modified by Vallieres *et al.*, we find using the Faddeev technique no 0^+ excited state below the two-body threshold. When the α-α force is made more attractive, a 0^+ excited state eventually appears. It is not an Efimov state.

[NUCLEAR STRUCTURE ^{12}C, examining 0^+ excited state for Efimov characteristics through α-α force.]

When three boson systems are investigated for the presence of Efimov states,[1,2] a likely candidate in the nuclear domain is ^{12}C. This nucleus can be treated as a bound system of three α particles which may harbor an excited 0^+ bound state with Efimov-state characteristics. Indeed, it is well known that there exists an excited 0^+ state in ^{12}C near the two-body threshold. Hence, the question arises: Is it an Efimov state? In this paper we report the results of a study using the Faddeev method which bears on this question.

The potential chosen for this analysis is

$$V = V_{\text{nuclear}} + V_{\text{Coulomb}}, \tag{1}$$

where

$$V_{\text{Coulomb}} = A(e^{-\mu r}/r) \quad (\text{with } \mu \ll 1) \tag{2}$$

and

$$V_{\text{nuclear}} = A_1 \exp[-(\alpha_1(r)^2] + A_2 \exp[-(\alpha_2 r)^2] \tag{3}$$

with

$$A = 4 \times 1.4397 \text{ MeV fm} = 4e^2,$$

$$\mu = 0.01,$$

$$A_1 = 360 \text{ MeV}, \quad \alpha_1 = 0.7 \text{ fm}^{-1},$$

$$A_2 = -130 \text{ MeV}, \quad \alpha_2 = 0.475 \text{ fm}^{-1}.$$

This potential with the given parameters is the realistic α-α potential used recently by Vallieres, Coelho, and Das[8,9] in their work on ^{12}C. It should be noted, though, that this potential does support a bound state at -1.75 MeV whereas ^8Be is not stable.

The Coulomb interaction has been modified by a shielding factor with a screening parameter μ. Since we use the Faddeev method in momentum space, this modification is chosen to avoid the difficulties associated with the momentum transform of

$$V = \frac{4e^2}{r}. \tag{4}$$

By allowing μ to be very small, Eq. (2) becomes an acceptable approximation to (4).

The momentum representation of the above potential is given by[1,6,7]

$$v_{ll'}(p,k) = (-1)^{l'-l} \frac{2}{\pi} \int_0^\infty j_l(pr) V(r) j_{l'}(kr) r^2 dr \tag{5}$$

For $l = l' = 0$

$$v_{00}(p,k)_{\text{Coul}} = \frac{A}{2\pi pk} \ln\left(\frac{1+\rho}{1-\rho}\right), \tag{6}$$

where

$$\rho = \frac{2pk}{p^2 + k^2 + \mu^2} \tag{7}$$

and

$$\begin{aligned} v_{00}(p,k)_{\text{nucl}} = &\frac{A_1}{\sqrt{\pi}\, pk\alpha_1} \exp(-p^2 + k^2/4\alpha_1^2) \sinh\left(\frac{pk}{2\alpha_1^2}\right) \\ &+ \frac{A_2}{\sqrt{\pi}\, pk\alpha_2} \exp(-p^2 + k^2/4\alpha_2^2) \\ &\times \sinh\left(\frac{pk}{2\alpha_2^2}\right). \end{aligned} \tag{8}$$

The numerical analysis of the 0^+ bound states of ^{12}C using this potential follows the Faddeev-unitary pole expansion (UPE) method of Lim, Duffy, and Damert.[1,4,5] For the repulsive core strength, $A_1 = 360$ MeV, we found only one 0^+ bound state (-5.8 MeV) for the 3-α system below the two-body threshold (see Table I). This is a bit underbound as compared with the Vallières hyperspherical calculation (-6.6 MeV) and with the experimental value (-7.27 MeV). Our figure would be improved by adding more terms to the UPE approximation as well as $l \neq 0$ states. However, the essential features of Efimov states can be determined by an s-wave analysis[3] and since that is the purpose of this paper, we used the UPA (one-term UPE).

By lowering A_1 to 300 MeV we were also able

TABLE I. Two- and three-body binding energies corresponding to various chosen potential core strengths.

Potential core strength A_1 (MeV)	Two-body binding energy (MeV)	Three-body binding energy Ground (MeV)	Excited
200	−7.5	−26.5	−10.0 MeV
250	−5.5	−18	−5.85 MeV
300	−3.2	−10.5	−3.4 MeV
330	−2.45	−7.9	Above two-body threshold
350	−1.95	−6.4	Above two-body threshold
360	−1.75	−5.8	Above two-body threshold
380	−1.35	−4.7	Above two-body threshold
400	−1.0	−3.7	Above two-body threshold
420	−0.7	−2.85	Above two-body threshold
460	−0.25	−1.5	Above two-body threshold
480	−0.1	−1.0	Above two-body threshold
490	−0.02	−0.75	Above two-body threshold

to produce a 3-α 0^+ excited state. Here the 2-α system is bound by 3.2 MeV—quite a distance from the resonance region. In light of this it is unlikely that the 3-α excited state would be an Efimov state.

The Efimov characteristics of this state can be tested by deepening the potential well or, equivalently, by reducing the repulsive core height. When this is done a true Efimov state will shortly disappear into the continuum, whereas a normal excited state will continue to increase in binding energy. Indeed, for a two-body system near resonance, the number of Efimov states in the corresponding three-body system can be approximated by[2]

$$N = \frac{1}{\pi} \ln \left| \frac{a}{r_0} \right| , \tag{9}$$

where a is the scattering length and r_0 is the effective range of the α-α system.

If the two-body system is bound, deepening the potential well or reducing the core height decreases a which, then, decreases N. Eventually, when N becomes much less than 1, all Efimov states will disappear whereas a normal excited state will remain.

Thus, we tested the behavior of the 0^+ excited state by further reducing the height of the core. Our results (Fig. 1) show that this state is a normal excited state. Indeed, that the two-body bound state is not very close to the continuum (at $A_1 = 300$ MeV) suggests that these results are as expected.

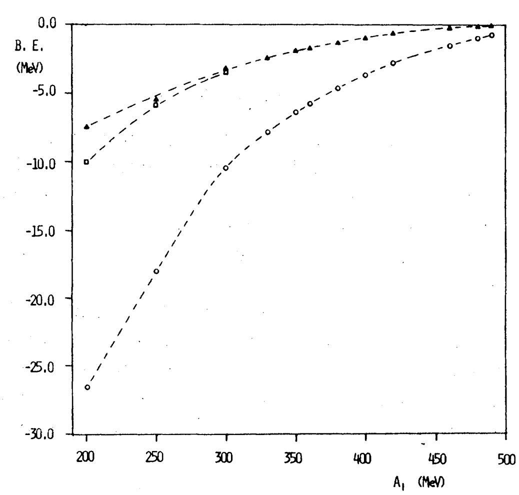

FIG. 1. Plot of binding energy vs repulsive strength A for the three-body ground (○) and 0^+ excited (□) states and the two-body threshold (△).

TABLE II. Two-body scattering length, effective range, and estimate of Efimov states for given potential core strengths.

Potential core strength A_1 (MeV)	a (Å)	r_0 (Å)	N
300	4.2	2.06	0.227
360	4.9	2.44	0.222
420	6.0	2.77	0.246

We also tested Eq. (9). We calculated N by first finding the scattering length, a, from the zero-energy radial wave function.[10] The value of r_0 follows from

$$\frac{1}{\gamma}=\frac{1}{a}+\frac{1}{2}\frac{r_0}{\gamma^2}, \qquad (10)$$

where $\gamma=\hbar/(2MB)^{1/2}$ and B is the binding energy and M is the reduced mass. As can be seen from Table II the computed values of N agree with our direct confirmation that the 0^+ excited state at $A_1=300$ MeV is not of the Efimov type.

One might expect the possibility of generating Efimov states by increasing A_1, causing the two-body binding energy to approach zero. However, with the potential considered in this paper, the Coulomb repulsion at large r far outweighs the nuclear attraction,[2] and overwhelms the long range attractive effect in the three-body system which manifests the Efimov states. We did increase A_1, determined the corresponding two- and three-body binding energies (Fig. 1) and, indeed, found no Efimov state.

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