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Quantitative resolution of ^{234}Th anomaly

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An earlier penetrability calculation through a triple-humped barrier for ^{234}Th nucleus by Sharma and Leboeuf failed to reproduce the right order of magnitude for penetrability. It is shown here that one can obtain the correct penetrability by assuming that Back *et al.* actually determined the parameters of the second saddle, third minimum, and the third saddle for ^{234}Th from an analysis of their (t,pf) fission data. The height of the first barrier is assumed to be that given by microscopic calculations for this nucleus.

[NUCLEAR REACTIONS, FISSION Fission penetrability calculated through triple-humped potential barrier in ^{234}Th, quantitative resolution of thorium anomaly along the lines suggested by Möller and Nix.]

For thorium isotopes, the calculated[1] first saddle and second minimum of the double-humped potential barrier are about 3 MeV lower than the experimental values[2,3] commonly attributed to them. This discrepancy constitutes the well-known "thorium anomaly" in the fission literature. However, Möller and Nix[4] have suggested a possible resolution of this anomaly in terms of a third asymmetric minimum in the fission barriers for thorium isotopes. Following this suggestion Bhandari[5] has recently calculated the penetrability for a triple-humped potential barrier in the WKB approximation and has suggested plausible potential shapes for thorium isotopes qualitatively consistent with their known subbarrier fission characteristics. A similar exact calculation of penetrability in terms of parabolic cylinder functions has been reported more recently by Sharma and Leboeuf,[6] who have also compared the fission penetrabilities for ^{234}Th calculated in double-hump and triple-hump models with those observed.[7] The authors conclude that although the penetrability calculated in the triple-humped model does help in reproducing the observed subbarrier fission resonance structure, its magnitude is considerably lower than that calculated in the double-humped model. The purpose of this communication is to point out that the low values of the penetrability obtained by Sharma and Leboeuf[6] were due to their choice of barrier parameters which are not consistent with the thorium anomaly mentioned above. We have used barrier shapes similar to those suggested by Bhandari[5] earlier and find a satisfactory explanation of the subbarrier fission characteristics of ^{234}Th.

The penetrability through a triple-humped barrier has been calculated in the WKB approximation and also exactly in terms of parabolic cylinder functions. The potential barrier has been parameterized by smoothly joining five parabolas and is given by

$$V(\epsilon) = E_i \pm \tfrac{1}{2}\mu w_i^2(\epsilon - \epsilon_i)^2,$$

where the $+ve$ sign applies for $i=2$ and 4, and the $-ve$ sign applies for $i=1$, 3, and 5. E_i represents the maxima and minima of the potential, $\hbar w_i$ their respective curvature parameters, and E_i the locations of extrema on the deformation axis. $V(\epsilon)$ is taken to be zero at $\epsilon = 0$. μ is the inertial mass parameter assumed to be constant for all values of ϵ and has the dimensions of moment of inertia; as ϵ, the distortion parameter is dimensionless. The value of μ used in the calculation is

$$\mu = 0.054A^{5/3}\hbar^2 \text{ MeV}^{-1}.$$

The details of the penetrability calculation in the WKB approximation have been reported earlier[5] and the exact calculation in terms of the parabolic cylinder functions is similar to that reported by Cramer and Nix[8] for a double-humped barrier and further extended by Sharma and Leboeuf[6] for a triple-humped barrier.

The experimental data on fission probability for ^{232}Th (t,pf) ^{234}Th of Back *et al.*[7] are shown in Fig. 1. The fission penetrabilities through a double-humped barrier whose parameters have been determined by a statistical model analysis of the experimental data by Back *et al.*[7] are also shown in Fig. 1, along with the fission penetrabilities through a triple-humped barrier whose parameters have been proposed by Sharma and Leboeuf.[6] As can be seen, the theoretical penetrabilities through the proposed triple-humped barrier of Sharma and Leboeuf are considerably lower than the observed ones. This discrepancy has been attributed to the

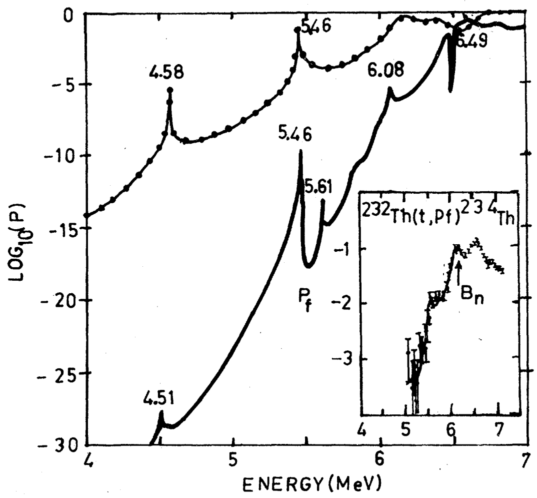

FIG. 1. A logarithmic plot of fission penetrability versus energy for the double-humped barrier of Back *et al.*[7] (shown by dotted lines; barrier parameters are $E_1 = 6.15$ MeV, $E_2 = 3.10$ MeV, $E_3 = 6.50$ MeV, $\hbar w_1 = 1.0$ MeV, $\hbar w_2 = 1.0$ MeV, and $\hbar w_3 = 0.75$ MeV) and the triple-humped barrier proposed by Sharma and Leboeuf (Ref. 6) (shown by solid curve; barrier parameters are $E_1 = 6.15$ MeV, $E_2 = 3.10$ MeV, $E_3 = 6.50$ MeV, $E_4 = 5.5$ MeV, $E_5 = 5.8$ MeV, $\hbar w_1 = 1.0$ Mev, $\hbar w_2 = 1.0$ MeV, $\hbar w_3 = 0.33$ MeV, $\hbar w_4 = 0.23$ MeV and $\hbar w_5 = 0.31$ MeV). The experimental data on fission probability P_f for ^{234}Th shown at bottom right is taken from Back *et al.* (Ref. 7) Figure after Sharma and Leboeuf (Ref. 6). B_n indicates the neutron binding energy.

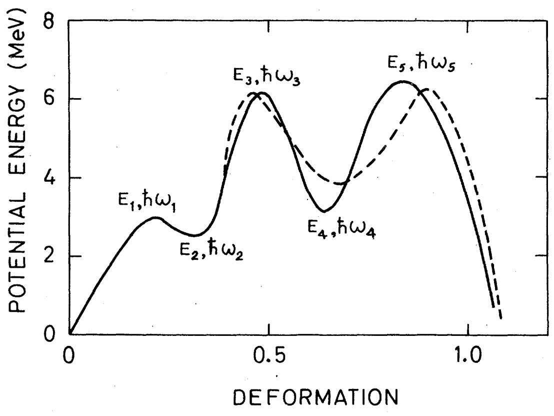

FIG. 2. Triple-humped barriers used for the calculation of fission penetrability in ^{234}Th. The solid curve corresponds to the following parameters: $E_1 = 3.0$ MeV, $E_2 = 2.5$ MeV, $E_3 = 6.15$ MeV, $E_4 = 3.1$ Mev, $E_5 = 6.5$ MeV, $\hbar w_1 = 0.5$ MeV, $\hbar w_2 = 1.2$ MeV, $\hbar w_3 = 1.0$ MeV, $\hbar w_4 = 1.0$ MeV, $\hbar w_5 = 0.75$ MeV, while the dashed line corresponds to $E_1 = 3.0$ MeV, $E_2 = 2.5$ MeV, $E_3 = 6.15$ MeV, $E_4 = 3.85$ MeV, $E_5 = 6.25$ MeV, $\hbar w_1 = 0.5$ MeV, $\hbar w_2 = 1.2$ MeV, $\hbar w_3 = 1.2$ MeV, $\hbar w_4 = 0.5$ MeV, $\hbar w_5 = 1.0$ MeV.

larger overall thickness of the triple-humped barrier in comparison with the double-humped barrier. It is important to note that any set of triple-humped barrier parameters proposed for ^{234}Th should be able to reproduce roughly the same order of magnitude for penetrabilities as the double-humped barrier of Back *et al.*; since otherwise the fission probability fit will be completely thrown off. Thus the comparison should now be made between the penetrabilities of the dotted line in Fig. 1 and those through any new triple-humped barrier that will be proposed for ^{234}Th. Following the suggestion made by Moller and Nix[4] for the possible resolution of the thorium anomaly, we have assumed that Back *et al.*[7] have actually determined the parameters of the second saddle, third minimum, and third saddle for the nucleus ^{234}Th from an analysis of their (t,pf) fission data. We have accordingly taken their parameters for that part of the triple-humped barrier (solid line in Fig. 2) and have fixed the first saddle approximately 3 MeV lower than the second saddle on the assumption that the theoretically calculated first saddle height is more or less correct.[4] Since the first saddle is very low and the second well rather shallow, the subbarrier resonance phenomena will be largely due to the vibrational states in the third well only. Even if the second well were much deeper, the features observed by Back *et al.*[7] near the fission threshold would remain unaffected. For energies $E > E_4$, the effective potential is only a double-humped barrier and the analysis done by Back *et al.*[7] then provides a satisfactory explanation for their observed subbarrier fission characteristics of ^{234}Th. The results of the penetrability calculation using WKB as well as exact methods for the barrier shown by the solid line in Fig. 2 are shown in Fig. 3.

By a suitable variation of the barrier parameters one can also increase the number of resonant states in the third minimum. This is shown in Fig. 2 (dashed line) where we have changed the value of third well curvature from 1.0 to 0.5 MeV and have also correspondingly altered other parameters to obtain a penetrability of roughly the same order of magnitude as in Fig. 3. The results of the penetrability calculation for such a barrier are shown in Fig. 4. A value of 0.5 MeV for second well (third well here) curvature is consistent with the recent evidence[9] for actinide nuclei. The triple-humped barrier penetrabilities as shown in Figs. 3 and 4 are of the same order of magnitude as the penetrability calculated by Sharma and Leboeuf[6] in their double-humped model using parameters which have already been used successfully by Back *et al.*[7] in analyzing their subbarrier fission data for ^{234}Th. A higher energy for the first saddle as assumed by Sharma and Leboeuf[6] will naturally result in very low penetrabilities; also such a poten-

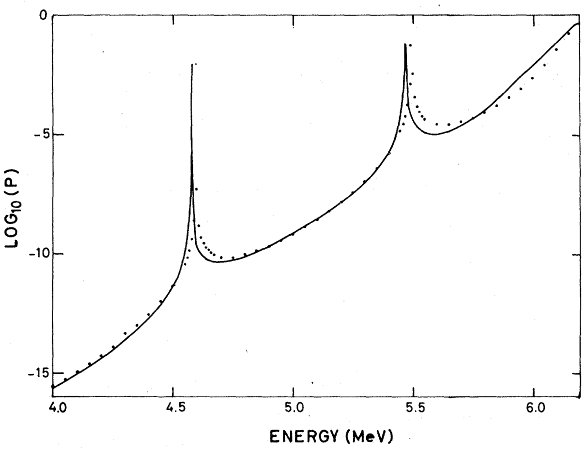

FIG. 3. A logarithmic plot of the fission penetrability in ^{234}Th calculated for the triple-humped barrier shown by the solid line in Fig. 2. The solid curve shows the results of the exact calculations and the dotted curve that of the WKB calculations. The penetrability shown in this figure should be compared with the dotted curve in Fig. 1.

tial shape will not be consistent with the known thorium anomaly as mentioned earlier.

The apparent differences between the WKB and the exact calculation are due to the fact that we have for simplicity neglected the effect of the first saddle and second well on the incident wave in the WKB calculation. We have thus considered a free incident wave for all energies $E > E_1$ in the WKB calculation, but have included the effects of the first saddle and second minimum on the incident wave in the exact calculation. Consequently, the exact penetrability is slightly lower than that calculated in the WKB approximation at low values of the incident energy. The second well of the triple-humped barrier shown by solid line in Fig. 2 is more anharmonic than that shown by dashed line in the same figure. The spacing between energy levels near the top of the well gets reduced due to such anharmonic effects. Consequently, the energy shift at the resonances between the WKB and the exact results is also enhanced, as seen in Fig. 3.

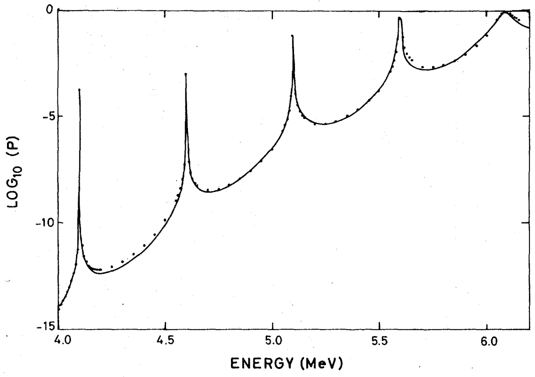

FIG. 4. A logarithmic plot of the fission penetrability calculated for the triple-humped barrier shown by the dashed line in Fig. 2. The solid curve shows the results of the exact calculations and the dotted curve that of the WKB calculations. The penetrability shown in this figure should be compared with the dotted curve in Fig. 1.

In conclusion, we can state that the barrier shapes of a triple-humped potential used in the present work are consistent with the known thorium anomaly as well as other recent evidences[10-12] in favor of a third well at a deformation much larger than that of the second well. A calculation of the penetrability through such triple-humped barriers results in a satisfactory explanation of the subbarrier fission characteristics of ^{234}Th and provides a quantitative resolution of the thorium anomaly along the lines suggested by Möller and Nix[4].

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