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Recipe for the treatment of isospin mixing in compound nucleus reactions

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Previous studies of isospin mixing in reactions have been restricted to the case of overlapping levels, and have produced a variety of cross-section formula. We point out that, despite some differences, these formulas are in essential agreement; further that, by comparison with results in the case of isolated levels, one can naturally establish a general formula that is expected to be valid for all situations.

[NUCLEAR REACTIONS Isospin mixing in nuclear reactions.]

The subject of isospin mixing effects in compound nucleus reactions has been discussed in at least six papers,[1-6] all specialized to the case of overlapping resonances (for each isospin involved). The reason for this specialization was probably that the relevant data had this feature. For example, some papers concentrate on explaining the departure from unity of the ratio $\sigma(p, p')\sigma(\alpha, \alpha')/\sigma(p, \alpha')\sigma(\alpha, p')$, where all four processes involve the same compound nucleus at about 20 MeV excitation; for the targets used (masses 50 to 120), this excitation implies overlapping resonances for both isospin sequences.

Isospin mixing effects are certainly not limited in practice to the case of overlapping compound levels. In fact, recently[7-9] the subject has been raised in connection with (p, γ) reactions for proton energies $\lesssim 5$ MeV and target masses 20 to 60, where compound levels are usually nonoverlapping (for given spin, isospin, and parity). This situation (of discrete levels) is one for which the compound nucleus cross section can be evaluated. It turns out, not surprisingly, that the result differs from what one gets if one blindly applies the formulas derived in the case of overlapping levels. However, there are striking similarities in the structure of the formulas which encourage one to search for a bridging formula to cover all cases. This is the main subject of the present work. An incidental subject is to compare the previous papers with each other. It turns out that despite initial indications to the contrary, all papers give essentially the same result.

We begin by surveying previous formulas for overlapping levels; afterwards the case of discrete levels will be treated, and comparison made with the overlapping case to establish a general formula.

We will restrict the discussion to the case of two isospins, the smaller isospin $T_<$ for the dense background states and the larger one $T_>$ for the analog states. The original paper[1] gave the following form of cross section $\sigma_{\alpha\beta}$ between channels α, β:

$$\sigma_{\alpha\beta} = \frac{(\tau_{\alpha 1} + \mu\tau_{\alpha 2})(\tau_{\beta 1} + \mu\tau_{\beta 2})}{\tau_1 + \mu\tau_2} + (1-\mu)\frac{\tau_{\alpha 2}\tau_{\beta 2}}{\tau_2}, \tag{1}$$

where

$$\tau_1 \equiv \sum_\alpha \tau_{\alpha 1}, \quad \tau_2 \equiv \sum_\alpha \tau_{\alpha 2}.$$

Here we follow convention and suppress the spin-statistical and kinematic factors in the cross section. $\tau_{\alpha 1}$ is the transmission function for channel α and the levels of unmixed isospin $T_<$ while $\tau_{\alpha 2}$ is the same for the $T_>$ levels. The assumption of overlapping levels implies that the total transmissions τ_1, τ_2 are both $\gg 1$. μ is a parameter reflecting isospin mixing, and will be discussed below.

The most recent paper[5] gives a form which at first sight is completely different, viz.,

$$\sigma_{\alpha\beta} = \frac{\tau_{\alpha1}\tau_{\beta1}}{\tau_1}\left(1 - \frac{\tau_2}{\tau_1}\nu\right) + (\tau_{\alpha1}\tau_{\beta2} + \tau_{\alpha2}\tau_{\beta1})\frac{\nu}{\tau_1} + \frac{\tau_{\alpha2}\tau_{\beta2}}{\tau_2}(1-\nu). \quad (2)$$

However, it turns out, upon algebraic manipulation, that *forms (1) and (2) are identical* provided that the parameters ν and μ are related thus:

$$\frac{1}{\nu} = \frac{1}{\mu} + \frac{\tau_2}{\tau_1}. \quad (3)$$

Other papers[2-4,6] yield a formula which again appears very different, viz.,

$$\sigma_{\alpha\beta} = \frac{T_{\alpha1}T_{\beta1}}{T_1} + \frac{T_{\alpha2}T_{\beta2}}{T_2}, \quad (4)$$

where $T_{\alpha1}$ is defined as

$$T_{\alpha1} \equiv (1-\chi p_2)\tau_{\alpha1} + p_1\chi\tau_{\alpha2}, \quad (5)$$

and similarly for $T_{\alpha2}$ on reversing 1 and 2. T_1 is $\sum_\alpha T_{\alpha1}$ and p_1 is $\rho_1(\rho_1+\rho_2)^{-1}$, where ρ_1 and ρ_2 are level densities of levels 1 and 2. χ is a mixing parameter. Again one finds that form (4) is equivalent to (2) [or (1)] if the mixing parameters χ and ν are related[10] in a certain way, viz.,

$$[\tau_2\tau_1 + \chi(p_2\tau_1 - p_1\tau_2)(\tau_1 - \tau_2) - \chi^2(p_2\tau_1 - p_1\tau_2)^2]\nu = [(p_2\tau_1 + p_1\tau_2) - (p_2^2\tau_1 + p_1^2\tau_2)\chi]\chi\tau_1. \quad (6)$$

Thus *all proposed forms (1), (2) and (4) of* $\sigma_{\alpha\beta}$ *are equivalent if the mixing parameters* μ, ν, *and* χ *are related appropriately*, i.e., by (3) and (6). This is not surprising when one notes that form (2) follows generally from the assumptions that $\sigma_{\alpha\beta}$ $(=\sigma_{\beta\alpha})$ is quadratic in transmissions of channels α and β and that $\sum_\beta \sigma_{\alpha\beta}$ equals $(\tau_{\alpha1}+\tau_{\alpha2})$. The differences between the three classes of theory lie not in the forms of $\sigma_{\alpha\beta}$ as a function of transmissions and mixing parameter, but in the identity of the mixing parameter. We now discuss this.

First we note that the general condition that $\sigma_{\alpha\beta}$ is symmetrical in 1 and 2 imposes a condition on the mixing parameters, viz. χ, $\tau_2\nu$ and $(\tau_1\mu^{-1}+\tau_2)$ are symmetric. This condition is satisfied by the forms we now present. The forms given in the cited references are

$$\mu = \Gamma_2^\downarrow(\Gamma_2^\uparrow + \Gamma_2^\downarrow)^{-1}, \quad (7)$$

$$\nu = [(\Gamma_2^\uparrow/\Gamma_2^\downarrow) + (\tau_2/\tau_1) + 1]^{-1}, \quad (8)$$

$$\chi = (\Gamma_2^\downarrow + \Gamma_1^\downarrow)[\Gamma_2^\downarrow + \Gamma_1^\downarrow + \Gamma_2^\uparrow + \Gamma_1^\uparrow]^{-1}, \quad (9)$$

where the new quantities are $(m = 1, 2)$

$$\Gamma_m^\uparrow \equiv \tau_m(2\pi\rho_m)^{-1}, \qquad \Gamma_m^\downarrow \equiv 2\pi|H'_{12}|^2\rho_1\rho_2/\rho_m, \quad (10)$$

and $|H'_{12}|^2$ is the mean square coupling matrix element between levels of the two types.

When we check forms (7) and (8) against relation (3) we find that (3) is satisfied, thereby showing that the first two approaches[1,5] are identical. When checking (8) and (9) against (6), we find that (6) is not satisfied [except in the limit $\rho_2 \ll \rho_1$ or in the limit $(\Gamma_2^\downarrow + \Gamma_1^\downarrow) \ll (\Gamma_2^\uparrow + \Gamma_1^\uparrow)$].

On comparing the three kinds of derivation leading to formulas (1), (2), and (4), we find that all are to some extent intuitive except for the derivation[5] leading to (2) with (8). This is based on the exact solution of a certain statistical model whose input assumptions are consistent with the other approaches. Therefore we can assume this to be correct, and can use it to judge the other approaches. Thus we conclude that (1) with (7) is correct, but that (4) with (9) is not. The use of (4) is correct only if χ of (9) is replaced by χ taken from (6), with ν from (8).

Finally, on this subject of three kinds of approach, a comment should be made on the exact model solution.[5] Upon detailed inspection, one finds that certain quantities differ from their counterparts in other papers, viz., $\Gamma_2^\downarrow$ contains an extra term not present in (10), and the transmissions contain mixing effects. These differences arise from external (Robson) mixing. This is not apparent at first sight, because certain simplifying assumptions have been made which obscure this connection. When these assumptions are relaxed,[11] it can be readily shown that the additional term in $\Gamma_2^\downarrow$ is precisely the familiar Robson contribution, and also that the transmissions are those corresponding to Robson external mixing (with no internal mixing).

Now we turn to the situation of isolated levels. In order to guarantee that all levels are isolated, one requires $\tau_1 < 1$, $\tau_2 < 1$, and $\Gamma_2^\downarrow < \Gamma_2^\uparrow$. In this situation, we have

$$\sigma_{\alpha\beta} = \left\langle \frac{\tau_\alpha\tau_\beta}{\tau} \right\rangle, \quad (11)$$

where $\tau \equiv \sum_\alpha \tau_\alpha$ and the brackets denote energy averaging over the fluctuations arising from the energy dependence of the τ's,

$$\tau_\alpha = \tau_{\alpha1} + \sum_A \frac{\Gamma_A^\downarrow \Gamma_{A\alpha}^\uparrow}{(E_A - E)^2 + \frac{1}{4}\Gamma_A^{\downarrow 2}}, \quad (12)$$

where $\Gamma_A^\downarrow$ and $\Gamma_A^\uparrow$ have mean values $\Gamma_2^\downarrow$ and $\tau_2/2\pi\rho_2$, respectively. If we assume that analogs A are uniformly spaced with uniform values of $\Gamma_A^\downarrow$ and $\Gamma_{A\alpha}^\uparrow$, one can evaluate $\sigma_{\alpha\beta}$ in (11). The remarkable thing is *that the resulting form of* $\sigma_{\alpha\beta}$ *is exactly that of (2)*, but with

$$\nu = \left[\left(1 + \frac{\tau_2}{\tau_1}\right)^2 + \frac{4\tau_2}{\tau_1}(e^{\pi\Gamma_2^\downarrow/D_2} - 1)^{-1}\right]^{-1/2}. \quad (13)$$

Of course, the assumption of uniform values is unnecessary when $\Gamma_2^{\downarrow} < D_2$. The fact that both extremes (large natural widths $\tau_1, \tau_2 \gg 1$, small natural widths $\tau_1, \tau_2 \ll 1$) give the same structure of $\sigma_{\alpha\beta}$ is a strong indication that (2) is generally valid. The only remaining question to settle is the functional form of ν, which can depend on τ_1, τ_2, ρ_1, ρ_2, and $|H_{12}'|^2$ [or on any other choice of five parameters equivalent to these through (10)]. When we compare forms (8) and (13) for ν, the natural choice of a bridging formula is

$$\nu = \left[\left(1 + \frac{\tau_2}{\tau_1} + \frac{\Gamma_2^{\uparrow}}{\Gamma_2^{\downarrow}} \right)^2 + \frac{4\tau_2}{\tau_1} (e^{\pi\Gamma_2^{\downarrow}/D_2} - 1)^{-1} \right]^{-1/2} . \quad (14)$$

Note that ν lies between the limits 0 and $(1 + \tau_2/\tau_1)^{-1}$, achieved in the small and large mixing limits ($\Gamma_2^{\downarrow} \to 0, \infty$), respectively. Since there are no known systematic methods for evaluating compound cross sections in general situations, the only way to check (14) is by computer experiments of the type done by Moldauer[12,13] in other contexts where systematic methods do not exist. The program for the present problem is quite clear: one makes the computer construct an R matrix containing two parts, R_1 and R_2. The R-matrix amplitudes $\gamma_{\lambda\alpha}$ are chosen from Gaussian distributions of zero mean and given mean square value. Now one specifies the mean square Coulomb matrix element $H_{12}'^2$ between the two kinds of level, again using a Gaussian distribution with zero mean. Diagonalization gives a new set of R-matrix levels, whence the computer can calculate $\sigma_{\alpha\beta}$. Thereby one can check numerically whether $\sigma_{\alpha\beta}$ really does have the form (2) with ν given by (14).

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