# Application of quasi-two-body scaling to $p + A \rightarrow p + X$

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Previously unpublished data on $p + A \rightarrow p + X$, by Cochran *et al.*, covering a wide range of angles and atomic numbers, are analyzed to probe the validity of the single scattering model and "quasi-two-body scaling."

[NUCLEAR REACTIONS Inclusive cross section, structure function, high momentum components, scaling.]

We have recently obtained hitherto unpublished data[1] on the interaction of 730 MeV protons ($p+A \rightarrow p+X$) with a large range of nuclei and over a large range of angles which are ideally suited to studying the validity of the single scattering model[2] and of "quasi-two-body scaling"[3] (QTBS).

In this model the inclusive cross sections per nucleon are related to the differential cross sections for $p$-$p$ scattering on free stationary protons and to the structure function $G(k)$, which is a functional of the ground state nuclear wave function,[4] by

$$d\sigma/d^3q = C(p, k_{\min})G(k_{\min})/|\vec{p}-\vec{q}|, \qquad (1)$$

with

$$k_{\min} = |\vec{p}-\vec{q}| - |\vec{p}'|. \qquad (2)$$

Here $\vec{p}$ is the momentum of the incident proton, $\vec{p}'$ is its final momentum, and $\vec{q}$ is the momentum of the detected proton. $k_{\min}$ is the minimum momentum of the recoiling $(A-1)$ nucleus that allows production of a proton of momentum $\vec{q}$. $d\sigma/dq^3$ is the inclusive cross section, $d^3\sigma/q^2dqd\Omega_q$. In the region of energies covered by the Cochran *et al.* data, elastic $p$-$p$ scattering is expected to play a dominant role. In addition, inelastic scattering would require higher values of $k_{\min}$ for the same observed $q$. Thus we employ the value of $C(p, k_{\min})$ obtained from elastic scattering as the best approximation.[3]

$$C(p, k_{\min}) = [s(s-4m_p^2)/32\pi^2 p m_p E_q] \times [d\sigma(\vec{k}_{\min} \rightarrow \vec{q})/dt]. \qquad (3)$$

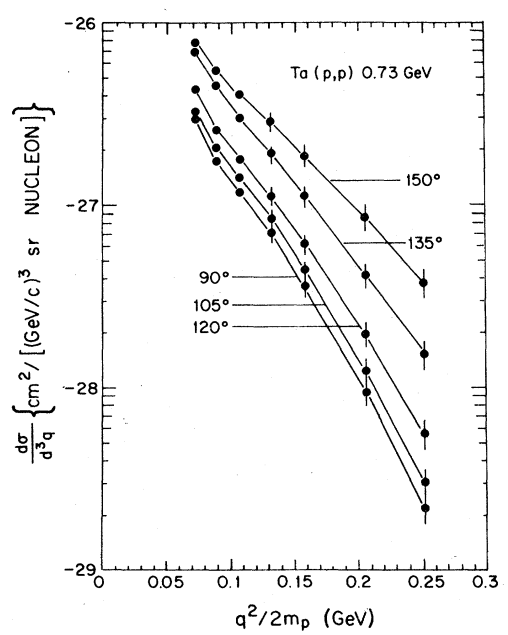


FIG. 1. $d\sigma/d^3q$ per nucleon vs $q^2/2pm$; $p$ + Ta $\rightarrow p + X$; 730 MeV incident protons.

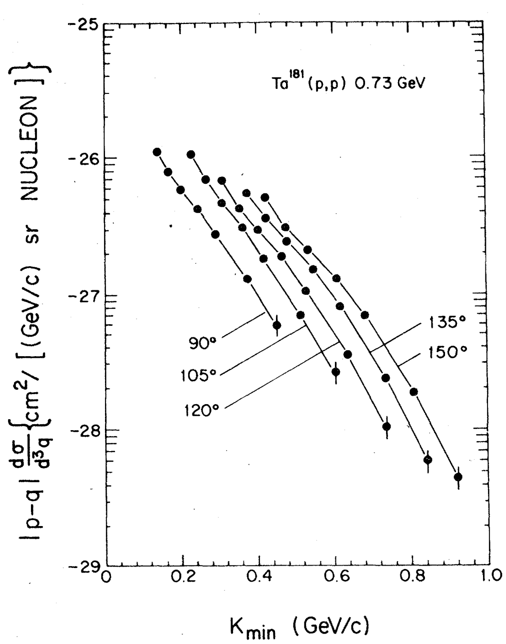


FIG. 2. $|p-q|$ $d\sigma/d^3q$ per nucleon vs $k_{\min}$; $p$ + Ta $\rightarrow p + X$.

 

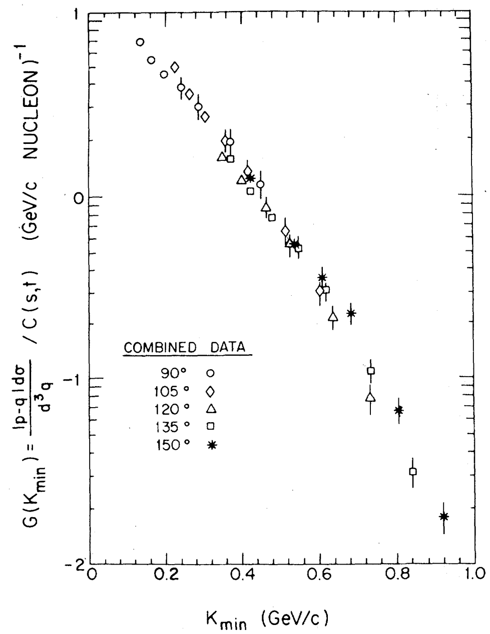


FIG. 3 $G(k) = |\vec{p}-\vec{q}|\, d\sigma/d^3q/C(p,k_{min})$ per nucleon vs $k_{min}$; $P + \mathrm{Ta} \to p + X$.

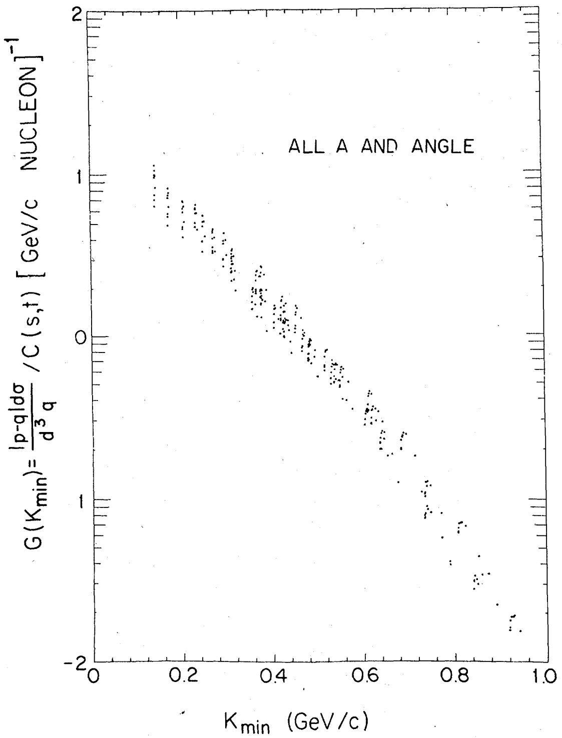


FIG. 5. $G(k) = |\vec{p}-\vec{q}|\, d\sigma/d^3q/C(p,k_{min})$ per nucleon vs $k_{min}$; same targets and angles as in Fig. 4. $G(k)$ falls rapidly but is not represented by a single exponential.

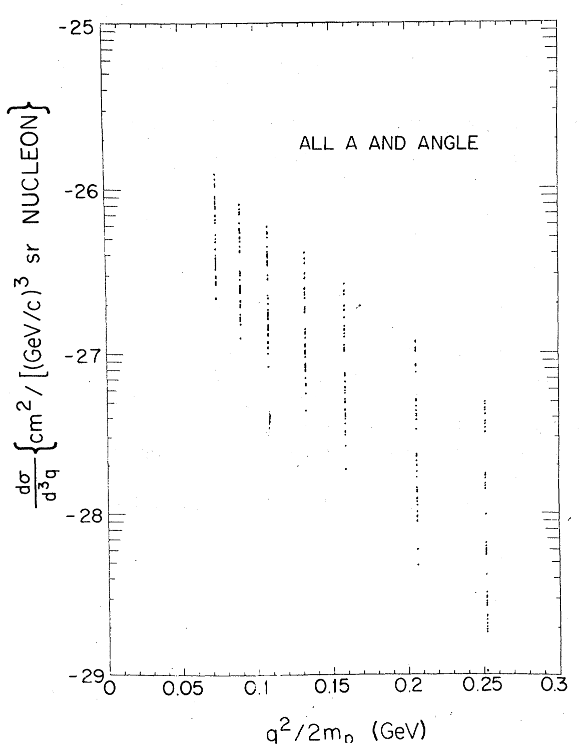


FIG. 4. $d\sigma/d^3q$ per nucleon vs $q^2/2pm$; $\theta = 90° \cdots 150°$; Be, C, Al, Ti, Cu, Ag, Ta, Pb, and Th. All experimental points are plotted to show the values of the cross section from the lowest to the highest momenta.

To use Eq. (3) we have employed interpolations of the data on $p$-$p$ scattering,[5] since not all values of $d\sigma/dt$ at the appropriate values of $s(k)$ and $t(k)$ have been measured. $s$ and $t$ are the Mandelstam variables for a bound nucleon, where the (momentum, energy) four-vector is $[k, m_A - (k^2 + m_{A-1}{}^2)^{1/2}]$.

A basic assumption of the model is that $d\sigma/dt$ varies slowly with $k_{min}$ as compared with $G(k_{min})$. This condition is satisfied for angles from 90° to 150° (lab) but not for the forward angles for which the factorization of Eq. (1) does not apply.

Figure 1 shows the differential cross section nucleon vs $q^2/2m_p$ for $^{181}$Ta at the laboratory angles 90°, 105°, 120°, 135°, and 150°. These cross sections show large angular variations, falling off more rapidly at the largest backward angle. Figure 2 shows the same data, but this time $|\vec{p} - \vec{q}|\, d\sigma/d^3q$/nucleon is plotted vs $k_{min}$, i.e., $C$ has been arbitrarily set to unity in Eq. (1). The shapes are more nearly the same at each angle but the magnitudes differ. Finally, Fig. 3 shows the same data, but now the variation of $C$ with angle and momentum obtained from Eq. (3) has been employed to present $|\vec{p} - \vec{q}|(d\sigma/d^3q)/C(p, k_{min}) \equiv G(k_{min})$

vs $k_{min}$. This figure shows the operation of QTBS when the $p$-$p$ scattering information is properly employed. It is clear that the $s$ and $t$ dependence with laboratory angle are needed to obtain a universal structure function $G(k)$.

In order to illustrate the $A$ independence of the scaling we show in Fig. 4 a plot of all the data at all angles for Be, C, Al, Ti, Cu, Ag, Ta, Pb, and Th, for $d\sigma/d^3q$ per nucleon vs $q^2/2m_p$, and similarly in Fig. 5, a plot of all the values of $G(k_{min})$ over this same range of $A(9\rightarrow 232)$ vs $k_{min}$. These "scatter" plots are meant to show at a glance the operation of QTBS and the approximate $A^1$ dependence of the magnitude of $G(k)$.

In Fig. 6 we compare the 730 MeV data of Cochran *et al.* at 150° with the recent higher momentum range data at 800 MeV of Frankel *et al.*[6] at 158° to show the internal consistency of the two experiments.

In this work we have attempted to show for values of $C(p, k_{min})$ and $k_{min}$ characteristic of proton production from nuclei, that a common structure function $G(k)$ can be extracted from the data. In this connection the reader is referred to a similar demonstration by Frankel and Frati,[7] this time for antiproton production, of the validity of Eq. (1). In the antiproton case, $k_{min}$ and $C(k_{min})$ are entirely different functions of the observed antiproton variables. Yet these authors have observed QTBS with a similar universal function $G(k)$. The straight line shown in Fig. 6 extends up to the same value of $k$ studied in the antiproton data. Clearly $G(k)$ is not a pure exponential. The line in Fig. 6 follows $\exp(-k/k_0)$ with $k_0 \cong 110$ MeV/$c$. $k_0$ $= 100 \pm 10$ MeV/$c$ was the best straight line fit to the $\bar{p}$ data.

Other tests of QTBS with the Cochran *et al.* data appear in the Los Alamos Conference proceedings, LA-6926-C, August 1977.

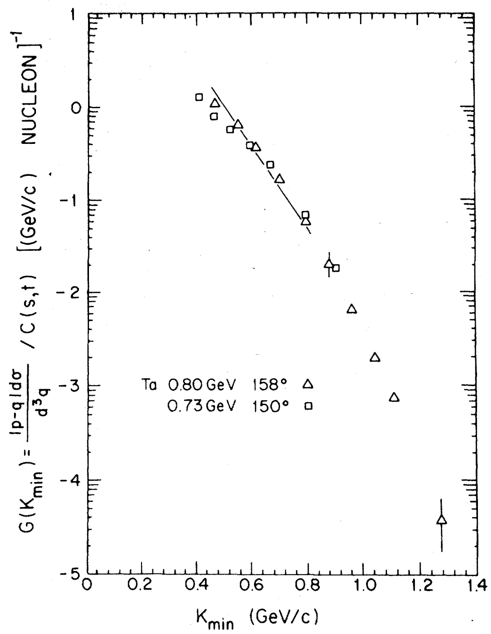


FIG. 6. $G(k)$ vs $k_{min}$: 730 MeV, $\theta = 150°$ (Ref. 1), and 800 MeV, $\theta = 158°$ (Ref. 6), showing $G(k)$ over a combined range from low to high internal momenta. The straight line shows $\exp(-k/k_0)$ with $k_0 = 110$ MeV/$c$ (see text).

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