Target residue mass and charge distributions in relativistic heavy ion reactions

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(Received 8 March 1978)

Calculations of the mass and charge distributions for the heavy target residues from relativistic heavy ion reactions are carried out for the reaction of ^{12}C with ^{238}U, ^{208}Pb, ^{197}Au, Ag, and Cu and compared with experimental data. The primary product distributions are calculated using the abrasion-ablation model. Nuclear charge distributions are calculated using either a stochastic model or a model based upon the zero-point oscillations of the giant dipole resonance. Standard statistical deexcitation calculations are used to calculate secondary product distributions. The results show that some of the principal features of the residue mass and charge distributions can be accounted for with the simple assumptions of the abrasion-ablation model and the assumption that product charge distributions are due to the sudden nature of the interaction and the zero-point oscillations of the giant dipole resonance.

[NUCLEAR REACTIONS Calculated $\sigma(Z,A)$ for 25.2 GeV $^{12}C+^{238}U$, ^{208}Pb, ^{197}Au, Ag, and Cu; comparison with data, abrasion-ablation, giant dipole resonance, relativistic heavy ion reactions, target residue mass distribution, charge distribution.]

I. INTRODUCTION

The first results from the study of targetlike residues produced in the interaction of relativistic heavy ions (RHI) with heavier targets have recently become available.[1-4] In these studies, the mass and charge distributions of the residues produced in 2.1 GeV/A ^{12}C induced reactions have been measured. In addition, several other studies are underway to characterize these distributions with a wide variety of projectiles, targets, and projectile energies. In this paper we report the results of calculations of the residue mass and charge distributions using simple models of the RHI interactions. By comparing these calculations with experimental data, we hope to gain insight into which features of the RHI interaction are most responsible for the residue mass and charge distributions.

II. MODELS USED IN THE CALCULATIONS

A. Mass distribution

The abrasion-ablation model of Bowman *et al.*[5] is used to calculate the mass distribution of the targetlike residues produced in relativistic heavy ion reactions. In this model, the target and projectile nuclei are assumed to be sharp spheres that make "clean cuts" through one another during the relativistic heavy ion reaction. The number of nucleons removed from the target nucleus, and therefore the corresponding size of the target residue left behind, is calculated as a function of impact parameter by calculating the intersecting volume of the target and projectile nuclei, and each impact parameter is weighted by its geometrical probability. The neutron/proton ratio of the removed nucleons is assumed to be the same as that of the target nucleus.

Each target residue is assumed to have an excitation energy given by multiplying the nuclear surface energy coefficient (~0.9–0.95 MeV/fm^2) by the "excess" surface area of the residue. This "excess area" is the surface area of the residue immediately after the collision (typically the residue has a "bite" taken out of it) less the surface area of a sphere of equivalent volume.[5] A standard statistical deexcitation calculation involving multiple particle emission with provision for neutron emission-charged particle emission-fission competition is then carried out to construct the *secondary* product residue mass distribution from the primary distribution and the excitation energy of each primary species.

B. Calculational methods employed

The number of nucleons removed a from a spherical target nucleus of mass number A_1 and radius R_1 struck at impact parameter b by a spherical nucleus of mass number A_2 and radius R_2 has been approximated as[6]

$$a(\nu,\beta)=A_1F(\nu,\beta), \tag{1}$$

where F is a function (given below) of the dimensionless parameter ν, specifying the relative sizes of the two nucleii and the dimensionless parameter β, specifying the impact parameter,

$$\nu=\frac{R_1}{R_1+R_2}, \quad \beta=\frac{b}{R_1+R_2}. \tag{2}$$

Swiatecki[6] (also see Gosset *et al.*[7]) has given the following formulas for the function $F(\nu,\beta)$ for those cases where $A_1>A_2$:

$$F_I=[1-(1-\mu^2)^{3/2}][1-(\beta/\nu^2]^{1/2}, \quad 0\leq b<R_1-R_2 \tag{3}$$

$$F_{II}=\tfrac{3}{4}(1-\nu)^{1/2}\left(\frac{1-\beta}{\nu}\right)^2-\frac{1}{8}\left\{\frac{3(1-\beta)^{1/2}}{\nu}-\frac{[1-(1-\mu^2)^{3/2}][1-(1-\mu)^2]^{1/2}}{\mu^3}\right\}\left(\frac{1-\beta}{\nu}\right)^3,$$

$$R_1-R_2\leq b\leq R_1+R_2. \tag{4}$$

(The abbreviation $\mu=1/\nu-1=R_2/R_1$ has been used.) These two regions correspond to (I) a cylindrical hole being gauged through A_1 and (II) a cylindrical channel being gauged through the side of A_1.

The same geometry problem of calculating the intersecting volume of target and projectile nuclei can be solved by numerical integration techniques, and has been done by Westfall.[7] In this solution the lens-shaped volume that is the region of overlap of the two sharp spheres at a given impact parameter is reduced to the sum of the overlap areas of a series of two-dimensional disks. We term the formulation of Swiatecki[6,7] [Eqs. (3) and (4)] and that of Westfall[7] as the *analytic* and *numerical* formulations, respectively.

Once we have calculated the number of nucleons removed a at any given impact parameter b, we can calculate the *primary* residue production cross section simply as

$$\sigma(A_1-a)=\pi\{[b(a-0.5)]^2-[b(a+0.5)]^2\}, \tag{5}$$

taking advantage of knowing the inverse function $b(a)$. We have chosen not to write an explicit function for $b(a)$ but rather to evaluate Eq. (1) for 500 evenly spaced values of β and do linear interpolations between evaluations.

We have developed a computer program (available on request from the authors) to calculate the primary residue production cross sections using the LBL CDC 7600 computer. Figure 1 shows the primary residue cross section as a function of residue mass number for the $^{12}C+^{238}U$ system. For comparison purposes we show the results from the analytic and numerical calculations of $a(\nu,\beta)$, as well as the ratio of the numerical to the analytic results. This ratio shows two features of the relation between the two methods of calculations: (1) They agree quite well in the region of grazing collisions. (2) There is a sharp discontinuity in the analytical formulation as the outer surface of the projectile crosses the sharp edge of the target nucleus and the projectile is completely eclipsed by the target. This leads to a significant error in the analytic formulation in this region. The analytic formulation does have the advantage of requiring less computer time [1.074 central processing unit (CPU) sec vs 23.841 (CPU) sec for the $^{12}C+^{197}Au$ case] and is useful for treating the most peripheral collisions or looking at gross features of the distributions. We have used the more accurate numerical formulation in all calculations in this paper.

As a lower limit to the excitation energy of the

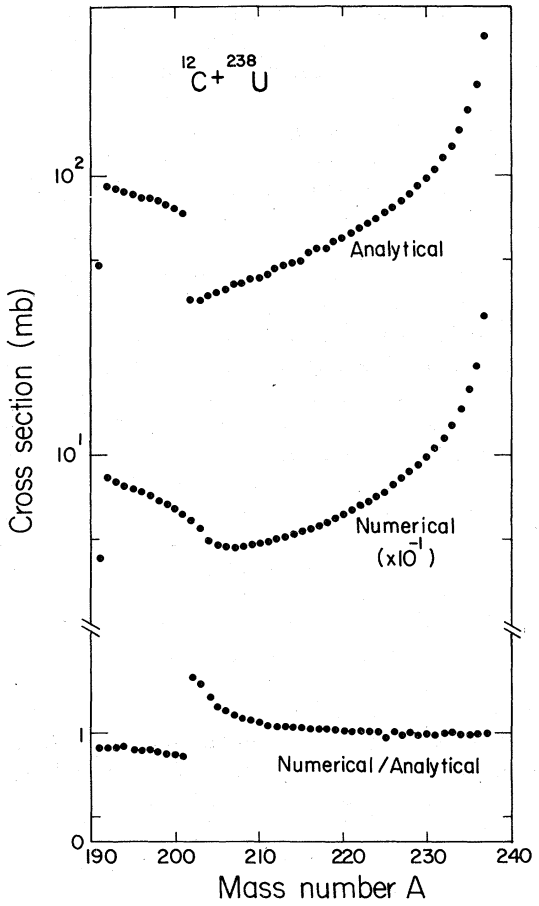

FIG. 1. Calculated cross sections as a function of target residue mass number A for the $^{12}C+^{238}U$ system. For a discussion of the analytical and numerical formulations, see text.

primary target residues, Bowman, Swiatecki, and Tsang[5,6] suggest that the energy tied up in the extra surface area of these residues can be written

$$E = 4\pi R_1^2\{1+P-[1-F(\nu,\beta)]^{2/3}\}E_s, \tag{6}$$

where E_s is the nuclear surface energy coefficient, taken to be 0.9–0.95 MeV/fm^2, and:

$$P_I = \left(\frac{(1-\mu^2)^{1/2}}{\nu}-1\right)[1-(\beta/\nu)^2]^{1/2} \tag{7}$$

and

$$P_{II} = \frac{(\mu\nu)^{1/2}}{8}\left(\frac{1}{\mu}-2\right)\left(\frac{1-\beta}{\nu}\right)^2 - \frac{1}{8}\left[\frac{1}{2}\left(\frac{\nu}{\mu}\right)^{1/2}\left(\frac{1}{\mu}-2\right) - \frac{[(1-\mu^2)/\nu^{1/2}-1](2\mu-\mu^2)^{1/2}}{\mu^3}\right]\left(\frac{1-\beta}{\nu}\right)^3. \tag{8}$$

Again, the subscripts I and II refer to the same regions of overlap as for the function F. While this analytic formulation for the excess surface areas has not been checked against a numerical prescription, it was felt that since the excitation energy given by this approach represented a lower limit, it was adequate to use this analytic expression for the excitation energy of the primary residues.

The deexcitation of the primary residues was calculated using a modified version of the computer code OVERLAID ALICE,[8] which traced the course of the neutron-fission-charged particle emission competition. The angular momentum of the primary residues was assumed to be $10\hbar$ or less.[2] The OVERLAID ALICE code evaluates the Weisskopf Ewing evaporation formula[9] with multiple particle emission. The Bohr-Wheeler model is used to evaluate the fission cross sections competing with particle emission.[10] Neutron, proton, and α-particle emission were allowed in addition to fission as deexcitation modes for the primary residues. In a fraction of the total cross section (26% for the most severe case of $^{12}C+^{238}U$) the primary residues are predicted to have excitation energies greater than 200 MeV. Since it is dubious that an evaporation model is applicable in these cases,[11] the primary residue was assumed to deexcite by a "fast, pre-equilibrium cascade" until its excitation energy was less than 200 MeV. Based upon evidence from Monte Carlo calculations of intranuclear cascades in high energy reactions, the pre-equilibrium cascade was assumed to remove ~10 MeV per emitted nucleon[12,13] with a ratio of emitted neutrons to protons in the cascade of 1.8–2.0[14] for the heavy target systems.

C. Charge distributions

Up to now we have not said anything about the dispersion of the number of neutrons and protons in the primary or secondary residues [other than the previously outlined calculations, which do give the average (Z,A) of each primary or secondary species]. We have investigated two prescriptions for calculating the charge dispersions of the primary residues. The first of these, due to Rasmussen *et al.*[15] suggests that the probability of forming a target residue with a given Z and A can be expressed in terms of the total cross section for that mass number A, by the hypergeometric expression

$$\sigma(Z,A) = \left[\binom{Z_1}{z}\binom{N_1}{n}\Big/\binom{A_1}{a}\right]\sigma(A), \tag{9}$$

where the primary residue mass number $A = A_1 - a$, Z_1, N_1, and A_1 refer to the number of protons, neutrons, and nucleons originally in the target, and z, n, and a refer to the number of protons, neutrons, and nucleons removed from the target in the interaction. This expression, of course, simply calculates the dispersion in the number of neutrons and protons removed from the target as equivalent to the relative number of ways of distributing neutrons and protons in an

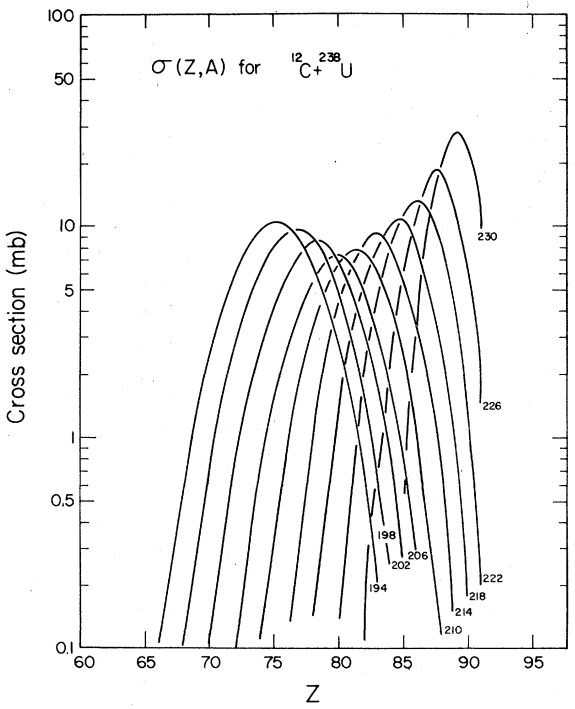

FIG. 2. Calculated primary product charge distributions for target residues from the interaction of $^{12}C + ^{238}U$ using the hypergeometric model.

assembly of a nucleons.

Figure 2 shows some representative primary product charge distributions for the $^{12}C + ^{238}U$ system calculated using the hypergeometric model. There are two important features of the nuclear charge dispersions that are evident in Fig. 2. First, the fact that the peak cross section for each distribution follows the shape of the mass distribution (see Fig. 1) falling to a minimum near mass 206, and then rising up again for those masses furthest removed from the target. The second feature is that the charge distributions are generally increasing in width as one removes more and more mass from the target.

As an alternative model for the charge dispersions based upon different physical considerations, we have developed the idea that in a "clean-cut" sudden interaction such as postulated in the abrasion-ablation model, the fluctuations in the number of swept-out target protons can arise from zero point vibrations of the giant dipole resonance (GDR) of the target nucleus. The GDR has been described as an out-of-phase vibration of the neutrons against the protons.[16] Myers *et al.* have recently treated the GDR in terms of the droplet model of the nucleus.[17] In this treatment they derived a harmonic oscillator (HO) potential to describe the motion of the neutrons against the protons. The HO potential can be approximated by the expression

$$V(\alpha) \approx \tfrac{1}{2} C \alpha^2 , \tag{10}$$

with

$$C = \frac{2JA}{(1+u)} , \qquad u = \frac{3J}{QA^{1/3}} , \tag{11}$$

where A is the mass number of the target nucleus and the zero point frequency is given by

$$\omega_0 = \left(\frac{8J}{m r_0^2}\right)^{1/2} \frac{1}{A^{1/3}} (1+u)^{-1/2} . \tag{12}$$

The terms J and Q are droplet model coefficients and have the nominal values of 25.76 and 11.9 MeV, respectively,[10] while the nuclear radius parameter r_0 is taken to be 1.18 fm, and the nucleon mass m is taken to be 938.9 MeV, the average of the proton and neutron rest masses.

Myers *et al.*[17] have shown that the relative displacement of centers of the neutron and proton spheres d in the Goldhaber-Teller mode of the GDR can be written

$$d = \left(\frac{u}{u+1}\right) \alpha R , \tag{13}$$

where R is the radius of the vibrating nucleus. With this equation and the frequency from Eq. (12), one can readily solve for the displacement of the neutrons from the protons at the classical turning point (where $T = 0$) of the zero-point oscillation d_{ctp} by

$$d_{ctp} = \frac{u}{u+1} R \left(\frac{2E_0}{C}\right)^{1/2} , \tag{14}$$

where $E_0 = \frac{1}{2}\hbar\omega_0$,

$$d_{ctp} = \frac{3.704}{A^{1/2}} \left(\frac{u^2}{(1+u)^3}\right)^{1/4} . \tag{15}$$

The distribution in displacements of the neutrons relative to the protons can be obtained from the displacement expectation values using the wave function for the lowest state of a harmonic oscillator. Such displacements follow a Gaussian distribution with width parameter σ_{disp}, given by

$$\sigma_{disp} = \frac{d_{ctp}}{\sqrt{2}} = \frac{2.619}{A^{1/2}} \left(\frac{u^2}{(1+u)^3}\right)^{1/4} . \tag{16}$$

We say that the dispersion in the number of target protons removed in the "instantaneous clean-cut" of the RHI interaction is given by

$$\sigma_Z = \sigma_{disp} \left(\frac{da}{db}\right) \frac{Z_1}{A_1} , \tag{17}$$

where (da/db) is the rate of change of the number of nucleons removed with impact parameter. We use this σ_Z parameter in a Gaussian charge dis-

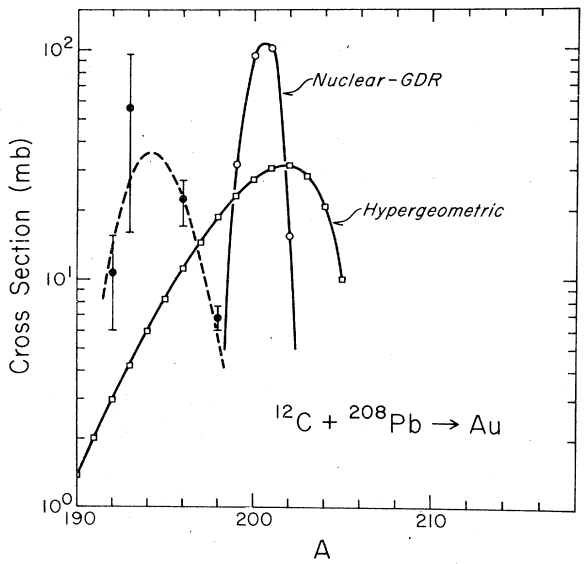

FIG. 3. A comparison of the primary product distribution calculated using the GDR and hypergeometric models of the Au isotopes formed in the reaction of ^{12}C with ^{208}Pb. Also shown are the experimental data of Ref. 2.

persion expression of the form

$$\sigma(Z,A)=\left(\frac{1}{(2\pi\sigma_Z{}^2)^{1/2}}\exp\left\{\frac{-[Z-a(Z_1/A_1)]^2}{2\sigma_Z{}^2}\right\}\right)\sigma(A) \tag{18}$$

to calculate the cross section for producing a primary residue species (Z,A) where $A=A_1-a$. The excitation energy of each isobar is calculated as described above, and the OVERLAID ALICE calculations are done to predict the secondary product charge dispersions.

In Fig. 3 we compare the predictions of the hypergeometric model and the giant dipole resonance model for yields of Au isotopes formed in the 25.2 GeV ^{12}C + Pb reaction with experimental data.[2] The calculated distributions represent *primary* product distributions, before the deexcitation process, but one can already see that the hypergeometric model predicts unusually large widths to the isotopic distributions, in clear variance with the experimental data. Since the hypergeometric model allows for unphysical possibilities such as removing all a nucleons as neutrons alone or protons alone, this prescription gives an upper limit to the primary neutron-proton dispersion. For small numbers of nucleons removed from the target (such as studied by Rasmussen *et al.*[15] in the interaction of ^{4}He and ^{12}C projectiles with ^{40}Ca targets), this model appears to work satisfactorily but appears to give only upper limits for the width of the charge dispersions when larger amounts of mass are removed from the target. Because of this feature, secondary product charge dispersions were not calculated for the hypergeometric model.

III. RESULTS

The secondary product mass distributions calculated for the interaction of ^{12}C with U, Pb, Au, Ag, and Cu using the model described in Sec. II B are shown in Fig. 4, along with experimental data[1-4] for the interaction of 25.2 GeV ^{12}C with these targets. (Also shown are the calculated primary product distributions prior to any particle emission or fission.) For the case of the reaction of 25.2 GeV ^{12}C with Au, an additional secondary product distribution as predicted by Monte Carlo calculations of the intranuclear cascade model by Gabriel *et al.*[18] is shown. In this cascade calculation, each nucleon within the projectile is allowed to interact with the target nucleons individually. No collective interactions were allowed and angular momentum conservation was not included. Pion production was allowed via the isobar model and the primary residues were allowed to evaporate neutrons and charged particles.

Examination of Fig. 4(c) shows that the Monte Carlo intranuclear cascade model does not correctly describe the secondary product mass distribution. The model predicts that the ^{12}C ion will interact with the target nucleus in a manner similar to that of a relativistic proton. We take this failure of the cascade model to indicate the "collective nature" of the ^{12}C-nucleus interaction wherein the ^{12}C nucleons are not acting as individual particles but as a single entity.

In surveying the broad spectrum of target systems represented in Fig. 4, we are impressed with how well the general features of the *very heavy* residue distributions are predicted by the simple abrasion-ablation model. (Sharp rises in cross section predicted by this model for those residues would come from the most central collisions, which are at the limit of applicability of this model.[5]) We believe that the success of the simple abrasion-ablation model in describing the residue mass distributions from peripheral interactions (region of the analytic model) is due to the fact that the shape of this portion of the mass distribution is governed strictly by the geometrical weighting of various impact parameters. Implicit in this conclusion is the notion that the ^{12}C-nucleus interaction is a "sudden" interaction of the entire ^{12}C nucleus with the target nucleus. In the heaviest nucleus U the RHI fission cross section can simply be accounted for as being the difference between the primary and secondary residue distributions, i.e., those primary residues that deexcited by fission. Closer examination of this component of the cross section reveals the bulk of the fission events correspond to residues from very large impact parameters, resulting from only 5–25 nucleons removed from the target. This feature is in agreement with other experimental data on RHI induced fission.[1]

Figures 5 and 6 show the calculated secondary product charge distributions using the GDR model for the Sc nuclides formed from the reaction of 25.2 GeV ^{12}C with Cu and the Au nuclides formed from the reaction of 25.2 GeV ^{12}C with Pb, respectively. These secondary product distributions are compared with the experimental data of Cumming *et al.*[4] and Loveland *et al.*,[2] respectively. The Sc distributions are amazingly well fitted by the GDR model (which has no free parameters in it). The Au distributions are not described as well by the GDR model (although still described creditably) due to two features: (a) The appearance of a sawtooth in the calculated secondary product distribution (due to odd-even effects in the neutron evaporation process) not seen in the experimental data. (b) Some apparent underestimation of the excitation energy of the targetlike products leading to

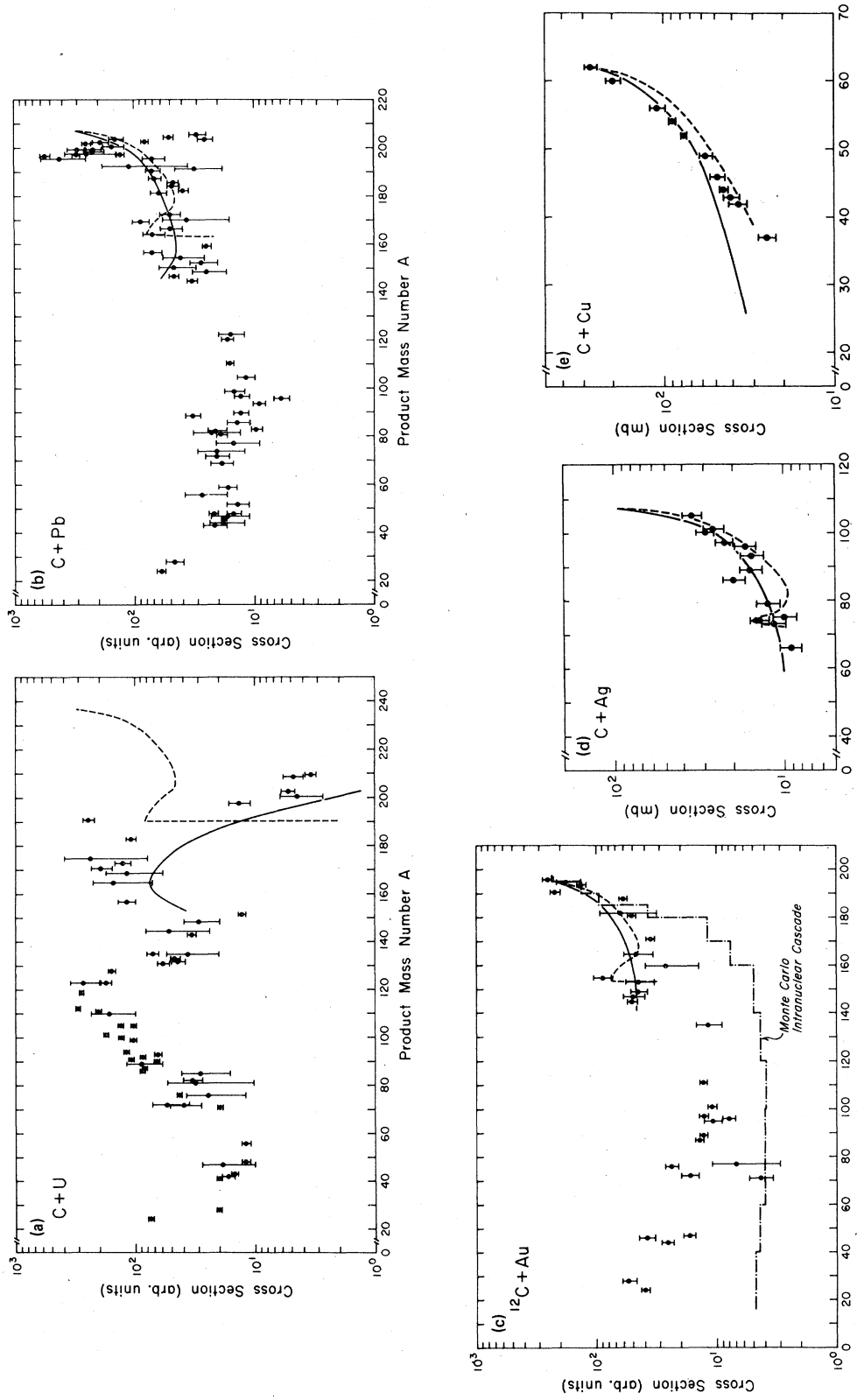

FIG. 4. Calculated secondary product mass distributions (solid line) for the reaction of ^{12}C with (a) U, (b) Pb, (c) Au, (d) Ag, and (e) Cu. The experimental data are from Refs. 1–4. The dashed lines indicate the calculated primary product mass distributions. Also shown in (c) is a Monte Carlo cascade calculation of the secondary product distribution.

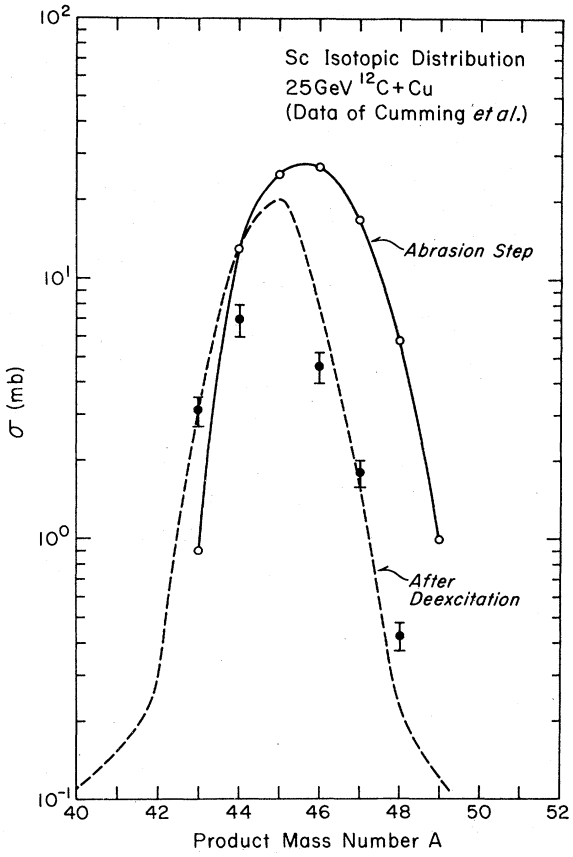

FIG. 5. Calculated Sc secondary product distribution (solid line) for the reaction of ^{12}C with Cu using the GDR model. Experimental data are those of Cumming *et al.* (Ref. 4). Dashed line is the calculated primary product distribution from the GDR model.

a calculated mean n/p ratio of the secondary residues greater than that seen experimentally.

The general success of the GDR model in accounting for the widths of the charge dispersions seems to point to the "quick, clean-cut" nature of the RHI interaction. Were extremely large amounts of excitation energy being deposited in these encounters, our evaporation calculations would tell us the charge dispersions would be much broader than observed.

Our GDR model is largely based on the Goldhaber-Teller mode[16] of the GDR and does neglect any broadening of the charge dispersions due to the Steinwedel-Jensen acoustic mode.[19] Myers *et al.*[17] have shown that the GDR is mainly a Goldhaber-Teller mode but does contain an essential admixture of the Steinwedel-Jensen mode, which becomes more important for heavier nuclei. Thus we must view with some caution the success of the GDR model in treating the Au charge dispersion.

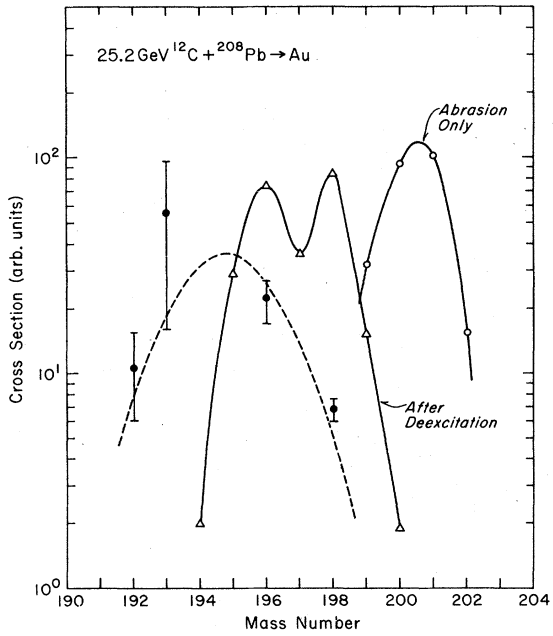

FIG. 6. Same as Fig. 5 except the products are Au isotopes, the reaction is ^{12}C+Pb, and the experimental data are those of Loveland *et al.* (Ref. 2).

IV. CONCLUSIONS

What have we learned from these calculations? It would appear that the gross features of the heavy secondary residue mass and charge distributions from RHI interactions are well described by the simple ideas of the parameter-free abrasion-ablation and GDR models of the mass and charge distributions, respectively, in which the RHI-nucleus interaction is a "quick, clean-cut" of the entire projectile acting as a single particle through the target, leading to smaller excitation energies. It will be interesting to direct further measurements of the heavy target residue mass and charge dispersions towards exploring the linear and angular momentum of the residues and to explore the energy dependence of these interactions.

ACKNOWLEDGMENTS

We would like to acknowledge fruitful discussions with W.J. Swiatecki, G.D. Westfall, J.O. Rasmussen, R. Donangelo, and L. Oliveira. We wish to especially thank Tony Gabriel of the Oak Ridge National Laboratory for performing the Monte Carlo calculations. One of us (WDL) gratefully acknowledges sabbatical leave support from Oregon State University during part of the period in which this work was conducted. This work was supported by the Divisions of High Energy and Nuclear Physics, U.S. Department of Energy.

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