Inelastic scattering of 40 MeV protons from ^{24}Mg. II. Microscopic calculations for positive parity states

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Proton inelastic scattering data at 40 MeV bombarding energy are compared to microscopic distorted-wave Born-approximation calculations for positive parity states in ^{24}Mg utilizing shell-model wave functions spanning the full $2s1d$ shell basis. Both empirical forces and forces derived from free nucleon-nucleon potentials are used in the calculations. Except for four transitions for which strong coupling effects are evident, the agreement between theory and experiment is quite good. Enhancement factors extracted for the natural parity transitions are consistent with the effective charges obtained from electromagnetic transition rates. Levels corresponding to the giant $M1$ resonance in ^{24}Mg have been resolved in the present experiment. The fact that the renormalization factor between theory and experiment is close to unity for the 10.713 MeV 1^+; $T = 1$ state indicates that little or no renormalization of the two-body force is necessary for this inelastic transition. It is argued that little renormalization is also involved for magnetic-type inelastic transitions of multi polarities higher than $M1$.

[NUCLEAR REACTIONS ^{24}Mg(p,p') at 40 MeV to positive parity states. Microscopic DWBA analysis with the full $2s1d$ basis wave functions. Deduced collective enhancement factors.]

I. INTRODUCTION

Due to recent progress in nuclear shell-model calculational techniques,[1] the wave functions of the 2s1d-shell nuclei calculated in the complete 2s1d-shell basis are presently available.[2] The predictions of these calculations have been compared with the traditional spectroscopic observables such as static and transition electromagnetic moments, beta decay log ft values and single nucleon spectroscopic factors. Another test of these wave functions is in the calculation of the nucleon inelastic scattering cross sections using a microscopic reaction theory. Sophisticated microscopic calculations of inelastic proton scattering are now possible, including finite range effects and exchange contributions and allowing for tensor and spin-orbit terms in the effective two-body interaction. In previous work in this mass region, calculations have been restricted to nuclei possessing fairly simple structure. They proved quite successful in interpreting the transitions to the members of the particle-hole multiplets in the closed-shell ^{16}O (Ref. 3) and ^{40}Ca (Refs. 4,5) nuclei. Some transitions in nuclei with a few nucleons beyond the closed ^{16}O core have been also analyzed assuming simple shell-model[6] and Hartree-Fock[7] wave functions.

Nuclei near the middle of the sd-shell like ^{24}Mg provide a particularly severe test of the shell-model wave functions. Therefore the main purpose of the present work was to determine whether the wave functions of Chung and Wildenthal[2] calculated in the full sd-shell basis can account for the ^{24}Mg(p,p') cross sections at $E_p = 40$ MeV. A second purpose was to investigate core polarization effects. The concept of the core polarization has been introduced[8] in order to account for the participation of configurations from outside of the model space in the transitions. Core contributions are taken into account by renormalizing the operators acting within the model space. This is accomplished by introducing effective charges into the electromagnetic operators and enhancement factors into the effective nucleon-nucleon force[9,10] used to calculate the inelastic cross sections. The cases most thoroughly studied at present are the core polarization effects involved in the transitions to the low-lying collective 2^+ and 3^- states (even-even target nuclei only are considered throughout the present paper). Core contributions are constructively coherent with the contributions of the valence particles for these electric-type inelastic scattering transitions, resulting in cross sections enhanced over the shell-model estimates.

Particular attention has been paid in the present work to extraction of the cross sections for the transitions to the unnatural parity states. These transitions involving the matrix elements of the spin operator will be termed magnetic-type following the terminology introduced in Ref. 11. These magnetic-type transitions have been much less studied in the past than electric-type transitions. It is of interest to establish whether the proportionality of the excitation strengths previously observed[12] between the electromagnetic and the isoscalar electric-type inelastic excitations holds also for the magnetic-type transitions. This is expected on the basis of the similarity of the matrix elements involved in their description.[11] For example, bare-nucleon g-factors in the magnetic moment operator and the full sd-shell basis wave functions are sufficient to reproduce the magnetic moments of the A = 17–25 nuclei.[13] One of the purposes of the present work is to determine whether the bare two-body force suffices to reproduce the 1^+, T = 1 cross sections seen in the present data.

For the analogs of the giant M1 resonance in ^{28}Si, an approximate proportionality of the charge-exchange (t, He) reaction cross sections to the reduced M1 excitation strengths has been reported.[14] However, the conclusions from the study of the 1^+, T = 1 states with nucleon induced inelastic scattering and charge-exchange reactions may be more convincing, since contributions from two-step stripping and pick-up processes are known to be important for reactions with mass-3 projectiles.

The angular distributions for the inelastic scattering of 40 MeV protons from the positive parity states of ^{24}Mg in the excitation energy range up to 11 MeV are discussed in the present work. The high resolution achieved with the MSU cyclotron and the Enge split-pole spectrograph permitted the extraction of cross sections for many more states than previously observed in inelastic scattering on ^{24}Mg. The details of the experimental method and the spectra are presented in a previous publication[15] in which the natural parity transitions were discussed in the framework of the macroscopic collective model. Some questions regarding the cross sections relevant for the present analysis which were not mentioned in Ref. 15 are discussed in Section III.

For the negative parity states of ^{24}Mg no shell-model wave functions exist at present. The difficulty consists in taking into account both the excitations from the 1p-shell and into the 2p1f-shell. Work is presently underway[16] to calculate the wave functions for these states with an open-shell RPA approach.

II. MICROSCOPIC DWBA CALCULATIONS

Microscopic DWBA calculations were performed with the code DWBA-70 of Schaeffer and Raynal[17] which utilizes the helicity formalism.[18] The knock-on exchange contributions to the reaction amplitude due to antisymmetrization between the incident and target nucleons were taken into account in all cross section calculations. A realistic effective interaction[19] derived from nucleon-nucleon potentials was used in the present calculations presented in Figures 2 and 3. This force is the sum of central, tensor, and spin-orbit components specified in the form of a superposition of Yukawa potentials of four different ranges. The tensor and spin-orbit components are particularly important for unnatural-parity transitions. Several choices for the specific individual terms in the interaction are given in Ref. 19. The present calculations utilized the sum of the interactions labeled 2, 5, 9, 14, 16, 17, and 20 in Table I of that paper. This interaction involves central-even components derived from the Hamada-Johnston[20] potential, but test calculations (not presented here) showed similar results with the corresponding terms derived from the Reid[21] potential. An indication of the agreement of these calculations and the data will be given by the normalization factors, N, required to fit the magnitudes of the transitions to the various sd-shell states.

Alternate calculations are also presented below (in Figure 4) for selected, natural-parity, T = 0 transitions with a simplified empirical force consisting only of a central interaction in the form of a Serber mixture. For these transitions the non-central forces are less important and this simplified interaction then permits the extraction of enhancement factors[9,10] to be compared with electromagnetic effective charges. It is also of interest to compare the strength of this force required for this light nucleus with that used earlier for medium[22] and heavy mass[23] nuclei.

The spectroscopic information is fed into the program in the form of the expectation values

$$\langle J_f T_f \| (a_i^\dagger a_{i'})_{JT} \| J_i T_i \rangle$$

between the initial state with spin $J_i = 0$ and isospin $T_i = 0$ and the final state with $J_f = J$ and $T_f = T$, J and T being the total angular momentum and isospin transferred in the reaction. The operator $a_{i'}$ annihilates the nucleon in the single-particle state $i' \equiv n'\ell'j'$, whereas $a_i^\dagger$ creates the nucleon in the state $i \equiv n\ell j$. Final angular momentum values up to 5 are allowed by the one-step DWBA theory for the even-even 2s1d shell nuclei.

The wave functions from which the $(a^\dagger a)$ coefficients were calculated span the entire 2s1d shell-model basis. They were derived from diagonalization of an empirical Hamiltonian whose matrix elements were obtained from fitting to the experimental energy levels in the A = 18–24 mass region. A complete account of the methods used in these calculations and the results are presented in Ref. 2.

In the DWBA calculations two prescriptions were used to calculate the bound-state wave functions. The results presented in Sec. IV were obtained with wave functions calculated by binding $1d_{5/2}$, $2s_{1/2}$, and $1d_{3/2}$ nucleons in Woods-Saxon wells of diffuseness a = 0.75 fm and radii r_0 = 1.33 fm, 1.25 fm, and 1.40 fm, respectively. Similar results were obtained with harmonic-oscillator wave functions assuming that the oscillator frequency $\hbar\omega$ is calculated according to $\hbar\omega = 45A^{-1/3} - 25A^{-2/3}$. Both of these sets of parameters of the single-particle potentials were previously[24] used to calculate the inelastic electron scattering form factors for the 2_1^+ states of some selfconjugate sd-shell nuclei and gave momentum transfer dependences in good agreement with the data.

The optical potential used to generate the distorted waves was obtained by fitting the optical-model parameters to the experimental ^{24}Mg(p,p) elastic scattering cross sections at 40 MeV (potential labeled SPH in Table I of Ref. 15).

III. IDENTIFICATION OF THE STATES OF $(sd)^8$ CONFIGURATION IN ^{24}Mg

In making a comparison between the predictions of the microscopic (p,p') calculations and the observed angular distributions it is necessary to associate each model state with a particular state in ^{24}Mg. The experimentally known positive parity states of ^{24}Mg and the shell-model states of the configuration $(sd)^8$ with energies up to about 11.2 MeV are shown in Fig. 1. Most of the experimental level energies and spin-parity assignments are from Endt and Van der Leun's compilation.[25] Included are the following modifications resulting from more recent measurements. The 10.713 MeV member of the 10.713–10.731 MeV doublet has been assigned 1^+ T = 1 (Refs. 26 and 27). At 11.017 MeV, there is probably a closely spaced doublet, consisting of a previously[25] known 2^+ and a newly found 5^+ T = 1 (Ref. 28) state. The assignments for the 9.300 (0^+), 9.528, 10.328, and 10.578 MeV states are tentative (in parentheses) as discussed in our previous work.[15] We do not find evidence of a 2^+ state at 9.283 MeV. Instead, this state appears to be a (4^-) state and therefore is not included in Fig. 1. The second member of the 9.300 MeV doublet, formerly[25] considered as $(3,4)^+$, has been given a definite $J^\pi = 4^+$ assignment.[29,30] Comparison of the experimental[31] spectroscopic factors for the ^{25}Mg(^{3}He,α)^{24}Mg reaction with those predicted[2] for the states of $(sd)^8$ confirmed most of the previous assignments (except for the 5_1^+ and $0_{2,3}^+$ states for which an intrinsically small strength is predicted and 6^+ states whose excitation is forbidden) and in addition suggests tentative assignments indicated by broken lines in Fig. 1. The most reliable predictions of the model are for the lowest few eigenvectors with a given spin and isospin. Therefore, the identification of the counterparts of such states, as for example, the 2_2^+; T = 1 and 3_1^+; T = 1, is important for the present shell-model test. The one-nucleon pickup strength for the states around 11 MeV is predicted to be concentrated in the 5^+; T = 1 and the 3^+; T = 1 states. Experimentally, it is found[31] that around 10 MeV the most strongly excited states are those at 10.822 MeV and 11.017 MeV. Since the latter is presently known to be 5^+; T = 1 (Ref. 28), the assignment of 3^+; T = 1 for the 10.822 MeV state seems highly plausible. Comparison of the spectroscopic factors also strongly favors the indicated assignment for the 2_1^+; T = 1 and 2_2^+; T = 1 states. It is noteworthy that the 1_2^+ and 2_2^+; T = 1 states experience a similar Thomas shift throughout the A = 24 isotriplet. They are separated by 7 keV in ^{24}Na (Ref. 32), 18 keV in ^{24}Mg, and about 20 keV in ^{24}Al (Ref. 33).

Around 9 MeV the obvious one-to-one correspondence between the calculated and experimental 2^+ states is lost

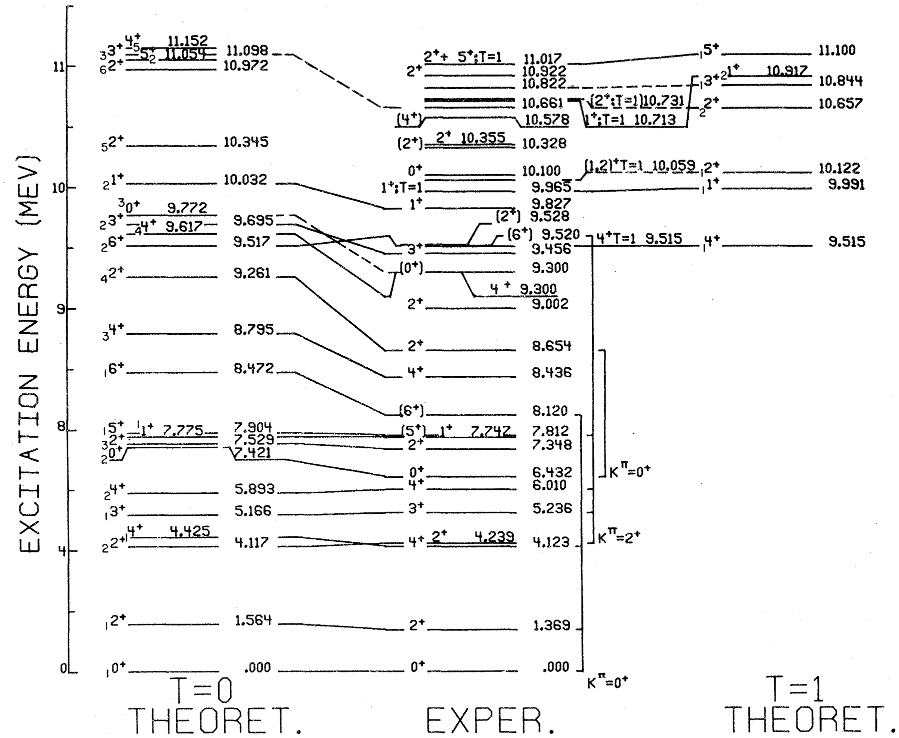

FIG. 1. Experimental and theoretical level schemes for the positive parity states in ^{24}Mg up to an excitation energy of about 11.0 MeV. The experimental 0$^+$ 10.682 MeV and (0$^+$) 10.160 MeV states not relevant for the present discussion have been omitted. Broken lines indicate tentative assignments discussed in the text.

(see Fig. 1). There are three candidates for the state corresponding to the 2_4^+ model state – the states at 8.654, 9.002, and 9.528 MeV. Spectroscopic factor[31] and γ-decay properties[34,35] indicate that the 8.654 MeV state is the most likely choice. However, the shell-model predicts[2] a weak 7% E2 branch to the ground state, while an upper limit of 3% for this transition has been established by experiment.[34,35] This conclusion is confirmed by the (p,p') reaction (see Sec. IV A) which also indicates that the ground-state transition rate for the 8.654 MeV state is over-predicted by the model. The 9.002 MeV 2^+ state has[31] a significant $\ell_n = 2$ parentage to the ground state of ^{25}Mg, therefore it is quite probable that it also belongs to (sd)^8. Since the properties of the 2_4^+ model state are only partly reflected by the (sd) wave functions and the other two 2^+ states seen around 9 MeV do not find counterparts in the calculated spectrum, it is doubtful whether the wave functions for the higher 2^+ states are reliably predicted by the model. We have rather arbitrarily assumed that the 11.017 2^+ state is a counterpart of the 2_6^+ model state; however, the predicted shapes of the $2_4^+ - 2_6^+$ states are rather similar. It would also be useful to test the predictions for the higher 4_3^+ and 4_4^+ states with the (p,p') reaction. Unfortunately, we were not able to resolve the 8.436 MeV 4^+ state from its strongly excited 8.438 MeV 1^- neighbour. In addition, only a very crude estimate of the 4^+ contribution to the unresolved doublet at 9.300 MeV can be made.

IV. DISCUSSION

In Figs. 2 and 3 the experimental data are compared with the microscopic DWBA calculations using the realistic effective interaction described in Sec. II. Indicated on the plots are the overall factors, N, used to normalize theory to the data. A label along with each of the theoretical curves indicates the model state (see Fig. 1) used to calculate the corresponding structure coefficients.

The shape of the predicted angular distributions is in fairly good agreement with the data in almost all the cases shown for which an unambiguous connection of model state and experimental level can be made. The normalization factors are between 1.0 and 5.0 except for the 0^+ states and states for which multistep processes are expected to contribute. The inelastic scattering cross sections, however, vary over 3 orders of magnitude from the weakest (1^+ 7.747 MeV) to the strongest excited (2^+ 1.369 MeV) states.

A. Multistep Excitation Processes

The type of multistep process responsible for the rather large normalizations for the 4^+ 4.123 MeV, 3^+ 5.236 MeV, and 5^+ 7.812 MeV states is reasonably well understood. In addition to rather weak direct transitions from the ground state, these states are fed by the two-step processes due to the enhanced in-band E2-like inelastic transitions. The classification of the low-lying states of ^{24}Mg into rotational bands is shown in Fig. 1. The 4^+ 4.123 MeV second excited ground state band member is mostly excited by the two-step process through the 2^+ first excited band member.[15] In the $K^{\pi} = 2^+$ band, the excitations of the unnatural parity 3^+ 5.236 MeV and 5^+ 7.812 MeV states are expected to be enhanced due to the

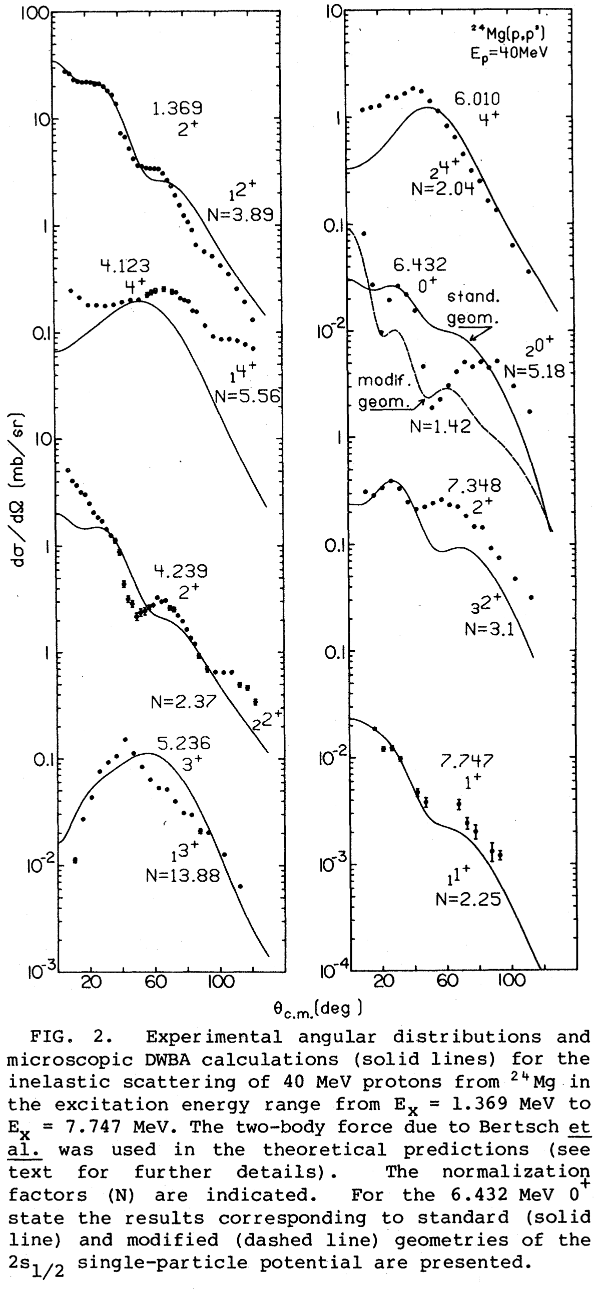

FIG. 2. Experimental angular distributions and microscopic DWBA calculations (solid lines) for the inelastic scattering of 40 MeV protons from ^{24}Mg in the excitation energy range from E_x = 1.369 MeV to E_x = 7.747 MeV. The two-body force due to Bertsch et al. was used in the theoretical predictions (see text for further details). The normalization factors (N) are indicated. For the 6.432 MeV 0^+ state the results corresponding to standard (solid line) and modified (dashed line) geometries of the $2s_{1/2}$ single-particle potential are presented.

quadrupole couplings: 2^+(4.238 MeV) → 3^+(5.236 MeV), 4^+(6.010 MeV) → 3^+(5.236 MeV) and 4^+ (6.010) MeV)→ 5^+ (7.812 MeV).

In Sec. III the 8.654 MeV state was examined as a possible candidate for the 2^+_4 model state. The (p,p') reaction tests the presence of a weak ground-state branch for which only an upper limit was established in the γ-ray work.[34,35] A rather weak cross section and a nearly isotropic yield is observed for the 8.654 MeV state (see Fig. 3). For orientation, the 2^+_4 predicted cross section is normalized to the data at forward angles yielding a normalization coefficient N = 1.2 much smaller than those observed for the lower 2^+ states. It seems that the shell-model strongly overestimates the ground state transition rate with the multistep mechanism giving the main contribution to the excitation of this state. Since there is little or no direct access to the 8.654 MeV state from the ground state and since an enhanced E2 transition to the 0^+_2 6.432 MeV state was observed,[35] the present results corroborate the suggestion[35] that these two states form a rotational band. The search[36] for higher band members has not yet been successful.

A microscopic coupled-channel code which includes knock-on exchange contributions does not exist at present. The main difficulty is associated[11] with the non-local coupling kernels which arise due to antisymmetrization. The importance of antisymmetrization is best illustrated by the fact that the differential cross sections obtained for the 2^+ state are lower by a factor of 5 when the effect of antisymmetrization is excluded. If such codes were developed it would be possible to determine whether the shell-model correctly predicts the strength of the $0^+_1 \to 4^+_1$, $0^+_1 \to 3^+_1$, and $0^+_1 \to 5^+_1$ transitions. For other states presented in Figs. 2 to 4, the direct one-step excitation apparently dominates, although one cannot exclude the possibility that coupled-channel effects might also contribute to these states. However, in the following discussion the assumption is made that the two-body force and the wavefunctions are the factors determining the cross sections.

B. Enhancement factors vs. effective charges in the electric-type transitions.

By using a simplified interaction as discussed in Sec. II and two additional assumptions,[9,10] it is possible to relate the effective charges involved in the electric transitions and the "enhancement factors" in the corresponding inelastic excitations. One assumption is that the collective enhancement in both is due entirely to T = 0 isospin transfer. The second is that the ratios of the inelastic single-particle amplitudes to the corresponding electric emplitudes are independent of the single-particle quantum numbers. The spin and isospin independent term of the effective interaction acquires the value $e_o V_o$, where e_o is the electromagnetic isoscalar effective charge (in units of the proton charge) and V_o is the bare interaction strength, provided the above assumptions are fulfilled. The Serber exchange mixture $(V_o:V_\sigma:V_\tau:V_{\sigma\tau}$ = −3:1:1:1, where the notation has its standard[19] meaning) and a Yukawa form factor with a range μ = 1.37 fm was used to extract the isoscalar enhancement factors for selected T = 0 states, those at 1.367, 4.238, 6.010, and 7.348 MeV. V_o was adjusted to reproduce the 2^+_1 1.369 MeV angular distribution with an effective charge, $e_o(2^+_1)$ = 1.88 obtained from the electromagnetic data.[37,38,39] The adjusted depth V_o = −16.31 MeV differs only slightly from those used in other mass regions (V_o = −15.0 MeV, μ = 1.37 fm in Ref. 22 and V_o = −12.6 MeV, μ = 1.4 fm in Ref. 23). In the remaining cases V_o was kept fixed at this value, while e_o was adjusted to give best agreement with the data. The angular distributions thus obtained are presented in Fig. 4 and the extracted enhancement factors, denoted as ε_o, are collected in Table I. A comparison with electromagnetic effective charges indicates that this method yields enhancement factors for these inelastic transitions consistent with those observed in the corresponding electric transitions. The phenomenological Serber interaction adopted here thus appears to be useful over a wide range of atomic masses. Formerly[14] an electromagnetic effective charge $e_o(2^+_1)$ = 2.1 was obtained from (e,e') data using wave functions calculated in a truncated basis. Increasing the shell-model space to include the complete 2s1d basis resulted, as expected, in a reduced effective charge.

The method of normalization used in the section, i.e. the extraction of an enhancement factor differs from that

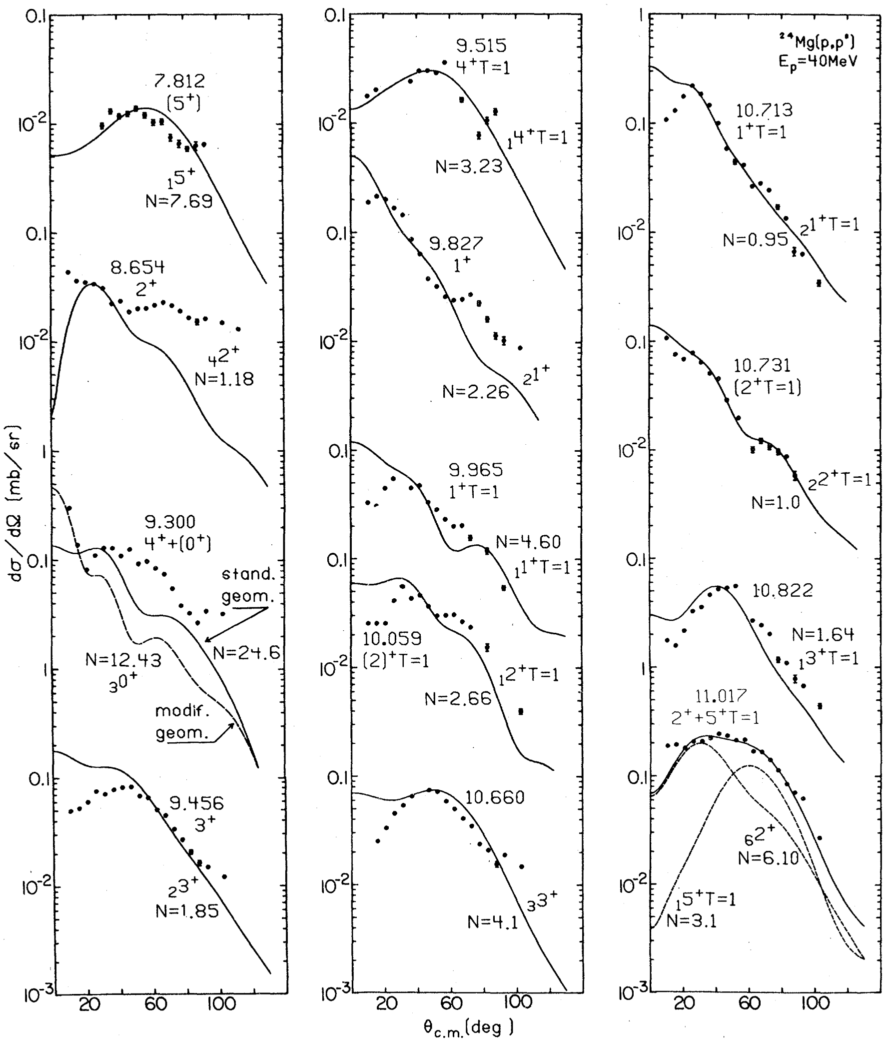

FIG. 3. Experimental and theoretical cross sections in the excitation energy range from $E_x = 7.812$ to $E_x = 11.017$ MeV. Other details are same as in Fig. 2.

used previously with the realistic effective interaction calculations where an overall normalization factor, N, was given. The enhancement factor ε_0, as given in Table I, renormalizes only the spin-isospin independent term of the central interaction, since this term is assumed to be responsible for the correlations leading to enhancements of the electric transitions.[9,10] The differences in ratios of the ε_0 values in Table I for these states and the corresponding ratios of the $N^{1/2}$ values can be attributed to differing relative contributions of the tensor and spin-orbit terms in the interaction.

There is a noteworthy difference between the experimental angular distributions for the 1.369 MeV and 4.239 MeV 2^+ states and the angular distribution for the 7.348 MeV state (see Fig. 4), the former two decreasing more rapidly with angle. This indicates qualitatively that the transition density for the latter state is concentrated at smaller radii. Shell-model calculations suggest that this is the result of cancellation in the surface region of the density for the transitions between the $2s_{1/2}$ and the d orbitals (dashed lines in the insets to Fig. 4) and the density for the transitions between the d orbitals (dashed-and-dot lines). Since the secondary maximum at around 65° is more emphasized in the experimental than in the predicted angular distributions for both empirical (Fig. 4) and realistic (Fig. 2) forces, the cancellation for the 7.348 MeV is probably more complete, i.e. $s \leftrightarrow d$ transitions contribute more heavily than assumed by the model. For the 1.369 MeV and 4.239 MeV states the partial densities add constructively yielding the resulting density (solid lines) with a surface-peaked shape. It is gratifying that proton inelastic scattering allows one to differentiate between vibrations, which in terms of the macroscopic collective model may be termed as surface (1.369 MeV and 4.239 MeV states) and compressional (7.348 MeV state).

C. Magnetic-type inelastic transitions

Only very recently measurements have been undertaken[14,43] to test whether the magnetic-type inelastic cross sections are proportional to the reduced electromagnetic excitation strengths, with both of the studied cases involving the giant M1 resonance. In Ref. 14,

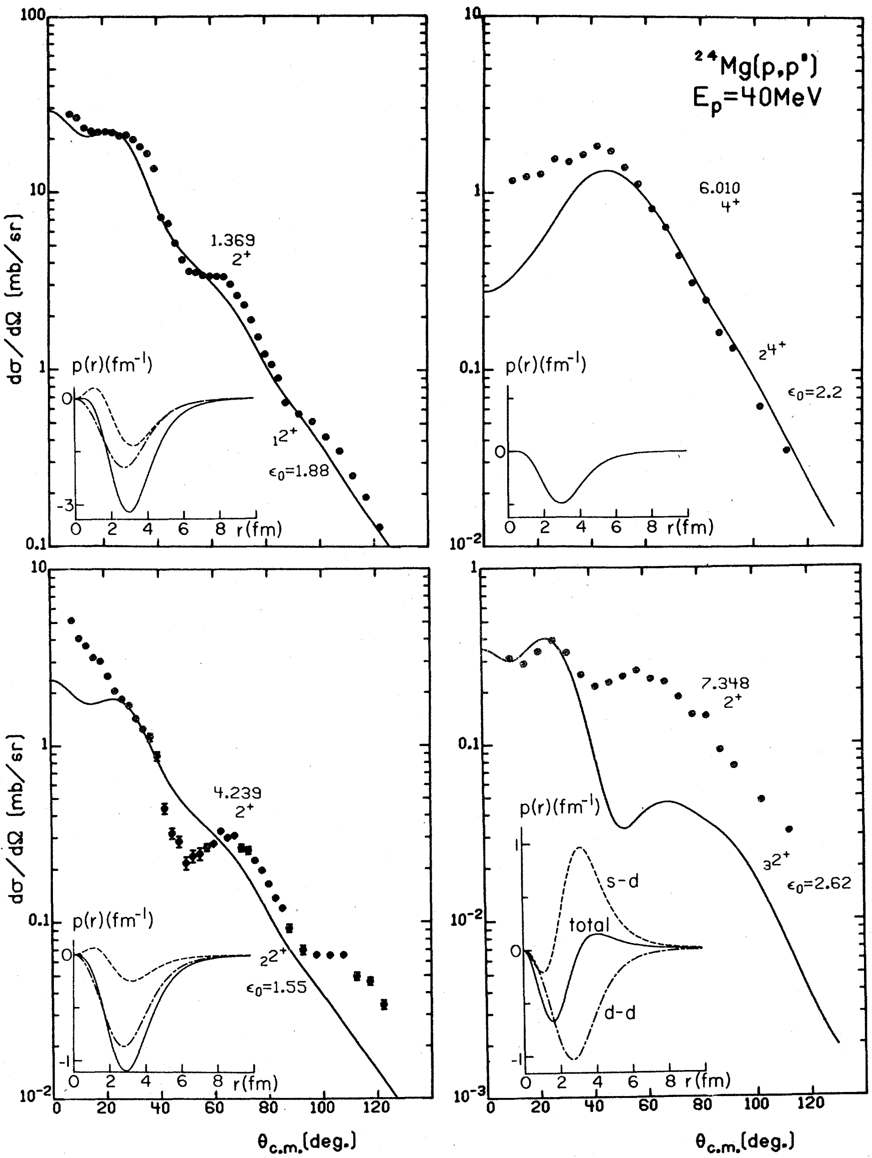

FIG. 4. Experimental and microscopic DWBA angular distributions for selected natural parity, $T=0$, states in ^{24}Mg. The isoscalar enhancement factors (ϵ_0) are indicated. An empirical two-body force consisting of Serber mixture was used to calculate the theoretical predictions. In the insets the microscopic transition densities for the transitions between s and d orbitals (dashed lines), between d orbitals (dash-and-dot) and total densities (solid lines) are presented.

the charge exchange (t,^{3}He) reaction to the isobaric analogs of the M1 resonance in ^{28}Si demonstrated an approximate proportionality of the M1 and inelastic strengths. In ^{24}Mg the 1^+; $T=1$ states at 9.965 MeV and 10.713 MeV form the giant M1 resonance in this nucleus, exhausting the bulk of the ground-state strength.[39,41] The total (p,p') cross section ratio for these states is 2.4, which is close to the B(M1) ratio of 2.42. The previous[43] attempt to demonstrate this equivalence with the (^{3}He, ^{3}He') reaction was not successful, since the 30 keV resolution used in Ref. 43 was not sufficient to resolve the 10.713–10.731 MeV doublet. The transition densities for the 9.965 MeV and 10.713 MeV states are somewhat different as can be seen from the differences in the shapes of the angular distributions (see Fig. 3) – the 10.713 MeV cross sections decreasing more rapidly with angle. These features are quite well reproduced by the present microscopic DWBA calculations. Normalization factors close to unity result from the comparison of both the electromagnetic transition strengths (Table I) and the (p,p') cross sections (Fig. 3) for the 10.713 MeV state with the theoretical predictions. Since bare-nucleon g factors are used to calculate the M1 rates, no renormalization of the spin-isospin part of the effective two-body interaction in the case of the (p,p') reaction is implied. In the case of the 9.965 MeV state the magnitudes of the normalization factors for the M1 and (p,p') transition are similar, indicating that shortcomings of the wave functions may be responsible for the underestimation of the theoretical transition rates in both cases. It has been demonstrated in

TABLE I. Ground-state experimental and theoretical[a] transitions rates for selected positive-parity states in ^{24}Mg.

	E_x (MeV)	Model[a] state	$B(L;0^+\to L)$ e^2 fm^{2L} Experiment	Theory[a]	$\bar{B}_{ex}/B_{th}$	e_o^{ex} [b]	ε_o^{ex} [c]	Reference
A. Electric-type transitions	1.369	2_1^+	416 ±21	475	0.88	1.88	1.88[d]	37
			420 ±20					38
			420 ±25					39
			421 ±12					42
	4.239	2_2^+	24.0±4.0	51.8	0.45	1.35	1.55	41
			22.8±3.7					40
			36.0±4.0					39
	6.010	4_2^+	$(4.2\pm1.0)\times10^4$	4.42×10^4	0.95	1.95	2.20	39
	7.348	2_3^+	8.01±0.8	2.61	3.1	3.50	2.62	41

	E_x (MeV)	Model State	$B(L;0_1^+\to L)$ e^2fm^{2L} Experiment	Theory[a]	$\bar{B}_{ex}/B_{th}$	N^c	Reference
B. Magnetic-type transitions	9.827	1_2^+	$(3.14\pm0.27)\times10^{-3}$	9.13×10^{-5}	34.0	2.26	41
	9.965	$1_1^+(T=1)$	$(1.38\pm0.20)\times10^{-2}$	6.35×10^{-3}	2.11	4.60	39
			$(1.30\pm0.44)\times10^{-2}$				
	10.713	$1_2^+(T=1)$	$(3.11\pm0.40)\times10^{-2}$	3.88×10^{-2}	0.88	0.95	39
			$(3.71\pm0.16)\times10^{-2}$				41

[a] Shell-model (Chung and Wildenthal[2]), assuming isoscalar effective charge ($e_o=e_p+e_n$) $e_o=2.0$ for electric transitions and bare-nucleon g-factors for magnetic transitions.

[b] Experimental isoscalar effective charge $e_o^{ex}=2.0\times(B_{ex}/B_{th})^{1/2}$

[c] Experimental (present work) isoscalar enhancement factors (ε_o) for electric-type inelastic transitions and normalization factors (N) for magnetic-type transitions.

[d] V_o was normalized so that $e_o^{ex}=\varepsilon_o^{ex}$ for this state.

Ref. 13 that using the bare-nucleon g factors in the M1 operator reproduces very well all known magnetic moments in the A = 17–25 region. The configuration mixing built into the wave functions is sufficient to reproduce the data. Thus the observation that the 10.713 MeV transition rate closely matches the experimental rate is expected on more general grounds.

The question whether the magnetic operators of higher multipolarities are renormalized, i.e. the question of magnetic core-polarization effects of higher multipolarities, remains largely open at present. The electromagnetic transition rates have not been determined for either the 5^+; T = 1 11.017 MeV or the suggested 3^+; T = 1 10.822 MeV states. The shapes of these two angular distributions are rather well reproduced by the microscopic DWBA (see Fig. 3). The normalization factor close to unity for the 10.822 MeV transition may indicate that little renormalization is involved. If the ground state electromagnetic transition rates were measured one could decide whether the transition to the 5^+; T = 1 state corroborates this conclusion. The normalization factor N = 3.1 would indicate that the shell-model underestimates this transition rate. The high resolution, high momentum-transfer (e,e') work just started[44] will hopefully yield the necessary data for such a comparison.

Once equivalence of the magnetic and magnetic-type nucleon inelastic operators has been established, the inelastic scattering of nucleons would be useful in testing the transition rates of magnetic isoscalar transitions. Determination of these transition rates through the inelastic electron scattering is not reliable since they are sensitive to small $T_> = 1$ admixtures. For example, the transition to the 1^+ 9.826 MeV state appears underpredicted by a factor of 30 (see Table I) if compared with the (e,e') data. Inspection of Figs. 2 and 3 indicates that both the 7.747 MeV and 9.826 MeV 1^+ inelastic cross sections are within a factor of 2 of the shell-model predictions. The same is true for the 3^+ 9.456 MeV transition.

D. Monopole Transitions

The E0 transitions are sensitive to the differences in the root-mean-square (rms) radii $\langle r^2\rangle_j \equiv \langle j|r^2|j\rangle$ of the single-particle orbits participating in the transition because the transition rates must vanish if all $\langle r^2\rangle_j$ are equal. A similar sensitivity may be expected for the E0-like nucleon-induced inelastic transitions. For the 6.432 and 9.305 MeV 0^+ states the results of calculations utilizing both the standard (solid line, see Sec. II) and modified (dashed line) single-particle wave functions are presented in Figs. 2 and 3. The modification consists of decreasing the binding energy of the $2s_{1/2}$ orbitals to

$E_b = 2.0$ MeV by modifying the binding potential. This increases the rms radius of the $2s_{1/2}$ orbital so that it becomes 1.5 fm larger than the average of the rms radii of the 1d orbitals. Such a difference between the rms radii of the $2s_{1/2}$ and 1d orbitals was found by B.A. Brown et al.[45] when these were treated as free parameters determined from the fit to the E0 matrix elements in the $24 \leq A \leq 30$ mass region. The increase of the rms radius of the $2s_{1/2}$ orbital was interpreted[45] as a manifestation of the monopole core polarization. The angular distribution calculated using the modified $2s_{1/2}$ orbit gives better agreement with the data at forward angles (see Figs. 2 and 3). In addition, a smaller, more reasonable normalization factor is required than is needed with the unmodified wave functions. There is a long standing opinion[46] that the excited 0^+ states possess complex structure, possibly containing large admixtures of multiparticle $(1f,2p)^2$, $(1f,2p)^4$ and/or multihole $(1p)^{-2}$, $(1p)^{-4}$ components. The rapid rise of the cross section towards zero degrees and much of the oscillatory structure of the data (Fig. 2) can probably be reproduced only when these $2\hbar\omega$ and $4\hbar\omega$ components are explicitly taken into account[47] in the description of the initial and final states.

V. CONCLUSIONS

The shell-model wave functions, calculated in the full sd-shell basis, for the positive parity states of ^{24}Mg have been tested by comparing the predicted and experimental cross sections for the inelastic scattering of 40 MeV protons. The microscopic DWBA calculations were performed with a realistic effective interaction derived by Bertsch et al.[19] from the free nucleon-nucleon interactions. Selected isoscalar, natural parity transitions were also studied with an empirical interaction. Exchange contributions to the cross sections were included in both types of calculation.

The properties of a number of states in ^{24}Mg have been examined to establish the correspondence between the model and experimental states in the upper ($E_x > 9$ MeV) part of the spectrum where many spin-parity assignments are lacking. Several states (4^+ 4.122 MeV member of the ground state rotational band, 3^+ 5.236 MeV and 5^+ 7.812 MeV members of the $K^\pi = 2^+$ band and 2^+ 8.654 MeV member of the excited $K^\pi = 0^+$ band) seem to be populated to a large extent via multistep processes. For the other states the one-step excitation mechanism seems to dominate, and it has been assumed that the wave functions and the two-body force determine the cross sections. For the natural parity states collective enhancement factors have been extracted. These are closely correlated with the effective charges involved in the electromagnetic transitions indicating that the microscopic DWBA tests shell-model wave functions to a degree similar, for example, to inelastic electron scattering. The noteworthy differences between the large-angle dependence of the experimental angular distributions for the 1.369 MeV and the 4.239 keV 2^+_+ states and the angular distribution for the 7.348 MeV 2^+ state, qualitatively accounted for by the present calculations, is attributed by the shell-model to the destructive interference of the s$\leftrightarrow$d and d$\leftrightarrow$d transitions in the surface region. The ratio of the total cross sections to the 1^+; T = 1 states at 9.965 MeV and 10.713 MeV which form the giant M1 resonance in this nucleus agrees with the ratio of the M1 electromagnetic excitation strengths. Bare-nucleon g factors and bare-nucleon two-body force account well for the 10.713 MeV excitation. The fact that the M1 operator is not renormalized in the shell-model space considered is expected on more general grounds. A normalization factor close to unity is observed also for the 10.822 MeV transition, which was tentatively assigned 3^+; T = 1, suggesting that there is little core polarization involved in the magnetic-type nucleon-induced inelastic transitions of multipolarities higher than M1. A parallel study of these transitions by electromagnetic means would be highly desirable to test whether in the case of the 5^+; T = 1 transition, which requires a proton inelastic normalization larger than unity, the electromagnetic rates are also underpredicted.

For the monopole transitions studied the higher excited multiparticle and/or multihole configuration admixtures probably need to be explicitly taken into account in order to satisfactorily predict the angular distributions.

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