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Antisymmetrization effects and the form factor of the real part of the α-nucleus potential*

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Antisymmetrization effects in the α-nucleus interaction are investigated on the basis of a microscopic model in an one nucleon exchange approximation. It influences the form factor, increasing the halfway radius and decreasing the diffuseness as compared with the direct term of the potential only. Antisymmetrization preserves the shape of the potential which can be parametrized by a Woods-Saxon squared form. The phenomenological potential with the energy independent form factor of the above shape fits experimental data in a wide energy region.

[NUCLEAR REACTIONS $^{90}Zr(\alpha,\alpha)$, E=26.2, 40, 59.1, 79.5, 99.5, 118, and 166 MeV. Microscopic optical model analysis with antisymmetrization.]

I. INTRODUCTION

Data on the elastic scattering of α particles from some nuclei taken in a broad energy range have recently become available. Phenomenological and/or semimicroscopic optical model calculations performed for some of these data assumed a linear energy dependence of the depth of the real part of the potential with the slope of about -0.26 MeV/MeV.[1] A nonlinear energy dependence of the parameters of the radial shape of the real potential was necessary in some cases.[2]

It is well known that a local phenomenological potential equivalent to a nonlocal one should show an energy dependence of its depth.[3] On the other hand it is difficult to explain the energy dependence of its radial shape which can also be influenced by the nonlocality of the interaction.

The nonlocality of the optical potential is mainly due to the effect of the antisymmetrization of the total wave function when exchange of nucleons between colliding nuclei is taken into account. This phenomenon can only be investigated on the basis of such theories as derivation of the real part of the optical potential from the elementary nucleon-nucleon interactions. This kind of "fully microscopic" model has been proposed previously.[4] Several attempts to introduce antisymmetrization effects into the microscopic model[5,6] were concerned with the scattering of α particles in a limited energy range only and did not yield any definite conclusions as to the shape of the real part of the optical potential.

The aim of this work is to investigate the influence of the exchange terms of the form-factor of the real part of the potential.

II. ANTISYMMETRIZATION PROCEDURE

The Schrödinger equation describing the elastic channel, after taking into account the antisymmetrization between the projectile and the target nucleus, can be reduced to the following integro-differential form:

$$[E_\alpha - \hat{T}_{\alpha\text{-}T}(r) - V^{d}_{\alpha\text{-}T}(r) - iW(r)]\,\varphi(r) = \int d\vec{r}\,'\, V^{ex}_{\alpha\text{-}T}(\vec{r},\vec{r}\,')\varphi(\vec{r}\,')\,, \quad (2.1)$$

where E_α denotes the kinetic energy of incoming particles, $\hat{T}_{\alpha\text{-}T}$ the kinetic energy operator, $V^{d}_{\alpha\text{-}T}(r)$ the direct term of the real potential, $V^{ex}_{\alpha\text{-}T}(\vec{r},\vec{r}\,')$ the nonlocal part of the potential resulting from the antisymmetrization, and $W(r)$ is the imaginary potential.

The direct part of the potential $V^{d}_{\alpha\text{-}T}(r)$ was calculated according to the double folding formula described previously:[4]

$$V^{d}_{\alpha\text{-}T}(r) = \int\int d\vec{z}\,' d\vec{z}\,'' \rho_\alpha(\vec{z}\,')\rho_T(\vec{z}\,'')t(\vec{r}_{NN})\,, \quad (2.2)$$

where ρ_α, ρ_T are matter density distributions in the α particle and the target nucleus, respectively, and $t(\vec{r}_{NN})$ is an effective nucleon-nucleon interaction. Internal coordinates $\vec{z}\,'$ and $\vec{z}\,''$ of the nucleons in the α particle and target nucleus, respectively, and the relative distance $\vec{r}_{NN} = \vec{r} + \vec{z}\,' - \vec{z}\,''$ between two interacting nucleons are defined in Fig. 1.

Since the exchange part of the potential $V^{ex}_{\alpha\text{-}T}$ cannot be exactly calculated for composite projectiles, it was obtained in an approximate way by taking into account the one nucleon exchange only.

It we consider for a moment any one nucleon

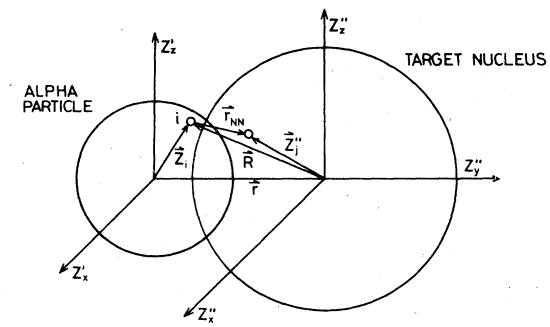

FIG. 1. Schematic representation of the coordinates of the α-target system.

from the α projectile (Fig. 1), the exchange potential term for the corresponding nucleon-target system can be written[7] as

$$\int d\vec{z}''\, V^{\mathrm{ex}}_{N-T}(\vec{R},\vec{z}'')\varphi(\vec{z}'') = \int d\vec{z}''\rho_T^{\mathrm{mix}}(\vec{R},\vec{z}'')t(\vec{R}-\vec{z}'')\varphi(\vec{z}'')\,, \quad (2.3)$$

where ρ_T^{mix} is a mixed density which can be calculated using a modification of the Slater approximation[8] due to Pandharipande:[9]

$$\rho_T^{\mathrm{mix}}(\vec{R},\vec{z}'') = [\rho_T(\vec{R})\rho_T(\vec{z}'')]^{1/2}\,\frac{3}{k_F r_{NN}} \times[\sin(k_F r_{NN}) - k_F r_{NN}\cos(k_F r_{NN})]\,, \quad (2.4)$$

where

$$k_F = \{\tfrac{3}{2}\pi^2\tfrac{1}{2}[\rho_T(\vec{R})+\rho_T(\vec{z}'')]\}^{1/3}\,. \quad (2.5)$$

In order to obtain the local equivalent approximation of the exchange potential (2.3), we take into account the short range of the two body interaction $t(\vec{R}-\vec{z}'')$, express the wave function of the relative motion in the form

$$\varphi(\vec{z}'') = \exp(r_{NN}\Delta)\varphi(\vec{R})\,. \quad (2.6)$$

Substituting (2.6) into Eq. (2.3) and performing the angular integration over the direction of $\vec{r}_{NN}$,[10] we obtain the local approximation in the form

$$d\vec{z}''\, V^{\mathrm{Ex}}_{N-T}(\vec{R},\vec{z}'')\varphi(\vec{z}'') = \varphi(\vec{R})\int d\vec{z}''\rho_T^{\mathrm{mix}}(\vec{R},\vec{z}'') \times t(\vec{R}-\vec{z}'')j_0(|\vec{r}_{NN}||\vec{k}|) = \varphi(\vec{R})V^{\mathrm{eq.\,ex}}_{N-T}(R)\,, \quad (2.7)$$

where k is the wave number of the incident nucleon taken at $\vec{R}_0 = \frac{1}{2}(\vec{R}+\vec{z}'')$.

As our incident nucleon is immersed in an α particle, its wave number k consists of two components, one corresponding to the average momentum in the α particle[22]

$$\langle k_{\mathrm{int}}\rangle = \left[\left\langle \frac{3}{5}\left(\frac{2m}{\hbar^2}\,\epsilon_F\right)\right\rangle\right]^{1/2} \quad (2.8)$$

and the other due to the motion in the field of the target nucleus for proton and neutron, respectively:

$$k_{p,\mathrm{ext}}(R_0) = \left\{\frac{2m}{\hbar^2}\,[\tfrac{1}{4}E_\alpha - V_{p-T}(R_0) + V_c(R_0)]\right\}^{1/2} \quad (2.9)$$

and

$$k_{n,\mathrm{ext}}(R_0) = \left\{\frac{2m}{\hbar^2}\,[\tfrac{1}{4}E_\alpha - V_{n-T}(R_0)]\right\}^{1/2}. \quad (2.10)$$

The total nucleon-target potential $V_{N-T}(R)$ contains both direct and exchange parts

$$V_{N-T}(R) = \int d\vec{z}''\rho_T(\vec{z}'')t(\vec{R}-\vec{z}'') + \frac{1}{2}\sum_{i=n,p}\int d\vec{z}''\,\rho_T^{\mathrm{mix}}(\vec{R},\vec{z}'')t(\vec{R}-\vec{z}'') \times j_0(|\vec{r}_{NN}||\vec{k}_i|)\,, \quad (2.11)$$

where $k_p^2 = k_{\mathrm{int}}^2 + k_{p,\mathrm{ext}}^2$ and $k_n^2 = k_{\mathrm{int}}^2 + k_{n,\mathrm{ext}}^2$ for proton and neutron, respectively. Its second part is energy dependent and should be calculated in a self-consistent way.

Folding V_{N-T} into the matter density distribution of the α particle, we obtain the total potential for the interaction of an α particle with a target nucleus[5,11]

$$V_{\alpha-T}(r) = \iint d\vec{z}'d\vec{z}''\,\rho_\alpha(\vec{z}')\rho_T(\vec{z}'')t(\vec{r}_{NN}) + \int d\vec{z}'\rho_\alpha(\vec{z}')\,\frac{1}{2}\sum_{i=n,p}\int d\vec{z}''\rho_T^{\mathrm{mix}}(\vec{R},\vec{z}'')t(\vec{R}-\vec{z}'')j_0(|\vec{r}_{NN}||\vec{k}_i|)]\,. \quad (2.12)$$

Calculations were made with the effective nucleon-nucleon interaction derived by Slanina and McManus[12] in the form

$$t(r_{NN}) = \left[\frac{Z}{A}\left(-3480\,\frac{\exp(-2.44r_{NN})}{2.44r_{NN}} + 4770\,\frac{\exp(-2.8r_{NN})}{2.8r_{NN}}\right) + \frac{N-Z}{A}\left(-237\,\frac{\exp(-1.7r_{NN})}{1.7r_{NN}} + 418\,\frac{\exp(-2.62r_{NN})}{2.62r_{NN}}\right)\right]\mathrm{MeV}\,. \quad (2.13)$$

TABLE I. Parameters of the direct potential (2.2) and total potential (2.12).

		U (MeV)	$D_1 A^{-1/3}$ (fm)	d_1 (fm)	$R_{1/2} A^{-1/3}$ (fm)	$t_{10\text{-}90}$ (fm)
Direct potential		183.78	1.288	1.284	1.035	4.736
Total	$E_\alpha = 46$ MeV	200.99	1.315	1.237	1.071	4.562
potential	$E_\alpha = 106$ MeV	193.37	1.310	1.249	1.064	4.606

The target matter point-density distribution ρ_T was taken to be of the Fermi shape with parameters obtained from electron scattering[13] and from the meson photoproduction.[14] The matter point-density distribution in the α particle ρ_α had the Gaussian form with parameters taken from the $\pi^\pm$ scattering.[15]

Direct (2.2) and total (2.12) α-target potentials are compared in Fig. 2. It can be seen from Table I that the halfway radius $R_{1/2}$ of the total potential is larger and the 10–90% distance $t_{10\text{-}90}$ is smaller than that of the direct potential.

Another important conclusion is that both the direct and the total potentials have a radial shape which is different from the Woods-Saxon form factor. They can be very well reproduced, however, by the square of the Woods-Saxon form factor,[16] which is also presented in Fig. 2.

It should be mentioned that the total α-target potential is deeper than the direct one, in agreement with the results of other authors.[5,6]

III. PHENOMENOLOGICAL REPRESENTATION

In order to represent the antisymmetrization effects in a phenomenological way the model potential was restricted to the direct term only with a suitable modification. As one can see from the preceding paragraph antisymmetrization effects influence the range and the diffuseness of the potential but they preserve its general radial shape. This shape can be well parametrized by the square of the Woods-Saxon form factor. This type of the form factor was recommended for phenomenological analyses in our earlier papers[17] and later proved in analyses made by other authors.[18]

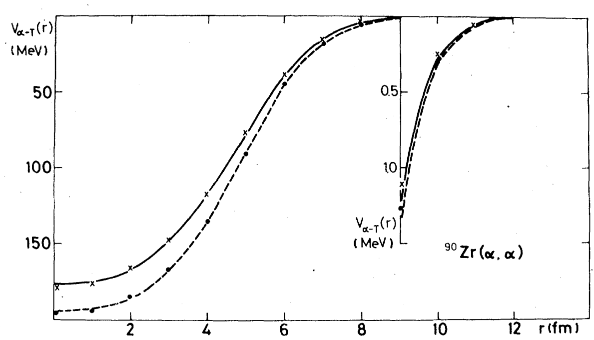

FIG. 2. The real part of the $^{90}Zr(\alpha,\alpha)$ potential. Direct potential (2.2) (solid lines) and total potential (2.12) (dashed line) for $E_\alpha = 46.0$ MeV. Crosses and dots represent fitted Woods-Saxon-squared potentials.

Data on elastic scattering of α particles from ^{90}Zr nuclei were chosen for the analysis since for this nucleus the most extensive measurements taken at the energies 26.2 MeV,[19] 40, 59.1, 79,5, 99.5, and 118 MeV,[20] and 166 MeV[27] are available.

The calculations were carried out using the α-target interaction in the form

$$V(r) = V_C(r) - Uf(r, D_1, d_1) - i\left[W_v f(r, D_2, d_2) - 4d_3 W_s \frac{d}{dr} f(r, D_3, d_3)\right], \tag{3.1}$$

where $V_C(r)$ is the Coulomb potential due to a uniformly charged sphere with a radius $R_C = 1.34 A^{-1/3}$ and

$$f(r, D_i, d_i) = \left[1 + \exp \frac{r - D_i}{d_i}\right]^{-2}. \tag{3.2}$$

The usual volume imaginary potential was supplemented by the surface term. This form of the imaginary potential was necessary to fit the $^{58,60}Ni(\alpha.\alpha)$ data in the large energy range scattering[28] and was accepted by analogy with the nucleon-nucleus calculations (see Ref. 30 and references quoted therein).

The energy dependence of the optical potential has been parametrized in the following way[28]:

$$U = A_1 + A_2 E_\alpha , \tag{3.3}$$

$$W_v = A_3 + A_4 \exp(-A_5 E_\alpha) , \tag{3.4}$$

$$W_s = A_6 + A_7 \exp(-A_8 E_\alpha) , \tag{3.5}$$

where E_α is the energy of the α particle. The parameters of the form factors were energy independent.

In the optical model analysis the GLOB code[29] was used. This code varied parameters of the model in order to minimize simultaneously the value of χ^2 for all of the angular distributions.

The halfway radius $R_{1/2}$ and the 10–90% distance

TABLE II. Optical model global best fit parameters.

Parameters	$^{90}Zr(\alpha,\alpha)$
A_1 (MeV)	177.17
A_2	−0.2189
$D_1A^{-1/3}$ (fm)	1.3854
d_1 (fm)	1.2259
A_3 (MeV)	20.49
A_4 (MeV)	−30.36
A_5 (MeV^{-1})	0.0222
$D_2A^{-1/3}$ (fm)	1.7333
d_2 (fm)	1.0009
A_6 (MeV)	0.7237
A_7 (MeV)	47.37
A_8 (MeV^{-1})	0.0184
$D_3A^{-1/3}$ (fm)	1.3261
d_3 (fm)	0.4068

$t_{10\text{-}20}$ describing the diffuseness of the potential in its surface part are connected with the parameters D and d in the following way:

$$R_{1/2}=r_{1/2}A_T^{1/3}=D+d\ln(\sqrt{2}-1)\,, \qquad (3.6)$$

$$t_{10\text{-}90}=d\ln[(\sqrt{10}-1)/(\sqrt{\tfrac{10}{9}}-1)]\,. \qquad (3.7)$$

A comparison of the experimental angular distributions for α-particle scattering with those calculated using the parameter set given in Table II is presented in Fig. 3 (solid line). The dashed line in Fig. 3 represents angular distributions calculated with the real part of the potential obtained from the formula (2.12) and the imaginary potential optimized by the search routine.

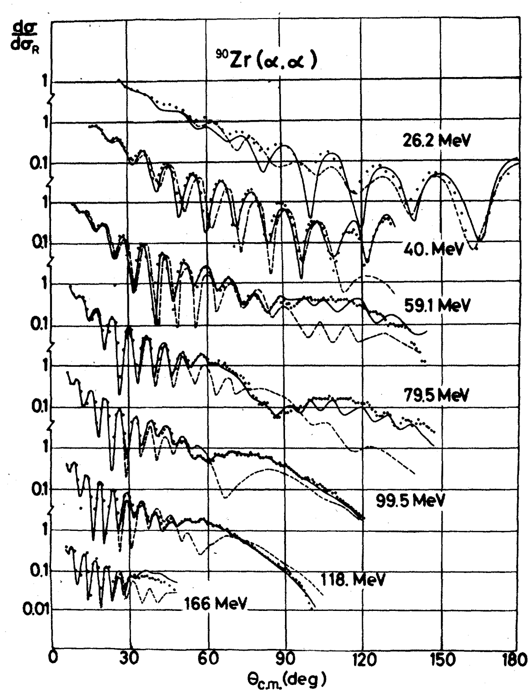

FIG. 3. Comparison of the experimental angular distributions for $^{90}Zr(\alpha,\alpha)$ scattering with distributions calculated with the optimum sets of parameters of the potentials given in Table II (solid line). The dashed line is explained in the text.

IV. DISCUSSION

Comparison of the phenomenological approach (Sec. III) with the results where one-nucleon exchange was involved (Sec. II) leads to the conclusion that the antisymmetrization effects are important in the whole energy region investigated. It can be seen from Tables I and II that the phenomenological best fit form factor has a larger radius and a smaller diffuseness than that of the microscopic direct part of the potential taken alone, 35% of the difference for the radius and 75% for the diffuseness can be accounted for by including the one-nucleon-exchange term.

The form factor of the microscopically derived potential varies slightly with the energy of the projectile. However, this variation is almost an order of magnitude smaller than differences between the parameters of the form factor of the direct potential and the parameters of the form factor of the total one. This energy dependence is too small to be found by phenomenological analysis. As was shown in Ref. 28, the best fit parameters do not indicate this systematic variation. Similar slight energy dependence of the form factor was found also for the nucleon-target interaction.[21]

The depth of the best fit potential decreases linearly with the energy of the α particle, according to the formula

$$U(E)=177(1-0.0012E_\alpha)\,. \qquad (4.1)$$

The parameters of this formula are in agreement with those found by other authors.[1] 60% of the value of the slope parameter of this energy dependence is explained by the one-nucleon-exchange term as can be seen from Table I.

There remains some discrepancy between the absolute depth of the potential calculated in the microscopic way and that obtained from fits to the experimental data. As can be seen from Tables I and II, the calculated microscopic potential is deeper than the one obtained in the phenomenological way. This is due to the fact that the phenomenological potential was obtained from fits to the experimental data which are sensitive to the outer region of the potential.[4] The difference can be removed when the density dependent effective nucleon-nucleon interaction[23] is used. The correction due to density dependence of the effective nucleon-nucleon interaction will be strongest for small distances between colliding nuclei.

As was shown by Thompson and Tang,[24] the inclusion of the antisymmetrization leads also to the appearance of the Majorana type interaction in the local phenomenological potential. It was shown by Kondo *et al.*[25] and by Majka[26] that the Majorana term mostly influences the angular distribution at the backward angles. Since the data used in this analysis do not cover that angular region at higher energies the Majorana term was neglected.

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