Evaluation of cross sections of the ^{6}Li(d,α)α reaction

R. G. Clark, H. Hora, P. S. Ray, and E. W. Titterton*
Department of Theoretical Physics, University of New South Wales, Kensington, Australia 2033
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Measured cross sections for the ^{6}Li(d,α)α reaction have been evaluated critically, covering the range 0.06–10 MeV and extrapolation carried out to the low energy region using Gamow plots. The reaction rate parameter $\langle\sigma v\rangle$ for thermonuclear reactions at equilibrium velocity distribution has been determined as a function of the plasma temperature T of the reacting nuclei from the experimental and extrapolated cross section values (T = 2 keV to 1.0 MeV).

[NUCLEAR REACTIONS ^{6}Li(d,α); E_d = 3 keV–1 MeV thermalized $\sigma(E_d)$ extrapolation, thermalized $\langle\sigma v\rangle$ optimized integration.]

I. INTRODUCTION

With the progress of inertial confinement for controlled thermonuclear reactions using compression by lasers[1-4] or particle beams,[5,6] the possibility of energy production not only from ^{2}H-^{3}H reactions, but also from ^{2}H-^{2}H and exotic reactions as ^{1}H-^{11}B or ^{2}H-^{6}Li and others are becoming interesting. One essential progress in this field is the discovery of the much shorter penetration of the charged reaction products in the hot and dense plasmas due to a collective model compared to the long ranges based on the Fokker-Planck equation.[7] The experimental proof is based on an agreement of the new theory with the penetration of 2 MeV electrons.[8] The resulting higher reheat for fusion reactions decreases the break even energies drastically at densities higher than 100 times the solid state, while ignition and self-burning results at surprisingly low temperatures (calculations by a general hydrodynamic code including reheat, depletion, and bremsstrahlung).[7,9]

For a better basis of the calculations of the ^{2}H-^{6}Li reaction branch leading to 2α with an energy release of 22.4 MeV, we have reexamined here the experimental cross sections. Using the best fitting theoretical plots, we have calculated the $\langle\sigma v\rangle$ values especially for low temperatures from $T=2$ keV, as there is evidence of a drastic decrease of the ignition conditions for self-burning.[9]

The question of resonances can be discussed on the basis of the result of Hirst, Johnstone, and Poole,[10] who reported no evidence for low energy resonances, in particular the 347 keV resonance previously reported by Whaling and Bonner.[11]

II. LOW ENERGY CROSS SECTION

Neglecting older data, we base our evaluation on the 90° differential cross sections reported by Hirst *et al.*[10] (E_D = 60–450 keV), as when converted to total cross section these data join smoothly to the higher energy results of McClenahan and Segel[12] as can be seen from Fig. 2.

To convert to total cross section, analysis was made of available reports on angular distribution. Good fits to angular distributions have been obtained using the Legendre polynomial expansion

$$Y(\theta)=Y(90°)[1+A(E)\cos^2\theta+B(E)\cos^4\theta].$$

It was found that $A(E)$ curves given by Antoufiev *et al.*[13] and Heydenburg *et al.*[14] are in good agreement for $E<1.75$ MeV. The large disagreement existing in their $B(E)$ curves does not effect the low energy angular distribution as both references find $B(E)=0$ for $E<1.25$ MeV. The $A(E)$ values given by Heydenburg for $E<450$ keV were used to convert to total cross section as follows:

$$\begin{aligned}\sigma&=\int Y(\theta)d\Omega\\&=2\pi Y(90°)\int_0^\pi[1+A(E)\cos^2\theta+B(E)\cos^4\theta]\sin\theta d\theta\\&=4\pi Y(90°)[1+A(E)/3+B(E)/5].\end{aligned}\qquad(1)$$

Using the values of A and B in the range of results reported by Hirst, we can use

$$\sigma=4\pi Y(90°)[1+A(E)/3].\qquad(2)$$

The small size of $A(E)$ values indicates that the reaction is virtually isotropic at low energy as found by Inglis[15] at $E=0.2$ MeV. It has been found theoretically[16] that, assuming the energies of the interacting nuclei are well below the top of the

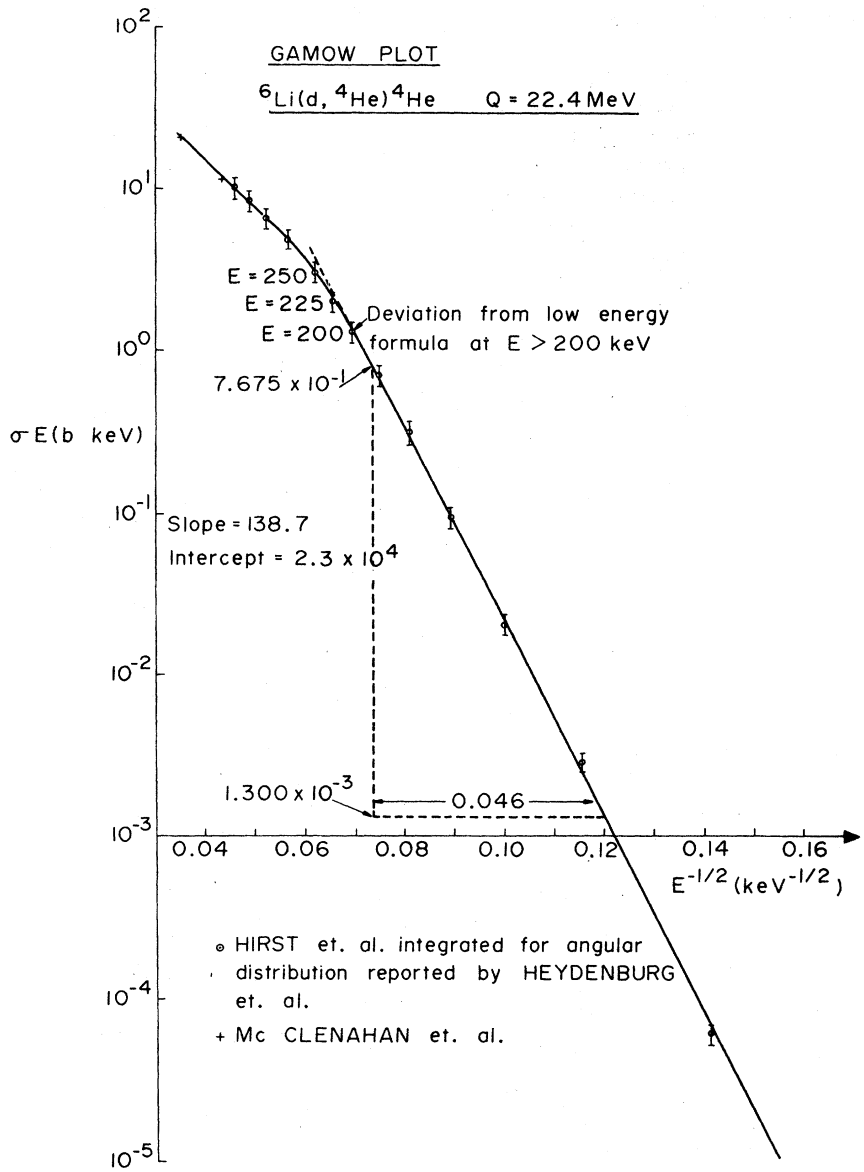

FIG. 1. Gamow plot for low energy cross sections σ of the ^{6}Li(d, α) α reaction, showing line of best fit, Eq. (5).

Coulomb barrier (height 0.28 Z_1Z_2 MeV) and further assuming zero relative angular momentum of reacting nuclei, the cross section for a bombarding charged particle mass m, energy E is given by the S wave Gamow form

$$\sigma(E) \approx \frac{S}{E} \exp \frac{-2^{3/2}\pi^2 m^{1/2} e^2 Z_1 Z_2}{hE^{1/2}} \quad (S = \text{constant}) \tag{3}$$

giving for the ^{2}H-^{6}Li reaction

$$\sigma(E) \approx S/E \exp(-133.2/E^{1/2}) \quad (E \text{ in keV}, \ \sigma \text{ in b}). \tag{4}$$

From the Gamow plot ($\ln\sigma E$ versus $E^{-1/2}$) (see Fig. 1) using total cross section calculated as in Eq. (2) from the data of Hirst *et al.* and Heydenburg *et al.*, it was found that experimental points were best-fitted by

$$\sigma(E) = \frac{2.3 \times 10^4}{E} \exp(-138.7/E^{1/2}) \tag{5}$$

in the energy range 60–200 keV. For $E > 200$ keV deviation from the Gamow plot straight line form Eq. (5) occurred as can be expected when the energy becomes less small in relation to the height of the Coulomb barrier. The departure of Eq. (5)

from the theoretical form of Eq. (4) is not very significant and is a consequence of the assumptions made in the theoretical derivation.

Extrapolation to energies below 60 keV can be carried out simply by extending the Gamow plot straight line form (5) down. However, as energy decreases and theoretical assumptions become increasingly valid, it is expected that the theoretical form (4) will apply. The problem is to decide at what energy this occurs. To this point reference is made to the experience found with the now well-known low energy ^{2}H-^{2}H cross section.[17] Theoretically this has the form

$$\sigma_{^2H^2H} = (S/E)\exp(-44.4/E^{1/2}), \qquad (6)$$

whereas from 13–100 keV ^{2}H-^{2}H experimental cross sections are best-fitted by

$$\sigma_{^2H^2H} = (288/E)\exp(-45.8/E^{1/2}). \qquad (7)$$

It is below 13 keV that the data are well represented by (6).

Converting this energy to a fraction of the height of the ^{2}H-^{2}H Coulomb barrier and applying this to the ^{2}H-^{6}Li reaction, it can be approximately inferred that the theoretical exponent will apply for $E < 40$ keV. The ^{2}H-^{6}Li cross section was consequently calculated to be well represented by the analytical forms (4) for $E = 1$ to 40 keV ($S = 9.63 \times 10^3$) and (5) for $E = 40$ to 200 keV.

III. TOTAL CROSS SECTION DATA

Summarizing the present status on total cross section data, the most recent reports for this reaction are given by McClenahan *et al.*[12] ($E_{^2H} = 0.5$–

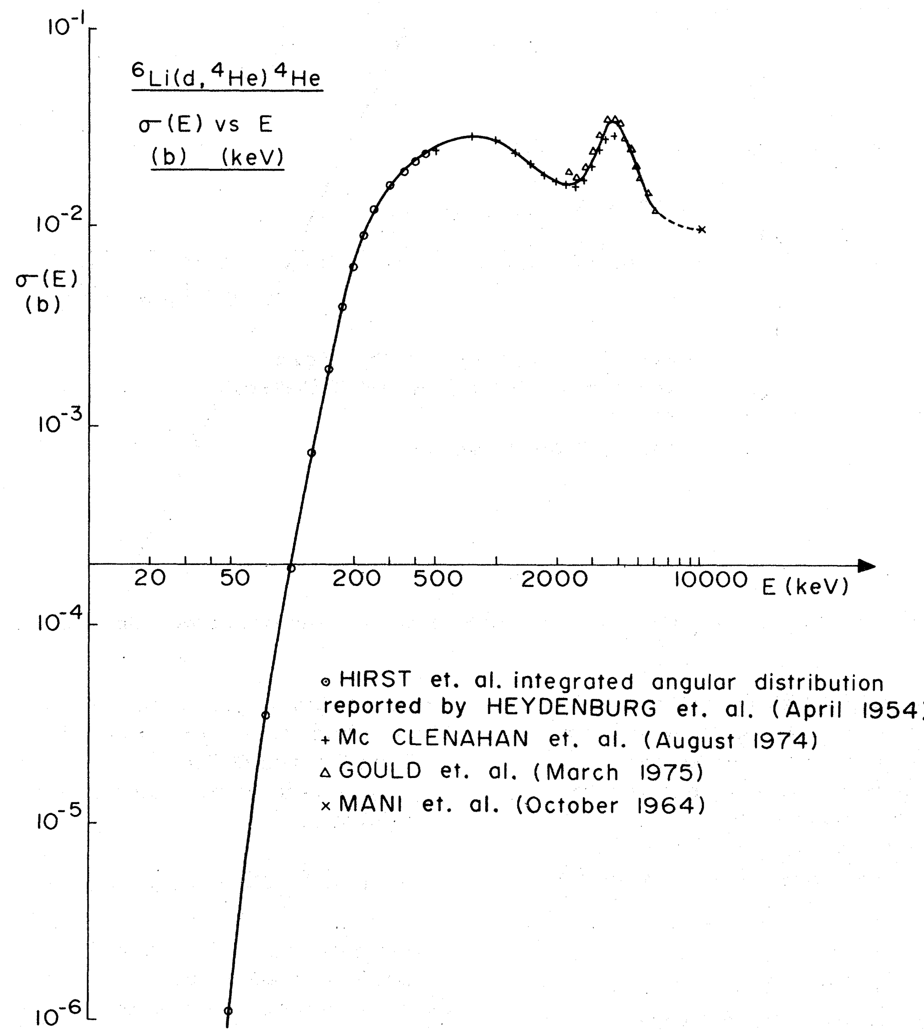

FIG. 2. Experimental values and best fit curves of cross sections σ of the ^{6}Li(d,α) α reaction.

3.4 MeV) and Gould and Joyce[18] ($E_{^2\mathrm{H}} = 2.25$–6.0 MeV). Both papers estimate errors at 15% and within this limit they show good agreement. Further, the total cross sections calculated from Hirst *et al.*[10] ($E_{^2\mathrm{H}} = 60$–450 keV) fit smoothly to the results of McClenahan. McClenahan also shows comparison with the total cross sections of Refs. 19–21, indicating agreement with Meyer, Pfeifer, and Staub[19] and disagreement with Bruno *et al.*,[20] and Jeronymo *et al.*[21] The shape of the cross section and angular distributions reported by Jeronymo were in agreement, however. These data have been used to establish a consistent cross section shown in Fig. 2. The high energy 9.5 MeV data point given by Mani *et al.*[22] has been included but must be regarded as purely approximate due to disagreement of their low energy results with those of McClenahan and Gould.

IV. REACTION RATE PARAMETER $\langle \sigma v \rangle$

Using the Maxwellian velocity distribution

$$dn = n(M/2\pi kT)^{3/2} \exp(-Mv^2/2kT)v^2 dv, \tag{8}$$

where dn is the number of particles relative to that of a given particle in the range v to $v+dv$, M being the reduced mass of interacting nuclei, the expression for (σv) can be derived from the relation[16]

$$\langle \sigma v \rangle = \frac{\int_0^\infty \sigma v dn}{\int_0^\infty dn}$$

leading to

$$\langle \sigma v \rangle = \frac{(8/\pi T)^{1/2} M^{3/2}}{m^2} \int_0^\infty \sigma \exp[-(M/m)(E/T)] \times (E/T) dE,$$

where m and E are the mass and energy of the bombarding nucleus in the laboratory system, respectively, and T is the "kinetic temperature" kT.

This integral has been evaluated numerically. Low energy cross section values were calculated from Eqs. (4) and (5) in the stated ranges of their validity. For energies greater than 200 keV the necessary cross section values were calculated by linear interpolation of input data point from the best-fit curves of Fig. 2. Linear interpolation is justified by the stated error of 15% in cross section values. A resulting plot of $\langle \sigma v \rangle$ versus T in the range $T = 2$ keV–1.0 MeV is shown in Fig. 3 in comparison with ^{2}H-^{2}H, ^{2}H-^{3}H and ^{2}H-^{3}He reaction rates. The accuracy of this plot will be increased for low values of T, which is the region of interest for controlled thermonuclear reactions.

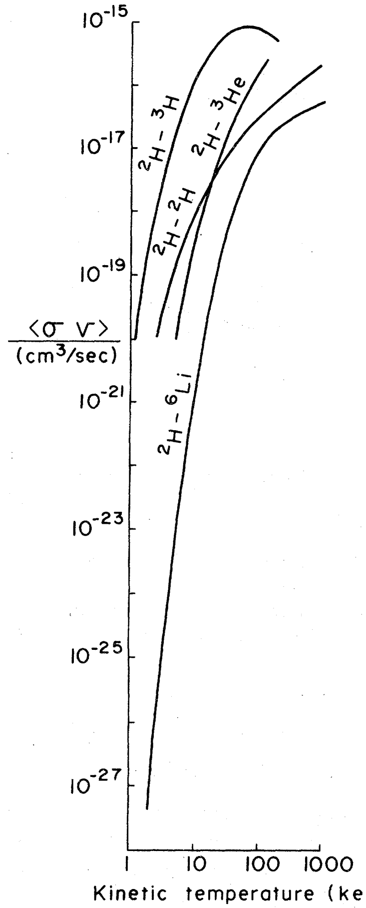

FIG. 3. $\langle \sigma v \rangle$ values calculated from cross sections σ derived from low energy Gamow extrapolations and best fit curves of Fig. 2 for the ^{6}Li$(d, \alpha)\,\alpha$ reaction. Comparison is shown with other fusion reactions.

V. DISCUSSION

Using Coulomb wave functions, Brennan[23] has derived a correction to the Gamow formula (3) expressed by a function $\phi(E)$ that allows a better approximation to cross section behavior at low energies. This has the form

$$\sigma(E) = 1/E \exp[\phi(E) - C/E^{1/2}], \tag{9}$$

where for the ^{2}H-^{6}Li reaction $C = 133.2$ (with σ in b, E in keV), E is the incident energy of the bombarding particle and it can be approximately assumed that $\phi(E) = K_1 + K_2 E$ (K_1, K_2 constant).[24] This correction is very small, however, in our case. A plot of $\phi(E)$ versus E using the cross section data of Hirst *et al.* integrated for angular distribution reported by Heydenburg *et al.* has been made for $E = 60$–200 keV (see Fig. 4) giving

$$\phi(E) = 9.18 + 0.0026E. \tag{10}$$

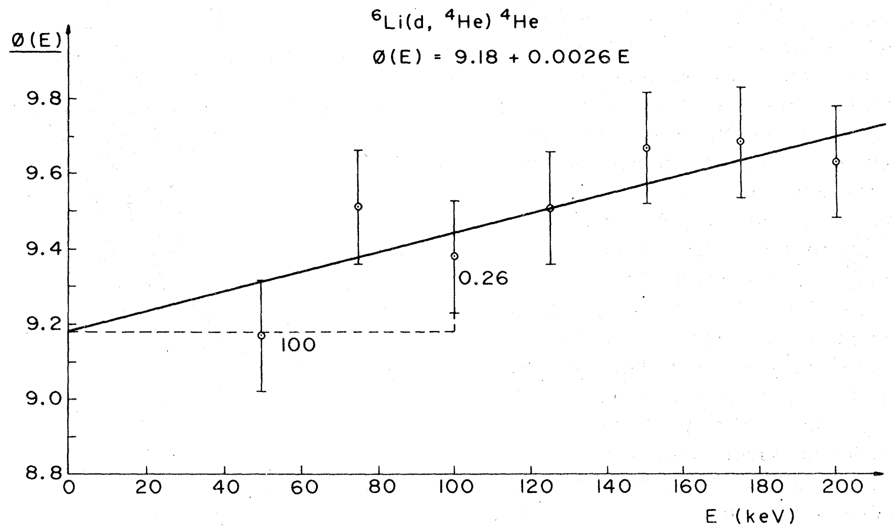

FIG. 4. Plot of $\phi(E)$ versus E to establish the low energy Brennan form Eq. (9) for the ^{6}Li(d, α) α reaction. Deviation of data points (calculated from the cross section results of Hirst integrated for Heydenburg angular distribution) from the line of best fit $\phi(E)=K_1+K_2E$ indicates the disparity of experimental error of data points with graphical scale required to evalute K_1 and K_2 and demonstrates that use of the Gamow plot (Fig. 1) is more realistic.

As such the Brennan approximation becomes

$$\sigma(E)=1/E\exp(9.18+0.0026E-133.2/E^{1/2}). \quad (11)$$

Considering the large experimental error in cross section values from which the $\phi(E)$ data points are deduced, and the consequent large deviation of some of these data points around the $\phi(E)$ straight line form (10) on the scale that has to be used to evaluate $\phi(E)$ with K_2 being so small, the accuracy of the determination of $\phi(E)$ is low. Our use of the Gamow form (5) as an approximation of Eq. (9) in extrapolating down to 40 keV and the consequent use of (4) for energies below this, as discussed in the text, is fully justified in terms of current experimental errors. Values of $\langle\sigma v\rangle$ obtained using (11) for $E=1-40$ keV in lieu of (4) are within $7\frac{1}{2}\%$ of those given in Fig. 3 at 2 keV and within $\frac{1}{2}\%$ at 5 keV. Indeed, the final result for the temperature dependence of the average cross section varies so strongly with temperatures that a few percent doubt in the cross section is not critical at this stage of reaction physics.

Previous calculations of $\langle\sigma v\rangle$ for this reaction have been carried out by Greene,[24] who used the low energy cross section data of Sawyer and Phillips[25] (30–250 keV) and the higher energy cross section data of Whaling and Bonner[26] (180–550 keV), Heydenburg *et al.*[14] (600–750 keV) and Jeronymo *et al.*[21] (0.92–4.6 MeV), assuming isotropy up to 550 keV. Comparison of these higher energy cross sections with the more recent results shown in Fig. 2 indicates the justification for the preference of the more recent data we have used here. The low energy cross section data of Sawyer is within 20% of the later results of Hirst. However, the results of Hirst fit more smoothly those of McClenahan and have been favored in our calculations.

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*Permanent address: Department of Nuclear Physics, Institute for Advanced Studies, Australian National University, Canberra, Australia, 2600.

[1]J. H. Nuckolls, in *Laser Interaction and Related Plasma Phenomena*, edited by H. Schwarz and H. Hora (Plenum, New York, 1974), Vol. 3B, p. 399.

[2]K. A. Brueckner (Ref. 1), p. 417.

[3]H. Hora, *Laser Plasmas and Nuclear Energy* (Plenum, New York, 1975); Sov. J. Quant. Electr. 6, 164 (1976); Nucl. Instrum. Methods 144, 18 (1977); R. W. B. Best, *ibid.* 144, 3 (1977).

[4]K. R. Manes, H. G. Ahlstrom, R. A. Haas, and J. F. Holzrichter, J. Opt. Soc. Am. 67, 717 (1977).

[5]F. Winterberg, Phys. Rev. 174, 212 (1968).

[6]J. H. Nuckolls, Opt. Comm. 18, 97 (1976).

[7]P. S. Ray and H. Hora, Nucl. Fusion 16, 535 (1976); Z. Naturforsch. 32A, 538 (1977); *Laser Interaction and Related Plasma Phenomena*, edited by H. Schwarz and H. Hora (Plenum, New York, 1977), Vol. 4B, p. 1081.

[8]E. Bagge and H. Hora, Atomkernenergie 24, 143 (1974).

[9]P. S. Ray and H. Hora, Z. Naturforsch. 33A, 976 (1978).

[10]F. Hirst, I. Johnstone, and M. J. Poole, Phil. Mag. 45, 762 (1954).

[11]W. Whaling, J. E. Evans, and T. W. Bonner, Phys. Rev. 75, 688 (1949).
[12]C. R. McClenahan and R. E. Segel, Phys. Rev. C 11, 370 (1975).
[13]Y. P. Antoufiev, M. M. El-Shesheni, H. R. Saas, Z. A. Saleh, and P. V. Sorokin, Nucl. Phys. 48, 299 (1963).
[14]N. P. Heydenburg, C. M. Hudson, D. R. Inglis, and W. D. Whitehead, Phys. Rev. 74, 405 (1948)
[15]D. R. Inglis, Phys. Rev. 74, 21 (1948).
[16]S. Gladstone and R. H. Lovberg, Controlled Thermonuclear Reactions (van Nostrand, Princeton, N.J., 1960).
[17]W. R. Arnold, J. A. Phillips, G. A. Sawyer, E. J. Stovall, and J. L. Tuck, Phys. Rev. 93, 483 (1954).
[18]C. R. Gould and J. M. Joyce, NBS Special Publication No. 425, 1975, Vol. II, p. 697.
[19]V. Meyer, W. Pfeifer, and H. H. Staub, Helv. Phys. Acta. 36, 465 (1963).
[20]G. Bruno, J. Decharge, A. Perrin, G. Surget, and C. Thibault, J. Phys. (N.Y.) 27, 517 (1966).
[21]J. M. F. Jeronymo, G. S. Mani, F. Picard, and A. Sadeghi, Nucl. Phys. 38, 11 (1962).
[22]G. S. Mani, R. M. Freeman, F. Picard, D. Redon, and A. Sadeghi, Proc. Phys. Soc. 85, 281 (1965).
[23]J. G. Brennan, Phys. Rev. 111, 1592 (1958).
[24]S. L. Greene, USAEC Report No. UCRL-70522, 1967 (unpublished).
[25]G. A. Sawyer and J. A. Phillips, Report No. LA-1578, 1953 (unpublished).
[26]W. Whaling and T. W. Bonner, Phys. Rev. 79, 258 (1950).