PHYSICAL REVIEW C VOLUME 18, NUMBER 3 SEPTEMBER 1978

Application of the quasiparticle formalism to neutron-deuteron scattering with local soft-core nucleon-nucleon potentials

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(Received 28 April 1978)

The quasiparticle approach which exactly reduces the three-body Faddeev formalism to an effective two-body theory is a general starting point for deriving approximation schemes. Practical methods, currently in use for solving the effective two-body equations, follow from it as special cases. A new procedure is proposed which is well suited for local nucleon-nucleon interactions containing repulsion. Its effectiveness is tested by calculating the triton binding energy and elastic neutron-deuteron scattering data for the soft-core potentials of Malfliet and Tjon. Particular emphasis is laid on extracting the influence of higher subsystem partial wave contributions of these potentials.

[NUCLEAR REACTIONS Three-body scattering theory, approximation methods and their convergence, numerical test in a three-nucleon model with local potential.]

I. INTRODUCTION

It is well known that the Faddeev equations[1] exhibit completely the basic principles underlying the three-body collision problem. They are, therefore, appropriate for mathematical investigations. Moreover, due to their uniqueness, they can be solved directly, though only with great numerical effort.

Other independent approaches[2,3] to the three-body problem emphasized the simplifying property of separable potentials, namely to lead to equations for quantities in which the internal momenta of the colliding clusters are integrated over. Such effective two-body equations which are of the multichannel Lippmann-Schwinger type put special emphasis on, and take into account explicitly, the composite particle aspect of the process under consideration, and are particularly suited for practical applications.

It was Lovelace[4] who exposed the relationship between both approaches by introducing separable subsystem amplitudes into the Faddeev equations. This procedure gave, via pole dominance arguments, a justification for considering separable potentials as a means to describe realistic composite particle collisions in an approximate but nevertheless relevant manner.

Generalizing these ideas it has been shown by Alt, Grassberger, and Sandhas[5] that the reduction of the Faddeev equations to effective two-body equations, which is so desirable from the physical and from the practical point of view, can be achieved without any approximation for arbitrary, in particular local, short-range potentials. (With some modifications this formalism is also applicable if, in addition, Coulomb forces are operative.[6]) This method (quasiparticle formalism), therefore, combines the generality of the Faddeev approach with the simplicity inherent in an effective two-body formulation of the three-body problem. The exact and closed form of the occurring effective potentials and Green functions represents an appropriate starting point for various approximation schemes, as e.g., the separable expansion method and the perturbative treatment of correction terms [quasi-Born approximations (QBA)]. This fact is being utilized in the following to derive a modification of the quasi-Born correction which is well suited for application to potentials containing repulsion.

The paper is organized as follows. In Sec. II we briefly recapitulate the quasiparticle concept. The latter is used in Sec. III for discussing the separable expansion method and the quasi-Born approximation from a unifying point of view. Their respective advantages are combined in a new approximation scheme which both is practical and provides a simple means for estimating the accuracy of the approximations. It is tested in Sec. IV in a three-nucleon calculation with the soft-core pair interactions of Malfliet and Tjon.[7] The numerical results for triton binding energy, neutron-deuteron scattering lengths and elastic cross sections are presented in Sec. V, and compared with those obtained by other methods. In Sec. VI

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we discuss in detail the contributions of the correction terms with particular emphasis on the influence of the interaction in higher subsystem angular momentum states.

II. THE QUASIPARTICLE APPROACH

One of the most powerful, physically motivated methods for applying the Faddeev equations[1], or the equivalent equations[5]

$$U_{\beta\alpha}(z)=\bar{\delta}_{\beta\alpha}G_0^{-1}(z)+\sum_\gamma \bar{\delta}_{\beta\gamma}T_\gamma(z)G_0(z)U_{\gamma\alpha}(z)\ , \tag{2.1}$$

is provided by reducing them to an effective two-body formalism. [We use the conventional notation $\bar{\delta}_{\beta\alpha}=(1-\delta_{\beta\alpha})$ and, for the free Green function, $G_0(z)=(z-H_0)^{-1}$.] To attain this goal, various approximation techniques have been proposed. All of them can easily be understood as special cases of the *exact* reduction scheme derived in Ref. 5, a fact which enables us to establish appropriate criteria for their respective accuracy. This scheme, moreover, represents a natural starting point for further approximation procedures, as will be exemplified in the next section.

We, therefore, begin by recalling the relevant formulas of the quasiparticle approach of Ref. 5. Let the given two-body transition operator T_γ be decomposed into two parts

$$T_\gamma=T_\gamma^s+T_\gamma' \quad (\gamma=1,2,3)\ . \tag{2.2}$$

Here T_γ^s consists of a sum of separable terms containing at least all bound-state and resonance poles of the γ subsystem,

$$T_\gamma^s=\sum_{r,s=1}^{N_\gamma}|\gamma r\rangle\Delta_{\gamma,rs}\langle\gamma s|\ , \tag{2.3}$$

and T_γ' is the nonpolar remainder. Note that the form factors $|\gamma r\rangle=|\gamma r;z\rangle$ in general depend on z (the explicit z dependence will, however, be suppressed in the following unless it is desired for clarity).

Introducing the splitting (2.2) into the Faddeev equations (2.1) leads to effective two-body equations of the Lippmann-Schwinger (LS) type, which in matrix notation read as

$$\underline{T}=\underline{V}+\underline{V}\underline{G}_0\underline{T}\ . \tag{2.4}$$

The effective amplitudes, potentials, and Green functions are defined by

$$\underline{T}_{\beta n,\alpha m}=\langle\beta n|G_0U_{\beta\alpha}G_0|\alpha m\rangle\ , \tag{2.5}$$

$$\underline{V}_{\beta n,\alpha m}=\langle\beta n|G_0U'_{\beta\alpha}G_0|\alpha m\rangle\ , \tag{2.6}$$

$$\underline{G}_{0;\beta n,\alpha m}=\delta_{\beta\alpha}\Delta_{\alpha,nm}\ , \tag{2.7}$$

with $U'_{\beta\alpha}$ fulfilling the Eq. (2.1) with T_γ replaced by T'_γ,

$$U'_{\beta\alpha}=\bar{\delta}_{\beta\alpha}G_0^{-1}+\sum_\gamma\bar{\delta}_{\beta\gamma}T'_\gamma G_0U'_{\gamma\alpha}\ . \tag{2.8}$$

The index n runs from 1 to N_β, the number of separable terms in T_β^s, and similarly m runs from 1 to N_α.

The decisive feature of this reduction is that all the operators (2.5)–(2.7) now act only on the plane wave states $|\vec{q}_\alpha\rangle$ describing the relative motion of the two-body cluster $|\alpha m\rangle$ and the third particle α. However, not all the quantities (2.5) are related to physical processes. In fact, only those amplitudes

$$\underline{T}_{\beta n,\alpha m}(\vec{q}'_\beta,\vec{q}_\alpha)=\langle\vec{q}'_\beta|\underline{T}_{\beta n,\alpha m}|\vec{q}_\alpha\rangle \tag{2.9}$$

describe the collision of an incoming two-body bound state $|\psi_{\alpha m}\rangle$ and a third particle α, leading to an outgoing configuration (β,n), for which on the energy shell $(z=E_{\alpha m}=E_{\beta n})$ the relation

$$G_0(E_{\alpha m})|\alpha m;E_{\alpha m}\rangle|\vec{q}_\alpha\rangle=|\psi_{\alpha m}\rangle|\vec{q}_\alpha\rangle\ , \tag{2.10}$$

and an analogous one for the final channel, holds.

Conventionally the splitting (2.2) of T_γ is based on a splitting of the corresponding potential[8] into a separable part and the rest

$$V_\gamma=V_\gamma^s+V_\gamma'=\sum_{r=1}^{N_\gamma}|\chi_{\gamma r}\rangle\lambda_{\gamma r}\langle\chi_{\gamma r}|+V_\gamma'\ . \tag{2.11}$$

In this case the "form factors" $|\gamma r\rangle$ of the amplitude (2.3) are related to the form factors $|\chi_{\gamma r}\rangle$ according to

$$|\gamma r\rangle=(1+T'_\gamma G_0)|\chi_{\gamma r}\rangle\ . \tag{2.12}$$

The elements of the matrix Δ_γ are determined by

$$(\Delta_\gamma^{-1})_{rs}=\delta_{rs}\lambda_{\gamma r}^{-1}-\langle\chi_{\gamma r}|(G_0+G_0T'_\gamma G_0)|\chi_{\gamma s}\rangle\ , \tag{2.13}$$

where T'_γ fulfils the LS equation

$$T'_\gamma=V'_\gamma+V'_\gamma G_0T'_\gamma\ . \tag{2.14}$$

From Eq. (2.12) it is apparent that there, as mentioned before, the form factor $|\gamma r\rangle$ depends on the energy z via $T'_\gamma(z)G_0(z)$ even if $|\chi_{\gamma r}\rangle$ is energy independent. We mention that it is advantageous to require for the form factors $|\chi_{\gamma r}\rangle$

$$\lambda_{\gamma r}\langle\chi_{\gamma r}|\psi_{\gamma s}\rangle=\delta_{rs},\quad \text{for } r=1,\ldots,N_\gamma \tag{2.15}$$

at the bound state energies $\hat{E}_{\gamma s}$, which guarantees that condition (2.10) is satisfied.

III. APPROXIMATION SCHEMES

Having recapitulated the reduction of the three-body equations to effective two-body equations in a completely general way, we are now going to describe commonly used approximation schemes as special cases of this formulation. Recalling the standard techniques from such a point of view, we are led to a new version which avoids their respective disadvantages.

A. Separable expansion method[9-15]

Choosing the number N_γ of separable terms in the decomposition (2.11) so large that V'_γ, and hence also T'_γ, can be neglected, Eqs. (2.12) and (2.8) simplify to

$$|\gamma r\rangle = |\chi_{\gamma r}\rangle \tag{3.1}$$

and

$$U'_{\beta\alpha} = \bar{\delta}_{\beta\alpha} G_0^{-1} , \tag{3.2}$$

respectively. The definition (2.6) of the effective potential, therefore, reduces to the familiar expression

$$\underline{V}_{\beta n,\,\alpha m} = \bar{\delta}_{\beta\alpha} \langle \chi_{\beta n} | G_0 | \chi_{\alpha m} \rangle . \tag{3.3}$$

It is graphically represented by the first diagram of Fig. 1. Similarly the inverse of the effective Green function (2.7) becomes

$$(\underline{G}_0^{-1})_{\beta n,\,\alpha m}{}^{-1} = \delta_{\beta\alpha}(\delta_{nm}\lambda_{\alpha m}{}^{-1} - \langle \chi_{\beta n} | G_0 | \chi_{\alpha m} \rangle) . \tag{3.4}$$

One well-known choice of the form factors $|\chi_{\gamma r}\rangle$, useful also for our later discussion, are the energy-dependent eigenfunctions of the two-body LS kernel (Sturmian functions),

$$V_\gamma G_0(z) | \chi_{\gamma r}(z) \rangle = \eta_{\gamma r}(z) | \chi_{\gamma r}(z) \rangle , \tag{3.5}$$

with $\eta_{\gamma r}$ being the corresponding eigenvalues. For such form factors, Eq. (2.3) simplifies to

$$T^s_\gamma(z) = \sum_{r=1}^{N_\gamma} | \chi_{\gamma r}(z) \rangle \lambda_{\gamma r}(z) [1 - \eta_{\gamma r}(z)]^{-1} \langle \chi_{\gamma r}(z) | , \tag{3.6}$$

with the strength parameter $\lambda_{\gamma r}(z)$ fixed by the condition

$$\lambda_{\gamma r}(z) \langle \chi_{\gamma r}(z) | G_0(z) | \chi_{\gamma s}(z) \rangle = \delta_{rs} \eta_{\gamma s}(z) . \tag{3.7}$$

Note that Eq. (3.7) reduces to (2.15) if the form factor $\chi_{\gamma s}(z)$ is related to a bound state wave function, and $z = \hat{E}_{\gamma s}$.

The advantage of having to deal with the simple driving terms (3.3), characteristic for the separable expansion method, has to be confronted with the disadvantage of a high dimension of the matrix equation (2.4). This becomes even more of a problem if higher partial wave contributions of V_γ are taken into account, because then a separable expansion has to be performed in all these angular momentum states. This shortcoming is avoided by the QBA procedure described in the following.

B. Quasi-Born approximation

The QBA[5] is complementary to the separable expansion method in the sense that the number N_γ of separable terms in T^s_γ [Eq. (2.3)], and hence also the dimension of the system (2.4), is kept as small as possible. However, in accordance with the discussion at the beginning of Sec. II, N_γ must not be smaller than the number of bound states and resonances in the subsystem γ. If the Hilbert-Schmidt expansion is used, the latter requirement means that the representation (3.6) of T^s_γ has to contain at least all those terms in which the *attractive* eigenvalues $\eta_{\gamma r}(z)$ (i.e., eigenvalues whose real parts are positive) come near to or even leave the unit circle in the energy region considered.

Then, in general, T'_γ is no longer negligible but may be taken into account perturbatively, i.e., by iterating Eq. (2.8)

$$U'_{\beta\alpha} = \bar{\delta}_{\beta\alpha} G_0^{-1} + \sum_\gamma \bar{\delta}_{\beta\gamma} \bar{\delta}_{\gamma\alpha} T'_\gamma + \cdots . \tag{3.8}$$

Inserting this expansion into the definition (2.6) and keeping in mind the relation (2.12), we obtain the quasi-Born series of the effective potential. The lowest order term $\underline{V}^{(0)}_{\beta n,\,\alpha m}$, called 0.QBA, is of the form (3.3), whereas in the next order (1.QBA) a correction term linear in T'_γ occurs:

$$\underline{V}^{(1)}_{\beta n,\,\alpha m} = \underline{V}^{(0)}_{\beta n,\,\alpha m} + \langle \chi_{\beta n} | G_0 \Big(\sum_\gamma T'_\gamma - \delta_{\beta\alpha} T'_\alpha \Big) G_0 | \chi_{\alpha m} \rangle . \tag{3.9}$$

The various terms which make up the 1.QBA are represented in diagrammatical form in Fig. 1.

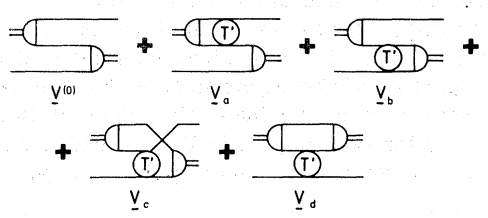

FIG. 1. Diagrammatical representation of the various terms of the effective potential (3.9) in 1.QBA. The first diagram $\underline{V}^{(0)}$ corresponds to the 0.QBA, and to the potential arising in the separable expansion method, Eq. (3.3).

We point out that the *additivity* of the correction terms in (3.9) has desirable consequences. For it enables us, after judging their magnitude relative to the driving terms $V^{(0)}$, to restrict ourselves to the important contributions when solving Eq. (2.4). Moreover, in this way a simple and practical criterion for estimating convergence is established.

Let us add some further comments. An essential aspect of the QBA is the *low dimension* of the matrix equation (2.4). However, care must be taken to get T'_γ small enough to guarantee sufficiently fast convergence of the quasi-Born series. For short-range potentials this is achieved with a number N_γ of separable terms in (2.3) which is much smaller than required by the separable expansion method. In the example of neutron-deuteron scattering discussed below, $N=1$ in every channel, accounting for the deuteron bound state and the singlet antibound state, turns out to be already sufficient. This fact shows another attractive feature of the QBA approach, namely that the majority of the occurring amplitudes is related to physically interpretable processes.

A further, very important advantage of the QBA is that *all subsystem partial waves* which have not been taken into account explicitly in T^s_γ enter the effective potential (3.9) automatically via T'_γ; e.g., if T^s_γ is chosen to act only in S states, then for the higher partial waves the rest amplitude $T'_{\gamma,l}$ is identical to the original $T_{\gamma,l}$ [compare the definition (2.2)]. Obviously this fact becomes decisive if many two-body angular momenta contribute nonnegligibly, as happens, e.g., in the case of the Coulomb repulsion in proton-deuteron scattering.[6]

C. Simple approximation of T'_γ in the effective potential

To calculate the 1.QBA (3.9) numerically is in many cases very cumbersome. Additional approximations are, therefore, often desirable.

We have already emphasized above that all *attractive* eigenvalues $\eta_{\gamma r}$ with $|\eta_{\gamma r}(z)| \geq 1$ have to be taken into account in the separable part V^s_γ of the splitting (2.11) for the quasi-Born series to converge. Thus, if the original potential V_γ does not support *repulsive* eigenvalues with absolute value near to or larger than unity, then the Born series for the rest amplitude T'_γ does converge. This fact suggests the further approximation

$$T'_\gamma \sim V'_\gamma \,, \tag{3.10}$$

which leads to manageable effective potentials

$$\underline{V}^{(1)}_{\beta n,\alpha m} = \underline{V}^{(0)}_{\beta n,\alpha m} + \sum_\gamma (1-\delta_{\beta\alpha}\delta_{\gamma\alpha})\langle \chi_{\beta n}| G_0 V'_\gamma G_0 |\chi_{\alpha m}\rangle \,, \tag{3.11}$$

and Green functions. They have been used successfully in model calculations for purely attractive Yukawa potentials.[17-22] (Equation (3.11) coincides with the exact first order formula arising in the second version of the quasiparticle method.[16])

However, potentials V_γ containing strong repulsion support also large repulsive eigenvalues, so that (3.10) is no longer justified. In this case we may proceed, e.g., by adding the separable terms corresponding to these repulsive eigenvalues to V^s_γ in Eq. (2.11). Thereby N_γ, and consequently also the number of coupled equations (2.4), increases again. This drawback is avoided in the subsequent approximation.

D. Improved approximation scheme

The method proposed here and applied in the following combines the advantages of the treatments discussed above. It is again based on the quasi-Born approach. But, instead of approximating T'_γ in the effective potential (3.9) by V'_γ which, as discussed in Sec. III C, is justified only in special situations, we split from the rest amplitude T'_γ *further* separable terms

$$T'_\gamma = T'^s_\gamma + T''_\gamma \tag{3.12}$$

before approximating the remaining rest T''_γ by its Born term.

In detail this may be done in complete analogy to (2.11), i.e., by splitting from V'_γ additional separable terms,

$$V'_\gamma = \sum_{r=N_\gamma+1}^{M_\gamma} |\chi_{\gamma r}\rangle \lambda_{\gamma r} \langle \chi_{\gamma r}| + V''_\gamma$$

$$= V'^s_\gamma + V''_\gamma \,. \tag{3.13}$$

We then obtain [compare (2.3), (2.12), and (2.13)]

$$T'^s_\gamma = \sum_{r,s=N_\gamma+1}^{M_\gamma} (1+T''_\gamma G_0)|\chi_{\gamma r}\rangle \Delta'_{\gamma,rs} \langle \chi_{\gamma s}|(1+G_0 T''_\gamma) \tag{3.14}$$

with

$$(\Delta'^{-1}_\gamma)_{rs} = \delta_{rs}\lambda_{\gamma r}^{-1} - \langle \chi_{\gamma r}| G_0 + G_0 T''_\gamma G_0 |\chi_{\gamma s}\rangle \,. \tag{3.15}$$

By choosing the number $(M_\gamma - N_\gamma)$ of separable terms in (3.13) large enough, the rest amplitude T''_γ defined by the LS equation

$$T''_\gamma = V''_\gamma + V''_\gamma G_0 T''_\gamma \tag{3.16}$$

can be neglected in (3.14) and (3.15), and may be replaced by its Born approximation in the second term of (3.12),

$$T_\gamma'' \sim V_\gamma'' \ . \tag{3.17}$$

[When $(M_\gamma - N_\gamma)$ is made so large that T_γ'' can be neglected completely in (3.12) and (3.14), we arrive at an alternative to the separable expansion method which avoids blowing up the dimension of the matrix equation (2.4). It has been applied successfully in Refs. 23, 24.] Then T_γ' takes the form

$$T_\gamma' = \sum_{r,s=N_\gamma+1}^{M_\gamma} |\chi_{\gamma r}\rangle \Delta_{\gamma,rs}'^{(0)} \langle \chi_{\gamma s}| + V_\gamma'' = V_\gamma' + \sum_{r,s=N_\gamma+1}^{M_\gamma} |\chi_{\gamma r}\rangle (\Delta_{\gamma,rs}'^{(0)} - \delta_{rs}\lambda_{\gamma r}) \langle \chi_{\gamma s}| \ , \tag{3.18}$$

where $\Delta_{\gamma,rs}'^{(0)}$ is given by (3.15) with $T_\gamma'' = 0$.

As compared to the simple approximation (3.10), additional terms occur in (3.18) which explicitly take into account the effect of the $(M_\gamma - N_\gamma)$ large eigenvalues (see in this respect Sec. III E). Inserting (3.18) in (3.9) we now get, instead of (3.11),

$$\underline{V}^{(1)}_{\beta n,\alpha m} = \underline{V}^{(0)}_{\beta n,\alpha m} + \sum_\gamma (1 - \delta_{\beta\alpha}\delta_{\gamma\alpha}) \sum_{r,s=N_\gamma+1}^{M_\gamma} \langle \chi_{\beta n}|G_0|\chi_{\gamma r}\rangle \Delta_{\gamma,rs}'^{(0)} \langle \chi_{\gamma s}|G_0|\chi_{\alpha m}\rangle + \sum_\gamma (1 - \delta_{\beta\alpha}\delta_{\gamma\alpha}) \langle \chi_{\beta n}|G_0 V_\gamma'' G_0|\chi_{\alpha m}\rangle \ . \tag{3.19}$$

If the second representation (3.18) of T_γ' is used, we obtain the correction terms in a form particularly suited for the following numerical considerations,

$$\underline{V}^{(1)}_{\beta n,\alpha m} = \underline{V}^{(0)}_{\beta n,\alpha m} + \sum_\gamma (1 - \delta_{\beta\alpha}\delta_{\gamma\alpha}) \langle \chi_{\beta n}|G_0 V_\gamma' G_0|\chi_{\alpha m}\rangle + \sum_\gamma \sum_{r,s=N_\gamma+1}^{M_\gamma} (1 - \delta_{\beta\alpha}\delta_{\gamma\alpha}) \langle \chi_{\beta n}|G_0|\chi_{\gamma r}\rangle (\Delta_{\gamma,rs}'^{(0)} - \delta_{rs}\lambda_{\gamma r}) \langle \chi_{\gamma s}|G_0|\chi_{\alpha m}\rangle \ . \tag{3.20}$$

Note that the first two terms on the right-hand side of (3.20) coincide with the expressions (3.11).

Summarizing, we therefore conclude that by the present approach the simple but often not satisfying approximation (3.11) is improved with little additional work. For, the added terms are not much more complicated than the effective potential $\underline{V}^{(0)}$ of the separable expansion method. Nonetheless, the advantages of the QBA, not to blow up the dimension of the integral equation (2.4), and to automatically incorporate the higher partial wave contributions of the potential V_γ via V_γ'' or V_γ', are preserved.

E. Question of convergence

For the investigation of the convergence of the methods discussed above, it is advantageous to choose Sturmian functions as defined in (3.5), with $\lambda_{\gamma r} = -\eta_{\gamma r}$ (ideal choice[8]).

In the separable expansion method (Sec. III A) the adequate convergence criterion is to choose the number N_γ of separable terms in (2.11) so large that T_γ' in (2.2) can be neglected. According to (3.6) this is achieved if

$$|\eta_{\gamma r}(z)[1 - \eta_{\gamma r}(z)]^{-1}| \ll 1, \quad \text{for } r > N_\gamma \ , \tag{3.21}$$

in the energy region under consideration. As compared to the conventional assumption

$$|\eta_{\gamma r}(z)| \ll 1, \quad \text{for } r > N_\gamma \ , \tag{3.22}$$

requirement (3.21) gives different weight to attractive and repulsive eigenvalues.

Conditions (3.21) and (3.22) both represent criteria on the two-body level, but they neglect the three-body context in which they are used. This shortcoming is avoided by starting from the quasi-Born expression (3.19) which in the "ideal choice" reads

$$\underline{V}^{(1)}_{\beta n,\alpha m} = \underline{V}^{(0)}_{\beta n,\alpha m} + \sum_\gamma (1 - \delta_{\beta\alpha}\delta_{\gamma\alpha}) \sum_{r=N_\gamma+1}^{M_\gamma} \langle \chi_{\beta n}|G_0|\chi_{\gamma r}\rangle \frac{-\eta_{\gamma r}}{1 - \eta_{\gamma r}} \langle \chi_{\gamma r}|G_0|\chi_{\alpha m}\rangle + \sum_\gamma (1 - \delta_{\beta\alpha}\delta_{\gamma\alpha}) \langle \chi_{\beta n}|G_0 V_\gamma'' G_0|\chi_{\alpha m}\rangle \ . \tag{3.23}$$

This representation shows that besides (3.21) two additional aspects are relevant for the convergence. First, the magnitude of the products $\langle\chi_{\beta n}|G_0|\chi_{\gamma r}\rangle\langle\chi_{\gamma r}|G_0|\chi_{\alpha m}\rangle$ appearing in the second term on the r.h.s. of (3.23) decisively influences the size of the correction relative to the driving term $\underline{V}^{(0)}$. Second, cancellations occur in general if the $(M_\gamma - N_\gamma)$ eigenvalues are partly attractive and partly repulsive. Calculating the triton binding energy and the doublet scattering length in a model based on the S-wave part of the Malfliet-Tjon (MT) potential the practical importance of these aspects has been demonstrated previously.[24]

In the following numerical investigation we incorporate also the third term in expression (3.23) which is mainly determined by the higher subsystem partial wave contributions. Again the additivity of the correction terms allows us to judge their respective importance.

IV. MODEL FOR NEUTRON-DEUTERON SCATTERING

In order to test the method proposed in Sec. III D we investigate the soft-core potentials of Malfliet and Tjon[7]

$$V_n(r) = g_n^A \exp(-\mu_n^A r)/r - g_n^R \exp(-\mu_n^R r)/r \quad (4.1)$$

($n = d, s$ for the triplet $^3S_1$ and the singlet $^1S_0$ state). Their S-wave parts have been widely used in three-nucleon calculations by means of Padé,[26] and separable expansion (UPE)[27,28] techniques, and of pole approximations (UPA),[27,29,30] applied to the Faddeev equations. Some results have been presented already in Ref. 25.

When applying the quasi-Born approach we have to take into account in the separable part V_γ^s of Eq. (2.11) the effect of the large attractive eigenvalues corresponding to the deuteron and the singlet S state; i.e., denoting these two eigenvalues by η_n ($n = d, s$) and the corresponding UPA eigenfunctions by $|\chi_n\rangle$ we have

$$V_n^s = -|\chi_n\rangle\eta_n\langle\chi_n| \ . \quad (4.2)$$

In contrast to the purely attractive Yukawa potentials which could be handled in Refs. 17–22 by the method discussed in Sec. III C, the MT potentials are known to have in both the triplet and the singlet channel one repulsive eigenvalue with absolute value becoming larger than 1. Therefore, the improved approximation of Sec. III D has to be used, with the sum over r and s in Eq. (3.20) reducing to a single term.

The two attractive and the two repulsive form factors $\chi_n(\vec{p}) = \langle\vec{p}|\chi_n\rangle$ needed have been given in Ref. 27 in the form[31]

$$\chi_n(\vec{p}) = \sum_{\nu=1}^{N} \frac{C_\nu^n}{\vec{p}^2 + (\beta_\nu^n)^2} \ , \quad \beta_\nu^n = \kappa_n + \nu/\alpha \ , \quad (4.3)$$

with $N = 15$. We refer to this work for the values of the constants C_ν^n, κ_n, and α. However, these expressions were not appropriate for use in the quasiparticle method due to the enormous amount of computer time which would be needed. They have, therefore, been fitted to the same analytical expressions (4.3) but now allowing also the constants β_ν^n to be varied. Satisfactory fits could be achieved in this way with $N = 4$.

Since our nucleon-nucleon potentials are purely central, the total spin S of the three-nucleon system, together with the total isospin I and their corresponding three-components, are conserved. Therefore the equations to be solved reduce, after symmetrization, to the system of integral equations

$$\underline{T}_{nm}^{SI}(\vec{q}', \vec{q}) = \underline{V}_{nm}^{SI}(\vec{q}', \vec{q}) + \sum_{r,s} \int d^3q'' \, \underline{V}_{nr}^{SI}(\vec{q}', \vec{q}'') \times \underline{G}_{0;rs}(q''^2) \underline{T}_{sm}^{SI}(\vec{q}'', \vec{q}) \ , \quad (4.4)$$

where n, m, and r take only the values d and s. Details of the effective potential $\underline{V}^{SI}$ are given in the Appendix. Its graphical representation is shown in Fig. 1. We stress once more that Eqs. (4.4), albeit being valid for *local* potentials, have in common with the equations of the simple separable potential model (= 0.QBA) that they are twice coupled for $S = \frac{1}{2}$, and reduce to a single one for $S = \frac{3}{2}$, and that they become one-dimensional after partial wave decomposition.

V. NUMERICAL RESULTS

The Eqs. (4.4) have been transformed into matrix equations by the Gauss-Legendre quadrature rules and solved numerically by matrix inversion. For the triton binding energy this amounted to the evaluation of a zero of the determinant of $(1 - \underline{V}\underline{G}_0)$. In order to solve the scattering equations in the elastic region we used the same procedure as in a previous calculation. It consists in transforming the system of singular integral equations via a matrix version[21] of the Kowalski-Noyes[32] trick into a system of nonsingular integral equations plus some singular integrals. In this way the accuracy obtained for a fixed number of integration points is much higher than without this transformation. From variation of the number of mesh points we estimate the accuracy of our results to be better than $\frac{1}{2}\%$.

In Table I we present our results in 1.QBA for the triton binding energy B_t, and for the doublet (^2a) and the quartet (^4a) scattering lengths. For

TABLE I. Triton binding energy, and doublet and quartet scattering lengths in 1.QBA for the MT potentials. Comparison is made with the 0.QBA and with results obtained with different methods which use only the S-wave parts of the interactions.

	1.QBA	S waves only	0.QBA
B_t (MeV)	8.72	8.3 [a] 8.58 [b] 8.4 [c] 8.56 [d]	8.63
^2a (fm)	0.71	0.9 [a] 0.90 [b] 0.82 [c] 1.2 [d]	0.94
^4a (fm)	6.38	6.35 [a] 6.387 [b] 6.44 [c] 6.37 [d]	6.394

[a] Reference 7.
[b] Reference 27.
[c] Reference 28.
[d] Reference 33.

comparison we include the corresponding values obtained by using only the S-wave part of the local MT potentials (via Padé[7] and separable expansion techniques[27,28,33] and the pole approximation to it (0.QBA = separable potential model = UPA).

For B_t and ^2a it has been shown previously[24,34] that the present formalism, when applied to the *S-wave* part of the MT potentials, reproduces the UPE results to high accuracy. This has two important consequences. First of all it implies that the higher than first order QBA terms are practically negligible. Second, it follows that the difference between the UPE results of Table I and the 1.QBA for the full MT potentials are due to the higher partial waves of the latter arising from V_γ'' in Eq. (3.19)

Let us add the remark that our 1.QBA value for the triton binding energy is in good agreement with a calculation employing the hyperspherical approach,[35] which gives $B_t = 8.77$ MeV.

In Fig. 2 we have plotted the doublet contribution to the differential cross section $d\sigma^{(1/2)}/d\Omega$ for neutron-deutron scattering, at a neutron laboratory energy E_n^L of 2.45 MeV, in 0.QBA and 1.QBA for the MT potentials. For comparison we also show the result obtained by using simple S-wave Yamaguchi potentials fitted to the same low-energy nucleon-nucleon data. The latter, being more attractive, yield a lower cross section, in particular in the forward direction. In Table II we compare the four lowest scattering phase shifts in 0.QBA and 1.QBA with those of Kloet and Tjon (KT), for $E_n^L = 3.27$ MeV. Recalling that their calculation uses only the S-wave part of the MT potentials, the good agreement with the 1.QBA, which automatically incorporates the higher subsystem partial waves, allows us to infer that these produce almost negligible effects.

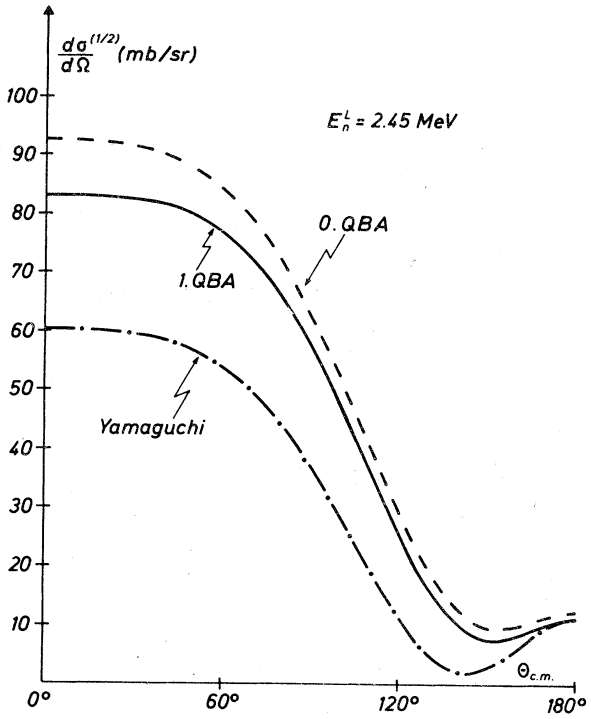

FIG. 2. Doublet contribution to the differential cross sections in 0.QBA and in 1.QBA for the MT potentials. For comparison the result for a Yamaguchi potential is included.

In Fig. 3 we present the quartet contribution to the differential cross section for $E_n^L = 2.45$ MeV, in 0.QBA and 1.QBA for the MT potentials and for a Yamaguchi potential. As for $S = \frac{1}{2}$, also here the latter yields a cross section which is lower in forward and backward direction. Table III contains the dominant quartet phase shifts at a neutron laboratory energy of 3.27 MeV. Evidently the 1.QBA corrections originating from the other subsystem S-wave eigenvalues (compare 0.QBA and KT) as well as those from the higher subsystem partial waves (compare KT and 1.QBA)

TABLE II. Neutron-deuteron doublet phase shifts at 3.27 MeV for the MT potentials. Values are in degrees.

$^{2S+1}\delta_L$	0.QBA	1.QBA	Kloet, Tjon[26]
$^2\delta_0$	144.6	146.6	146.4
$^2\delta_1$	−6.19	−5.97	−6.0
$^2\delta_2$	2.63	2.71	2.66
$^2\delta_3$	−0.53	−0.53	−0.53

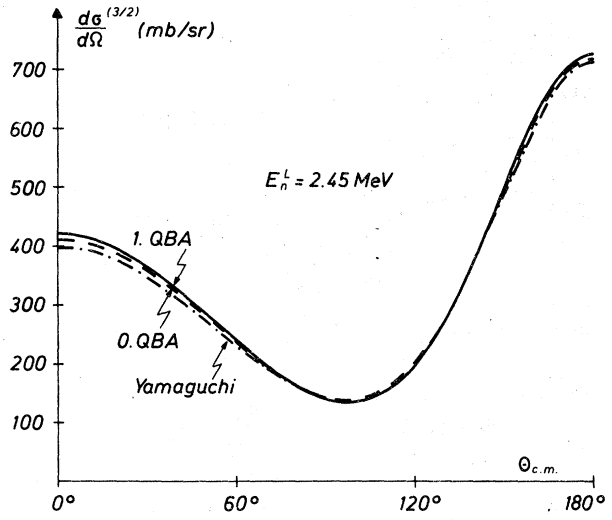

FIG. 3. As in Fig. 2, but for the quartet channel.

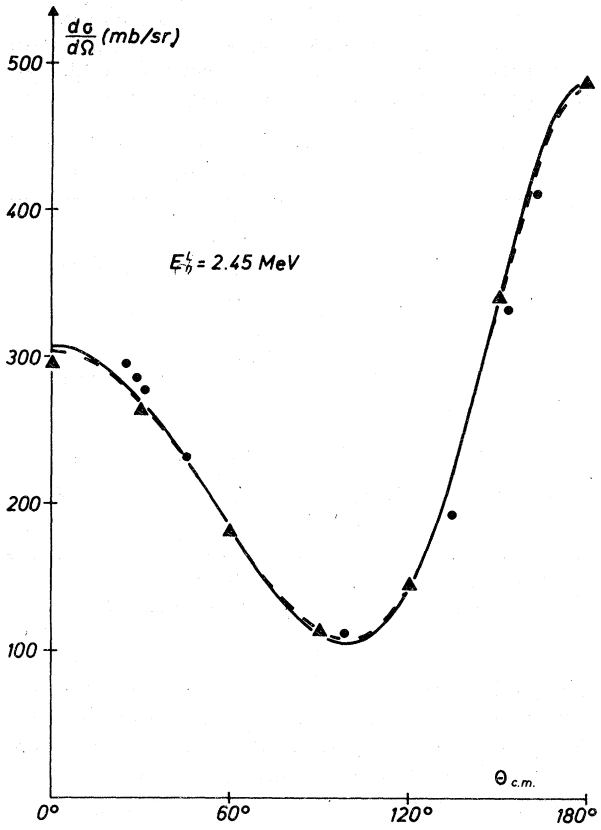

FIG. 4. Full differential cross section for neutron-deuteron scattering in 0.QBA (dashed line) and 1.QBA (full line). Also shown are the calculated values of Ref. 26 (triangles) and the experimental results of Ref. 36 (dots).

are small. The same conclusion holds for the full cross sections (Figs. 4 and 5), too. Since, therefore, the latter show only a small sensitivity, no reduction of the discrepancy between theory and experiment in the forward direction is achieved by incorporating the $l>0$ partial waves of the MT potentials, at least at the energies considered.

VI. DISCUSSION

Let us first investigate the relative contributions of the various graphs of Fig. 1, to three-nucleon quantities. This will be exemplified by means of the most important doublet and quartet phase shifts at $E_n^L = 2.45$ MeV. In Fig. 6 we have displayed in graphical form what happens when we start in Eq. (4.4) with the potential $\underline{V}^{(0)}$ and add to it successively the contributions of $(\underline{V}_a + \underline{V}_b)$, then of $\underline{V}_c$ and, finally, of $\underline{V}_d$, thereby producing the 1.QBA results. $\underline{V}_0$ is the same effective potential which produces the 0.QBA. However, the effective propagator used here is the one calculated in 1.QBA [Eq. (2.13) with T'_γ as shown in Eq. (3.18)]. Inspection of this figure reveals some interesting aspects.

TABLE III. Neutron-deuteron quartet phase shifts at 3.27 MeV for the MT potentials. Values are in degrees.

$^{2S+1}\delta_L$	0.QBA	1.QBA	Kloet, Tjon[26]
$^4\delta_0$	106.6	106.8	107.6
$^4\delta_1$	26.9	27.36	27.3
$^4\delta_2$	−4.66	−4.64	−4.66
$^4\delta_3$	1.08	1.09	1.09

(I) The doublet phase shifts for given angular momentum are more strongly influenced than the corresponding quartet phase shifts.
(II) The doublet and quartet P-wave phase shifts are most sensitive to the various parts of the effective potential.
(III) The various graphs contribute differently to the S-wave phase shifts and to the ones with $L>0$. In particular, the largest effect for $^2\delta_0$ and $^4\delta_0$ originates from $\underline{V}_a$ and $\underline{V}_b$ which is then canceled in part by the contributions of $\underline{V}_c$ and $\underline{V}_d$ both of which have the same sign. In contrast, for the doublet and quartet P- and D-wave phase shifts, where the two terms $\underline{V}_a$ and $\underline{V}_b$ are less important, the term $\underline{V}_d$ gives the largest contribution, which, however, is reduced appreciably by the one of $\underline{V}_c$.

Thus the relatively small 1.QBA corrections of the phase shifts, leading to the small corrections of the cross sections observed in Sec. V, originate in a complicated cancellation mechanism.

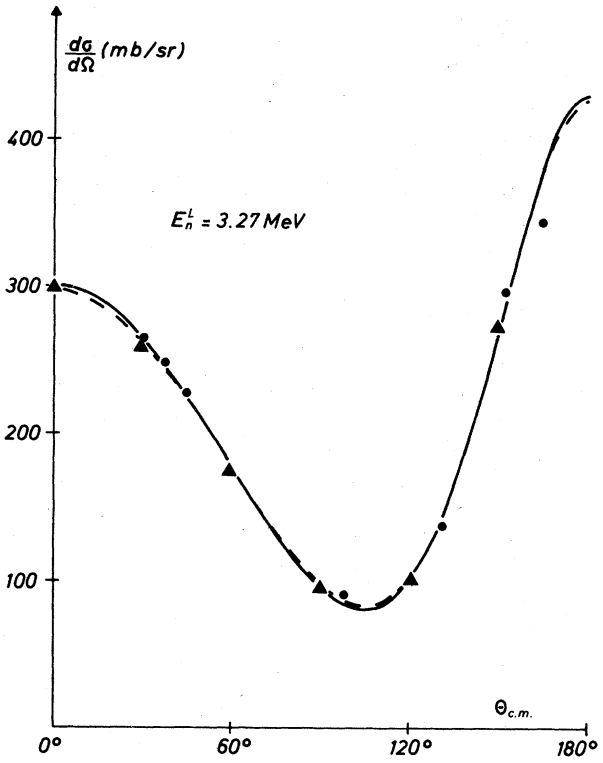

FIG. 5. Same as in Fig. 4, but for higher neutron bombarding energy E_n^L.

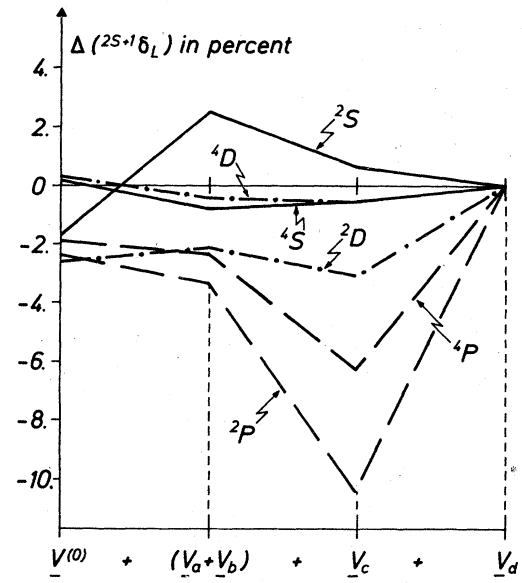

FIG. 6. Contributions of the various parts of the effective potential (3.20) (see also Fig. 1) to the lowest doublet and quartet neutron-deuteron phase shifts, in percents of their final 1.QBA values. The first number results from the 0.QBA term $\underline{V}^{(0)}$. Adding to $\underline{V}^{(0)}$ the terms ($\underline{V}_a$+ $\underline{V}_b$) the second value is obtained, and so on. In order to guide the eye the corrections to each phase shift have been connected with lines.

This cancellation can be attributed to three causes.

(i) As we know from Refs. 27–30, the effect of the full S-wave part of the MT potentials is already well reproduced by the UPA = 0.QBA for energies below, and not too far above, the breakup threshold. This is due to the fact that the contributions corresponding to the higher attractive and repulsive S-wave eigenvalues almost cancel each other in three-nucleon quantities. Consequently, any difference between the 0.QBA and the 1.QBA neutron-deuteron phase shifts should be attributed essentially to the higher subsystem partial waves.

(ii) These, however, are also suppressed. For, in the terms $\underline{V}_a$ and $\underline{V}_b$ the S-wave form factors project the $l \neq 0$ parts of the two-body potential onto zero. Furthermore, the Pauli principle requires that for odd nucleon-nucleon angular momenta $\underline{V}_c$ and $\underline{V}_d$ are equal but of opposite sign,[37] thus annulling each other (this is, of course, true only in the simple model for the nucleon-nucleon forces investigated here).

(iii) As a consequence of points (i) and (ii) the first subsystem partial wave which may contribute appreciably to the sum $\underline{V}_a+\underline{V}_b+\underline{V}_c+\underline{V}_d$, is a D wave. But at the low energies which we are working at, such high subsystem angular momenta are unimportant.

These considerations demonstrate that the smallness of the 1.QBA corrections is due to the remarkable quality of the UPA for the S-wave part of potentials containing repulsion, due to the Pauli principle and due to the insignificance of the D-wave part of the MT potential.

It is interesting to contrast this situation with the one arising for purely attractive Yukawa potentials.[19-22] Points (ii) and (iii) mentioned above still remain valid. But the cancellation among the subsystem S-wave contributions, discussed in point (i), can no longer happen since all eigenvalues are of the same sign (attractive). We, therefore, expect an appreciable modification of the 0.QBA (UPA) when going over to the 1.QBA. Indeed, it turns out that for the binding energy this modification is by one order of magnitude larger than for the MT potentials. The same holds true for the differential cross section, as illustrated in Fig. 7. Here only the dominant quartet contributions are plotted in 0.QBA and 1.QBA, both for the Yukawa and the MT potentials. More details can be inferred from Table IV in which the real and imaginary parts of the partial wave amplitudes for the relative angular momenta $L = 0, 1, 2, 3$ between the deuteron and the third nucleon are shown. It is remarkable that for even L neither the 1.QBA corrections nor the difference

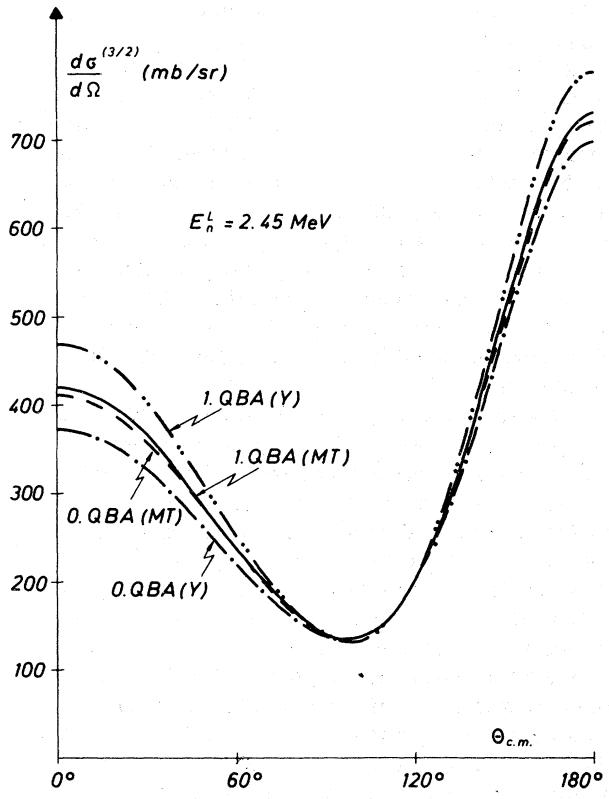

FIG. 7. Quartet contribution to the differential cross section in 0.QBA and 1.QBA for the MT potentials (MT) as compared to the corresponding results for Yukawa potentials (Y).

between the two potentials produce noticeable effects. This result was to be anticipated from the Pauli principle. It implies that the cross section in the minimum which is predominantly determined by the even-L amplitudes (the odd-L Legendre polynomials vanish at 90°) is the same for all potentials. On the other hand we have pronounced changes in the odd-L amplitudes when correcting the 0.QBA Yukawa results by the 1.QBA, and when going over from the Yukawa to the MT potentials. Since the $L=1$ amplitudes essentially determine the forward, and also to a large extent the backward, peak of the cross section, the differences in these regions are mainly a consequence of the sensitivity of the neutron-deuteron P state to the potentials employed. At higher energies the odd-L amplitudes can be expected to play an important role also at the cross section minimum which is shifted away from 90° towards larger angles.

Concluding we can say that the quasi-Born approach is not only an efficient method when applied to potentials acting in few partial waves, but is particularly effective for genuinely local potentials.

One of the authors (E.O.A) acknowledges grant of computer time at the Rechenzentrum der Universität Mainz and the RHRK-Kaiserslautern.

APPENDIX

The analytic expressions corresponding to the various terms of the effective potential $\underline{V}^{(1)}$ are similar to those already given in Refs. 17 and 21, so they will not be repeated here.

The identity of the nucleons leads to the form

$$\underline{V}^{(1)}_{nm} = \underline{V}^{(1)}_{\alpha n,\alpha m} + 2\underline{V}^{(1)}_{\beta n,\alpha m} \quad (\beta \neq \alpha) \tag{A1}$$

for the symmetrized effective potential. Here the nondiagonal part consists of the sum of $\underline{V}^{(0)}$, $\underline{V}_a$, $\underline{V}_b$, and $\underline{V}_c$ of Fig. 1. But since the diagonal contribution equals two times the term $\underline{V}_d$ [compare the definition (3.9)], we can separate out a common factor of 2 and write symbolically for the symmetrized effective potential

$$\underline{V}^{(1)}_{nm} = 2(\underline{V}^{(0)}_{nm} + \underline{V}_{a,nm} + \underline{V}_{b,nm} + \underline{V}_{c,nm} + \underline{V}_{d,nm}) \ . \tag{A2}$$

The effects of including the spin and isospin of the nucleons can be summarized by specifying the spin-isospin matrices which multiply the various contributions to $\underline{V}^{(1)}$ as calculated by neglecting spin and isospin. Looking carefully at Fig. 1

TABLE IV. Real and imaginary parts of the neutron-deuteron partial wave scattering amplitudes at $E_n^L = 2.45$ MeV (in fm). Compared are the 0.QBA and the 1.QBA, both for purely attractive Yukawa and for the MT potentials.

	Yukawa				Malfliet-Tjon			
	0.QBA		1.QBA		0.QBA		1.QBA	
L	Re$\underline{T}_L$	Im$\underline{T}_L$	Re$\underline{T}_L$	Im$\underline{T}_L$	Re$\underline{T}_L$	Im$\underline{T}_L$	Re$\underline{T}_L$	Im$\underline{T}_L$
0	0.779	−1.734	0.774	−1.738	0.761	−1.753	0.762	−1.747
1	−0.752	−0.321	−0.832	−0.415	−0.782	−0.354	−0.791	−0.365
2	0.126	−0.007 7	0.129	−0.008 0	0.128	−0.007 9	0.128	−0.007 9
3	−0.0261	−0.000 33	−0.0285	−0.000 39	−0.0262	−0.000 33	−0.0263	−0.000 33

it becomes clear that, with respect to spin-isospin recoupling, the terms $\underline{V}^{(0)}$, $\underline{V}_a$, and $\underline{V}_b$ are equivalent, as are the terms $\underline{V}_c$ and $\underline{V}_d$. We thus expect two different sets of spin-isospin matrices which we denote by Λ and $\tilde{\Lambda}$.

Let S (I) be the total spin (isospin) of the three-nucleon system. Then we write for the first group of potential terms

$$\underline{V}^{(0)SI}_{nm} = \Lambda^{SI}_{nm}\underline{V}^{(0)}_{nm}\,,$$
$$\underline{V}^{SI}_{a,nm} = \Lambda^{SI}_{nm}\underline{V}_{a,nm}\,, \tag{A3}$$

and similarly for $\underline{V}^{SI}_b$, where the quantities $\underline{V}$ without spin-isospin indices S, I are those calculated for spinless particles. The indices n, m characterize the subsystems in the initial and the final state taking the values $n, m = d, s$ corresponding to the deuteron and the singlet S state. The numerical values are well known

$$\Lambda^{3/2\ 1/2}_{nm} = \begin{pmatrix} -\frac{1}{2} & 0 \\ 0 & 0 \end{pmatrix}, \quad \Lambda^{1/2\ 1/2}_{nm} = \tfrac{1}{4}\begin{pmatrix} 1 & -3 \\ -3 & 1 \end{pmatrix}. \tag{A4}$$

Some attention is required for the terms $\underline{V}^{SI}_c$ and $\underline{V}^{SI}_d$ because in the intermediate state the two nucleons which interact via T' must be arranged into a subsystem with values of subsystem spin and isospin which match those of T'. And this pair must then combine with the third particle to give the desired values for S and I. We, therefore, split $\underline{V}^{SI}_c$ and $\underline{V}^{SI}_d$ into two parts in order to distinguish whether the action of T' takes place in the intermediate d or s channel

$$\underline{V}^{SI}_c = \underline{V}^{SI}_c(d) + \underline{V}^{SI}_c(s) \tag{A5}$$

and analogously for $\underline{V}^{SI}_d$. If we now define the coupling matrices $\tilde{\Lambda}(d)$ and $\tilde{\Lambda}(s)$ as

$$\underline{V}^{SI}_{c,nm} = \tilde{\Lambda}^{SI}_{nm}(d)\underline{V}_{c,nm}(d) + \tilde{\Lambda}^{SI}_{nm}(s)\underline{V}_{c,nm}(s)\,, \tag{A6}$$

and similarly for $\underline{V}^{SI}_d$ (with the *same* $\tilde{\Lambda}$ matrices), then it is evident that $\tilde{\Lambda}(d)$ and $\tilde{\Lambda}(s)$ are determined by Λ as

$$\tilde{\Lambda}^{SI}_{nm}(d) = \Lambda^{SI}_{nd}\Lambda^{SI}_{dm}$$
and
$$\tilde{\Lambda}^{SI}_{nm}(s) = \Lambda^{SI}_{ns}\Lambda^{SI}_{sm}\,. \tag{A7}$$

The numerical values can thus be trivially obtained from Eq. (A4).

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