

Quasiparticle-phonon coupling model calculations for odd- A Nb isotopes

K. Krishan and S. Sen

Saha Institute of Nuclear Physics, Calcutta 700009, India

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The structures of the odd- A Nb isotopes $^{93,95,97}\text{Nb}$ are studied in a semimicroscopic model which couples the proton quasiparticle motion to the quadrupole and the octupole vibrations of the neighboring even core. Inclusion of the octupole vibration of the core improves the $l = 3$ spectroscopic strength distribution of the earlier theoretical studies. There is a satisfactory agreement between the calculated and experimental level spectra, spectroscopic factors, and electromagnetic transition rates.

NUCLEAR STRUCTURE $^{93,95,97}\text{Nb}$, calculated levels J , π , $B(E2)$, quasiparticle-phonon coupling.

I. INTRODUCTION

The odd- A Nb nuclei have been extensively studied by many workers¹⁻⁶ using the effective shell model approach, with ^{88}Sr or ^{90}Zr as an inert core. These shell model calculations could account for the qualitative features of the level spectra but in detailed comparison with the experiment, there are serious discrepancies, e.g., a $\frac{11}{2}^+$ or $\frac{13}{2}^+$ state is predicted to be the first positive parity excited state, whereas the observed one is $\frac{5}{2}^+$ or $\frac{7}{2}^+$. Moreover, for many odd- A Nb nuclei a satisfactory comparison was not forthcoming for want of sufficient experimental data. As the neutron stripping experiments on even Zr isotopes⁷ reveal the importance of the $3s_{1/2}$, $2d_{3/2}$, and $1g_{7/2}$ neutron orbitals, it seems quite logical not to confine the neutrons in the case of Nb nuclei, only to the $2d_{5/2}$ orbital (a feature of all the shell model calculations done so far) for the residual $n-p$ interactions can make an appreciable contribution to the low lying spectra. Recently Bindal, Youngblood, and Kozub⁸ performed the proton pickup experiments on $^{96,98,100}\text{Mo}$ isotopes and interpreted the resulting $^{95,97,99}\text{Nb}$ spectra in terms of the core-coupling model of Thankappan and True,⁹ using quasiparticle formalism. Qualitatively, there is a good agreement with the experiment. However, the $l = 3$ spectroscopic strength distributions in the $^{95,97}\text{Nb}$ nuclei, are in serious disagreement with the experiment. This poor agreement in the case of $l = 3$ strength distribution may be thought of due to the neglect of the octupole vibration by Bindal *et al.*⁸ in their calculations. It has been shown in the case of odd- A In nuclei¹⁰ that the mixing of the negative parity states generated by coupling the octupole phonon to the $1g_{9/2}$ particle state and the other negative parity states, can cause a redistribution of the single-particle

strengths. In the present work we have calculated the level structure of the $^{95,97}\text{Nb}$ nuclei using the quasiparticle-phonon coupling model which couples the motion of the even Mo core with that of the single proton quasiparticle. The phonon space considered in the present work spans quadrupole states (up to two quadrupole phonons) and one octupole phonon state, whereas the quasiparticle motion is confined to $1f_{5/2}$, $2p_{3/2}$, $2p_{1/2}$, and $1g_{9/2}$ orbitals. The validity of the phonon-particle model, which involves a number of parameters, can be rigorously checked only if the electromagnetic data are reproduced along with level spectrum. So, for checking the validity of the model and for the general interest in the systematics of the nuclei in the $A = 90$ region, we have included ^{93}Nb in the present calculations because for this isotope there are several experimentally observed electromagnetic transition rates. From the systematics point of view the present work is an extension of our previous work on odd- A Rb and Y nuclei.^{11,12}

The odd- A Nb nuclei, however, can be approached through the proton stripping experiments on even Zr nuclei. As shown by the unified model calculations by Paradellis, Hontzeas, and Blok¹³ for Zr isotopes, the vibrational character of the even- A Zr isotopes is not so good as to use them as vibrational cores for the Nb nuclei.

II. MODEL

The total Hamiltonian of the core-particle coupled system is given by

$$H = H_c + H_{s.p.} + H_{int},$$

where $H_{s.p.}$ is the usual single-particle shell model Hamiltonian and H_c describes the core vibrations. The basis states, which are the eigenfunctions of

$H_c + H_{s.p.}$, can be written as $\{[N_2 V_2 L_2, N_3 V_3 L_3]^L (nl \frac{1}{2})^J\}^{JM}$, where $(N_\lambda V_\lambda L_\lambda)$ is the totally symmetric state of phonons of multipolarity λ coupled to angular momentum L and V represents all the additional quantum numbers necessary for complete specifications of the state; L_2 and L_3 are coupled to the angular momentum L of the core which is then coupled to j of the particle or hole to form the resultant J .

The core-particle interaction is given by

$$H_{int} = \sum_{\lambda}^{2,3} k_{\lambda}(\nu) \sum_{\mu=-\lambda}^{\lambda} \alpha_{\lambda\mu}^* Y_{\lambda\mu}(\theta_p, \phi_p),$$

$Y_{\lambda\mu}$ are the spherical harmonics corresponding to the particle coordinates, $\alpha_{\lambda\mu}$ is the usual combination of phonon creation and destruction operators for the core. The radial integrals of $k(\nu)$ are of the form used by Lande and Brown,¹⁴ viz.,

$$k_{\lambda}(\nu) = K_{\lambda} \left[\left(\frac{m\omega_0}{\hbar} \right)^{1/2} r_p \right]^{\lambda}.$$

k_{λ} 's are treated as the adjustable parameters and $m\omega_0/\hbar$ is the harmonic oscillator constant. Including the pairing effects the matrix elements of the interaction Hamiltonian in the basis defined earlier are^{10, 14}

$$\begin{aligned} & \langle \{[N_2 V_2 L_2, N_3 V_3 L_3]^L (nl \frac{1}{2})^J\}^{JM} | H_{int} | \{[N'_2 V'_2 L'_2, N'_3 V'_3 L'_3]^L (n'l' \frac{1}{2})^{J'}\}^{JM} \rangle \\ &= \sum_{\lambda}^{2,3} X_{\lambda} \langle nl | [(m\omega_0/\hbar)^{1/2} r_p]^{\lambda} | n'l' \rangle (-1)^{J-1/2+\lambda} \frac{1}{2} \{ [L][L'] [j][j'] \}^{1/2} \\ & \times \left(\frac{1 + (-1)^{l+l'+\lambda}}{2} \right) \left(\delta_{\lambda,2} + \delta_{\lambda,3} (-1)^{L+L'+L_3+L'_3} \right) \begin{pmatrix} j' & \lambda & j \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} L & L' & \lambda \\ L_{\lambda}' & L_{\lambda} & L_{\eta} \end{pmatrix} (U_{j'} U_j - V_{j'} V_j) \delta_{N_{\eta} N'_{\eta}} \delta_{L_{\eta} L'_{\eta}} \delta_{V_{\eta} V'_{\eta}} \\ & \times [\delta_{N_{\lambda}, N'_{\lambda}+1} (-1)^{L_2+L_3} \langle N_{\lambda} L_{\lambda} | b_{\lambda}^{\dagger} | N'_{\lambda} L'_{\lambda} \rangle + \delta_{N'_{\lambda}, N_{\lambda}+1} (-1)^{L'_2+L'_3} \langle N'_{\lambda} L'_{\lambda} | b_{\lambda}^{\dagger} | N_{\lambda} L_{\lambda} \rangle]. \end{aligned}$$

X_2 and X_3 are the quadrupole and octupole interaction strengths, respectively, and $\eta=2$ when $\lambda=3$ and vice versa.

III. CHOICE OF PARAMETERS

There are a number of parameters involved in the calculations, viz., U_j factors, quasiparticle energies, and interaction strength parameters X_2 and X_3 . The U_j factors for all the isotopes have been fixed from the experimental data. Instead of taking U_j factors directly from the pickup data⁸ we have determined them by renormalizing the pickup and stripping data (assuming a single step process) on even Mo isotopes in a manner similar to that of Comfort, Duray, and Braithwaite.¹⁵ The renormalization process proceeds as follows:

The proton transfer spectroscopic factors satisfy the sum rules

$$\sum \frac{2J_f+1}{2J_i+1} (C^2S)_{<} + \sum \frac{2J_f+1}{2J_i+1} (C^2S)_{>} = \text{proton holes in the target,} \quad (1a)$$

$$\sum (C^2S')_{>} = \text{proton particles in the target} \quad (1b)$$

for stripping and pickup reactions, respectively.

The subscripts \langle and \rangle correspond to the $T_{<}$ and $T_{>}$ final states. For each shell model orbit j , the

right hand sides of (1a) and (1b) should add up to $(2j+1)$. In (1a) the first term is measured in proton stripping experiment, whereas the second one corresponds to the neutron stripping strength. Since we are interested only in the $2p_{1/2}$, $2p_{3/2}$, $1f_{5/2}$, and $1g_{9/2}$ orbitals, the second term of (1a) is always zero for $A > 90$ Mo isotopes. Therefore it follows that

$$\sum (2J_f+1) (C^2S)_{<} + \sum (C^2S')_{>} = 2j+1, \quad (2)$$

since $J_i=0$ for even-even nuclei. However, the experimental quantities may need to be normalized so as to satisfy the relation (2).

Therefore we write Eqs. (2) as

$$\alpha x_j + \beta y_j = 2j+1, \quad (3)$$

where $x_j = \sum (2J_f+1) (C^2S)_{<}$, $y_j = \sum (C^2S')_{>}$ and α, β are the normalization factors. A least square fitting procedure is used to obtain the values of α and β . This procedure need not preserve the sum $\sum_j (2j+1) = 22$, but in practice the discrepancy is small. From the normalized strengths thus determined, the occupation probabilities V_j^2 were found and were used in the subsequent calculations.

For ⁹³Nb, the single-particle strength for $1f_{5/2}$ orbital, in pickup data,¹⁶ is about 60%, whereas the stripping data¹⁷ do not show up any strength at all. The normalization of the experimental

TABLE I. Parameter values used in this calculation.

Nucleus	Core	V_j^a				Quasiparticle energies ^b (MeV)			Quadrupole coupling strength	Octupole coupling strength
		$1g_{9/2}$	$2p_{1/2}$	$2p_{3/2}$	$1f_{5/2}$	$2p_{1/2}$	$2p_{3/2}$	$1f_{5/2}$	X_2 (MeV)	X_3 (MeV)
⁹³ Nb	⁹⁴ Mo	0.544	0.904	0.877	0.762	0.05	1.15	1.35	0.5	0
⁹⁵ Nb	⁹⁶ Mo	0.469	0.728	0.871	0.948	0.20	0.88	1.20	0.7	0.1
⁹⁷ Nb	⁹⁸ Mo	0.469	0.728	0.871	0.948	0.90	1.40	1.50	0.4	0.1

^aThese are the normalized values discussed in the text.

^bThese energies are relative to the $1g_{9/2}$ orbital.

data carried out including $1f_{5/2}$ orbital produced strengths far off from the sum rule limit (3). Therefore the $1f_{5/2}$ orbital was dropped from the least square fitting procedure and the values of the occupation probabilities thus obtained are listed in Table I. For ⁹⁵Nb, the proton stripping and pickup data are taken from the work of Medsker *et al.*¹⁸ and Bindal *et al.*,⁸ respectively. The resulting values of V_j^2 are listed in the Table I. In these two cases, the sum rules are maintained by the renormalized strengths. The values of α and β obtained in the two cases are 0.85, 1.022 and 1.23, 0.87, respectively. For want of sufficient data, the same procedure could not be applied for the ⁹⁷Nb case. However, the pickup⁸ data reveal almost the same amount of the single-particle strengths as observed in ⁹⁵Nb and therefore the values of V_j^2 used in the calculations for ⁹⁷Nb are the same as those for ⁹⁵Nb.

In the Coulomb excitation experiments¹⁹ on ⁹³Nb, a number of positive parity states are found to be excited near the one phonon energy of the core nucleus ⁹⁴Mo, whereas in ⁹⁵Nb and ⁹⁷Nb no positive parity state, except the ground state, is found to be excited in the pickup reactions.⁸ So in ⁹³Nb, the strength parameters are fixed by first calculating the positive parity spectrum. Since this calculation involves only one positive parity orbital $1g_{9/2}$ and the relative positions of

the positive parity levels are found to be insensitive to the variation of X_3 , only X_2 is to be adjusted to reproduce the positive parity spectrum. The X_2 value thus found is then used to calculate the negative parity spectrum by adjusting X_3 and quasiparticle energies ($\epsilon_{5/2}, \epsilon_{3/2}, \epsilon_{1/2}$). However, in ⁹⁵Nb and ⁹⁷Nb, in the absence of any positive parity excited state, all the parameters are simultaneously adjusted. The best values of the parameters determined in the present work are given in Table I. It is to be mentioned here that in the case of ⁹³Nb, the value of X_3 is found to be very small ~ 0 and as such its effect on the subsequent calculations for this isotope is neglected.

The static electromagnetic moments and $E2$ and $M1$ transition rates for several states in ⁹³Nb have been calculated with following values of effective g factors and effective charges, viz., $g_l = 1.0$, $(g_s)_{\text{eff}} = 4.0$, $e_p = 1.0$, and $eZ(\hbar\omega_p/2C_2)^{1/2} = 2.4 e_p$. The calculated results along with the experimental values are given in Table II. Below we discuss the results, individually, for each of the isotopes in the light of recent experimental data.

IV. RESULTS AND DISCUSSIONS

⁹³Nb

Experimental level scheme, based on the pickup reaction,¹⁶ Coulomb excitation,¹⁹ and $(n, n\gamma)$ experiments²⁰ is shown in Fig. 1 along with the calculated one. The experimentally observed negative parity levels and their spectroscopic factors are correctly reproduced. In the pickup reaction,¹⁶ a level is excited at 1320 keV with probable spin-parity $\frac{3}{2}^-$, $\frac{5}{2}^-$. In the calculated spectrum there are two levels in this region with spin-parity $\frac{3}{2}^-$ and $\frac{5}{2}^-$ and their spectroscopic factors are very close to the experimental values. The observed level at 1320 keV may be a doublet with spin-parities $\frac{3}{2}^-$ and $\frac{5}{2}^-$. The group of positive parity levels observed below 1 MeV excitation in the Coulomb excitation experiments¹⁹ is reproduced. It is observed that inclusion of the dipole-

TABLE II. Calculated and experimental $B(E2)$ values in ⁹³Nb.

Present work			Experiment ^a		
E (keV)	J^π	$B(E2)$ ($e^2 b^2$)	E (keV)	J^π	$B(E2)$ ($e^2 b^2$)
742	$\frac{5}{2}^+$	0.0193	808	$\frac{5}{2}^+$	0.0162
823	$\frac{7}{2}^+$	0.0166	744	$\frac{7}{2}^+$	0.0180
854	$\frac{11}{2}^+$	0.0443	978	$\frac{11}{2}^+$	0.0127
964	$\frac{13}{2}^+$	0.0433	950	$\frac{13}{2}^+$	0.0247

^aReference 19.

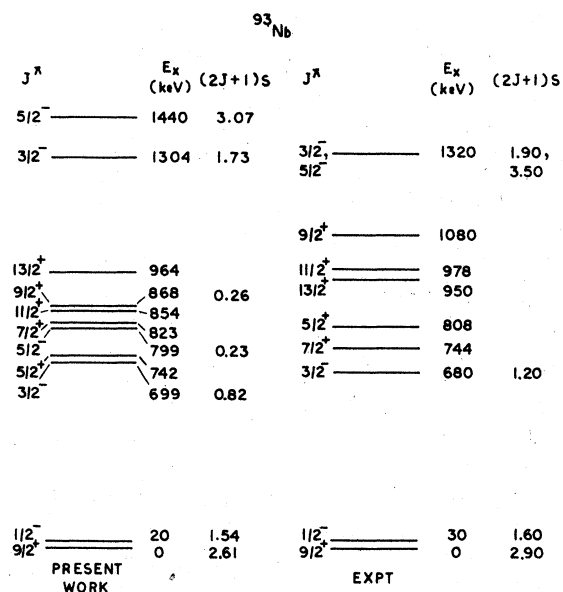


FIG. 1. Calculated and experimental level schemes of ⁹³Nb. The spectroscopic factors are given on the right hand side of the level schemes. The experimental spectrum is taken from Refs. 16, 19, and 20.

dipole interaction terms⁹ gives better agreement with respect to the level ordering. The spectrum shown in the Fig. 1 is calculated with $\eta = -0.012$. However, in detailed comparison one can see several discrepancies. The level ordering of the $\frac{5}{2}^+$ and $\frac{7}{2}^+$ states has just been reversed. The calculated $B(E2)$ values for these two levels are in good agreement with the experimental values. However, if we assign spin-parity $\frac{5}{2}^+$ and $\frac{7}{2}^+$ to the observed levels at 744 and 808 keV, respectively, then we get an excellent agreement with the calculated values with respect to the excitation energies as well as $B(E2)$ values. On the basis of available experimental data²¹ this possibility cannot be ruled out altogether. The calculated $\frac{1}{2}^+$ state is close to the experimental one and the calculated $B(E2)$ value is also not far off from the observed value. Comparatively, the excitation energy and the $B(E2)$ value for the $\frac{11}{2}^+$ state are not well reproduced.

⁹⁵Nb

The overall agreement between the calculated and experimental levels schemes as well as the

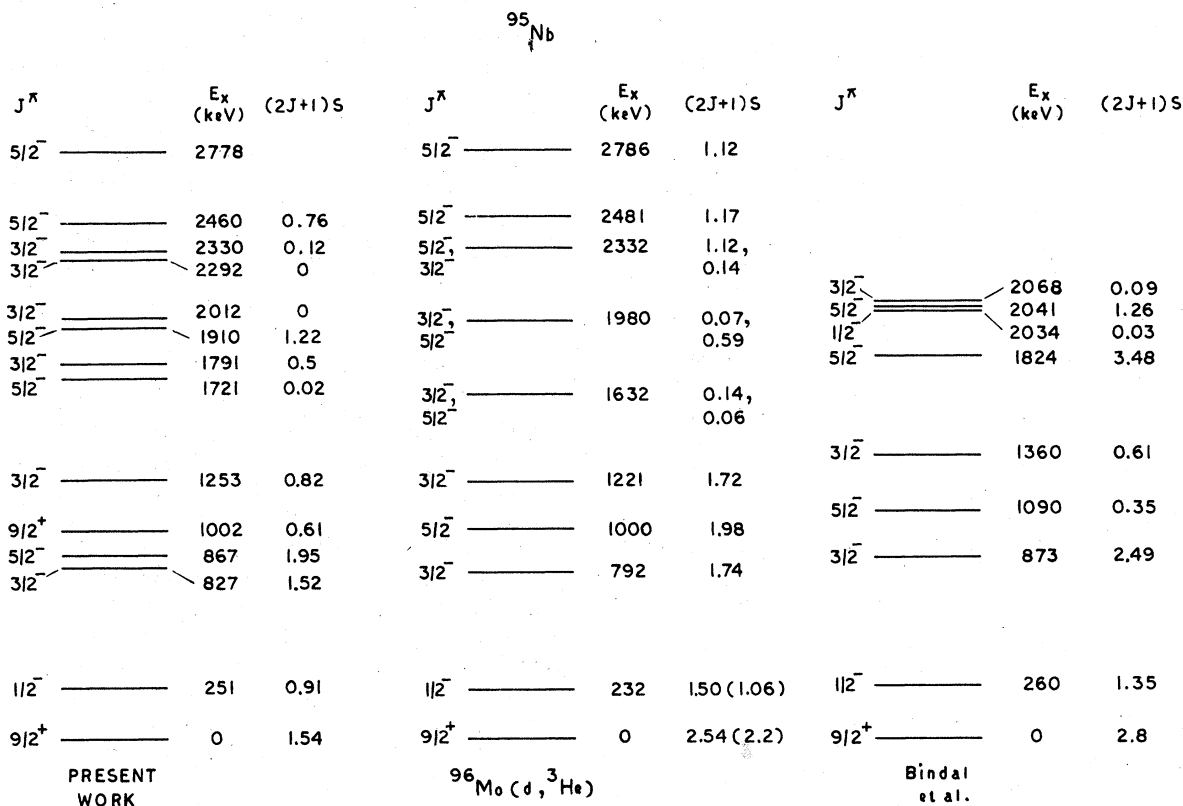


FIG. 2. Calculated and experimental level spectra of ⁹⁵Nb. The level scheme calculated by Bindal *et al.* (Ref. 8) is also shown. The ⁹⁶Mo(d, ³He) data are taken from Ref. 8.

spectroscopic factors of different levels can be considered to be quite good (Fig. 2). The spectroscopic factors of the ground and first excited states shown in parentheses in Fig. 2 (levels observed in $^{96}\text{Mo}(d, \text{He}^3)$ reaction⁸) are the renormalized values. The calculated spectroscopic factors are in better agreement with the renormalized values than with the experimental values. The observed levels at 792, 1000, and 1221 keV and their spectroscopic factors are more or less correctly reproduced. On the basis of the spectroscopic factor considerations, the observed levels at 1980 and 2332 keV should have spin-parities $\frac{5}{2}^-$ and $\frac{3}{2}^-$, respectively. There are two calculated levels at 1721 ($\frac{3}{2}^-$) and 1791 ($\frac{3}{2}^-$) keV and any one of them may correspond to the observed level at 1632 keV having spin-parity $\frac{3}{2}^-$ or $\frac{5}{2}^-$. The present work predicts a $\frac{5}{2}^-$ level at 1324 keV with an appreciable single-particle strength which is not observed experimentally. It would be interesting to search for this level in future experimental investigations. It can be seen from

Fig. 2 that the agreement with the experimental level scheme as well as the $l=3$ spectroscopic strength distribution achieved in the present work is better than that obtained in the calculations done by Bindal *et al.*⁸

^{97}Nb

The experimental level scheme observed in the pickup reaction⁸ is shown in Fig. 3 along with level spectra calculated in the present work and by Bindal *et al.*⁸ The excitation energies as well as the spectroscopic factors of the observed levels at 750 ($\frac{1}{2}^-$), 1251 ($\frac{3}{2}^-$), 1438 ($\frac{5}{2}^-$), and 1764 ($\frac{3}{2}^-$) keV are more or less correctly reproduced in the present work. From the spectroscopic factor considerations, the calculated levels at 1779 ($\frac{5}{2}^-$), 2205 ($\frac{3}{2}^-$), and 2560 ($\frac{3}{2}^-$) keV may correspond to the observed levels at 2090 ($\frac{5}{2}^-$), 2244 ($\frac{3}{2}^-$), and 2550 ($\frac{3}{2}^-$) keV, respectively. The calculated $\frac{5}{2}^-$ level at 2600 keV may correspond to the observed 2948 ($\frac{3}{2}^-$, $\frac{5}{2}^-$) keV level. The agreement between the calculated level scheme as well as the $l=3$

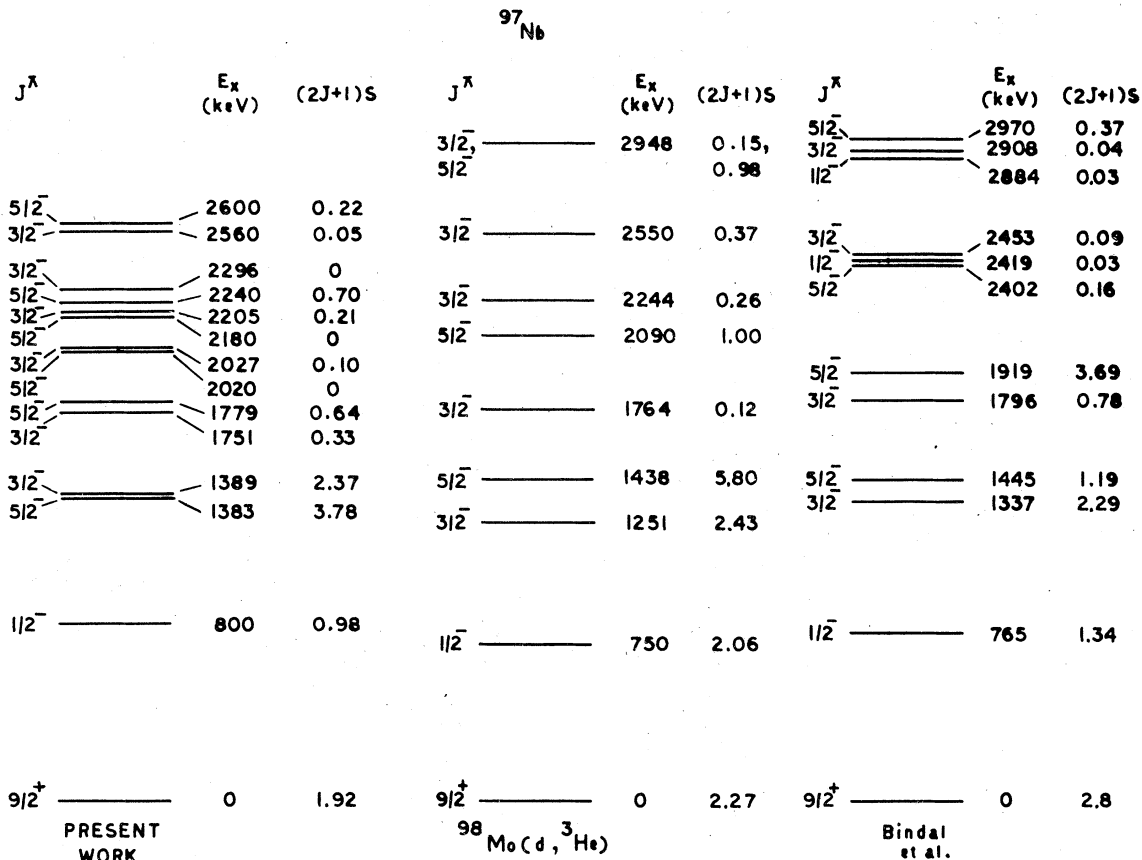


FIG. 3. Calculated and experimental level schemes of ^{97}Nb . The level scheme calculated by Bindal *et al.* (Ref. 8) is also shown. The $^{98}\text{Mo}(d, \text{He}^3)$ data are taken from Ref. 8.

spectroscopic distribution of Bindal *et al.*⁸ and the experimental data is not so good. However, in a recent work Bindal *et al.*²² reported their calculations for ⁹⁷Nb isotope with modified values of the model parameters and obtained better agreement.

In addition to the levels discussed above there are some more levels predicted by the present work. The calculated $\frac{3}{2}^-$ level at 2028 keV is found to arise mainly from the coupling of a proton in the $1g_{9/2}$ orbital to an octupole phonon and this may be the reason that this level is not observed in the (*d*, ³He) work.⁸ However, this may correspond to a ($\frac{1}{2}^-, \frac{3}{2}^-$) level at 2107 keV observed in the decay¹⁸ of ⁹⁷Zr. Two levels at 2180 ($\frac{3}{2}^-$) and 2296 ($\frac{3}{2}^-$) keV are predicted with no single-particle strengths. It has been observed that the total $g_{9/2}$ single-particle strength is exhausted in the 0 ($\frac{3}{2}^+$) and 1202 ($\frac{3}{2}^+$) keV levels. The nonexcitation of 1202 ($\frac{3}{2}^+$) keV level in the pickup experiment⁸ may be ascribed to its small spectroscopic strength ~ 0.07 . However, there is a calculated level at 2240 ($\frac{5}{2}^-$) keV with an appreciable single-particle strength which is not observed in the proton pickup experiment. The identification of this level in the future experimental investigations may be quite interesting.

V. CONCLUSIONS

The present calculation is quite successful in interpreting the available experimental data of the odd-*A* Nb nuclei. The inclusion of the octupole vibration of the core seems to be necessary to reproduce the observed *l*=3 spectroscopic strength distributions. Although, in ⁹⁵Nb and ⁹⁷Nb the present calculation can reproduce the proton pickup reaction data⁸ very well, yet the model could not be tested in more detail in the absence of electromagnetic data.

It would be quite interesting to verify experimentally the few predictions we have made in the odd-*A* Nb nuclei on the basis of the present investigations. In the light of our previous work on odd-*A* Rb and Y nuclei^{11,12} and the present calculations, it can be concluded that the coupling of the single proton motion to the collective vibrations of the core is the principal mode of excitation for the low lying states of the odd proton nuclei in the *Z* = 40 and *A* = 90 region.

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