Perturbation formalism for the complex poles and widths of the transition matrix with an application to intermediate structure phenomena*

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The complex poles and widths of the transition T matrix are determined by the trajectory equations which consist of a set of first order nonlinear differential equations. A hierarchy of approximate solutions to the trajectory equations is developed by iterative methods. The results of this formalism are compared with exact solutions for the case of some strongly interacting pairs of resonances in two iron isotopes. In the presence of intermediate structure the average neutron reaction cross section is interpreted in terms of a resonant strength function which exhibits peaks at neutron energies corresponding to "doorways" levels.

[NUCLEAR REACTIONS Complex poles and widths, intermediate structure.]

I. INTRODUCTION

The study of nuclear structure from neutron cross section information is performed by suitable parametrizations of the cross section data in terms of either R-matrix parameters (Wigner and Eisenbud[1]) or the poles and residues of the collision matrix (Kapur and Peierls,[2] Humblet and Rosenfeld[3]). In the first approach, one goes from the R matrix, via Wigner's level matrix, to the calculation of the collision matrix, and then the cross section. In the latter formalism the collision matrix itself is expanded in partial fractions with residues and poles which are related to the R-matrix parameters.[4]

The R-matrix parameter set is of a more fundamental nature than the collision matrix parameters, because the former set is directly related to general properties of the nuclear Hamiltonian. In contrast the Kapur-Peierls and Humblet-Rosenfeld formalisms lead directly to the cross section expression, dispensing with the inversion of large level matrices.[5]

Clearly there are two ways by which one can proceed. That is, one can perform the level matrix inversion by some approximation technique to arrive at the collision matrix, or alternatively one works out approximate relations between the R-matrix parameters and the poles and residues of the collision matrix. In either instance the convergence of the perturbation method depends on the value of the degree of level interference θ_R defined as the ratio of level coupling to level spacing. For nuclei in the actinide region, θ_R is rarely over 0.1. In this region the level interference effects are well described by the addition of a level interference correction to the Breit-Wigner resonance formula.[6] Application of the Bethe approximations to the calculation of the neutron ^{238}U total cross section has been discussed by de Saussure, Olsen, and Perez.[7] Perturbative calculations of the collision matrix parameters from the corresponding set of R-matrix parameters have been performed by Harris[8] in the actinide region.

However, recent total neutron cross section measurements in iron by Pandey *et al.*,[9] have brought up evidence as to the existence of highly interfering resonances ($\theta_R \approx 1$), hence providing incentive for the development of improved perturbation approaches for the calculation of the collision matrix parameters.

As will be shown later in this work, the main idea is to improve the convergence of the perturbation series expansion by starting from a perturbed system which already contains the diagonal elements of the interaction matrix, rather than from the usual unperturbed configuration. In this manner the convergence criterion is determined by the ratio of the off-diagonal interaction matrix elements to the spacing of the complex poles of the "diagonal" initial system. One acquires in this way a damping factor proportional to the difference of the level widths of the interacting levels. In the language of the propagator formalism, this is equivalent to a partial summation of the perturbation series.

We shall also consider the application of the present formalism to the study of intermediate structure phenomena of Robson,[10] Weigmann,[11] Lynn,[12] and Lane[13] and derive formulas for average reac-

tion cross sections especially geared towards the interpretation of intermediate structure in both the fission and capture cross sections of the heavy nuclides (see Difilippo *et al.*,[14] Perez and de Saussure[15]).

Finally a word about nomenclature: in this work we shall use the concept of the transition T matrix rather than the collision matrix (U matrix) because the former matrix is devoid of inconvenient factors arising from the hard sphere scattering functions. The complex poles in both matrices are of course the same, and the widths (or their combinations thereof to form the U-matrix residues) are easily related to one another.

II. GENERAL THEORY

In this section we shall use a formalism previously developed by Perez and de Saussure,[16] to obtain expressions for the partial widths and complex poles of the transition T matrix.

Let $\{\phi_{\nu c}\}$ be an eigenset of R-matrix states which satisfy real, momentum independent boundary conditions at the nuclear surface, i.e.,

$$\frac{d}{dr}\phi_{\nu c}(r)\Big|_{a_c} = (b_c/a_c)\phi_{\nu c}(a_c)\,, \tag{1}$$

where a_c is the channel radius and b_c the boundary condition numbers of R-matrix theory.[17] In terms of these R-matrix states one defines, in the usual manner, the reduced level widths, partial widths, and level shift factors as follows:

$$\gamma_{\nu c} = (A_c/a_c^{1/2})\phi_{\nu c}(a_c)\,, \tag{2}$$

$$\Gamma_{\nu\nu' c} = 2P_c\gamma_{\nu c}\gamma_{\nu' c} = (\Gamma_{\nu c}\Gamma_{\nu' c})^{1/2}\,, \tag{3}$$

$$\Delta_{\nu\nu' c} = -(S_c - b_c)\gamma_{\nu c}\gamma_{\nu' c}\,, \tag{4}$$

$$\Gamma_{\nu\nu'} = \sum_c \Gamma_{\nu\nu' c}\,, \tag{5}$$

$$\Delta_{\nu\nu'} = \sum_c \Delta_{\nu\nu' c} \tag{6}$$

with

$$A_c = (\hbar^2/2M_c)^{1/2}\,, \tag{7}$$

where the channel subindex c includes the pertinent set of quantum numbers, M_c is the channel mass, P_c the channel penetration factor, and S_c the channel shift factors and where ν is the level subindex.

The transition matrix is given in terms of the complex widths and poles of the collision matrix by the expansion

$$T_{cc'} = i\sum_\nu D_\nu^{-1} g_{\nu c} g_{\nu c'} \tag{8}$$

with

$$D_\nu = \epsilon_\nu - E\,, \tag{9}$$

$$\epsilon_\nu = E'_\nu - \tfrac{1}{2}i\Gamma'_\nu\,, \tag{10}$$

where E'_ν and $\frac{1}{2}\Gamma'_\nu$ are the real and imaginary components of the complex poles ϵ_ν. The complex widths are given in terms of the R-matrix states by

$$g_{\nu c} = \eta_c \sum_{\nu'} \beta_{\nu\nu'}\phi_{\nu' c}(a_c)\,, \tag{11}$$

where $\beta_{\nu\nu'}$ are modal amplitudes, to be computed later, and

$$\eta_c = A_c(2P_c/a_c)^{1/2}\,. \tag{12}$$

The modal amplitudes $\beta_{\nu\nu'}$ and complex poles satisfy the following set of first-order differential equations[16]:

$$\frac{d}{d\tau}\beta_{\nu\nu'} = \sum_{\nu''\neq\nu'} W_{\nu''\nu'}^{-1} P_{\nu'\nu''}\beta_{\nu''\nu}\,, \tag{13}$$

$$\frac{d}{d\tau}\epsilon_\nu = -P_{\nu\nu}\,, \tag{14}$$

where τ indicates any parameter in the Hamiltonian or boundary conditions describing the nuclear system. We have also defined

$$W_{\nu\nu'} = \epsilon_\nu - \epsilon_{\nu'}\,, \tag{15}$$

and the matrix $\underline{P}$

$$P_{\nu\nu'} = \sum_{\nu'',\nu'''} \beta_{\nu\nu''}\Omega_{\nu''\nu'''}\beta_{\nu'''\nu'}\,, \tag{16}$$

where we have introduced the interaction matrix $\underline{\Omega}$:

$$\Omega_{\nu\nu'} = -\sum_{c,c'}\Big[\int dr\,\delta(r-a_c)\delta_{cc'}\phi_{\nu c}(r)\Big(\frac{d}{d\tau}B_c\Big) \times \phi_{\nu' c'}(a_{c'}) - \frac{d}{d\tau}V_{\nu\nu'}^{c,c'}(\tau)\Big]. \tag{17}$$

The first term in the right-hand side of Eq. (17) corresponds to the variation in the boundary conditions and the second term describes the effect of switching on of the interaction channel potential's matrix elements. The matrix $\underline{P}$ itself satisfies a differential equation,[16] i.e.,

$$\frac{d}{d\tau}P_{\nu\nu'} = \sum_{\nu''\neq\nu} W_{\nu''\nu}^{-1}P_{\nu\nu''}P_{\nu''\nu'} + \sum_{\nu''\neq\nu'} W_{\nu''\nu'}^{-1}P_{\nu\nu''}P_{\nu''\nu'}\,, \tag{18}$$

which turns out to be very convenient for the calculation of the modal amplitudes $\beta_{\nu\nu}$ and the complex poles ϵ_ν. In Ref. 16 it was shown that the complex widths $g_{\nu c}$ also satisfy a set of differential equations

$$\frac{d}{d\tau}g_{\nu c} = \sum_{\nu'\neq\nu} W_{\nu'\nu}^{-1}P_{\nu\nu'}g_{\nu' c}\,. \tag{19}$$

Hence the complex widths of the transition matrix can be obtained, either by solving Eq. (13) for the modal amplitudes $\beta_{\nu\nu'}$ followed by utilization of Eq. (11) or by direct solution of the trajectory Eqs. (19).

Let us now examine the physical meaning of the previous developments. The initial set of R-matrix states satisfies the boundary conditions (1), whereas the appropriate eigenset for the collision matrix $\{\tilde{\phi}_{\nu c}\}$ must satisfy the complex momentum dependent boundary conditions

$$\frac{d}{dr}\tilde{\phi}_{\nu c}(r)\Big|_{a_c} = -(B_{1c}/a_c)\tilde{\phi}_{\nu c}(a_c)\,, \tag{20}$$

where the eigenfunctions $\tilde{\phi}_{\nu c}(a_c)$ are related to the complex widths by the relation

$$g_{\nu c} = \eta_c \tilde{\phi}_{\nu c}(a_c) \tag{21}$$

and

$$B_{1c} = -(A_c^{\,2}/a_c)L_c\,, \tag{22}$$

$$L_c = S_c + iP_c\,. \tag{23}$$

The change from the boundary conditions (1) to the U-matrix boundary conditions (20) is carried out by the use of the boundary condition function $B_c(\tau)$ defined as

$$B_c(\tau_1,\tau_2) = [\tau_1(B_{1c}-B_{0c})+B_{0c}](\theta_D+\tau_2\theta_N)\,, \tag{24}$$

where θ_D and θ_N are operators such that

$$\int dr\, F(r)\phi_{\nu c}(r)\theta_D\phi_{\nu' c}(r) = 0 \quad (\nu\neq\nu')\,, \tag{25}$$

$$\int dr\, F(r)\phi_{\nu c}(r)\theta_N\phi_{\nu' c}(r) = 0 \quad (\nu=\nu')\,, \tag{26}$$

where $F(r)$ is an arbitrary function.

In analogous fashion one introduces the channel interaction potentials

$$V^{c,c'}_{\nu\nu'}(\tau_1,\tau_2) = \int dr\, \phi_{\nu c}(r) V^{c,c'}(r,\tau_1,\tau_2)\phi_{\nu' c}(r) \tag{27}$$

with

$$V^{c,c'}(r,\tau_1,\tau_2) = \tau_1 V^{c,c'}(r)(\theta_D+\tau_2\theta_N)\,. \tag{28}$$

For $\tau_2=0$, one identifies τ_1 with τ in the trajectory equations where now only the diagonal elements of the interaction matrix Ω will be present in the equations. The initial condition in this case will correspond to $\tau_1=\tau=0$, in which case the function $B_c(0,0)$ reduces to B_{0c}, and $V^{c,c}(0,0,r)=0$. This situation corresponds to a configuration of uncoupled R-matrix states. Now while one keeps $\tau_2=0$, the change of $\tau_1=\tau$ between 0 and unity defines a family of intermediate configurations between the initial set of uncoupled R-matrix states and the final configuration of still uncoupled T-matrix states (only the diagonal elements of Ω are considered) satisfying the correct boundary conditions (20). In the next step one keeps $\tau_1=1$ and one identifies τ_2 with τ. In this instance the parameter τ turns on the off-diagonal elements. By the time that τ reaches unity, one obtains the final coupled T-matrix states. By means of the introduction of the two arbitrary parameters τ_1 and τ_2 we have defined two initial value problems which we shall study now in detail.

Trajectory equations for the diagonal case

In this instance, $\tau_2=0$ and $\tau_1=1$. Upon taking derivatives with respect to τ in Eqs. (24) and (28), one obtains

$$\frac{d}{d\tau}B_c(\tau,0) = (B_{1c}-B_{0c})\theta_D\,, \tag{29}$$

$$\frac{d}{d\tau}V^{c,c'}(r,\tau,0) = V_1^{c,c'}(r)\theta_D\,. \tag{30}$$

The results (29) and (30) are then introduced in Eq. (17) for the interaction matrix. After use is made of Eqs. (22) and (23) and the relations (2) up to (6), one obtains

$$\Omega_{\nu\nu} \equiv \omega_{\nu\nu} = -(\Delta_{\nu\nu}+V_{1\nu\nu})+i/2\Gamma_{\nu\nu}\,, \tag{31}$$

$$\Omega_{\nu\nu'} = 0 \quad (\nu\neq\nu')\,. \tag{32}$$

Equations (31) and (32) determine the interaction matrix. The initial conditions correspond to uncoupled R-matrix states. Hence, the complex widths and poles become real and equal to the square root of the partial channel widths and the R-matrix poles, respectively, i.e.,

$$g^{(D)}_{\nu c}(0) = \Gamma_{\nu c}^{\,1/2}\,, \tag{33}$$

$$\epsilon^{(D)}_{\nu}(0) = E_\nu\,. \tag{34}$$

Next from Eqs. (33) and (11) one concludes that the initial values of the modal amplitudes $\beta_{\nu\nu'}$ are

$$\beta^{(D)}_{\nu\nu'}(0) = \delta_{\nu\nu'}\,, \tag{35}$$

which after utilization of Eq. (16) shows that the initial values for the matrix elements $P^{(D)}_{\nu\nu'}(0)$ are

$$P^{(D)}_{\nu\nu'}(0) = \omega_{\nu\nu}\delta_{\nu\nu'}\,. \tag{36}$$

Now inspection of Eq. (18) for the evolution of the P matrix reveals that for the present diagonal case, all the successive derivatives of the P matrix vanish; hence, this matrix remains invariant and equal to its initial value. As for the modal amplitudes $\beta_{\nu\nu'}$, the lack of level coupling keeps them invariant and equal to their initial values. In view of these results for $\tau_1=\tau=1$, the solution of the "diagonal" problem yields ($\tau_1=1,\tau_2=0$)

$$g_{\nu c}(1,0)=\Gamma_{\nu c}^{1/2}, \tag{37}$$

$$\beta_{\nu\nu'}(1,0)=\delta_{\nu\nu'}, \tag{38}$$

$$\epsilon_\nu(1,0)=\epsilon_\nu^{(D)}(0)-\int_0^1 d\tau\, P_{\nu\nu}(\tau)=\epsilon_\nu^{(D)}(0)-\omega_{\nu\nu}, \tag{39}$$

where in obtaining the result (39) use was made of the trajectory Eq. (14). Clearly, the results (37) and (39) reproduce the single-level Breit-Wigner approximation.

Trajectory equations for the nondiagonal case

In this instance we have $\tau_1=1$ and $\tau_2=\tau$. From Eqs. (17), (24), (27), and (28) the interaction matrix is now given by

$$\Omega_{\nu\nu'}=-\left(\sum_c \phi_{\nu c}(a_c)B_{1c}\phi_{\nu' c}(a_c)+V_{\nu\nu'}\right) \quad (\nu\neq\nu'), \tag{40}$$

$$\Omega_{\nu\nu}=0, \tag{41}$$

with

$$V_{\nu\nu'}=\sum_{c,c'} V_{1\nu\nu'}^{c,c'}. \tag{42}$$

Next after use is made of Eqs. (22) and (23) and the set of results (2) up to (6) one obtains

$$\Omega_{\nu\nu'}=-V_{\nu\nu'}+S_{\nu\nu'}+\tfrac{1}{2}i\Gamma_{\nu\nu'} \tag{43}$$

with

$$S_{\nu\nu'}=\sum_c S_c\gamma_{\nu c}\gamma_{\nu' c}. \tag{44}$$

The initial value of the P matrix is obtained from its definition [Eq. (16)], the values (41) and (43) of the interaction matrix Ω, and the initial values of the modal amplitudes given by Eq. (35). One obtains

$$P_{\nu\nu'}(0)=\Omega_{\nu\nu'} \quad (\nu\neq\nu'), \tag{45}$$

$$P_{\nu\nu}(0)=0. \tag{46}$$

In summary, after identification is made of the parameter τ_2 with τ while keeping $\tau_1=1$, one has to solve the trajectory Eqs. (14), (18), and (19) subject to the initial conditions (37), (39), (45), and (46).

III. APPROXIMATE SOLUTIONS FOR THE TRAJECTORY EQUATIONS FOR THE COMPLEX WIDTHS AND POLES OF THE TRANSITION MATRIX

The initial value problem posed in the previous section can be solved numerically by any of the various techniques currently available, such as the Runge-Kutta technique[18] and the Lie series method,[19] among others. However, for either low or even moderately strong level interference, it is both convenient and instructive to develop approximate perturbation formulas for the complex poles and widths of the T matrix.

Starting from Eqs. (14), (18), and (19) one expands the poles and widths in Taylor series around an arbitrary $\tau=\tau_0$, i.e.,

$$\epsilon_\nu(\tau)=\epsilon_\nu(\tau_0)-(\tau-\tau_0)P_{\nu\nu}(\tau_0)-\tfrac{1}{2}(\tau-\tau_0)^2\dot{P}_{\nu\nu}(\tau_0), \tag{47}$$

$$g_{\nu c}(\tau)=g_{\nu c}(\tau_0)+(\tau-\tau_0)\dot{g}_{\nu c}(\tau_0)+\tfrac{1}{2}(\tau-\tau_0)^2\ddot{g}_{\nu c}(\tau_0), \tag{48}$$

where the Taylor series was truncated at the second-order terms and where the dot notation indicates the operator $d/d\tau$. Also from Eq. (19)

$$\ddot{g}_{\nu c}(\tau_0)=\sum_{\nu'\neq\nu}\{W_{\nu,\nu'}^{-1}(\tau_0)[g_{\nu' c}(\tau_0)\dot{P}_{\nu\nu'}(\tau_0)+P_{\nu\nu'}(\tau_0)\dot{g}_{\nu' c}(\tau_0)]-W_{\nu,\nu'}^{-2}(\tau_0)P_{\nu\nu'}(\tau_0)g_{\nu' c}(\tau_0)\theta_{\nu\nu'}(\tau_0)\} \tag{49}$$

with

$$\theta_{\nu\nu'}(\tau_0)=P_{\nu\nu}(\tau_0)-P_{\nu'\nu'}(\tau_0). \tag{50}$$

For the calculation of $\epsilon_\nu(\tau_0)$, $g_{\nu c}(\tau_0)$, and $P_{\nu\nu'}(\tau_0)$, we convert Eqs. (14), (18), and (19) into a set of coupled Volterra equations:

$$\epsilon_\nu(\tau)=\epsilon_\nu(0)-\int_0^\tau d\tau' P_{\nu\nu}(\tau'), \tag{51}$$

$$g_{\nu c}^{(\tau)}=g_{\nu c}(0)+\sum_{\nu'\neq\nu}\int_0^\tau d\tau'\left\{\left[W_{0\nu',\nu}+\int_0^{\tau'}d\tau''\theta_{\nu\nu'}(\tau'')\right]^{-1}P_{\nu\nu'}(\tau')g_{\nu' c}(\tau')\right\}, \tag{52}$$

$$P_{\nu\nu'}(\tau)=P_{\nu\nu'}^{(0)}-\int_0^\tau d\tau'\left\{\left[W_{0\nu',\nu}+\int_0^{\tau'}d\tau''\theta_{\nu\nu'}(\tau'')\right]^{-1}P_{\nu\nu'}(\tau')\theta_{\nu\nu'}(\tau')\right\}+\sum_{\nu''\neq\nu,\nu'}\int_0^\tau d\tau' P_{\nu\nu''}(\tau')P_{\nu''\nu'}(\tau')\left\{\left[W_{0\nu,\nu''}+\int_0^{\tau'}d\tau''\theta_{\nu\nu''}(\tau'')\right]^{-1}+\left[W_{0\nu'',\nu'}+\int_0^{\tau'}d\tau''\theta_{\nu'',\nu''}(\tau'')\right]^{-1}\right\}, \tag{53}$$

where

$$W_{0\nu',\nu} = \epsilon_{\nu'}(0) - \epsilon_{\nu}(0) \tag{54}$$

and where $\epsilon_\nu(0)$, $g_{\nu c}(0)$, and $P_{\nu\nu'}(0)$ are given by the initial conditions (37), (39), (45), and (46).

The first-order approximation is obtained by setting $\tau' = 0$ in the integrands of Eqs. (52) and (53). In this case we have, in view of the initial conditions of the problem,

$$P^{(1)}_{\nu\nu'}(\tau) = \Omega_{\nu\nu'} + \tau \sum_{\nu'' \neq \nu,\nu'} \Omega_{\nu\nu''}\Omega_{\nu''\nu'}(W^{-1}_{0\nu''\nu} + W^{-1}_{0\nu''\nu'}), \tag{55}$$

$$g^{(1)}_{\nu c}(\tau) = \Gamma_{\nu c}^{1/2} + \tau \sum_{\nu' \neq \nu} \Omega_{\nu\nu'}\Gamma_{\nu' c}^{1/2} W^{-1}_{0\nu',\nu}. \tag{56}$$

Next, from Eq. (55) for $\nu = \nu'$, one obtains

$$P^{(1)}_{\nu\nu}(\tau) = 2\tau \sum_{\nu' \neq \nu} \Omega^2_{\nu\nu'} W^{-1}_{0\nu',\nu}. \tag{57}$$

Finally, introduction of the result (57) into (51) yields

$$\epsilon^{(1)}_{\nu}(\tau) = \epsilon_\nu(0) - \tau^2 \sum_{\nu' \neq \nu} W^{-1}_{0\nu',\nu}\Omega^2_{\nu',\nu}. \tag{58}$$

In order to obtain the second-order approximation for the complex widths, one introduces the results (55) and (56) into the Volterra equation (52). One obtains

$$g^{(2)}_{\nu c}(\tau) = \Gamma_{\nu c}^{1/2} + \sum_{\nu' \neq \nu} \int_0^\tau d\tau' \{(\Omega_{\nu\nu'}\Gamma_{\nu' c}^{1/2} + \tau' A_{\nu',\nu} + \tau'^2 B_{\nu',\nu})[W_{0\nu',\nu}(1 + \tau'^2\alpha_{\nu',\nu})]^{-1}\}, \tag{59}$$

where

$$A_{\nu',\nu} = W^{-1}_{0\nu\nu'}\Omega_{\nu\nu'}\Gamma_{\nu c}^{1/2} + \sum_{\nu'' \neq \nu,\nu'} W^{-1}_{0\nu''\nu'}\Omega_{\nu\nu'}\Omega_{\nu',\nu''}\Gamma_{\nu'' c}^{1/2} + \Gamma_{\nu' c}^{1/2} \sum_{\nu'' \neq \nu,\nu'} (W^{-1}_{0\nu''\nu} + W^{-1}_{0\nu''\nu'})\Omega_{\nu\nu''}\Omega_{\nu''\nu'}, \tag{60}$$

$$\begin{aligned} B_{\nu',\nu} = {} & \Gamma_{\nu c}^{1/2} \sum_{\nu'' \neq \nu,\nu'} \Omega_{\nu\nu'}\Omega_{\nu\nu''}\Omega_{\nu''\nu'} W^{-1}_{0\nu\nu'}(W^{-1}_{0\nu''\nu} + W^{-1}_{0\nu''\nu'}) \\ & + \sum_{\nu'' \neq \nu,\nu'} \Omega_{\nu''\nu'}\Omega_{\nu\nu''}\Omega_{\nu''\nu'} W^{-1}_{0\nu''\nu'}(W^{-1}_{0\nu''\nu} + W^{-1}_{0\nu''\nu'})\Gamma_{\nu'' c}^{1/2} \\ & + \sum_{\nu'' \neq \nu,\nu'} \sum_{\nu''' \neq \nu'',\nu',\nu} \Omega_{\nu''\nu'}\Omega_{\nu\nu'''}\Omega_{\nu'''\nu'} W^{-1}_{0\nu''\nu'}(W^{-1}_{0\nu'''\nu} + W^{-1}_{0\nu'''\nu'})\Gamma_{\nu'' c}^{1/2} \end{aligned} \tag{61}$$

with

$$\alpha_{\nu',\nu} = \theta_{0\nu\nu'}/W_{0\nu',\nu}, \tag{62}$$

$$\theta_{0\nu\nu'} = \sum_{\nu'' \neq \nu} W^{-1}_{0\nu''\nu}\Omega^2_{\nu\nu''} - \sum_{\nu'' \neq \nu'} W^{-1}_{0\nu''\nu'}\Omega^2_{\nu'\nu''}. \tag{63}$$

The final expression for $g^{(2)}_{\nu c}$ is obtained from Eq. (59):

$$\begin{aligned} g^{(2)}_{\nu c}(\tau) = \Gamma_{\nu c}^{1/2} + \sum_{\nu' \neq \nu} \Bigg\{ & \left(\frac{\Omega_{\nu\nu'}\Gamma_{\nu' c}^{1/2}}{\alpha_{\nu',\nu}^{1/2} W_{0\nu',\nu}}\right) \tan^{-1}(\tau\alpha_{\nu',\nu}^{1/2}) + \frac{1}{2}\left(\frac{A_{\nu',\nu}}{\alpha_{\nu',\nu} W_{0\nu',\nu}}\right) \ln(1 + \alpha_{\nu',\nu}\tau^2) \\ & + \frac{B_{\nu',\nu}}{W_{0\nu',\nu}}[\alpha^{-1}_{\nu',\nu}\tau - \alpha^{-3/2}_{\nu',\nu}\tan^{-1}(\alpha_{\nu',\nu}^{1/2}\tau)] \Bigg\}. \end{aligned} \tag{64}$$

For the calculation of the poles one needs to obtain an expression for $P^{(2)}_{\nu\nu}(\tau)$ to be inserted in Eq. (51) for the complex poles. We have from Eq. (53) for $\nu = \nu'$

$$P^{(2)}_{\nu\nu}(\tau) = 2 \sum_{\nu' \neq \nu} \int_0^\tau d\tau' [W^{(1)}_{\nu',\nu}]^{-1} [P^{(1)}_{\nu\nu'}(\tau')]^2, \tag{65}$$

where in the integrand of (53) we introduced the pertinent first-order approximations. Upon integration in Eq. (65) one obtains

$$P_{\nu\nu}^{(2)}(\tau)=2\sum_{\nu'\neq\nu}\left\{\left(\frac{\Omega_{\nu\nu'}{}^2}{W_{0\nu'\nu}\alpha_{\nu'\nu}{}^{1/2}}\right)\tan^{-1}(\alpha_{\nu'\nu}{}^{1/2}\tau)+\left(\frac{L_{\nu'\nu}}{W_{0\nu'\nu}\alpha_{\nu'\nu}}\right)\ln(1+\alpha_{\nu'\nu}\tau^2)\right.$$
$$\left.+\frac{M_{\nu'\nu}}{W_{0\nu'\nu}\alpha_{\nu'\nu}}[\tau-\alpha_{\nu'\nu}{}^{-1/2}\tan^{-1}(\alpha_{\nu'\nu}{}^{1/2}\tau)]\right\}, \tag{66}$$

where

$$L_{\nu'\nu}=\sum_{\nu''\neq\nu,\nu'}\Omega_{\nu\nu'}\Omega_{\nu'\nu''}\Omega_{\nu''\nu}(W_{0\nu''\nu}{}^{-1}+W_{0\nu''\nu'}{}^{-1}), \tag{67}$$

$$M_{\nu'\nu}=\sum_{\nu''\neq\nu,\nu'}\Omega_{\nu\nu''}{}^2\Omega_{\nu''\nu'}{}^2(W_{0\nu''\nu}{}^{-1}+W_{0\nu''\nu'}{}^{-1})^2$$
$$+\sum_{\nu''\neq\nu,\nu'}\sum_{\nu'''\neq\nu'',\nu,\nu'}\Omega_{\nu\nu''}\Omega_{\nu''\nu'}\Omega_{\nu'\nu'''}\Omega_{\nu'''\nu}(W_{0\nu''\nu}{}^{-1}+W_{0\nu''\nu'}{}^{-1})(W_{0\nu'''\nu}{}^{-1}+W_{0\nu'''\nu'}{}^{-1}). \tag{68}$$

Next insertion of the result (66) into Eq. (51) yields the second-order approximation to the complex poles

$$\epsilon_\nu^{(2)}(\tau)=\epsilon_\nu(0)-2\sum_{\nu'\neq\nu}W_{0\nu'\nu}{}^{-1}\left\{\left(\frac{\Omega_{\nu\nu'}{}^2}{\alpha_{\nu'\nu}{}^{1/2}}\right)[\tau\tan^{-1}(\alpha_{\nu'\nu}{}^{1/2}\tau)-\tfrac{1}{2}\alpha_{\nu'\nu}{}^{-1/2}\ln(1+\alpha_{\nu'\nu}\tau^2)]\right.$$
$$+\frac{L_{\nu\nu'}}{\alpha_{\nu'\nu}}[\tau\ln(1+\alpha_{\nu'\nu}\tau^2)-2\tau+2\alpha_{\nu'\nu}{}^{-1/2}\tan^{-1}(\alpha_{\nu'\nu}{}^{1/2}\tau)]$$
$$\left.+\frac{M_{\nu\nu'}}{W_{0\nu'\nu}\alpha_{\nu'\nu}}[\tfrac{1}{2}\tau^2-\alpha_{\nu'\nu}{}^{-1/2}\tau\tan^{-1}(\alpha_{\nu'\nu}{}^{1/2}\tau)+\tfrac{1}{2}\alpha_{\nu'\nu}{}^{-1}\ln(1+\alpha_{\nu'\nu}\tau^2)]\right\}. \tag{69}$$

The calculation of the off-diagonal matrix elements $P_{\nu\nu'}^{(2)}$ to second order is carried out in similar fashion to obtain

$$P_{\nu\nu'}^{(2)}(\tau)=\Omega_{\nu\nu'}[1-\ln(1+\alpha_{\nu'\nu}\tau^2)]-2X_{\nu\nu'}[\tau-\alpha_{\nu'\nu}{}^{-1/2}\tan^{-1}(\alpha_{\nu'\nu}{}^{1/2}\tau)]$$
$$+\sum_{\nu''\neq\nu,\nu'}\left(\Omega_{\nu\nu''}\Omega_{\nu''\nu'}[\alpha_{\nu''\nu}{}^{-1/2}W_{0\nu''\nu}{}^{-1}\tan^{-1}(\alpha_{\nu''\nu}{}^{1/2}\tau)+\alpha_{\nu''\nu'}{}^{-1/2}W_{0\nu''\nu'}{}^{-1}\tan^{-1}(\alpha_{\nu''\nu'}{}^{1/2}\tau)]\right.$$
$$+\tfrac{1}{2}Y_{\nu''\nu}^{\nu\nu''}[W_{0\nu''\nu}{}^{-1}\alpha_{\nu''\nu}{}^{-1}\ln(1+\alpha_{\nu''\nu}\tau^2)+W_{0\nu''\nu'}{}^{-1}\alpha_{\nu''\nu'}{}^{-1}\ln(1+\alpha_{\nu''\nu'}\tau^2)]$$
$$+Z_{\nu\nu'}^{\nu''}\{(W_{0\nu''\nu}\alpha_{\nu''\nu})^{-1}[\tau-\alpha_{\nu''\nu}{}^{-1/2}\tan^{-1}(\alpha_{\nu''\nu}{}^{1/2}\tau)]$$
$$\left.+(W_{0\nu''\nu'}\alpha_{\nu''\nu'})^{-1}[\tau-\alpha_{\nu''\nu'}{}^{-1/2}\tan^{-1}(\alpha_{\nu''\nu'}{}^{1/2}\tau)]\}\right), \tag{70}$$

where

$$X_{\nu\nu'}=\theta_{\nu\nu'}\sum_{\nu''\neq\nu,\nu'}\Omega_{\nu\nu''}\Omega_{\nu''\nu'}(W_{0\nu''\nu}{}^{-1}+W_{0\nu''\nu'}{}^{-1}), \tag{71}$$

$$Y_{\nu'\nu}^{\nu''}=\Omega_{\nu\nu''}\sum_{\nu'''\neq\nu'',\nu'}\Omega_{\nu''\nu'''}\Omega_{\nu'''\nu'}(W_{0\nu'''\nu''}{}^{-1}+W_{0\nu'''\nu'}{}^{-1})+\Omega_{\nu''\nu'}\sum_{\nu'''\neq\nu'',\nu}\Omega_{\nu\nu'''}\Omega_{\nu'''\nu''}(W_{0\nu'''\nu}{}^{-1}+W_{0\nu'''\nu''}{}^{-1}), \tag{72}$$

$$Z_{\nu'\nu}^{\nu''}=\sum_{\nu'''\neq\nu,\nu''}\sum_{\nu^{i}\nu_{\nu''\nu'}}\Omega_{\nu\nu'''}\Omega_{\nu'''\nu''}\Omega_{\nu''\nu i\nu}\Omega_{\nu i\nu_{\nu'}}(W_{0\nu'''\nu}{}^{-1}+W_{0\nu'''\nu''}{}^{-1})(W_{0\nu i\nu_{\nu''}}{}^{-1}+W_{0\nu i\nu_{\nu'}}{}^{-1}). \tag{73}$$

The result (70) gives an expression for $P_{\nu\nu'}^{(2)}$ which coincides as expected with Eq. (66) for $\nu=\nu'$.

Introduction of the results (64), (66), (69), and (70) evaluated at $\tau=\tau_0$ are then inserted in Eqs. (47), (48), and (49) for the calculation of the complex widths and poles. We obtain in this manner a hierarchy of perturbation formulas for the complex widths $g_{\nu c}$ and complex poles ϵ_{ν_1}.

The convergence of the present perturbation method depends on the condition

$$\theta_R=\left|\frac{\Omega_{\nu'\nu}}{W_{0\nu'\nu}}\right|<1 \tag{74}$$

for the ratio of the off-diagonal elements of the interaction matrix to the spacing of the poles in the complex plane.

IV. COMPLEX POLES AND WIDTHS OF THE T MATRIX IN THE PRESENCE OF INTERMEDIATE STRUCTURE

In this section we shall derive expressions for the complex widths and poles of the T matrix for

the special case of two subsets of interacting states, a case which represents the common feature of all the intermediate structure phenomena. In accordance with the current physical interpretation[13,20] of the intermediate structure, this effect appears whenever there are two subsets of eigenstates which are coupled via a residual force. In the spirit of this hypothesis, starting with a ν-level set, one considers it being split into two subsets: (i) the λ-level subset which includes the "normal" compound nucleus levels and (ii) the μ-level subset which includes those states characterized by Lane[13] as "special" eigenstates.

These special states are endowed with the following properties (Lane[13]): (a) Their level widths Γ_μ are larger than the average level width of the compound nucleus states; (b) the levels in the μ subset are not eigenfunctions of the total nuclear Hamiltonian but of a slightly different Hamiltonian, which deviates from the former by a residual interaction, which couples the levels belonging to different subsets; (c) the "special" levels satisfy the boundary conditions of R-matrix theory, Eq. (1).

One faces then a situation in which both the boundary conditions and the Hamiltonian have been perturbed. In consequence one can apply the formalism developed in the previous section.

Under the assumption that both the residual interaction and the level interference effects are weak, it suffices to utilize the first-order expressions (56) and (58) for the complex widths and poles, respectively. To this end after setting $\tau = 1$ in both Eqs. (56) and (58), one splits the summations involved in the above expressions into the two level subsets to obtain

$$g_{\lambda c} = \Gamma_{\lambda c}^{1/2} + \sum_{\lambda' \neq \lambda} W_{0\lambda'\lambda}^{-1} \Omega_{\lambda\lambda'} \Gamma_{\lambda' c}^{1/2} + \sum_{\mu \neq \lambda} W_{0\mu\lambda}^{-1} \Omega_{\mu\lambda} \Gamma_{\mu c}^{1/2}, \tag{75}$$

$$g_{\mu c} = \Gamma_{\mu c}^{1/2} + \sum_{\mu' \neq \mu} W_{0\mu'\mu}^{-1} \Omega_{\mu\mu'} \Gamma_{\mu' c}^{1/2} + \sum_{\lambda \neq \mu} W_{0\lambda\mu}^{-1} \Omega_{\lambda\mu} \Gamma_{\lambda c}^{1/2}, \tag{76}$$

$$\epsilon_\lambda = \epsilon_{D\lambda} - \sum_{\lambda' \neq \lambda} W_{0\lambda'\lambda}^{-1} \Omega_{\lambda'\lambda}^{\ 2} - \sum_{\mu \neq \lambda} W_{0\mu\lambda}^{-1} \Omega_{\mu\lambda}^{\ 2}, \tag{77}$$

$$\epsilon_\mu = \epsilon_{D\mu} - \sum_{\mu' \neq \mu} W_{0\mu'\mu}^{-1} \Omega_{\mu\mu'}^{\ 2} - \sum_{\lambda \neq \mu} W_{0\lambda\mu}^{-1} \Omega_{\lambda\mu}^{\ 2}, \tag{78}$$

where from Eq. (39) we made

$$\epsilon_{D\nu} = \epsilon_\nu(1,0) = \epsilon_\nu^D(0) - \omega_{\nu\nu} \quad (\nu = \lambda, \mu). \tag{79}$$

Next we write the complex widths and poles in the form

$$g_{\lambda c} = g_{R\lambda c} + i g_{I\lambda c}, \tag{80}$$

$$\epsilon_\lambda = E'_\lambda - \frac{i}{2}\Gamma'_\lambda \tag{81}$$

with similar expressions for the widths and poles of the μ-level subset. By separation of real and imaginary parts in Eq. (76) up to (78) one obtains

$$g_{R\lambda c} = g_{1R\lambda c} - \sum_\mu \Gamma_{\mu c}^{1/2} [\tilde{V}_{\mu\lambda}(E_\mu - E'_\lambda) - \tfrac{1}{4}\Gamma_{\mu\lambda} \times (\Gamma_\mu - \Gamma_\lambda)] |W_{0\mu\lambda}|^{-2}, \tag{82}$$

$$g_{I\lambda c} = g_{1I\lambda c} + \frac{1}{2}\sum_\mu \Gamma_{\mu c}^{1/2} [\Gamma_{\mu\lambda}(E_\mu - E_\lambda) - \tilde{V}_{\mu\lambda} \times (\Gamma_\mu - \Gamma_\lambda)] |W_{0\mu\lambda}|^{-2}, \tag{83}$$

$$g_{R\mu c} = g_{1R\mu c} - \sum_\lambda \Gamma_{\lambda c}^{1/2} [\tilde{V}_{\lambda\mu} \Gamma_{\lambda c}(E_\lambda - E_\mu) - \tfrac{1}{4}\Gamma_{\lambda\mu})] |W_{0\lambda\mu}|^{-2}, \tag{84}$$

$$g_{I\mu c} = g_{1I\mu c} + \frac{1}{2}\sum_\lambda \Gamma_{\lambda c}^{1/2} [\Gamma_{\lambda\mu}(E_\lambda - E_\mu) - \tilde{V}_{\lambda\mu} \times (\Gamma_\lambda - \Gamma_\mu)] |W_{0\lambda\mu}|^{-2}, \tag{85}$$

where

$$\tilde{V}_{\mu\lambda} = \tilde{V}_{\lambda\mu} = V_{\mu\lambda} - S_{\lambda\mu}, \tag{86}$$

$$g_{1R\lambda c} = \Gamma_{\lambda c}^{1/2} - \sum_{\lambda' \neq \lambda} \Gamma_{\lambda' c}^{1/2} [S_{\lambda\lambda'}(E_{\lambda'} - E_\lambda) + \tfrac{1}{4}\Gamma_{\lambda\lambda'} \times (\Gamma_{\lambda'} - \Gamma_\lambda)] |W_{0\lambda\lambda'}|^{-2}, \tag{87}$$

$$g_{1I\lambda c} = \frac{1}{2}\sum_{\lambda' \neq \lambda} \Gamma_{\lambda' c}^{1/2} [\Gamma_{\lambda\lambda'}(E_{\lambda'} - E_\lambda) - S_{\lambda\lambda'}(\Gamma_{\lambda'} - \Gamma_\lambda)] |W_{0\lambda\lambda'}|^{-2}, \tag{88}$$

with similar expressions to Eqs. (87) and (88) for $g_{1R\mu c}$ and $g_{1I\mu c}$ by just replacing the λ subindices by μ subindices. For the poles one obtains

$$E'_\lambda = E_{1\lambda} - \sum_\mu [I_{\mu\lambda}(E_\mu - E_\lambda) + \tfrac{1}{2} J_{\mu\lambda}(\Gamma_\mu - \Gamma_\lambda)] |W_{0\mu\lambda}|^{-2}, \tag{89}$$

$$\Gamma'_\lambda = \Gamma_{1\lambda} + \sum_\mu [I_{\mu\lambda}(\Gamma_\mu - \Gamma_\lambda) - 2 J_{\mu\lambda}(E_\mu - E_\lambda)] |W_{0\mu\lambda}|^{-2}, \tag{90}$$

$$E'_\mu = E_{1\mu} - \sum_\lambda [I_{\lambda\mu}(E_\lambda - E_\mu) + \tfrac{1}{2} J_{\lambda\mu}(\Gamma_\lambda - \Gamma_\mu)] |W_{0\lambda\mu}|^{-2}, \tag{91}$$

$$\Gamma'_\mu = \Gamma_{1\mu} + \sum_\lambda [I_{\lambda\mu}(\Gamma_\lambda - \Gamma_\mu) - 2 J_{\lambda\mu}(E_\lambda - E_\mu)] |W_{0\lambda\mu}|^{-2}, \tag{92}$$

where

$$\Gamma_{\mu\lambda}=\Gamma_{\lambda\mu}=\sum_c(\Gamma_{\mu c}\Gamma_{\lambda c})^{1/2}, \tag{93}$$

$$I_{\mu\lambda}=I_{\lambda\mu}=\tilde{V}_{\mu\lambda}{}^2-\tfrac{1}{4}\Gamma_{\mu\lambda}{}^2, \tag{94}$$

$$J_{\lambda\mu}=J_{\mu\lambda}=\tilde{V}_{\mu\lambda}\Gamma_{\mu\lambda}, \tag{95}$$

$$|W_{0\mu\lambda}|^2=(E_\mu-E_\lambda)^2+\tfrac{1}{4}(\Gamma_\mu-\Gamma_\lambda)^2, \tag{96}$$

$$E_{1\lambda}=E_\lambda+\Delta_{\lambda\lambda}-\sum_{\lambda'\neq\lambda}[I_{\lambda\lambda'}(E_{\lambda'}-E_\lambda)+\tfrac{1}{2}J_{\lambda\lambda'}(\Gamma_{\lambda'}-\Gamma_\lambda)]\times|W_{0\lambda'\lambda}|^{-2}, \tag{97}$$

$$\Gamma_{1\lambda}=\Gamma_\lambda+\sum_{\lambda'\neq\lambda}[I_{\lambda\lambda'}(\Gamma_{\lambda'}-\Gamma_\lambda)-2J_{\lambda\lambda'}(E_{\lambda'}-E_\lambda)]|W_{0\lambda'\lambda}|^{-2} \tag{98}$$

with similar expressions for $E_{1\mu}$ and $\Gamma_{1\mu}$ and where we defined

$$I_{\lambda\lambda'}=S_{\lambda\lambda'}{}^2-\tfrac{1}{4}\Gamma_{\lambda\lambda'}{}^2, \tag{99}$$

$$J_{\lambda\lambda'}=-S_{\lambda\lambda'}\Gamma_{\lambda\lambda'}. \tag{100}$$

Inspection of Eq. (82) up to (92) reveals some interesting features. For example, let us rewrite Eq. (82) for the real part of the complex widths, belonging to the compound nucleus states. We have (after neglecting level interference effects)

$$g_{R\lambda c}=\Gamma_{\lambda c}{}^{1/2}-\sum_\mu\left[\frac{\tilde{V}_{\mu\lambda}(E_\mu-E_\lambda)-\frac{1}{4}\Gamma_{\mu\lambda}\Gamma_\mu}{(E_\mu-E_\lambda)^2+\frac{1}{4}\Gamma_\mu{}^2}\right]\Gamma_{\mu c}{}^{1/2}, \tag{101}$$

where we neglected Γ_λ versus Γ_μ. In a similar manner the imaginary part will be given according to Eq. (83) by

$$g_{I\lambda c}=\frac{1}{2}\sum_\mu\left\{\frac{[\Gamma_{\mu\lambda}(E_\mu-E_\lambda)-\tilde{V}_{\mu\lambda}\Gamma_\mu]\Gamma_{\mu c}{}^{1/2}}{(E_\mu-E_\lambda)^2+\frac{1}{4}\Gamma_\mu{}^2}\right\}. \tag{102}$$

Clearly the coupling between the two subset of levels results from an interplay between "internal mixing" due to the residual force and the "external mixing" which arises from level interference terms between the two level subsets.[13]

The former is proportional to the matrix elements $V_{\lambda\mu}$ while the latter depends on the "off-diagonal" level widths $\Gamma_{\mu\lambda}$ and the factors $S_{\lambda\mu}$. The maxima exhibited by the reduced widths at $E_\mu=E_\lambda$ are damped by the quantity $(\Gamma_\mu/4)^2$ in the denominator, an effect already discussed by Lane[13] as a reduction in effective mixing power. The mixing between the two subsets of levels results [Eq. (76)] in the generation of a partial decay width for the special levels to channels where $\Gamma_{\mu c}$ was initially zero.

Within the same approximations used above for the complex widths, the complex poles can be written from Eqs. (89) and (90) in the form

$$E'_\lambda=E_\lambda+\tilde{\Delta}_{\lambda\lambda}, \tag{103}$$

$$\Gamma'_\lambda=\Gamma_\lambda+\phi_\lambda. \tag{104}$$

These results indicate that the R-matrix poles of the compound nucleus levels are shifted by a resonant shift factor

$$\tilde{\Delta}_{\lambda\lambda}=\Delta_{\lambda\lambda}-\sum_\mu\left[\frac{\frac{1}{2}J_{\mu\lambda}\Gamma_\mu+I_{\mu\lambda}(E_\mu-E_\lambda)}{(E_\mu-E_\lambda)^2+\frac{1}{4}\Gamma_\mu{}^2}\right]. \tag{105}$$

The level widths in turn acquire an extra term,

$$\phi_\lambda=\sum_\mu\left[\frac{I_{\mu\lambda}\Gamma_\mu-2J_{\mu\lambda}(E_\mu-E_\lambda)}{(E_\mu-E_\lambda)^2+\frac{1}{4}\Gamma_\mu{}^2}\right], \tag{106}$$

which will also peak at $E_\mu=E_\lambda$.

The terms proportional to the factor $(E_\mu-E_\lambda)$ introduce an asymmetry in both the resonant shift factor $\tilde{\Delta}_{\lambda\lambda}$ and level width ϕ_λ. Finally the following sum rules for the complex poles and widths are direct results of Eq. (75) up to (78):

$$\sum_\lambda g_{\lambda c}+\sum_\mu g_{\mu c}=\sum_\lambda\Gamma_{\lambda c}{}^{1/2}+\sum_\mu\Gamma_{\mu c}{}^{1/2}, \tag{107}$$

$$\sum_\lambda\epsilon_\lambda+\sum_\mu\epsilon_\mu=\sum_\lambda\epsilon_{D\lambda}+\sum_\mu\epsilon_{D\mu}. \tag{108}$$

V. REACTION CROSS SECTION IN THE PRESENCE OF INTERMEDIATE STRUCTURE

In this section we shall derive expressions for the reaction cross section in the presence of intermediate structure. The reaction cross section between channels c and c' is given for a particular spin and parity by

$$\sigma_{cc'}^{(J^\pi,l)}=\frac{\pi}{k_c^2}g_J|T_{cc'}^{(J^\pi,l)}|^2 \tag{109}$$

with

$$k_c=\left(\frac{\hbar^2E_c}{2M_c}\right)^{1/2}, \tag{110}$$

where M_c, E_c, and g_J are the channel mass energy and the statistical spin factor, respectively, and where for clarity sake we shall drop the superindices (J^π,l) in the developments that follow. In the present instance the transition T matrix can be written from Eq. (8) in the form

$$T_{cc'}=i\left(\sum_\lambda\frac{g_{\lambda c}g_{\lambda c'}}{\epsilon_\lambda-E}+\sum_\mu\frac{g_{\mu c}g_{\mu c'}}{\epsilon_\mu-E}\right). \tag{111}$$

Introduction of Eq. (111) into Eq. (109) yields, after some manipulations, the result

$$\sigma_{cc'}=\frac{\pi g_J}{k_c^2}\left[\sum_\lambda\frac{\frac{1}{2}G_\lambda^{cc'}\Gamma'_\lambda+H_\lambda^{cc'}(E'_\lambda-E)}{(E'_\lambda-E)^2+\frac{1}{4}(\Gamma'_\lambda)^2}+\sum_\mu\frac{\frac{1}{2}G_\mu^{cc'}\Gamma'_\mu+H_\mu^{cc'}(E_\mu-E)}{(E'_\mu-E)^2+\frac{1}{4}(\Gamma'_\mu)^2}\right]. \tag{112}$$

The symmetric and asymmetric parts of the U-

matrix residues R_λ, R_μ are represented by $G_\lambda, (G_\mu)$ and $H_\lambda, (H_\mu)$, respectively.[4] They are given in the present formulation by the expressions

$$R_\lambda^{cc'} = H_\lambda^{cc'} - iG_\lambda^{cc'}$$
$$= 2\sum_{\lambda'} g_{\lambda c} g_{\lambda c'} g^*_{\lambda' c} g^*_{\lambda' c'} / (\epsilon^*_{\lambda'} - \epsilon_\lambda)$$
$$+ 2\sum_{\mu} g_{\lambda c} g_{\lambda c'} g^*_{\mu c} g^*_{\mu c'} / (\epsilon^*_{\mu} - \epsilon_\lambda), \quad (113)$$

$$R_\mu^{cc'} = H_\mu^{cc'} - iG_\mu^{cc'}$$
$$= 2\sum_{\mu'} g_{\mu c} g_{\mu c'} g^*_{\mu' c} g^*_{\mu' c'} / (\epsilon^*_{\mu'} - \epsilon_\mu)$$
$$+ 2\sum_{\lambda} g_{\mu c} g_{\mu c'} g^*_{\lambda c} g^*_{\lambda c'} / (\epsilon^*_{\lambda} - \epsilon_\mu). \quad (114)$$

It is apparent from Eq. (112) that the reaction cross section splits into two components arising from contributions by the compound levels and by the special states. The line shapes are asymmetric by virtue of the terms in $H_\lambda^{c,c'}$ and $H_\mu^{c,c'}$. Also the coupling between the two subsets of levels introduces resonant terms in the residues R_λ and R_μ as expressed by Eqs. (113) and (114). We shall now look in more detail into the structure of the complex residues R_λ and R_μ.

For this purpose, we again neglect level interference terms within each subset of levels and assume $\Gamma_{\mu c}^{1/2} = 0$, i.e., only exit channel doorways are considered. Then from the general Eqs. (75) and (76) one obtains

$$g_{\lambda c} = \Gamma_{\lambda c}^{1/2}, \quad (115)$$

$$g_{\lambda c'} = \Gamma_{\lambda c'}^{1/2} + \sum_{\mu} W_{0\mu\lambda}^{-1} \Omega_{\mu\lambda} \Gamma_{\mu c'}^{1/2}, \quad (116)$$

$$g_{\mu c} = \sum_{\lambda} W_{0\lambda\mu}^{-1} \Omega_{\mu\lambda} \Gamma_{\lambda c}^{1/2}, \quad (117)$$

$$g_{\mu c'} = \Gamma_{\mu c'}^{1/2} + \sum_{\lambda} W_{0\lambda\mu}^{-1} \Omega_{\mu\lambda} \Gamma_{\lambda c'}^{1/2}. \quad (118)$$

Further, we assume $g_{\mu c}$ to be negligible and obtain after introduction of the above equations into the relations (113) and (114) for the residues the result

$$G_\lambda^{cc'} = 2\frac{\Gamma_{\lambda c}}{\Gamma'_\lambda}\Big\{\Gamma_{\lambda c'} - 2\sum_{\mu} (\Gamma_{\lambda c'} \Gamma_{\mu c'})^{1/2}$$
$$\times [\tilde{V}_{\mu\lambda}(E_\mu - E_\lambda) + \tfrac{1}{4}\Gamma_{\mu\lambda}\Gamma_\mu](W_{0\mu\lambda})^{-2}$$
$$+ \sum_{\mu} (W_{0\mu\lambda})^{-2} \Gamma_{\mu c} (\tilde{V}_{\mu\lambda}^2 + \tfrac{1}{4}\Gamma_{\mu\lambda}^2)\Big\}, \quad (119)$$

$$H_\lambda^{c,c'} = 0, \quad (120)$$

$$R_\mu^{cc'} = 0, \quad (121)$$

where we used the expressions (43) and (86).

In view of these results and Eqs. (103), (104), and (112) one obtains for the cross section the result

$$\sigma_{cc'} = \frac{\pi}{k_c^2} g_J \sum_{\lambda} \frac{\Gamma_{\lambda c} \tilde{\Gamma}_{\lambda c'}}{(E'_\lambda - E)^2 + \frac{1}{4}(\Gamma'_\lambda)^2}. \quad (122)$$

Hence, neglecting level interference effects within each subset of levels, the cross section given by Eq. (122) exhibits the customary Breit-Wigner line shape with total level widths Γ'_λ and effective R-matrix poles E'_λ given by Eqs. (104) and (103), respectively. The effective partial level width $\tilde{\Gamma}'_{\lambda c}$ is given from Eqs. (119) and (112) by

$$\tilde{\Gamma}_{\lambda c'} = \Gamma_{\lambda c'} + \langle D_\lambda \rangle \theta_{\lambda, c'} \quad (123)$$

with

$$\theta_{\lambda, c'} = \sum_{\mu} |W_{0\mu\lambda}|^{-2} \{\Gamma_{\mu c'} \Gamma^{\downarrow}_{\mu\lambda} - 2[\xi_{\mu\lambda, c'}(E_\mu - E_\lambda) + Y_{\mu\lambda, c'} \Gamma_\mu]\}, \quad (124)$$

where $\langle D_\lambda \rangle$ is the average level spacing of the compound nucleus levels and we define the following parameters:

the spreading level width

$$\Gamma^{\downarrow}_{\mu\lambda} = (\tilde{V}_{\mu\lambda}^2 + \tfrac{1}{4}\Gamma_{\mu\lambda}^2)/\langle D_\lambda \rangle, \quad (125)$$

the internal mixing partial width

$$\xi_{\mu, \lambda, c} = (\Gamma_{\lambda c} \Gamma_{\mu c})^{1/2} \left(\frac{\tilde{V}_{\mu\lambda}}{\langle D_\lambda \rangle}\right), \quad (126)$$

the external mixing partial width

$$Y_{\mu, \lambda, c} = (\Gamma_{\lambda c} \Gamma_{\mu c})^{1/2} (\Gamma_{\mu\lambda} / \langle D_\lambda \rangle). \quad (127)$$

The results (123) and (124) indicate that there is an asymmetry in the magnitude of the effective level partial widths which is introduced by the location of the compound nucleus level relative to the energy eigenvalue of the unperturbed special level E_μ. The sign of the asymmetric contribution depends on the combination of signs attached to $\Gamma_{\lambda c'}^{1/2}$ and $\Gamma_{\mu c'}^{1/2}$, as well as on the sign of the factor $(E_\mu - E_\lambda)$. The combination of damping and asymmetry effects could drastically reduce the size of some of the compound nucleus resonances interacting with a given "special" level. This effect could in principle provide an explanation for the observation[14] that some of the subthreshold fission resonances in the (^{238}U+n) compound nucleus, clustering around 721 eV (which are of the correct spin and parity to interact with the 721 eV class II level) exhibit vanishingly small fission widths. We consider next the case in which the compound nucleus resonances are no longer resolved. In this instance the information available is the average cross section defined as

$$\langle E^{1/2}\sigma_{cc'}\rangle = q_c E^{-1/2}\frac{g_J}{\Delta_\lambda}\sum_\lambda\left[\int_\Delta dE\frac{\Gamma_{\lambda c}\tilde{\Gamma}_{\lambda c'}}{(E'-E)^2+\frac{1}{4}(\Gamma_\lambda')^2}\right], \tag{128}$$

$$q_c = \pi\left(\frac{\hbar^2}{2M_c}\right) \tag{129}$$

where the energy interval Δ_λ, given by

$$\Delta_\lambda = N_\lambda\langle D_\lambda\rangle, \tag{130}$$

is chosen such that it comprises a large number N_λ of compound nucleus levels, but is smaller than the broad cross section features due to intermediate structure effects (i.e., $\Delta_\lambda < \Gamma_\mu$). The result of the integration in Eq. (128) with the assumption that $[(E_\lambda'-E)/\Gamma_\lambda'] \gg 1$, when evaluated at the limits of integration, yields

$$\langle E^{1/2}\sigma_{cc'}\rangle = \frac{\sigma_0}{\langle D_\lambda\rangle}E^{-1/2}\left\langle\frac{\Gamma_{\lambda c}\tilde{\Gamma}_{\lambda c'}}{\Gamma_\gamma'}\right\rangle \tag{131}$$

with

$$\sigma_0 = 2\pi q_c g_J. \tag{132}$$

In order to recast the result (131) in terms of the averages of the parameters within the bracket in the right-hand side of (131) we introduce the statistical fluctuation factor (see for example Schmidt[21]). One obtains, after use is made of Eqs. (123) and (104) and of the fact that in the unresolved region the high density compound levels allow us to replace E_λ by the neutron energy E, the result

$$\langle E^{1/2}\sigma_{cc'}\rangle = \frac{\sigma_0}{\langle D_\lambda\rangle}E^{-1/2}\times\left[\frac{\langle\Gamma_{\lambda c}\rangle(\langle\Gamma_{\lambda c'}\rangle R_1+\langle D_\lambda\rangle\langle\theta_{\lambda c'}\rangle R_2)}{\langle\Gamma_\lambda\rangle+\langle\phi_\lambda\rangle}\right] \tag{133}$$

with

$$\langle\theta_{\lambda c'}\rangle = \sum_\mu |W_{0\mu}|^{-2}[\Gamma_\mu U_{\mu c'} - K_{\mu c'}(E_\mu - E)], \tag{134}$$

$$\langle\phi_\lambda\rangle = \sum_\mu |W_{0\mu}|^{-2}[\langle I_{\mu\lambda}\rangle\Gamma_\mu - 2\langle J_{\mu\lambda}\rangle(E_\mu - E)], \tag{135}$$

$$|W_{0\mu}|^{-2} = (E_\mu - E)^2 + \tfrac{1}{4}\Gamma_\mu{}^2, \tag{136}$$

$$U_{\mu c'} = \frac{\langle\Gamma_{\mu\lambda}^{\downarrow}\rangle}{\Gamma_\mu}\Gamma_{\mu c'} - 2\langle\Gamma_{\mu\lambda c'}\rangle \ \text{(eV)}, \tag{137}$$

$$K_{\mu c'} = 2\langle\xi_{\mu\lambda c'}\rangle \ \text{(eV)}, \tag{138}$$

$$\langle I_{\mu\lambda}\rangle = \langle\tilde{V}_{\mu\lambda}{}^2\rangle - \tfrac{1}{4}\langle\Gamma_{\mu\lambda}{}^2\rangle \ \text{(eV)}^2, \tag{139}$$

$$\langle J_{\mu\lambda}\rangle = \langle\tilde{V}_{\mu\lambda}\Gamma_{\mu\lambda}\rangle \ \text{(eV)}^2. \tag{140}$$

The statistical fluctuation factors, R_1 and R_2, are given by

$$R_1 = \frac{\langle\Gamma_\lambda\rangle+\langle\phi_\lambda\rangle}{\langle\Gamma_{\lambda c}\rangle\langle\Gamma_{\lambda c'}\rangle}\int d\Gamma_{\lambda c}d\Gamma_{\lambda c'}d\Gamma_\lambda d\phi_\lambda \times\left[\frac{\Gamma_{\lambda c}\Gamma_{\lambda c'}}{(\Gamma_\lambda+\phi_\lambda)}\right]P_1(\Gamma_{\lambda c},\Gamma_{\lambda c'},\Gamma_\lambda,\phi_\lambda), \tag{141}$$

$$R_2 = \frac{\langle\Gamma_\lambda\rangle+\langle\phi_\lambda\rangle}{\langle\Gamma_{\lambda c}\rangle\langle\theta_{\lambda c}\rangle}\int d\Gamma_{\lambda c}d\theta_{\lambda c}d\Gamma_\lambda d\phi_\lambda\left(\frac{\Gamma_{\lambda c}\theta_{\lambda c}}{\Gamma_\lambda+\phi_\lambda}\right) \times P_2(\Gamma_{\lambda c},\theta_{\lambda c},\Gamma_\lambda,\phi_\lambda). \tag{142}$$

Although the probability distributions P_1 and P_2 are not known, one can arrive at an estimation of the statistical fluctuation factors by Monte Carlo techniques via the generation of ladders of resonances.

Upon introduction of the strength functions

$$S_{0c} = \Gamma_{\lambda c}/\langle D_\lambda\rangle, \tag{143}$$

$$S_{0c'} = \Gamma_{\lambda c'}/\langle D_\lambda\rangle. \tag{144}$$

The average cross section in Eq. (133) can be written in the form

$$\langle E^{1/2}\sigma_{cc'}\rangle = \sigma_0 E^{-1/2}\left[\frac{\langle\Gamma_{\lambda c'}\rangle}{\langle\Gamma_\lambda\rangle}\right]S_{cc'}(E), \tag{145}$$

where we introduced the generalized strength function

$$S_{cc'}(E) = S_{0c}M_{c'}(E) \tag{146}$$

and where the modulation factor $M_{c'}(E)$ is given by

$$M_{c'}(E) = M_{1c'}(E)/M_2(E) \tag{147}$$

with

$$M_{1c'}^{(E)} = R_1 + R_2\langle\theta_\lambda\rangle/S_{0c'}, \tag{148}$$

$$M_2^{(E)} = 1 + \langle\phi_\lambda\rangle/\langle\Gamma_\lambda\rangle. \tag{149}$$

The result (145) shows that in the presence of intermediate structure one can interpret the average cross section in terms of a "resonant" strength function $S_{cc'}(E)$, showing local enhancements of the cross section at bombarding energies in the vicinity of the resonance energy of the unperturbed "special" states. The fit of the experimental data to Eq. (145) will yield the parameters E_μ, Γ_μ, $U_{\mu c'}$, $K_{\mu c'}$, $\langle I_{\mu\lambda}\rangle$, and $\langle J_{\mu\lambda}\rangle$, for each "special" state. The last four parameters are related to the internal mixing parameters $V_{\mu\lambda}$ and the external mixing parameters $S_{\lambda\mu}$ and $\Gamma_{\mu\lambda}$ by the relations (137) up to (140). It is to be noticed that the asymmetric terms in the average cross section are controlled by the factors $K_{\mu c'}$ and $\langle J_{\mu\lambda}\rangle$ which depend in turn on cross terms between internal and external mixing.

The relations (137) up to (140) between the "observable" parameters $U_{\mu c'}$, $K_{\mu c'}$, $\langle I_{\mu\lambda}\rangle$, and $\langle J_{\mu\lambda}\rangle$ and the nuclear parameters of interest can be

simplified in extreme cases: (i) For pure internal mixing, $\langle J_{\mu\lambda}\rangle$ vanishes and one obtains

$$\langle \tilde{V}_{\mu\lambda}{}^2\rangle = \langle I_{\mu\lambda}\rangle , \tag{150}$$

$$\Gamma_{\mu c'} = \Gamma_\mu \left(\frac{U_{\mu c'}}{\langle I_{\mu\lambda}\rangle}\right)\langle D_\lambda\rangle , \tag{151}$$

$$\langle \tilde{V}_{\mu\lambda}\rangle = \pm\frac{1}{2}\frac{K_{\mu c'}}{\langle(\Gamma_{\lambda c}\Gamma_{\mu c'})^{1/2}\rangle}\langle D_\lambda\rangle . \tag{152}$$

(ii) For pure external mixing one obtains the following relations:

$$\langle I_{\mu\lambda}\rangle = -\tfrac{1}{4}\langle \Gamma_{\mu\lambda}{}^2\rangle , \tag{153}$$

$$\langle J_{\mu\lambda}\rangle = -\langle S_{\lambda\mu}\Gamma_{\mu\lambda}\rangle , \tag{154}$$

$$U_{\mu c'} = [\tfrac{1}{4}\langle \Gamma_{\mu\lambda}{}^2\rangle - 2\langle\Gamma_{\mu\lambda}\rangle\langle(\Gamma_{\lambda c'}\Gamma_{\mu c'})^{1/2}\rangle]/\langle D_\lambda\rangle , \tag{155}$$

$$K_{\mu c'} = -2\Gamma_{\mu c'}{}^{1/2}\langle \Gamma_{\lambda c'}{}^{1/2}\rangle\langle S_{\mu\lambda}\rangle/\langle D_\lambda\rangle . \tag{156}$$

The result (153) shows that for the pure external mixing case $\langle I_{\mu\lambda}\rangle$ should be negative. (iii) For the case in which the decay of the special level takes place mostly to a single channel c' one can write $\Gamma_\mu \approx \Gamma_{\mu c'}$, and the following relations are then derived in the presence of both internal and external mixing:

$$K_{\mu c'} = \pm 2\langle \Gamma_{\lambda c'}{}^{1/2}\rangle \Gamma_\mu{}^{1/2}\frac{\langle \tilde{V}_{\mu\lambda}\rangle}{\langle D_\lambda\rangle} , \tag{157}$$

$$\langle J_{\mu\lambda}\rangle = \langle \tilde{V}_{\mu\lambda}\Gamma_{\mu\lambda}\rangle , \tag{158}$$

$$U_{\mu c'} = \langle D_\lambda\rangle^{-1}(\langle \tilde{V}_{\mu\lambda}{}^2\rangle + \tfrac{1}{4}\langle \Gamma_{\mu\lambda}{}^2\rangle \mp 2\langle\Gamma_{\mu\lambda}\rangle\langle\Gamma_{\lambda c'}{}^{1/2}\rangle\Gamma_\mu{}^{1/2}) . \tag{159}$$

VI. DISCUSSION AND SOME APPLICATIONS OF THE PRESENT FORMALISM

In order to evaluate the formalism developed in this work for the complex widths and poles of the transition matrix we shall compare the results from the present formalism with exact calculations for the simple example of two interacting resonances. Comparison will also be made between the total neutron cross section computed from the T-matrix widths and poles, both exact and approximate, with the calculation of total cross sections by the Bethe approximation.[6] The purpose of this comparative study based on cross section calculations is twofold: First, one desires to know how well approximate values of the complex poles and residues will do in predicting cross sections, and second, one likes to find out the range of applicability of the Bethe formulation which is a widely utilized approximation in many cross section calculations.[7]

The total neutron cross section is given by

$$\sigma_T = g_n E^{-1} g_J R_e[(1 - U_{nn})] , \tag{160}$$

where U_{nn} is the collision matrix, which is related to the T matrix by

$$U_{nn} = e^{-2i\phi_n}(1 + T_{nn}) , \tag{161}$$

where for s-wave neutrons the hard sphere scattering factor U_n is

$$\phi_n = 2k_n a_n \tag{162}$$

with

$$k_n = \left(\frac{2M_n}{\hbar^2}E\right)^{1/2} \tag{163}$$

and, a_n, the channel radius. In turn the transition matrix element T_{nn} is given from Eq. (8) by

$$T_{nn} = i\sum_\nu \frac{g_{\nu n}{}^2}{\epsilon_\nu - E} . \tag{164}$$

Introduction of Eqs. (164) and (161) into (160) yields

$$\sigma_T = \sigma_P + \sigma_{0T}\sum_\nu |W_\nu|^{-2}[\tfrac{1}{2}\Gamma'_\nu G^T_\nu + (E'_\nu - E)H^T_\nu] \tag{165}$$

with

$$\sigma_P = 2g_n E^{-1} g_J \sin^2\phi_n , \tag{166}$$

$$G^T_\nu = \alpha_\nu\cos(2\phi_n) + \beta_\nu\sin(2\phi_n) , \tag{167}$$

$$H^T_\nu = \beta_\nu\cos(2\phi_n) - \alpha_\nu\sin(2\phi_n) , \tag{168}$$

$$|W_\nu|^2 = (E'_\nu - E)^2 + \tfrac{1}{4}(\Gamma'_\nu)^2 , \tag{169}$$

and where the real and imaginary parts of the T-matrix residues have been defined as

$$\alpha_\nu = \mathrm{Re}(g_{\nu,n}{}^2) = g_{\nu R,n}{}^2 - g_{\nu I,n}{}^2 , \tag{170}$$

$$\beta_\nu = \mathrm{Im}(g_{\nu n}{}^2) = 2g_{\nu R,n}g_{\nu I,n} . \tag{171}$$

Exact expressions for the residues α_ν, β_ν and the complex poles $\epsilon_\nu = E'_\nu - \frac{1}{2}\Gamma_\nu$ have been derived by Garrison[22] for the two level case.

The total cross sections in the Bethe approximation is written in the form[7]

TABLE I. Parameters of the various sets of two interacting resonances examined in the present study.

No.	E_1 (keV)	E_2 (keV)	Γ_{1n}[a] (keV)	Γ_{2n}[a] (keV)	θ_R[c]	Remarks
1	317.0	331.2	8.8	0.32	0.11	^{56}Fe
2	356.9	362.0	1.8	3.35	0.48	b
3	246.8	258.0	19.7	3.36	0.59	^{54}Fe
4	356.9	362.0	3.6	6.7	0.92	^{56}Fe
5	326.0	332.0	20.0	24.0	3.46	^{54}Fe

[a] Neutron capture is negligible.

[b] The neutron widths in this case are one half of the neutron widths in case 4.

[c] See Eq. (179) in text.

TABLE II. Comparison between the exact and perturbation calculations for the complex poles of two interacting nuclear resonances.

No.	E_1' T0	E_1' T1	E_1' TE	E_2' T0	E_2' T1	E_2' TE	Γ_1' T0	Γ_1' T1	Γ_1' TE	Γ_2' T0	Γ_2' T1	Γ_2' TE
1	317.05	317.04	317.04	331.15	331.16	331.16	8.83	8.83	8.83	0.28	0.29	0.29
2	357.2	357.2	357.2	361.71	361.71	361.70	1.71	1.71	1.70	3.41	3.42	3.43
3	248.25	247.73	247.69	256.55	257.07	257.11	21.82	21.15	21.26	1.17	1.84	1.73
4	358.07	358.03	358.07	360.83	360.87	360.83	2.90	2.77	2.31	7.35	7.48	7.94
5	···	328.73	328.73	···	329.27	329.27	···	0.83	0.83	···	42.96	42.96

$$\sigma_T = \sigma_P + \mathcal{G}_n E^{-1} g_J [A\cos(2\phi_n) - B\sin(2\phi_n)] + \mathcal{G}_n E^{-1} g_J \sum_\nu [\tfrac{1}{2}\Gamma_\nu G_\nu^{(B)} - (E_\lambda - E) H_\nu^{(B)}] \, |W_{0\nu}|^{-2} , \tag{172}$$

where

$$|W_{0\nu}|^2 = (E_\nu - E)^2 + \tfrac{1}{4}\Gamma_\nu{}^2 , \tag{173}$$

$$A = \tfrac{1}{2}\sum_\nu |W_{0\nu}|^{-2} \Gamma_{\nu n}\Gamma_\nu , \tag{174}$$

$$B = \sum_\nu |W_{0\nu}|^{-2} \Gamma_{\nu n}(E_\nu - E) , \tag{175}$$

$$G_\nu^{(B)} = \tfrac{1}{2}\sum_{\nu\neq\nu'} \rho_{\nu\nu'}{}^{-2} \Gamma_{\nu n}\Gamma_{\nu' n}(\Gamma_\nu + \Gamma_{\nu'}) , \tag{176}$$

$$H_\nu^{(B)} = \sum_{\nu\neq\nu'} \rho_{\nu\nu'}{}^{-2} \Gamma_{\nu n}\Gamma_{\nu' n}(E_\nu - E_{\nu'}) , \tag{177}$$

with

$$\rho_{\nu,\nu'}{}^2 = (E_\nu - E_{\nu'})^2 + \tfrac{1}{4}(\Gamma_\nu + \Gamma_{\nu'})^2 . \tag{178}$$

Inspection of Eq. (172) reveals that the Bethe approximation consists of the addition of a level interference term [the third term in the right-hand side of Eq. (172)] to the usual single level Breit-Wigner approximation.

For the purpose of the present comparison we have chosen five pairs of resonance parameters taken from the total iron cross section measurements of Pandey *et al.*[9] The details are given in Table I where the various sets have been arranged from low to high level interference. The degree of level interference is determined by the parameter

$$\theta_R = \Gamma_{12}/[(E_1 - E_2)^2 + \tfrac{1}{4}(\Gamma_{1n} - \Gamma_{1n})^2]^{1/2} . \tag{179}$$

Table II shows the results obtained for the complex poles, and Table III contains the results for the real part Λ_ν and imaginary part Ξ_ν of the squared neutron complex widths, $g_{\nu n}{}^2$. In both tables, the columns labeled "*T*0" exhibit the results obtained from the usual first-order perturbation theory of Adler and Adler.[4] The columns labeled "*T*1" and "*TE*" correspond to the present perturbation method and the exact results, respectively.

Case 1 is an example of relatively low level interference. There is in this instance agreement between the two perturbation formalisms and the exact results. The total cross section for this case is shown in Fig. 1. The Bethe approximation underestimates the broad peak and overestimates the narrow peak.

The examples corresponding to case 2 and case 3 in Table I represent situations of relatively high level interference. The first-order approximation begins to fail, and there are small deviations between the exact values for the "residues" Λ_ν and Ξ_ν and the results of the present perturbation approach. Nevertheless, the total cross section is still quite well represented, Figs. 2 and 3, by our perturbation method.

For case 4 the interference parameter is close to unity and the perturbation approach (*T*1) reaches its limit of validity. The residues still have the correct sign but exhibit substantial deviations from the exact calculation. One still has a reasonable description of the cross section, Fig. 4.

For the final example, case 5, the level interference parameter is larger than unity. In this instance the perturbation formalisms do not apply anymore. The results shown in the columns la-

TABLE III. Comparison between the exact and perturbation calculations for the squared neutron complex widths $g_{\nu n}{}^2$ of two interacting nuclear resonances.

No.	Λ_1 T0	Λ_1 T1	Λ_1 TE	Λ_2 T0	Λ_2 T1	Λ_2 TE	Ξ_1 T0	Ξ_1 T1	Ξ_1 TE	Ξ_2 T0	Ξ_2 T1	Ξ_2 TE
1	15.63	15.77	15.77	0.50	0.42	0.42	0.34	0.29	0.29	−0.34	−0.29	−0.29
2	2.69	2.52	2.45	5.39	6.07	6.13	1.96	1.98	2.07	−1.96	−1.98	−2.07
3	38.80	46.67	46.72	1.50	−0.36	−0.45	11.64	4.22	3.79	−11.64	−4.26	−3.79
4	3.46	−0.033	−1.68	9.75	16.80	18.85	7.86	6.94	5.46	−7.86	−7.45	−5.46
5	···	−1.44	−1.51	···	78.10	78.19	···	0.033	0.076	···	−0.20	−0.076

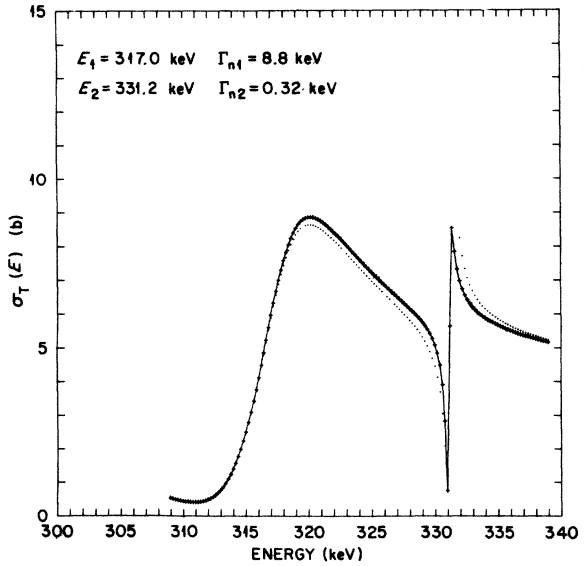

FIG. 1. The total cross section for two interacting resonances. The degree of interference is $\theta_R = 0.11$ (continuous line), exact treatment; (+), present perturbation theory; (···), Bethe approximation.

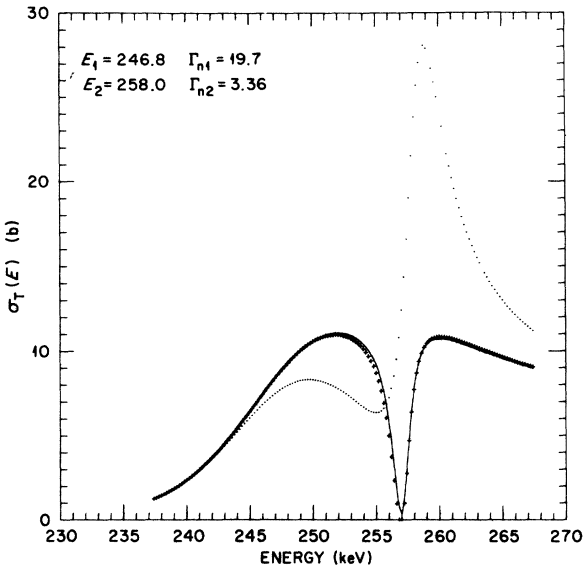

FIG. 3. The total cross section for two interacting resonances. The degree of interference is $\theta_R = 0.59$ (continuous line), exact treatment; (+), present perturbation theory; (···), Bethe approximation.

beled "$T1$" were obtained by a first-order Runge-Kutta solution of the differential Eqs. (14), (19), and (18). The agreement in regard to the complex poles is good. The small discrepancies observed in the residues are due to lack of convergence in the computational procedure. The results concerning the total cross section calculation are shown in Fig. 5. Notice the appearance of Lynn's[23] "quasiresonance" effects due to the high level of interference between the two resonances.

One concludes from the previous results that the present perturbation formalism is applicable with reasonably good accuracy up to values of 0.9 for the degree of level interference. First-order perturbation theory and the Bethe approximation are applicable for $\theta_R \leq 0.1$.

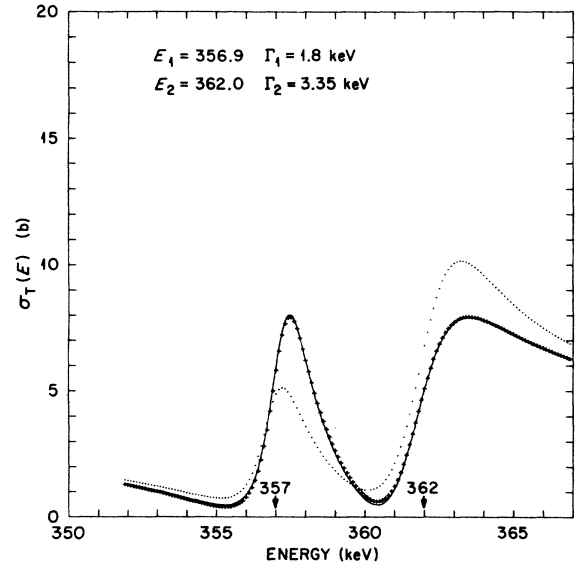

FIG. 2. The total cross section for two interacting resonances. The degree of interference is $\theta_R = 0.48$ (continuous line), exact treatment; (+), present perturbation theory; (···), Bethe approximation.

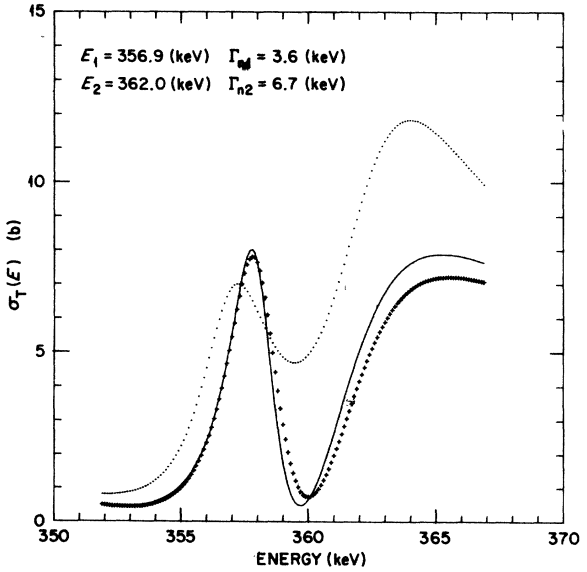

FIG. 4. The total cross section for two interacting resonances. The degree of interference is $\theta_R = 0.92$ (continuous line), exact treatment; (+), present perturbation theory; (···), Bethe approximation.

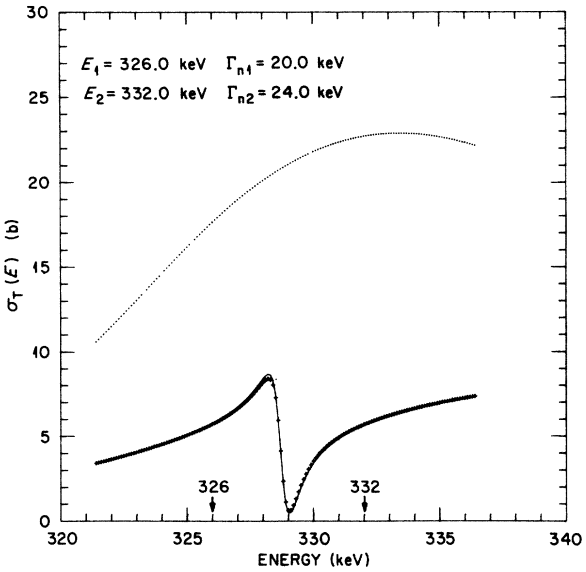

FIG. 5. The total cross section for two interacting resonances. The degree of interference is $\theta_R = 3.46$ (continuous line), exact treatment; (+), present theory; (···), Bethe approximation.

VII. CONCLUSIONS

On the basis of a previously developed formalism for the transition T matrix, one has constructed a perturbation approach for the calculation of the complex poles and widths of the T matrix, which takes into account both changes in the Hamiltonian of the system as well as in the boundary conditions. The new approach applies to cases of high level interference and correctly predicts the total cross section for pairs of highly coupled resonances.

The present formalism has also been applied to the study of intermediate structure phenomena. Lane's[13] line strength reduction effect, due to the finite lifetime of the "special" states, is automatically incorporated in the present theory as a damping factor. The expressions obtained for the complex widths and residues explicitly contain the contributions of "external" and "internal" mixing effects.

The results obtained for the partial level widths in the presence of intermediate structure show asymmetric effects depending on the location of the compound nucleus resonance relative to the position of the "special" levels and sign combinations of the R-matrix partial level widths, $\Gamma_{\nu c}^{1/2}$. It is suggested that the asymmetry effect coupled to statistical level width fluctuations may account for a substantial decrease in the strength of the fission doorway states.[14]

It has also been shown that the average cross section in the unresolved energy region can be written in terms of a generalized strength function which exhibits resonant behavior at the poles of the "special" levels.

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