Neutron-proton bremsstrahlung results at 200 MeV†

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Neutron-proton bremsstrahlung cross sections $d\sigma/d\Omega_p d\Omega_n$ and asymmetries at $E_{\rm lab} = 200$ MeV have been calculated using the Lomon-Feshbach and Hamada-Johnston models. The cross sections from these models differ by about 10% at $\theta_p = \theta_n = 30°$ after approximate adjustments are made for elastic scattering differences. The cross sections integrated over only the larger peak in the θ_γ distribution differ by about 16%. The calculations are based on an expansion of the electromagnetic current in powers of the photon energy rather than an expansion of the complete bremsstrahlung matrix element. The calculation includes the external emission amplitude to all orders in the photon energy and the electric dipole part of the internal emission amplitude. Higher order contributions to the internal emission amplitude are discussed. The numerical results and amplitude expressions are compared with those of other authors. We find numerical agreement with Brown and Franklin but not with Celenza, Gibson, Liou, and Sobel.

NUCLEAR REACTIONS *NN* bremsstrahlung amplitudes, nonlocal and exchange effects; *np* bremsstrahlung $d\sigma/d\Omega_p d\Omega_n$ and asymmetry calculated at $E_{\rm lab} = 200$ MeV with Lomon-Feshbach and Hamada-Johnston models.

I. INTRODUCTION

Nucleon-nucleon bremsstrahlung ($NN\gamma$) has long been studied as a possible source of information about the strong interaction. Most of the work has been focused on proton-proton bremsstrahlung because that experiment is easiest. Much of the theoretical work has been within the potential model (Schrödinger equation) because it was hoped that one could eliminate some of the phenomenological potentials which almost equally well fitted the elastic scattering data. This additional discriminatory power of $NN\gamma$ is due to its dependence on the half-off-energy-shell two-body *NN* amplitude.

Neutron-proton bremsstrahlung ($np\gamma$) calculations within the potential model have been performed by Brown and Franklin[1] and by Celenza, Gibson, Liou, and Sobel.[2] Calculations by McGuire and Pearce[3] were also based on the Schrödinger equation, although these authors used an *ad hoc* approximation for the half-off-energy-shell *NN* amplitude rather than computing it from a potential.

We present here calculations for the Lomon-Feshbach[4] and Hamada-Johnston[5] models. We find that for the kinematical regions we explore these models give $np\gamma$ cross sections which differ by about 15% or less, after adjustments for different elastic amplitudes. For Lomon-Feshbach models with different $^3S_1$-$^3D_1$ channel parameters the $np\gamma$ cross section computed with only the external emission amplitude, rather than the complete amplitude, is found to be proportional to the elastic cross section.

Our calculations with the Hamada-Johnston potential overlap those of Brown and Franklin[1] and Celenza *et al.*,[2] and we compare our results with theirs.

The $np\gamma$ calculations mentioned above are all approximate since one does not know the correct electromagnetic current for the interacting two-nucleon system. In addition to the sum of the individual free nucleon currents, one in general has a two-body interaction current which depends on the strong interaction dynamics.

The existence of the two-body current is especially clear in $np\gamma$ where it is large and is necessary to satisfy gauge invariance. The approximate forms of the two-body current used in most $np\gamma$ calculations are derived by imposing current conservation or gauge invariance and involve truncated expansions in powers of the photon energy.

In this paper we use an expansion of the current density $\vec{J}(\vec{k})$. We therefore avoid expanding the initial and final *NN* wave functions, in contrast to what would be the case if the bremsstrahlung matrix element were expanded. The expansion of the two-body current is one in powers of k times the range of the strong interaction[1] (no larger than m_π^{-1}), whereas one does not know the largest expansion parameter to be associated with the expansion of the matrix element. We also use an expansion of the one-body current which we justify by comparing with calculations which do not use it.

In the Coulomb gauge the bremsstrahlung amplitude can be written to first order in the charge as

$$\vec{\mathfrak{M}} = \langle \Psi_f^{(-)} | [\vec{J}_1(\vec{k}) + \vec{J}_2(\vec{k})] | \Psi_i^{(+)} \rangle \ , \tag{1a}$$

$$\vec{J}_i(\vec{k}) \equiv \int d^3x \, e^{-i\vec{k}\cdot\vec{x}} \vec{J}_i(\vec{x}) \ , \tag{1b}$$

where $\Psi_{f,i}^{(\pm)}$ are NN scattering states, $\vec{k}$ is the photon momentum, and $\vec{J}_1$ and $\vec{J}_2$ are the one-body and two-body current densities. There should be no confusion caused by using the same symbol for $\vec{J}(\vec{x})$ and its Fourier transform $\vec{J}(\vec{k})$. We take the one-body current and charge densities to have their usual forms

$$\vec{J}_1(\vec{x}) = \frac{e}{2m}\sum_\alpha Q_\alpha[\vec{p}_\alpha \delta^3(\vec{x}-\vec{r}_\alpha) + \delta^3(\vec{x}-\vec{r}_\alpha)\vec{p}_\alpha] - \sum_\alpha \vec{\mu}_\alpha \times \vec{\nabla}_x \delta^3(\vec{x}-\vec{r}_\alpha) , \tag{2a}$$

$$\rho_1(\vec{x}) = e\sum_\alpha Q_\alpha \delta^3(\vec{x}-\vec{r}_\alpha) , \tag{2b}$$

where Q_α is unity if nucleon α is a proton and zero if it is a neutron, and $\vec{\mu}_\alpha$ is the magnetic moment of nucleon α. The $\vec{p}_\alpha$ and $\vec{r}_\alpha$ are the momentum and position operators. We have assumed the nucleons have equal masses, m. We will refer to those parts of $\vec{J}_1$ which are proportional to $\vec{p}_\alpha$ as the convection current. For later use we define the matrix elements

$$\langle \vec{r}''\vec{R}'' | \vec{J}(\vec{x}) - \vec{J}^{(e)}(\vec{x}) | \vec{r}'\vec{R}'\rangle = \delta^3(\vec{R}''-\vec{R}')\vec{J}(\vec{r}'',\vec{r}',\vec{x}-\vec{R}') ,$$

$$\langle \vec{r}''\vec{R}'' | \rho(\vec{x}) | \vec{r}'\vec{R}'\rangle = \delta^3(\vec{R}''-\vec{R}')\rho(\vec{r}'',\vec{r}',\vec{x}-\vec{R}') , \tag{3}$$

where the external current is defined in terms of the total momentum operator $\vec{P}$ as

$$\vec{J}^{(e)}(\vec{x}) = \frac{1}{4m}[\vec{P}\rho(\vec{x}) + \rho(\vec{x})\vec{P}] ,$$

and, as usual, $\vec{r}' = \vec{r}_1' - \vec{r}_2'$, $\vec{R}' = \frac{1}{2}(\vec{r}_1' + \vec{r}_2')$. These expressions hold for the one-body and two-body densities. Translation invariance demands that the martix elements depend only on the variables $\vec{r}''$, $\vec{r}'$, $\vec{R}''-\vec{R}'$, and $\vec{x}-\vec{R}'$, and the Galilean transformation properties require that the dependence on $\vec{P}$ is only that which occurs in the external current.

The bremsstrahlung amplitude may be computed in two parts, called the external and internal emission amplitudes. The external emission amplitude

$$\vec{\mathfrak{M}}_E = \langle \Phi_f | \vec{J}_1(\vec{k}) | X_i^{(+)}\rangle + \langle X_f^{(-)} | \vec{J}_1(\vec{k}) | \Phi_i\rangle , \tag{4}$$

where $\Phi_{f,i}$ are noninteracting NN states and $X_{f,i}^{(\pm)} = \Psi_{f,i}^{(\pm)} - \Phi_{f,i}$, is the easiest to compute since each term of it is simply the product of a strong interaction T matrix element, a Green's function, and a matrix element of $\vec{J}_1$. The second part of most calculations is the internal emission amplitude, which itself has two constituents. First, one has the rescattering amplitude

$$\vec{\mathfrak{M}}_R = \langle X_f^{(-)} | \vec{J}_1(\vec{k}) | X_i^{(+)}\rangle , \tag{5}$$

which, although difficult, can be computed.[1] Second, one has the two-body current part

$$\vec{\mathfrak{M}}_2 = \langle \Psi_f^{(-)} | \vec{J}_2(\vec{k}) | \Psi_i^{(+)}\rangle . \tag{6}$$

The external currents $\vec{J}_1^{(e)}$ and $\vec{J}_2^{(e)}$ will not contribute to the amplitude if it is computed in the center of mass frame. We will, therefore, ignore the external currents from this point on. We will also refer to these amplitudes with a momentum conserving δ function removed:

$$\vec{\mathfrak{M}} = (2\pi)^3\delta^3(\vec{P}_f + \vec{k} - \vec{P}_i)\vec{M} ,$$

where $\vec{P}_f$ and $\vec{P}_i$ are the final and initial total NN momenta.

II. APPROXIMATIONS TO $\vec{M}_2$ AND $\vec{M}_R$

The amplitude $\vec{\mathfrak{M}}_2$ cannot be calculated exactly without introducing (and solving) a fundamental theory of the strong and electromagnetic interactions since one does not otherwise know $\vec{J}_2$ from theory. One does, however, know its divergence from the continuity relation provided that nonrelativistically the charge density has the one-body form in Eq. (2b); that is, if there is no ρ_2. The divergence of $\vec{J}_2$ is sufficient to obtain an approximation to $\vec{\mathfrak{M}}_2$ as will be shown below. The crucial point here is the observation that the one-body charge and current densities are not conserved if the nucleon-nucleon potential contains nonlocality or charge exchange terms, but that the additional term required for conservation is uniquely determined in the long wavelength limit.[6-11]

Our approximation to $\vec{\mathfrak{M}}_2$ is obtained by expanding $\exp(i\vec{k}\cdot\vec{R})\vec{J}_2(\vec{k})$, where $\vec{R}$ is the NN c.m. position operator, in powers of k. The $\exp(i\vec{k}\cdot\vec{R})$ guarantees that momentum conservation is preserved. The first term of this expansion is then written in terms of the divergence of $\vec{J}_2$.

We remove a momentum conserving δ function as follows. In terms of the coordinate space wave functions Eq. (6) reads

$$\vec{\mathfrak{M}}_2 = \int d^3r''d^3r'd^3R'd^3y\, e^{-i\vec{k}\cdot\vec{R}'} \times \Psi_f^{(-)*}(\vec{r}'',R')e^{-i\vec{k}\cdot\vec{y}}\vec{J}_2(\vec{r}'',\vec{r}',\vec{y})\Psi_i^{(+)}(\vec{r}',\vec{R}') . \tag{7}$$

By writing the wave functions as products,

$$\Psi_i^{(+)}(\vec{r}',\vec{R}') = e^{i\vec{P}_i\cdot\vec{R}'}\psi_i^{(+)}(\vec{r}') ,$$

$$\Psi_f^{(-)}(\vec{r}',\vec{R}') = e^{i\vec{P}_f\cdot\vec{R}'}\psi_f^{(-)}(\vec{r}') ,$$

we find

$$\vec{M}_2 = \int d^3r''d^3r'd^3y\, \psi_f^{(-)*}(\vec{r}'') \times e^{-i\vec{k}\cdot\vec{y}}\vec{J}_2(\vec{r}'',\vec{r}',\vec{y})\psi_i^{(+)}(\vec{r}') . \tag{8}$$

Since $\vec{J}_2(\vec{x})$ is associated with the strong interaction, its matrix element must vanish when $\vec{x}$ is far from the center-of-mass position of the two nucleons, that is, when y is large. Thus it is reasonable to introduce an expansion of the exponential function,

$$e^{-i\vec{k}\cdot\vec{y}} = 1 - i\vec{k}\cdot\vec{y} + \cdots \ , \tag{9}$$

which produces an expansion of $\vec{M}_2$ in powers of k times the range of the two-body current. Clearly the expansion is useful only if every term of it yields convergent integrals in Eq. (8). The Yukawa asymptotic behavior expected for $\vec{J}_2$ is certainly sufficient.[6] The error one incurs by keeping only the first term is probably fractionally largest for the longest range part of $\vec{J}_2$, although no definitive statement can be made without actual calculation. The approximation of keeping only the first term has been checked for the longest range (one-pion-exchange) part of $\vec{J}_2$ and was found to be good.[12]

If only the first term of the expansion, Eq. (9), is retained then current conservation allows us to approximate $\vec{M}_2$ compactly in terms of the potential. Upon replacing the $\exp(-i\vec{k}\cdot\vec{y})$ in Eq. (8) by unity we may partially integrate with respect to $\vec{y}$ to obtain $\vec{M}_2$ in terms of the divergence of $\vec{J}_2$:

$$\vec{M}_2^{E1} = -\int d^3r''d^3r'd^3y \times \psi_f^{(-)*}(\vec{r}'')\vec{y}\,\vec{\nabla}_y\cdot\vec{J}_2(\vec{r}'',\vec{r}',\vec{y})\psi_i(\vec{r}') \ , \tag{10}$$

where the superscript $E1$ is to remind us that this is the electric dipole approximation. From the continuity equations

$$\vec{\nabla}\cdot[\vec{J}_1(\vec{x}) + \vec{J}_2(\vec{x})] + i[H, \rho_1(\vec{x}) + \rho_2(\vec{x})] = 0 \ , \tag{11a}$$

$$\vec{\nabla}\cdot\vec{J}_1(\vec{x}) + i[H_0, \rho_1(\vec{x})] = 0 \ , \tag{11b}$$

and the explicit form of ρ_1, Eq. (2), we find, when $\rho_2 = 0$,

$$\vec{\nabla}_y\cdot\vec{J}_2(\vec{r}'',\vec{r}',\vec{y}) = -ie\langle\vec{r}''|[V, Q_1\delta^3(\vec{y}-\tfrac{1}{2}\vec{r}) + Q_2\delta^3(\vec{y}+\tfrac{1}{2}\vec{r})]|\vec{r}'\rangle \tag{12}$$

where H_0 is the kinetic energy operator $V = H - H_0$, and $\vec{r}$ is the relative coordinate operator. Combining Eqs. (10) and (12) yields the following result when $\rho_2 = 0$:

$$\vec{M}_2^{E1} = \tfrac{1}{2}(ie)\langle\psi_f^{(-)}|[V, \vec{\Omega}]|\psi_i^{(+)}\rangle , \tag{13}$$

where

$$\vec{\Omega} = (Q_1 - Q_2)\vec{r} \ .$$

Note that Eq. (13) vanishes for proton-proton bremsstrahlung since $\vec{\Omega}$ is proportional to the difference of the individual particle charges. If ρ_2 is nonzero then one must add the following to Eq. (13):

$$-i(\epsilon_i - \epsilon_f)\int d^3r''d^3r'\psi_f^{(-)*}(\vec{r}'') \times \int d^3y\,\vec{y}\rho_2(\vec{r}'',\vec{r}',\vec{y})\psi_i^{(+)}(\vec{r}') \ , \tag{14}$$

where ϵ_i and ϵ_f are the initial and final internal energies.

We now introduce an approximation to the rescattering amplitude. This approximation involves an expansion of the *one-body* current in powers of k. This simplifies calculation of the lowest order internal emission amplitude while leaving as a more difficult calculation a small, order k correction. We justify it ultimately by showing that it gives a small numerical error. Liou[13] has commented on the advantage of simultaneously considering both parts of the internal emission amplitude.

The approximate rescattering amplitude is obtained by using the following approximate expression for the one-body operator:

$$\int d^3x\, e^{-i\vec{k}\cdot\vec{x}}[\vec{J}_1(\vec{x}) - \vec{J}_1^{(e)}(\vec{x})]_{(c)} \approx e^{-i\vec{k}\cdot\vec{R}}\int d^3x[\vec{J}_1(\vec{x}) - \vec{J}_1^{(e)}(\vec{x})]_{(c)} \ , \tag{15}$$

where the subscript (c) means that the equation applies only to the convection current, and $\vec{R}$ is the NN c.m. position operator. Furthermore, we have the exact equation

$$\int d^3x[\vec{J}_1(\vec{x}) - \vec{J}_1^{(e)}(\vec{x})]_{(c)} = \tfrac{1}{2}(ie)[h_0, \vec{\Omega}] , \tag{16}$$

where $h_0 = p^2/m$ and $\vec{p} = \frac{1}{2}(\vec{p}_1 - \vec{p}_2)$. The rescattering amplitude, from Eq. (5), is then approximately

$$\vec{M}_R^{E1} = \tfrac{1}{2}(ie)\langle\chi_f^{(-)}|[h_0, \vec{\Omega}]|\chi_i^{(+)}\rangle \ . \tag{17}$$

Using Eq. (15) in the external emission amplitude would lead to serious error because $\vec{J}_1$ is not of finite range. Equation (15) is acceptable for use in the rescattering amplitude because of the (short range) strong interaction preceding and following the radiation. We have checked the accuracy of Eq. (17) by comparing it numerically with the exact result. This comparison is shown later.

We obtain an approximate internal emission amplitude by adding Eq. (13) and Eq. (17). Using the definition of the T matrix $V\psi = T(\epsilon)\phi$ and the relation $V\psi = (\epsilon - h_0)\chi$ we find

$$\vec{M}_R^{E1} + \vec{M}_2^{E1} = \tfrac{1}{2}(-ie)\{\langle\phi_f|\vec{\Omega}T(\epsilon_i)|\phi_i\rangle - \langle\phi_f|T(\epsilon_f)\vec{\Omega}|\phi_i\rangle + (\epsilon_i - \epsilon_f)\langle\chi_f^{(-)}|\vec{\Omega}|\chi_i^{(+)}\rangle\} \ , \tag{18}$$

The ϕ_i for $np\gamma$ is explicitly

$$|\phi_i\rangle = |\vec{p}_i s_i \mu_i\rangle(\xi_1 + \xi_0) + (-1)^{s_i}|-\vec{p}_i s_i \mu_i\rangle(\xi_1 - \xi_0) \ , \tag{19}$$

and similarly for ϕ_f, where $|\vec{p}s\mu\rangle$ is an eigenstate of relative momentum $\vec{p}$, spin s and spin projection μ. The ξ_I are proton-neutron isospin functions for isospin I. It is convenient to define the T matrix elements of definite isospin by

$$\langle\vec{p}_f s_f \mu_f | T_I(\epsilon) | \vec{p}_i s_i \mu_i\rangle = \langle\vec{p}_f s_f \mu_f | \xi_I^\dagger T(\epsilon)\xi_I[1+(-1)^{s_i+I+1}\Pi] | \vec{p}_i s_i \mu_i\rangle \ , \tag{20}$$

where Π is the parity operator. The internal emission amplitude can now be written from Eqs. (18)–(20) as

$$\vec{M}_I^{E1} \equiv \vec{M}_R^{E1} + \vec{M}_2^{E1} = \vec{M}_I^{(1)} + \vec{M}_I^{(2)} \ , \tag{21a}$$

$$\vec{M}_I^{(1)} = e\vec{\nabla}_{pf}\langle\vec{p}_f s_f \mu_f | [T_0(\epsilon_i)+T_1(\epsilon_i)] | \vec{p}_i s_i \mu_i\rangle + e\vec{\nabla}_{pi}\langle\vec{p}_f s_f \mu_f | [T_0(\epsilon_f)+T_1(\epsilon_f)] | \vec{p}_i s_i \mu_i\rangle \ , \tag{21b}$$

$$\vec{M}_I^{(2)} = \tfrac{1}{2}(-ie)(\epsilon_i - \epsilon_f)\langle\chi_f^{(-)}|\vec{\Omega}|\chi_i^{(+)}\rangle \ . \tag{21c}$$

This amplitude is sensitive to the short distance NN wave functions via the half-shell T matrix elements in Eq. (21b) and the state vectors in Eq. (21c). We have written the internal emission amplitude in terms of $\vec{M}_I^{(1)}$ and $\vec{M}_I^{(2)}$ to facilitate the discussion. We will also refer to a third part which is the error introduced by the approximation, Eq. (17), for the rescattering amplitude:

$$\vec{M}_{\Delta R} = \vec{M}_R(\text{exact}) - \vec{M}_R^{E1} \ . \tag{22}$$

The relation to the amplitudes used by other authors is as follows. The internal emission amplitude used by BF[1] is $\vec{M}_I^{(1)} + \vec{M}_I^{(2)} + \vec{M}_{\Delta R}$. This is just the sum of Eq. (13) and the exact rescattering amplitude. The internal emission amplitude used by Celenza *et al.* (CGLS)[2] is very similar to $M_I^{(1)}$. In our notation the CGLS amplitude is

$$\vec{M}_I^{(\text{CGLS})} = e\vec{\nabla}_{pf}\langle(\vec{p}_f - \tfrac{1}{2}\vec{k})s_f\mu_f | [T_0(\epsilon_i)+T_1(\epsilon_i)] | \vec{p}_i s_i \mu_i\rangle + e\vec{\nabla}_{pi}\langle\vec{p}_f s_f \mu_f | [T_0(\epsilon_f)+T_1(\epsilon_f)] \times | (\vec{p}_i + \tfrac{1}{2}\vec{k}) s_i \mu_i\rangle \ . \tag{23}$$

Numerical comparisons with these authors will be given below. In the notation suggested by McGuire[3] Eq. (21b) reads

$$\vec{M}_I^{(1)} = e\vec{\nabla}_{pf} T(p_i^2, p_i^2 - p_f^2, \hat{p}_i\cdot\hat{p}_f) + e\vec{\nabla}_{pi}\tilde{T}(p_f^2, p_f^2 - p_i^2, \hat{p}_i\cdot\hat{p}_f) \ . \tag{24}$$

The internal emission amplitude of McGuire and Pearce (within a factor of $-2e$ which is absorbed elsewhere), in the c.m. frame, is obtained by rewriting Eq. (24) in terms of derivatives with respect to the second and third arguments of T and $\tilde{T}$ and (1) replacing p_i and p_f by an average value except where they appear as vectors, (2) replacing the third argument of T and $\tilde{T}$ by a similar variable defined in Ref. 3 and (3) assuming $\theta_p = \theta_n$ where terms proportional to $\vec{p}_i\cdot\vec{p}_f$ may be dropped.

To this point we have included $\vec{J}_2$ only in the $E1$ approximation. The second term of the expansion, Eq. (9), cannot be included without specifying the solenoidal part of $\vec{J}_2$. This can be seen by using partial integration:

$$-\int d^3y\, i\vec{k}\cdot\vec{y}\vec{J}_2(\vec{r}'',\vec{r}',\vec{y}) = \tfrac{1}{2}i\int d^3y\,\vec{y}\vec{y}\cdot\vec{k}\,\vec{\nabla}\cdot\vec{J}_2(\vec{r}'',\vec{r}',\vec{y}) + i\vec{k}\times\vec{m}_2(\vec{r}'',\vec{r}') = \tfrac{1}{8}eQ_{\text{total}}\langle\vec{r}''|[V,\vec{r}\vec{r}\cdot\vec{k}]|\vec{r}'\rangle + i\vec{k}\times\vec{m}_2(\vec{r}'',\vec{r}') \ , \tag{25}$$

where eQ_{total} is the charge of the system and the $\vec{\nabla}$ is taken with respect to $\vec{y}$. In Eq. (25) $\vec{m}_2$ is the magnetic dipole moment

$$\vec{m}_2(\vec{r}'',\vec{r}') = \tfrac{1}{2}\int d^3y\,\vec{y}\times\vec{J}_2(\vec{r}'',\vec{r}',\vec{y}) = -\tfrac{1}{4}\int d^3y\, y^2\vec{\nabla}\times\vec{J}_2(\vec{r}'',\vec{r}',\vec{y}) \ ,$$

which is completely determined by the solenoidal part of $\vec{J}_2$. The two terms in Eq. (25) may be recognized as the electric quadrapole ($E2$) and magnetic dipole ($M1$) contributions. The quadrapole term clearly contributes only for a nonlocal potential, and is present for proton-proton and neutron-proton bremsstrahlung. It is useful at this time to record the first correction to the approximation given by Eq. (15, 16) for the one-body bremsstrahlung operator:

$$-\int d^3x\, i\vec{k}\cdot(\vec{x}-\vec{R})[\vec{J}_1(\vec{x}) - \vec{J}_1^{(e)}(\vec{x})]_{(c)} = \frac{-ie}{4m}Q_{\text{total}}(\vec{p}\,\vec{r}\cdot\vec{k} + \vec{r}\cdot\vec{k}\,\vec{p}) = \tfrac{1}{8}eQ_{\text{total}}([h_0,\vec{r}\vec{r}\cdot\vec{k}] + (2i/m)\vec{k}\times\vec{L}) \ , \tag{26}$$

where $\vec{L} = \vec{r}\times\vec{p}$. The $E2$ contribution to the rescattering amplitude is then

$$\vec{M}_R^{E2} = \tfrac{1}{8}eQ_{\text{total}}\langle\chi_f^{(-)}|[h_0,\vec{r}\vec{r}\cdot\vec{k}]|\chi_i^{(+)}\rangle \ . \tag{27}$$

From Eq. (25) we have

$$\vec{M}_2^{E2} = \tfrac{1}{8}eQ_{\text{total}}\langle\psi_f^{(-)}|[V,\vec{r}\vec{r}\cdot\vec{k}]|\psi_i^{(+)}\rangle \ . \tag{28}$$

Proceeding in a way analogous to the derivation of Eq. (18) we obtain

$$\vec{M}_I^{E2} = \vec{M}_2^{E2} + \vec{M}_R^{E2} = -\tfrac{1}{8}eQ_{\text{total}}[\langle\phi_f|\vec{r}\vec{r}\cdot\vec{k}T(\epsilon_i)|\phi_i\rangle - \langle\phi_f|T(\epsilon_f)\vec{r}\vec{r}\cdot\vec{k}|\phi_i\rangle + (\epsilon_i - \epsilon_f)\langle\chi_f^{(-)}|\vec{r}\vec{r}\cdot\vec{k}|\chi_i^{(+)}\rangle] \ . \tag{29}$$

Note that the last term is second order in k. Specializing to the $np\gamma$ case and using Eq. (19) and (20) gives

$$\vec{M}_I^{E2} = \tfrac{1}{4} e \vec{\nabla}_{p_f} \vec{k} \cdot \vec{\nabla}_{p_f} \langle \vec{p}_f s_f \mu_f | [T_0(\epsilon_i) + T_1(\epsilon_i)] | \vec{p}_i s_i \mu_i \rangle$$
$$- \tfrac{1}{4} e \vec{\nabla}_{p_i} \vec{k} \cdot \vec{\nabla}_{p_i} \langle \vec{p}_f s_f \mu_f | [T_0(\epsilon_f) + T_1(\epsilon_f)] | \vec{p}_i s_i \mu_i \rangle$$
$$- \tfrac{1}{8} e (\epsilon_i - \epsilon_f) \langle \chi_f^{(-)} | \vec{r}\, \vec{r} \cdot \vec{k} | \chi_i^{(+)} \rangle \quad . \qquad (30)$$

One may write (for $np\gamma$)

$$\vec{M}_I^{E1} + \vec{M}_I^{E2} = e \vec{\nabla}_{p_f} \langle (\vec{p}_f + \tfrac{1}{4}\vec{k}) s_f \mu_f | [T_0(\epsilon_i) + T_1(\epsilon_i)] | \vec{p}_i s_i \mu_i \rangle$$
$$+ e \vec{\nabla}_{p_i} \langle \vec{p}_f s_f \mu_f | [T_0(\epsilon_f) + T_1(\epsilon_f)] | (\vec{p}_i - \tfrac{1}{4}\vec{k}) s_i \mu_i \rangle$$
$$+ \vec{M}_I^{(2)} + \mathcal{O}(k^2) \ . \qquad (31)$$

III. NUMERICAL RESULTS

Our calculation shows that the rescattering amplitude is well approximated by its $E1$ part $\vec{M}_R^{E1}$. In Fig. 1 we have compared the cross section computed with the approximate rescattering amplitude, $\vec{M}_R^{E1}$, with that computed with the exact rescattering amplitude. The latter was obtained by computing, with the Hamada-Johnston[5] potential, the exact and approximate rescattering amplitudes and adding the difference to $\vec{M}_I^{E1}$ in Eq. (21). At $\theta_p = \theta_n = 30°$ replacing the exact $\vec{M}_R$ by $\vec{M}_R^{E1}$ decreases $d\sigma/d\Omega_p d\Omega_n$ by 3%. According to Ref. 1 completely omitting $\vec{M}_R$ decreases the cross section by 10%, so that our approximate form is rather accurate. Our result with the exact rescattering amplitude (and all NN partial waves having $J \leq 4$) is 34.3 μb/sr^2, which is in good agreement with the BF result of 34.6 μb/sr^2.

We have computed the cross section at three kinematical points for the Hamada-Johnston potential[5] and for several versions of the Lomon-Feshbach model.[4] The Lomon-Feshbach models differ from one another only in the $^3S_1 - {}^3D_1$ channel and are labeled by the deuteron d-state probability. The cross sections from our calculations are listed in Table I in columns under the amplitudes used in the calculations. The number η in this table is the ratio of the elastic differential cross section at 200 MeV, $\theta_{\text{lab}} = \pi/4 + \frac{1}{2}(\theta_n - \theta_p)$ for the model to the same quantity for the Hamada-Johnston (HJ) potential. This number allows one to compare approximately the agreement in the on-shell amplitudes of the models.

The Lomon-Feshbach (LF) $np\gamma$ cross sections are correlated with the elastic cross sections in an unexpected way. The difference between the cross sections given by any two of the LF models is mostly due to external emission. One finds that the ratio of the external emission cross sections for any two of the LF models is equal to the ratio of the elastic cross sections.

This behavior would follow if the $1/k$ dependence arising from the square of the external emission amplitude were to dominate. From the low-energy theorem one knows that the external plus internal emission unpolarized $np\gamma$ cross section is determined by the unpolarized elastic cross section if terms linear and higher order in the photon energy in an expansion of the cross section are dropped.[13-15] However, the internal emission amplitude is found to be almost independent of which LF model is used. (We found that the external and internal emission amplitudes add, in effect, almost incoherently in $d\sigma/d\Omega_p d\Omega_n$, the cross term contributing about 10% at $\theta_p = 30°$, $\theta_n = 10°$ and about 1% or less in the two symmetric cases. The cross term is more important in $d\sigma/d\Omega_p d\Omega_n d\theta_\gamma$ at most θ_γ.)

This correlation suggests that if the LF interaction were adjusted to agree with the HJ elastic cross section then the $np\gamma$ cross section would be increased by about 0.8 μb/sr^2 at $\theta_p = \theta_n = 30°$, 2.7 μb/sr^2 at $\theta_p = \theta_n = 38°$, and 0.5 μb/sr^2 at $\theta_p = 30°$, $\theta_n = 10°$. These values are obtained by applying a factor of $\eta(\text{HJ})/\eta(\text{LF})$ to the LF external emission cross sections. The LF cross sections would then be 11%, 7%, and 1%, respectively, greater than HJ.

Most of the difference in the cross sections occurs around the peak centered near $\theta_\gamma = -75°$. The cross section at $\theta_p = \theta_n = 30°$ integrated from $\theta_\gamma = -135°$ to $-30°$ is 14.3 μb/sr^2 for HJ and 16.3 μb/sr^2 for LF (7.55% d state) using $\vec{M}_E + \vec{M}_I^{E1}$. The corresponding numbers for $\vec{M}_E$ alone are 5.2 and 5.5 μb/sr^2. Thus, there is a 16% difference in the cross sections integrated over this peak, after ad-

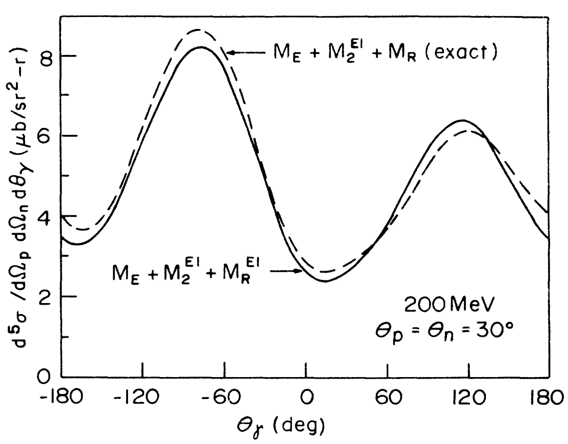

FIG. 1. Neutron-proton bremsstrahling cross section $d^5\sigma/d\Omega_p\, d\Omega_n\, d\theta_\gamma$ versus the laboratory photon angle. The solid curve was computed with $\vec{M}_E + \vec{M}_I^{E1}$; the dashed curved was computed with $\vec{M}_E + \vec{M}_I^{E1} + \vec{M}_{\Delta R}$. Positive (negative) values of θ_γ correspond to the photon on the final proton (neutron) side of the beam axis. The HJ potential with $J \leq 5$ was used.

TABLE I. Coplanar neutron-proton bremsstrahlung results at 200 MeV lab kinetic energy from the Hamada-Johnston and Lomon-Feshbach models including all partial waves through $J=4$. The LF models are distinguished by their deuteron d-state fraction. The number η is the ratio of the elastic differential cross section to that of HJ. The fifth, sixth, and seventh columns give the cross sections $d\sigma/d\Omega_p d\Omega_n$ in μb/sr^2 computed from the amplitudes shown. Columns BF and CGLS contain the cross sections from Refs. 1 and 2. $\mathcal{A}$ is the $np\gamma$ asymmetry.

θ_p (deg)	θ_n (deg)	Model	η	M_E	$M_E+M_I^{(1)}$	$M_E+M_I^{(1)}+M_I^{(2)}$	BF	CGLS	$\mathcal{A}$
30	30	HJ	1.00	14.5	32.0	33.4	34.6	30	0.204
		LF (7.55)	0.95	15.0	34.6	36.3			0.160
		LF (5.20)	0.84	13.2	32.5	34.0			0.146
		LF (4.57)	0.80	12.6	31.7	33.2			0.143
38	38	HJ	1.00	51.1	68.2	68.7	69.8	49	0.091
		LF (7.55)	0.95	51.2	69.8	70.7			0.075
		LF (5.20)	0.84	45.0	63.3	64.1			0.063
30	10	HJ	1.00	8.8	14.4	15.7			
		LF (7.55)	0.94	8.6	14.4	15.3			
		LF (5.20)	0.83	7.7	13.8	14.6			

justment for elastic scattering differences. An experiment which accepts events only in this region may be as useful as one which measures the entire θ_γ distribution, provided the θ_γ acceptance is well established.

The experimental data are not precise enough to prefer one of the models considered here. The experimental values at $(\theta_p, \theta_n) = (30°, 30°)$ and $(38°, 38°)$ are 35 ± 14 and 116 ± 20 μb/sr^2, respectively.[16] The latter value is in moderate disagreement with theory. To obtain agreement with this value would presumably require a potential which is much different from the ones used to date or a large two-body current which is itself conserved. Whether any (energy-independent) potential can produce this value without a $\vec{J}_2$ in addition to the one considered here is unknown. We may note that a similar situation exists at 130 MeV.[1,17]

Our result at $\theta_p = \theta_n = 38°$ using the HJ potential is consistent with that of BF[1] but not with that of CGLS[2]. One sees in Table I that the value found by BF is close to the result we find using $\vec{M}_E + \vec{M}_I^{(1)} + \vec{M}_I^{(2)}$, which indicates that $\vec{M}_{\Delta R}$ is very small. The result of CGLS at this angle is in serious disagreement with our results and those of BF. It is immediately clear that the term $\vec{M}_I^{(2)}$ cannot account for the discrepancy between CGLS and BF as has been suggested.[1] The internal emission amplitude of CGLS, given in Eq. (23), differs from $\vec{M}_I^{(1)}$ only in the points at which the derivatives are evaluated. Since $\frac{1}{2}k$ is about 0.1 fm^{-1} at 38° we would not expect that the difference between $\vec{M}_I^{(\text{CGLS})}$ and $\vec{M}_I^{(1)}$ could account for the large discrepancy in Table I. Therefore, we have computed the cross section at 38° using the CGLS amplitude. The result is 68.4 μb/sr^2, which, as expected, is close to the 68.2 μb/sr^2 found using $\vec{M}_I^{(1)}$.

The relativistic effects included by CGLS are significant but do not resolve this discrepancy. Taking the convection current to be $\frac{1}{2}(\vec{p}/E_b + \vec{p}/E_a)$, rather than $\vec{p}/m$, where E_b and E_a are the nucleon energies immediately before and after radiation, and including the relativistic spin corrections described by CGLS reduces the $\theta_p = \theta_n = 30°$ cross section from 32.0 to 28.0 μb/sr^2 and the 38° cross section from 68.2 to 61.0 μb/sr^2. In all of our calculations relativistic energy differences are used for the Green's functions in the external emission amplitude.

The last column of Table I contains our results for the neutron-proton asymmetry (for the proton target polarized normal to the scattering plane) computed from the amplitude $\vec{M} = \vec{M}_E + \vec{M}_I^{(1)} + \vec{M}_I^{(2)}$. The definition is

$$\mathcal{A} = \frac{\int d\theta_\gamma \,\mathrm{Tr}[(\vec{M}^\dagger \cdot \vec{M} - |\hat{k} \cdot \vec{M}|^2)\vec{\sigma}_p \cdot \hat{n}]}{\int d\theta_\gamma \,\mathrm{Tr}[\vec{M}^\dagger \cdot \vec{M} - |\hat{k} \cdot \vec{M}|^2]} ,$$

where $\vec{\sigma}_p \cdot \hat{n}$ is the component of the target proton spin normal to the scattering plane. The positive values of $\mathcal{A}$ indicate an excess of protons when $\vec{p}_p \times \vec{p}_n$ is in the normal direction, where $\vec{p}_p$ and $\vec{p}_n$ are the final state proton and neutron laboratory mo-

TABLE II. The ratios of the cross section $d\sigma/d\Omega_p d\Omega_n(\sigma)$ and the asymmetry ($\mathcal{A}$) computed with all $J \leq 5$ partial waves to those with all $J \leq 4$ partial waves, using the HJ potential.

θ_p (deg)	θ_n (deg)	σ	$\mathcal{A}$
30	30	0.973	0.989
30	10	1.014	···
38	38	0.998	0.979

menta. At $\theta_p = \theta_n = 30°$ the HJ and LF models differ by more than 20% in their asymmetry predictions.

The results in Table I were computed including all partial waves with $J \leq 4$ in the nucleon-nucleon T matrix elements and wave functions. In Table II we show the effect of adding the $J = 5$ partial waves to the HJ calculations. Limited computation time prevented our including the $J = 5$ partial waves in all the results of Table I. From Table II one can see that the effect on the cross section of including the $J = 5$ partial waves is small but at $\theta_p = \theta_n = 30°$ it is about the same magnitude as the effect of including $\vec{M}_I^{(2)}$ or $\vec{M}_{\Delta R}$.

IV. SUMMARY

The only approximation required to obtain Eq. (13), once we decide to employ a nonrelativistic potential model, is to keep only the lowest order term in the expansion of the two-body interaction current, and to neglect ρ_2. Ignoring the second order term in the expansion may be a good approximation since successive terms are expected to be smaller by factors of k times the range of $\vec{J}_2$, which is no larger than m_π^{-1}. In the case of single pion exchange, neglecting ρ_2 is equivalent to neglecting terms higher order in v/c. (This was shown, for example, in Ref. 6.)

By introducing an approximation to the rescattering amplitude we obtain an expression, Eq. (21), for the bremsstrahlung amplitude which is linear in the NN T matrix except for a term of order k which we find to be numerically small. This expression is easily related to the amplitudes used by other authors. The error introduced by this approximation is about 3% at $\theta_p = \theta_n = 30°$ and smaller at larger angles. Nevertheless, quantitative comparison with experimental results (when precise data are available) may require an exact calculation of the rescattering amplitude, particularly at angles smaller than 30°. We expect that it is not crucial to include the exact rescattering amplitude when comparing model calculations since the model dependence of the relevant T matrix elements is usually not large.

Where our results can be compared with those of BF[1] we find good agreement. We do not agree with CGLS,[2] especially at $\theta_p = \theta_n = 38°$.

More precise experiments are needed to provide a stringent test of current NN models and to help determine the interaction current. Careful measurements over a limited range in θ_γ would be useful. However, experiments would have to be very precise to distinguish the models considered here and in Ref. 1. Such precision has only recently become possible for $pp\gamma$ experiments. When additional $np\gamma$ experiments are performed, verification of the present large angle measurements would be very desirable.

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