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Proton distributions for ^{208}Pb and the single particle model

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Experimentally determined coefficients for a Fourier-Bessel description of the charge distribution of ^{208}Pb are used as geometrical constraints in a single particle model analysis. Calculations are made using local and nonlocal potentials to fit both these coefficients and the level positions.

[NUCLEAR STRUCTURE ^{208}Pb, calculated charge distribution, single-particle model. Fits to experimental Fourier-Bessel coefficients.]

The single-particle model has a long and honorable history in the study of the properties of nuclei.[1] Whilst the model in its simplest form does not contain the more sophisticated features usually included in Hartree-Fock and other similar calculations, it is nevertheless still of considerable interest, particularly in view of its relative computational simplicity which makes it popular in various analyses of experimental data. An obvious example of the wide use of this model is the calculation of form factors for nucleon-transfer reactions in the distorted wave Born approximation (DWBA) method.

The nucleus ^{208}Pb has been extensively studied using the single-particle model in its simplest form, i.e., noninteracting particles bound in a common potential well. One of the first detailed analyses was made by Rost[2] who obtained a good fit to the known positions of the single-particle and single-hole states using a local potential of Woods-Saxon form. These potentials were used by Elton[3] to calculate nucleon density distributions and the electron-scattering cross sections calculated using the proton distributions were found to be in very satisfactory agreement with the electron-scattering data available at that time. There are very good theoretical reasons for believing that the potential should be nonlocal, but the evidence available[4] suggests that local potentials are better able to reproduce the positions of the low-lying states. A nonlocal potential has been used[5] to fit a range of nuclei from ^{12}C to ^{208}Pb, but the level positions are only reproduced to within a few MeV, whereas in the other analyses[2-4] the average deviation between calculation and experiment is typically a few hundred keV.

It has so far been assumed that ^{208}Pb is a "good single-particle nucleus." However, recent calculations[6] have indicated that the level positions in this nucleus may be significantly shifted by core polarization effects. It is therefore of interest to consider, within the single-particle model, other properties of this nucleus such as proton density distributions which give information about an overall sum of nuclear wave functions. These *geometrical* aspects of nucleon density distributions obtained from the single-particle model have not been emphasized in earlier analyses. Whilst it has been known for some while that spectroscopic factors obtained from analyses of transfer reactions depend on the parameters used for the single-particle potentials, it has only recently been emphasized that the spectroscopic factors depend very strongly on the root mean square (rms) radii of the density distributions of the particular state concerned.[7] It is therefore of interest to combine geometrical information about the nucleon distribution with the traditional energy data in analyses of the single-particle model. Analyses by Elton and Swift[8] and by Gamba, Ricco, and Rottigni[9] for light nuclei have used a "nucleon geometry plus energy" approach but in both cases the emphasis has been more on fitting the overall electron-scattering cross sections than the *detailed* proton density distribution. In the present work we apply explicit geometrical constraints to the density distributions calculated using the single-particle model.

Euteneuer, Friedrich, and Voegler[10] have recently published a detailed "model-independent" analysis of electron-scattering data for ^{208}Pb. Their results are expressed in terms of coefficients for a Fourier-Bessel expansion of the charge distribution

$$\rho_{\mathrm{ch}}(r)=\begin{cases} Z\sum_{\nu=1}^{N} a_\nu j_0(q_\nu r), & r\le R \\ 0, & r>R, \end{cases} \tag{1}$$

where R is a suitably chosen cutoff radius, e.g., $R=12$ fm for ^{208}Pb, and $q_\nu=\nu\pi/R$. In the present work we used the first 11 a_ν coefficients as the geometrical constraints. The procedure was to calculate the proton distribution from the single-particle model, fold in the finite size of the proton

TABLE I. Parameters for the single-particle potential from least-squares fits to Fourier-Bessel coefficients. The adjusted potentials were obtained by also requiring a good fit to level positions for the hole states. A range of nonlocality of $\beta = 0.85$ fm was used in the second and third cases.

Model	V_n (MeV)	r_0 (fm)	a (fm)	V_s (MeV)	r_s (fm)	a_s (fm)	χ^2_{FB}	χ^2_E (MeV^2)
Local potential	56.80	1.247	0.832	6.2	1.1	0.6	279	73
(adjusted)	61.44	1.265	0.879	6.2	1.1	0.6	308	0.74
Local potential with Perey factor	61.60	1.249	0.805	6.2	1.1	0.6	286	0.58
(adjusted)	61.86	1.248	0.804	6.2	1.1	0.6	297	0.52
Nonlocal potential	72.85	1.200	0.688	8.0	1.1	0.6	274	78
(adjusted)	78.75	1.217	0.744	8.0	1.1	0.6	302	1.62

to obtain the charge distribution, and then from the latter calculate the corresponding Fourier-Bessel coefficients. These calculated coefficients were then compared with those measured by Euteneuer *et al.* and the parameters of the single-particle potential varied to obtain a best fit using the usual χ^2 minimization procedures. In this way the experimentally determined shape parameters of the charge distribution (together with their uncertainties) were used as the constraints in the single-particle model, in analogy to the conventional calculations where the constraints are the values of single-particle binding energies. Both local and nonlocal potentials were used in the present work. The local single-particle potential was of the form

$$V(r) = V_C(r) - V_n f_n(r) + V_s\left(\frac{\hbar}{m_\pi c}\right)^2 \frac{1}{r}\frac{d}{dr} \times f_{so}(r)\vec{L}\cdot\vec{\sigma} \ , \tag{2}$$

where

$$f(r) = \left[1 + \exp\left(\frac{r - r_0(A-1)^{1/3}}{a}\right)\right]^{-1}$$

with $V_C(r)$ the Coulomb potential, taken to be that of a uniformly charged sphere and where different radial parameters (r_0, a) are used for the central and spin-orbit terms in the nuclear potential.

For the nonlocal calculations a potential of the form

$$V_{NL}(\vec{r},\vec{r}') = V_0[\tfrac{1}{2}(\vec{r}+\vec{r}')]\frac{1}{\pi^{3/2}\beta^3} \times \exp\left[-\left(\frac{\vec{r}-\vec{r}'}{\beta}\right)^2\right] \tag{3}$$

was used. Instead of solving the full nonlocal wave equation, an equivalent local potential $V_L(r)$ was used[11] which is related to the nonlocal potential $V_0(r)$ by

$$V_0(r) = V_L(r)\exp\left(\frac{m\beta^2}{2\hbar^2}[V_L(r) - E - V_C(r)]\right) \tag{4}$$

and the full nonlocal wave function is then obtained from the "equivalent local" solution ψ_L as follows[12]:

$$\psi_{NL}(r) = \psi_L(r)/\left(1 + \frac{2m}{\hbar^2}\frac{\beta^2}{4}\left|V_L(r)\right|\right). \tag{5}$$

The factor multiplying ψ_L in this expression is usually known as the "Perey factor."

Three sets of calculations were made using (i) a local potential, (ii) a local potential but including the correction due to the Perey factor, using $\beta = 0.85$ fm, and (iii) a nonlocal potential with $\beta = 0.85$. The above model (ii), which includes only the nonlocality correction to the wave function, has been used[7,13] in several DWBA analyses of transfer reactions. Reasonable variations in the value of β did not cause any significant changes in the present results.

The results of fits using these models are shown in Table I where the values of the parameters of the potential and of χ^2_{FB} are given. Parameters of the spin-orbit potential were kept fixed during the fitting procedure and it was found by gridding that the values quoted are close to the optimum. As can be seen, all three models give almost equally good fits to the Fourier-Bessel coefficients. Also given in Table I are values of χ^2_E, the sum of squares of differences between calculated and experimental binding energies for hole states. It is found that whilst the positions of the hole states calculated using the local potential plus Perey factor [(ii) above] are in reasonably good agreement with the experimental values, those calculated using the other two models are too weakly bound, although the level ordering and relative spacings are reasonably correct. Adjustments were therefore made to the potential parameters so as to improve the level positions although giving only slightly worse fit to the Fourier-Bessel coefficients. The results of these analyses are also given in Table I where they are marked "(adjusted)." The differences between the various models are clearly observed in the values of V_n

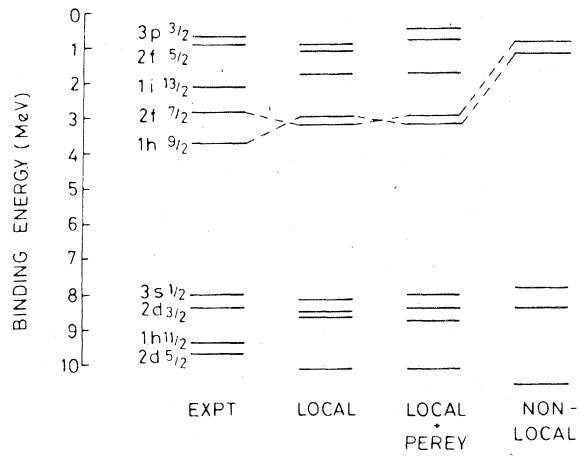

FIG. 1. Level positions of single-hole and single-particle states in ^{208}Pb for the three models. Only hole states were used when adjusting the parameters.

which, in the first and third models, had to be adjusted by 5–6 MeV in order to fit the binding energies. [For the nonlocal potential $V_0(r)$ is written as $V_n f_n(r)$.] Without these adjustments the levels have binding energies which are typically 4 MeV too small, thus resulting in most of the particle states being unbound. In Fig. 1 we show the level positions obtained with these adjusted potentials for all three models. Only the hole states were used in the fitting procedure but both the local potential and local potential with Perey factor models also give reasonably good representations of the positions of the particle states. However, the nonlocal model gives too wide a level spacing, a feature which has been noted previously.[14] It further is worth noting that only the model using a local potential with the Perey factor gives reasonably good agreement for the level positions when the Fourier-Bessel coefficients are used as the *sole constraints*. In the other two models reasonable fits to the level positions require further adjustments.

The difficulties with the nonlocal potential are not surprising in view of previous observations.[4,5,14] A significant feature is the large gap (10.3 MeV) between hole levels and particle levels, compared with the experimental value of ≈7 MeV. Hamamoto and Siemens[6] showed that core polarization effects could reduce the gap calculated in the single-particle model by 4 MeV. This could explain the results we obtain with the nonlocal potential. However, they also suggest[6] that effects on level positions due to core polarization and due to nonlocality will cancel out so that a local potential will best reproduce level positions, as is indeed observed. Such a cancellation of terms in the energies will probably still cause the wave functions to depart from pure single-particle wave functions, and this could explain why the local potential with the attenuation provided by the Perey factor was the most successful in fitting simultaneously both charge distribution and level positions.

TABLE II. Least-squares fits of analytical forms of charge and proton distributions to Fourier-Bessel coefficients.

Model	χ^2_{FB}
Fermi charge distribution	354
Fermi proton distribution	195
Parabolic Fermi charge distribution	310
Parabolic Fermi proton distribution	122
Modified Gaussian charge distribution	171
Modified Gaussian proton distribution	130

Finally, in order to assess the meaning of the values of χ^2_{FB} obtained in these calculations, we have also compared the measured charge distribution with simple analytical forms for either the charge distribution or for the proton distribution. In the latter case the charge distribution is obtained from the proton distribution by folding in the finite size of the proton. Three analytical forms were used: Fermi, parabolic Fermi, and modified Gaussian.[15] The procedure was, again, to fit the parameters of these distributions by performing a least-squares fit to the experimental Fourier-Bessel coefficients. The results are given in Table II. Note that in all three cases using the analytical form for the charge distribution gives a worse fit than when the analytical form is used for the proton distribution. It is interesting that the single-particle model gives a somewhat worse fit to the measured charge distribution than either a parabolic Fermi or modified Gaussian form for the proton distribution. Charge distributions obtained for these various forms have been compared and it was found that the single-particle model gives too large a central bump compared with the experimental values and that it also predicts too much structure in the central region. This feature has been discussed previously in various analyses of electron scattering data.[16]

In conclusion the single-particle model has been used to calculate the nuclear charge distribution for ^{208}Pb which fits the experimentally determined Fourier-Bessel coefficients. A local potential seems to be the most satisfactory when it is also required to fit the level positions and there is some evidence that including the Perey factor produces better results. The apparent failure of the nonlocal potential may be due to core polarization effects.

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