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Parity violating observables in low energy nucleon-nucleon scattering*

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We have calculated the parity violating observables in p-p and n-p scattering using a number of strong nucleon-nucleon potentials for a large class of weak parity violating potentials.

[NUCLEAR REACTIONS pp and np low energy scattering; calculated parity violating observables A_p, An, various strong and weak potentials.]

As a by-product of our recent calculation of the parity violating observables in the reaction $n+p\rightarrow d+\gamma$ (Refs. 1 and 2) we are able to give the parity violating observables for proton-proton and neutron-proton scattering at low energies, for a wide class of strong nucleon-nucleon potentials and weak parity violating nucleon-nucleon potentials. In this note we present results for the asymmetries or analyzing powers A_n and A_p for the scattering of longitudinally polarized neutrons and protons from protons. Details of the calculational technique may be found in Ref. 1.

Following Danilov[3] we write the amplitude for np scattering at low energies, $f^{(n)}(\vec{k}_n, \vec{k}_n')$, as

$$\begin{aligned} f^{(n)}(\vec{k}_n, \vec{k}_n') = & f_s^{(n)}(\vec{k}_n, \vec{k}_n)P_s + f_t^{(n)}(\vec{k}_n, \vec{k}_n')P_t \\ & + C^{(n)} f_t^{(n)}(\vec{k}_n, \vec{k}_n')(\vec{\sigma}_n + \vec{\sigma}_p)\cdot(\vec{k}_n + \vec{k}_n') \\ & + (\vec{\sigma}_p - \vec{\sigma}_n)\cdot[\lambda_t^{(n)} f_t^{(n)}(\vec{k}_n, \vec{k}_n')(\vec{k}_n' P_t + \vec{k}_n P_s) \\ & + \lambda_s^{(n)} f_s^{(n)}(\vec{k}_n, \vec{k}_n')(\vec{k}_n P_t + \vec{k}_n' P_s)] \, . \end{aligned} \quad (1)$$

In Eq. (1) $\vec{k}_n$ and $\vec{k}_n'$ are the initial and final neutron momenta in the c. m. frame, P_s and P_t are projection operators onto spin singlet and spin triplet states, $f_s^{(n)}$ and $f_t^{(n)}$ are the corresponding parity conserving (PC) scattering amplitudes, and $C^{(n)}$, $\lambda_t^{(n)}$, and $\lambda_s^{(n)}$ are parity nonconserving (PNC) amplitudes which are real at low energies. In principle they can be functions of $\vec{k}_n^{\,2} = \vec{k}_n'^{\,2}$ and $\vec{k}_n\cdot\vec{k}_n'$, but at low energies they become constants, proportional to the amplitudes for the PNC transitions ${}^3S_1+{}^3D_1\leftrightarrow{}^3P_1$, ${}^3S_1+{}^3D_1\leftrightarrow{}^1P_1$, and ${}^1S_0\leftrightarrow{}^3P_0$, respectively. We define the neutron analyzing power by[4]

$$A_n = (\sigma_\uparrow - \sigma_\downarrow)/(\sigma_\uparrow + \sigma_\downarrow) \, , \quad (2)$$

where $\sigma_\uparrow$ and $\sigma_\downarrow$ are the cross sections for incident neutrons of helicity $+\frac{1}{2}$ and $-\frac{1}{2}$, respectively, A_n is given in terms of the parameters of Eq. (1) by[5]

$$A_n = \frac{2k_n\{4C^{(n)}|f_t^{(n)}|^2 - 2\lambda_t^{(n)}|f_t^{(n)}|^2 - 2\lambda_s^{(n)}|f_s^{(n)}|^2\}}{3|f_t^{(n)}|^2 + |f_s^{(n)}|^2} \, . \quad (3)$$

Notice that, at least in the low energy region where $C^{(n)}$, $\lambda_t^{(n)}$, $\lambda_s^{(n)}$, $f_t^{(n)}$, and $f_s^{(n)}$ may be regarded as constants, A_n is proportional to k. This behavior is to be expected since A_n, at low energies, is proportional to the amplitude for mixing to a P state, and thus should be proportional to k_n. This argument breaks down at higher energies, when higher partial waves become important. At 15 MeV, where we will apply Eq. (3) the corrections due to higher partial waves amount to 20% or so.[6] It should be emphasized that C and λ_t already include transitions from the coupled ${}^3S_1+{}^3D_1$ state to ${}^1P_1$ and ${}^3P_1$, respectively. Omitting the tensor force coupling can change C and λ_t by almost 50%.[2,6]

In the proton-proton case the scattering amplitude $f^{(p)}(\vec{k}_1, \vec{k}_1')$ is simpler than Eq. (1) at low energies because the Pauli principle forces the spin-triplet amplitude to vanish. We have

$$\begin{aligned} f^{(p)}(\vec{k}_1, \vec{k}_1') = & \tilde{f}_s^{(p)}(\vec{k}_1, \vec{k}_1')p_s - \lambda_s^{(p)} f_s^{(p)} \\ & \times (\vec{k}_1, \vec{k}_1')(\vec{\sigma}_1 - \vec{\sigma}_2)\cdot(\vec{k}_1' P_s + kP_t) \, , \end{aligned} \quad (4)$$

with only one parameter $\lambda_s^{(p)}$ describing the parity violation at low energies. The analyzing power A_p is given by

$$A_p = -4\lambda_s^{(p)} k_1 \frac{\mathrm{Re}(f_s^{(p)} \tilde{f}_s^{(p)*})}{|f_s^{(p)}|^2} \, , \quad (5)$$

where we make a distinction between $\tilde{f}_s^{(p)}$, the total

TABLE I. Parity violating pv potential parameters. Two methods of calculation are used, the standard method, reviewed in Ref. 11, and the renormalization group method, reviewed in Ref. 9. We use $\sin\theta_C = 0.235$, $\sin^2\theta_W = 0.4$. The parameters are given numerically. Algebraic formulas may be found in Refs. 1, 9, and 11.

Potential no.	Weak Hamiltonian	Ref.	Method	A	H	K	L	I	I'
1	Cabibbo	a	Std	−0.17	0.95	0	0	0	0
2	d'Espagnat	b	Std	2.92	0.95	0.95	0.39	−0.55	−0.55
3	Weinberg-Salam	c, d	Std	3.47	0.95	0.40	−4.8	0	0
4	Cabibbo	a	Renorm. with no enhancement	0.010	0.98	−0.34	0.96	0	−0.05
5	Weinberg-Salam	c	Renorm. with enhancement	−0.77	1.22	0.67	−1.56	0.52	0.53

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amplitude including Coulomb scattering, and $f_s^{(p)}$ the purely nuclear amplitude. This distinction is potentially important at very low energies. For example, as has been emphasized by Lewis,[7] at 90° in the c.m. frame at an energy of 500 keV the total amplitude $\tilde{f}_s^{(p)}$ is very small because of destructive interference between Coulomb scattering and nuclear scattering, and a large enhancement of A_p is therefore possible. However, in this note we quote values of A_n and A_p at 15 MeV laboratory energy, corresponding to that of the only available experiment,[8] and at these energies Coulomb effects are relatively unimportant. The higher partial waves omitted in this calculation can modify the 15 MeV results by about 20%. Nevertheless, we quote results at this energy for illustrative purposes.

In these calculations we have used as a weak potential that of Ref. 1, namely

$$V = V_\rho + V_\pi , \tag{6}$$

with

$$V_\pi^{\Delta T=1} = (-\tfrac{1}{2} i \tau^{(1)} \times \tau^{(2)})_0 (\sigma^{(1)} + \sigma^{(2)}) \cdot [p, v_\pi(r)]_- , \tag{7a}$$

where

$$v_\pi(r) = \frac{g_\pi f_\pi}{4\pi\sqrt{2} m_N} \frac{e^{-m_\pi r}}{r} , \tag{7b}$$

$g_\pi^2/4\pi \approx 14$, and m_π is the pion mass. The notation employed is that $\sigma_\mu^{(i)}$ ($\tau_\mu^{(i)}$) represents the μth spherical component (i.e., $\mu = 0, \pm 1$) of the spinor (isospinor) acting on the ith particle. The bracket $[p, v(r)]_-$ denotes a commutator, and $[p, v(r)]_+$ will be used to denote an anticommutator, with $p = -i\nabla = \frac{1}{2}(p^{(1)} - p^{(2)})$ the conjugate relative momentum operator.

There is at present some controversy over the value of f_π, the weak PNC $NN\pi$ amplitude, which is implied by a particular form of the weak Hamiltonian. We do not want to enter this controversy here, simply setting

$$|f_\pi| = 2.73 \times 10^{-7} |A|$$

and listing in Table I the values of A implied by various models and various methods of calculation. (The sign of A is not determined by the standard analysis. We have arbitrarily chosen a particular sign.) This list is meant to be typical rather than exhaustive. We give our results in such a way that the reader can extract values of A_n and A_p for whatever weak interaction potential is preferred, provided that it can be parametrized in the way that ours is. Also listed in Table I are the parameters of the ρ exchange potential,

$$V_\rho = V_\rho^{\Delta T=0,2} + V_\rho^{\Delta T=1} , \tag{8}$$

where, neglecting the ρ-ω mass difference,

$$V_\rho^{\Delta T=1} = \left[\frac{I}{6\sqrt{3}}(\tau_0^{(2)}\sigma^{(1)} - \tau_0^{(1)}\sigma^{(2)}) + \tfrac{1}{2}\sqrt{3}I'(\tau_1^{(1)}\sigma^{(1)} - \tau_0^{(2)}\sigma^{(2)})\right]\cdot[p, v_\rho(r)]$$
$$+\left[\frac{1+\mu_v}{12\sqrt{3}}I + \tfrac{1}{4}\sqrt{3}\,(1+\mu_s)I'\right](\tau_0^{(1)}+\tau_0^{(2)})(i\sigma^{(1)}\times\sigma^{(2)})\cdot[p, v_\rho(r)]_-$$
$$+\lambda(-\tfrac{1}{2}i\tau^{(1)}\times\tau^{(2)})_0(\sigma^{(1)}+\sigma^{(2)})\cdot[p, v_\rho(r)]_-\,, \tag{9a}$$

$$V_\rho^{\Delta T=0,2} = [M(\tau)+\tfrac{1}{6}L](\sigma^{(1)}-\sigma^{(2)})\cdot[p, v_\rho(r)]_+ + [(1+\mu_v)M(\tau)+(1+\mu_s)\tfrac{1}{6}L](i\sigma^{(1)}\times\sigma^{(2)})\cdot[p, v_\rho(r)]_-\,, \tag{9b}$$

where

$$V_\rho(r) = -\frac{GG_A m_\rho^2}{4\pi\sqrt{2}m_N}\frac{e^{-m_\rho r}}{r}\,, \tag{9c}$$

$$M(\tau) = \tfrac{1}{2}H(\tau^{(1)}\cdot\tau^{(2)} - \tau_0^{(1)}\tau_0^{(2)}) + \tfrac{1}{2}K\tau_0^{(1)}\tau_0^{(2)}\,, \tag{10}$$

and μ_s (=0.88) and μ_v (=3.70) are the isoscalar and isovector anomalous nucleon magnetic moments. The last contribution to $V_\rho^{\Delta T=1}$ is the pseudotensor contribution with strength given by[10,11]

$$\lambda = \frac{5}{24}\frac{f_\pi}{f_\pi^{(\mathrm{Cabibbo})}}\left(\frac{m_\pi}{m_\rho}\right)^2\,, \tag{11}$$

in the absence of second class currents; its effects are generally negligible compared to the isovector one-pion exchange (OPE) potential of Eq. (7). If second class currents are present this term is modified as shown by Blin-Stoyle and Herczeg[12] and others.[13] The isoscalar and isotensor contributions to $V_\rho^{\Delta T=0,2}$ can be separated out by introducing the rank-two isotensor of zero projection

$$[\tau^{(1)}\times\tau^{(2)}]_0^{(2)} = -(\tfrac{1}{6})^{1/2}(\tau^{(1)}\cdot\tau^{(2)} - 3\tau_0^{(1)}\tau_0^{(2)})\,. \tag{12}$$

Thus, $M(\tau)$ contains the only isotensor contribution, and may be reexpressed in terms of the isoscalar $(\tau^{(1)}\cdot\tau^{(2)})$ and isotensor parts as

$$M(\tau) = \tfrac{1}{6}(2H+K)\tau^{(1)}\cdot\tau^{(2)} + (\tfrac{1}{6})^{1/2}(K-H)[\tau^{(1)}\times\tau^{(2)}]_0^{(2)}\,. \tag{13}$$

One of the features of interest in this reaction is the degree to which one can draw firm conclusions about the weak interaction parameters from the observations. This is governed by the extent to which the strong nucleon-nucleon interaction influences the observables. We investigate this problem by calculating with several different strong nucleon-nucleon potentials. These are as follows: (a) the Reid soft core potential[14] as modified by Peiper[15] (RPSC); (b) the supersoft core potential of Gogny, Pires, and de Tourreil[16] (GPD); (c) the separable potential of Sirohi and Srivastava[17] (SS). This potential does not fit the two-particle scattering data as well as we would like, as explained in Refs. 1 and 15, and the variation of this case from the others is as much a consequence of this variation in the phase shifts as it is of the soft core, separable nature of the potential. Finally, there are (d) the one boson exchange potentials of Gersten, Thompson, and Green[18] (GTG) and of the Bonn group Erkelenz, Holinde, and Machleidt,[19] (EHM). The features of these potentials which are of importance in determining the parity admixtures in the wave functions are discussed in detail in Ref. 1 and this discussion will not be reported here. Instead we simply quote the results of the calculations.

Table II gives the results for the parity violating amplitudes $C^{(n)}$, $\lambda_t^{(n)}$, and $\lambda_s^{(n)}$ and $\lambda_s^{(p)}$ in terms of the parity violating potential parameters. We write

TABLE II. Expansion parameters for the parity violating amplitudes.

Amplitude ($\times 10^8$)	RPSC	GPD	SS	GTG	EHM
	Strong potential				
C_1	−27.0	−24.9	−26.4	−23.9	−27.4
C_2	−0.92	−2.26	−5.48	−1.59	−1.06
μ	−8.60	−17.7	−133.5	−13.3	−9.45
ν	−1.90	−1.02	−5.67	−2.19	−2.08
η	+11.66	+23.3	+89.3	+16.00	+17.05
κ	−1.226	−2.51	−9.81	−1.64	−1.81
ρ	−0.355	−0.751	−3.01	−0.458	−0.531

TABLE III. Expansion parameters for parity violating observables at 15 MeV.

Parameter ($\times 10^8$)	RPSC	GPD	Strong potential SS	GTG	EHM
n_1	−1.03	−2.05	−6.83	−1.39	−1.56
n_2	−4.66	−9.32	−35.72	−6.40	−6.82
n_3	0.398	0.769	3.03	−0.526	0.576
n_4	−0.848	−0.782	−0.829	0.750	−0.860
n_5	−0.029	−0.071	−0.172	−0.050	−0.033
p_1	+4.08	+8.16	+31.26	+5.60	+5.97
p_2	0.429	0.879	3.43	0.574	0.634
p_3	0.124	0.263	1.054	0.160	0.186

$$m_N C^{(n)} = C_1 A + C_2(I' - \tfrac{1}{9}I)\,, \tag{14a}$$

$$m_N \lambda_t^{(n)} = \mu(2H+K) + \nu L\,, \tag{14b}$$

$$m_N \lambda_s^{(n)} = \eta(-2H+K) + \kappa L\,, \tag{14c}$$

$$m_N \lambda_s^{(p)} = -\eta K + \kappa L + \rho(9I'+I)\,, \tag{14d}$$

and tabulate C_1, C_2, μ, ν, η, and κ for the various strong potentials. This table permits the construction of A_n and A_p for the various strong potentials at any sufficiently low energy.

As an example we select 15 MeV laboratory energy and tabulate the appropriate expansion parameters for A_n and A_p at this energy. We write

$$A_n = n_1(2H+K) + n_2(K-H) + n_3 L + n_4 A + n_5(I' - \tfrac{1}{9}I)\,, \tag{15}$$

$$A_p = p_1 K + p_2 L + p_3(9I'+I)\,. \tag{16}$$

It is immediately apparent from Table III that the analyzing powers at low energy do not suffer from the amazing cancellations which occur when calculating P_γ, and that, if we ignore the SS case, the expansion parameters are determined to within a factor of 2. Remembering that $K = \frac{1}{3}[(2H+K) + 2(K-H)]$, we see that A_n is much more sensitive to the isotensor component of the potential than is A_p, a point which we have already noted elsewhere.[5,20]

Finally, to illustrate the use of Table III we present the values of A_n and A_p for the various weak potentials of Table I. These results are given in Tables IV and V.

The SS potential is markedly different from the other strong potentials in its predictions, but here we suspect that this is a consequence of the poor fit to the phase shifts. For the other strong potentials the result is quite stable, leading to the hope that measurements of A_n and A_p will provide useful information about the weak potential, and hopefully the weak Hamiltonian. However, the experiments on A_p have to be improved by about an order of magnitude in sensitivity before they probe the region of the predicted values.

TABLE IV. $10^8\,A_n$ for various parity violating (pvp) and parity conserving (pcp) potentials.

pvp \ pcp	RPSC	GPD	SS	GTG	EHM
1	2.61	5.09	21.10	3.57	3.66
2	−5.24	−7.79	−20.62	−5.92	−6.72
3	−4.65	−5.99	−13.48	−4.80	−5.59
4	4.86	9.72	38.99	6.70	7.02
5	−0.62	−1.88	−5.76	−1.07	−1.35

TABLE V. $10^8\,A_p$ for various parity violating and parity conserving potentials.

pvp \ pcp	RPSC	GPD	SS	GTG	EHM
1	0.0	0.0	0.0	0.0	0.0
2	3.36	6.66	25.24	4.66	4.90
3	−0.43	−0.96	−3.96	−0.52	−0.66
4	−1.03	−2.05	−7.81	−1.42	−1.50
5	2.72	5.47	21.17	3.70	3.99

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