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Vanishing closure correction in the second-order optical potential

G. A. Miller*
Department of Physics, University of Washington, Seattle, Washington 98195

N. Austern*†
Department of Physics, University of Washington, Seattle, Washington 98195
and Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, Pennsylvania 15260‡

M. Silver†
Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, Pennsylvania 15260
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The first-order correction to the closure approximation in the multiple-scattering calculation of the second-order optical potential is shown to vanish under rather general conditions.

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The closure approximation is commonly used to reduce the complication of higher-order terms in multiple-scattering theory. For example, a typical term in the second-order optical potential is

$$M_{12}= \langle\Phi_0|t_2Gt_1|\Phi_0\rangle , \qquad (1)$$

where Φ_0 is the ground state wave function of the target nucleus, and t_1, t_2 are effective interactions between the projectile and the target nucleons labeled 1 and 2. The full Green's function for the system is

$$G=(E^+ - h - H_N)^{-1} , \qquad (2)$$

where h is the projectile Hamiltonian and H_N is the Hamiltonian of the target nucleus, with $H_N\Phi_0=0$. Under the closure approximation, H_N in Eq. (2) is replaced by a constant,[1,2] interpreted as an average excitation energy. Usually this constant is chosen to equal the ground state energy, hence

$$H_N \rightarrow 0 , \qquad (3)$$

expressing the idea that the projectile scatters before the target nucleons have time to recoil. Under these approximations G reduces to

$$G \rightarrow g=(E^+ - h)^{-1} , \qquad (4)$$

and Eq. (1) becomes

$$M_{12}^0=\langle\Phi_0|t_2 g t_1|\Phi_0\rangle . \qquad (5)$$

One obvious correction to the closure approximation is derived by expanding Eq. (2) to first order in H_N, giving

$$\Delta M_{12}=\langle\Phi_0|t_2 g H_N g t_1|\Phi_0\rangle . \qquad (6)$$

Equation (6) seems nontrivial, because H_N does not commute with the t_j.

It is of interest to evaluate Eq. (6) under the familiar simplifying assumptions that the t_j are approximately local in the nucleon coordinates[3] [e.g., $t_j=e^{i(\vec{k}-\vec{k}')\cdot\vec{r}_j}\, t(\vec{k},\vec{k}')$, where k' and k are initial and final projectile momenta] and the interactions in H_N are local. In this case only the kinetic energy operator in H_N, a linear combination of single-particle operators, does not commute with t_j. Use of the property $H_N\Phi_0=0$ allows a variety of reduced expressions for ΔM_{12}; a typical one is

$$\Delta M_{12}=(-\hbar^2/2M)\langle\Phi_0|t_2 g^2[\nabla_1^2, t_1]|\Phi_0\rangle . \qquad (7)$$

We now observe that ∇_1^2 in the second term of the commutator operates on Φ_0 at the right hand end of Eq. (7), while ∇_1^2 in the first term commutes to the left through the other operators and operates on Φ_0 at the left. Because ∇_1^2 is even under time reversal, the two operations on Φ_0 at the left and at the right are equal, therefore the commutator vanishes, and we have the result

$$\Delta M_{12}=0 . \qquad (8)$$

The above analysis does not use any special aspects of the ∇^2 operator. It only uses the properties that ΔM_{12} is a diagonal matrix element and the kinetic energy is a one-body operator that is even under time reversal.

The same result[4] may be obtained using the momentum space double-commutation relations of Noble.[5]

Although the result $\Delta M_{12}=0$ is implicit in earlier articles,[4,6] the simplicity and generality of the above analysis does not seem to be known. Naturally, there are conditions that violate this theorem. For example, the t_j or the interactions in H_N may be nonlocal. Or the target nucleus may be very light, in which case recoil would effectively cause the wave functions at the two ends of the matrix element to be different. These wave functions also differ if inelastic processes should be of interest. Nevertheless, it is intriguing that an apparently straightforward correction to closure for elastic scattering tends to vanish.

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‡Permanent address.

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