Microscopic theory of the imaginary inelastic transition form factor

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A microscopic formulation of the nucleon-nucleus imaginary inelastic form factor using particle-hole intermediate states gives a result which is roughly the real form factor times a function consisting of intermediate particle- and hole-scattering terms. These terms have opposite signs, allowing the possibility of a radial dependence for the form factor similar to the collective model. Numerical results for 30 MeV ^{40}Ca$(n,n')^{40}$Ca (3^-, 3.73 MeV) scattering are compared with the collective model.

[NUCLEAR REACTIONS Inelastic scattering, microscopic calculation of imaginary form factor for ^{40}Ca$(n,n')^{40}$Ca(3^-, 3.73 MeV).]

In recent years a number of informative papers[1-4] have clarified the origin and nature of the imaginary part of the nuclear optical potential, which arises mainly from second-order scattering proceeding through open-channel intermediate states. However, for inelastic scattering very little is known about the imaginary part of the form factor from a microscopic point of view, although the real part is quite well understood. In the collective model the inelastic form factor is simply proportional to the derivative of the optical potential, and therefore has a real and an imaginary part. The data require[5-7] an imaginary inelastic interaction, and the collective model version seems satisfactory.

The problem with the microscopic theory of inelastic scattering has to do with the nature of the intermediate states. To elastic transitions, any strong intermediate state n contributes significantly in both steps of the transition $i \to n$ and $n \to i$, and all terms have the same phase since the absolute square of the matrix element appears. For inelastic scattering the intermediate state should have substantial coupling to both the initial and final state, but since the two matrix elements $i \to n$ and $n \to f$ are different, there is no guarantee that the phases of the various n terms are constructive. They could be random and therefore destructive.[8,9] We will show that for a wide range of final states there is no randomness of phase, and that substantial imaginary interaction is expected.

We make the following three assumptions: (1) The initial and final nuclear states are assumed to be related by creation and destruction operators as follows:

$$\Phi_f = \sum_{\bar{m}\underline{n}} A_{\bar{m}\underline{n}} a^\dagger_{\bar{m}} a_{\underline{n}} \Phi_i \,, \tag{1}$$

where $\bar{m}$ refers to an unoccupied state (particle), and $\underline{n}$, refers to an occupied state (hole) based on the target. Equation (1) would hold for the Tamm-Dancoff approximation (TDA) collective states as well as for simple single-particle transitions. (2) The intermediate states can be taken as simple particle-hole doorway states based on the ground state. Actual nuclear states will be linear combinations of these and more complicated configurations, not excited in this order. Energies will be shifted, but the calculation is not extremely sensitive to the energy of the intermediate Green's function anyway. (3) The identity of the projectile with target nucleons is neglected, although rough account is taken of exchange effects in determination of the interaction strength constant.

For elastic scattering any intermediate particle-hole state can contribute, since it can always be destroyed in the second step. By contrast, for inelastic scattering the first step must always take the target nucleus "half way" to the final state; that is, either the intermediate particle or hole must be in the final configuration. Thus we find two types of important intermediate states for each particle-hole configuration mn in Eq. (1). They are the particle-scattering type, and the hole-scattering type, Figs. 1(a) and 1(b), respectively. From the fermion anticommutation relations it follows that the two contributions, Figs. 1(a) and 1(b), *have opposite signs*.

In order to understand the main characteristics of the inelastic form factor, we have tried to formulate our calculation as simply as possible. A zero-range force of strength C_0 has been used and we have avoided making angular momentum couplings of each step, which, although straightforward, obscure the interpretation. We obtain the following simple result for the second-order in-

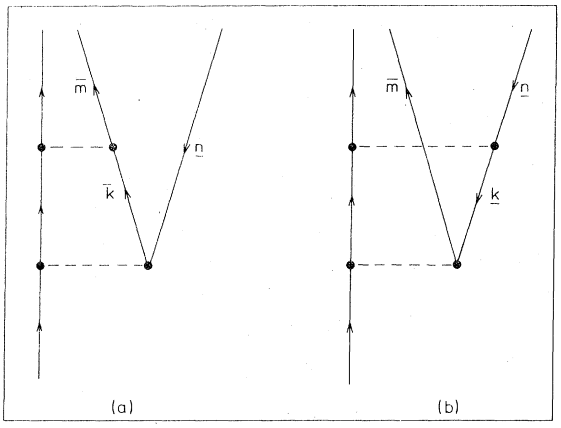

FIG. 1. Diagrams showing the (a) particle-scattering and (b) hole-scattering second-order interactions contributing to the imaginary inelastic form factor.

elastic form factor:

$$F^{(2)}=C_0\sum_{\bar{m}\underline{n}}A_{\bar{m}\underline{n}}\varphi^{\dagger}_{\bar{m}}(\vec{r}')\varphi_{\underline{n}}(\vec{r})G_{\bar{m}\underline{n}}(\vec{r}',\vec{r})\,, \tag{2}$$

where a spin-matrix element must still be taken between single-particle functions $\varphi^{\dagger}_{\bar{m}}(\vec{r}')$ and $\varphi_{\underline{n}}(\vec{r})$, and

$$\begin{aligned}G_{\bar{m}\underline{n}}&=G^{p}_{\underline{n}}-G^{h}_{\bar{m}}\\&=C_0\sum_{\bar{k}\lambda}\frac{2j_{\bar{k}}+1}{2}\,\frac{2\lambda+1}{(4\pi)^2}\\&\quad\times[R_{\bar{k}}(r)R_{\bar{k}}(r')g_{\lambda\underline{n}\bar{k}}(r,r')-R_{\underline{k}}(r)R_{\underline{k}}(r')\\&\quad\times g_{\lambda\bar{m}\underline{k}}(r,r')]P_{l_{\bar{k}}}(\hat{r}\cdot\hat{r}')P_{\lambda}(\hat{r}\cdot\hat{r}')\,.\end{aligned} \tag{3}$$

In Eq. (3) P_l is the Legendre polonomial, $R_k(r)$ is the radial wave function taken from a harmonic oscillator, and $g_{\lambda\bar{m}\underline{k}}$ is the projectile radial Green's function as defined, for example, in Ref. 1. Discretized Woods-Saxon[10] wave functions were also tried for $R_k(r)$, but in the region where the form factor was significantly large it did not make any important changes. Were it not for the nonlocality, and the fact that $G_{\bar{m}\underline{n}}$ depends on m and n, the mn sum of the function multiplying $G_{\bar{m}\underline{n}}$ in Eq. (2) would be just the first-order form factor $F^{(1)}$ for a zero-range spin-independent interaction. We will now try to justify as a rough approximation the use of a $\langle G\rangle$ averaged over m and n, on which we will make a local approximation to give the following simple result.

$$Im\tilde{F}^{(2)}=F^{(1)}\langle Im\tilde{G}\rangle\,, \tag{4}$$

where $\tilde{F}^{(2)}$ and $\tilde{G}$ denote the local equivalents of $F^{(2)}$ and G. To get Eq. (4) we have neglected the variation of $\varphi_m(\vec{R}-\frac{1}{2}\vec{s})\varphi_n(\vec{R}+\frac{1}{2}\vec{s})$ with $\vec{s}$, where $\vec{R}=\frac{1}{2}(\vec{r}+\vec{r}')$ and $\vec{s}=\vec{r}-\vec{r}'$. With harmonic oscillator functions the main dependence of $\varphi_m(\vec{r}')\varphi_n(\vec{r})$ on the coordinate $\vec{s}$ is through the factor $\exp(-\frac{1}{4}\alpha^2 s^2)$, which varies by 10 to 15% in the nonlocality length of G.

The functions $G^{p}_{\underline{n}}$ and $G^{h}_{\bar{m}}$ in Eq. (3) are peaked at locality $\vec{r}=\vec{r}'$, due to the peaking of both the Green's functions and the sums over particle or hole intermediate states[these sums would give $\delta^3(\mathbf{r}-\mathbf{r}')$ if the Green's function g were energy independent, but Eq. (3) is far from that limit]. The functions G have typically a nonlocality length[in which $G(\vec{R}-\frac{1}{2}\vec{s},\ \vec{R}+\frac{1}{2}\vec{s})$ drops by $1/e$] varying from $\approx$0.8 fm at low values of R to about 1.4 fm at R = 5 fm. The nonlocality is nearly spherical but sometimes not quite; in these cases we have used an rms value. We approximate the nonlocality by a Gaussian but with a variable nonlocality length obtained from our calculations of G. The local approximation[11] is then made on this Gaussian function following Ref. 2.

The function $G_{\bar{m}\underline{n}}$ depends on m and n only through the single-particle energies at which the single-particle Green's function is evaluated. When a particular channel is open, $\mathrm{Im}G^{p}_{\underline{n}}$ and $\mathrm{Im}G^{h}_{\bar{m}}$ do not depend very sensitively on energy. As a rough approximation we replace $\mathrm{Im}G^{p}_{\underline{n}}$ and $\mathrm{Im}G^{h}_{\bar{m}}$ with values averaged with a weighting equal to the probability of a particular single-hole or single-particle state in the microscopic final-state wave function. Thus we obtain Eq. (4).

A result very similar to Eq. (2) is obtained for the optical potential, except that the *hole-scattering term is missing*:

$$W^{(2)}=C_0\sum_{\underline{n}}\varphi^{\dagger}_{\underline{n}}(\vec{r}')\varphi_{\underline{n}}(\vec{r})G^{p}_{\underline{n}}(\vec{r}',\vec{r})\,. \tag{6}$$

The similarity between Eqs. (6) and (2) demonstrates the lack of randomness of the intermediate states in the inelastic-scattering case, since in both Eqs. (6) and (2) one has functions $G^{p}_{\underline{n}}$ and $G^{h}_{\bar{m}}$ with steady phases (in the local approximation) times a density or transition density.

The local functions $\mathrm{Im}\tilde{G}_p$ and $\mathrm{Im}\tilde{G}_h$ have everywhere the sign of $-C_0$, but they enter Eq. (3) with opposite signs. Because $\tilde{G}_h$ is summed over occupied states, it is confined to the region of the nucleus; on the other hand, since $\tilde{G}_p$ is summed over unoccupied single-particle states, it has a finite value both inside and outside of the nucleus. $\tilde{G}_p$ will be larger relative to $\tilde{G}_h$ at higher energies because of an increasing number of particle states and a decreasing number of high-energy hole states. Thus one can imagine a situation in which at low-energy $\mathrm{Im}\tilde{G}$ is negative inside the nucleus and positive outside. This function multiplying a surface-peaked collective real form factor can

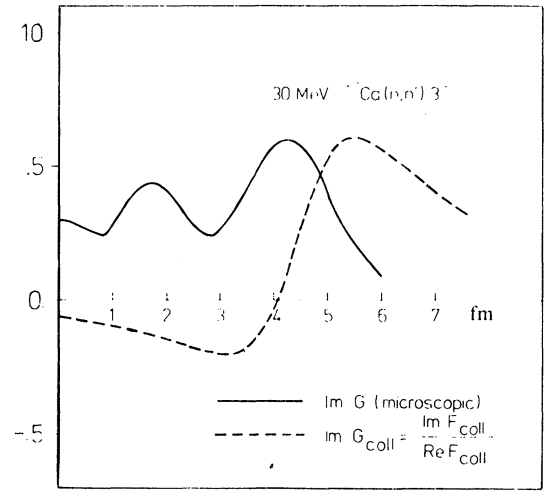

FIG. 2. Microscopic and collective functions G relating approximately real and imaginary form factors (see text).

give a result like the collective model prescription of taking the derivative of the imaginary potential consisting of a volume plus surface term. At higher energies $\tilde{G}_p$ will dominate, and the sign of the imaginary form factor will always be opposite that of the transition density, also resembling qualitatively the energy dependence of the derivative of the optical potential. In the high-energy limit $\tilde{G}_h$ can be neglected in Eq. (4), giving agreement with the impulse approximation and the "frivolous model" of Satchler.[5] In the collective model the magnitude of the imaginary potential is proportional to that of the real part, since the strength of each part is determined by the deformation parameter β. In our result the strengths are also related since the imaginary form factor contains approximately the real one as a factor.

Calculations have been carried out for $^{17}O(n,n')$-$^{17}O(\frac{1}{2}^-, 0.86\ \text{MeV})$ and for $^{40}Ca(n,n')^{40}Ca(3^-, 3.73$ MeV) at 30 MeV incident energy with very similar results. The intermediate projectile Green's function was calculated using a complex Woods-Saxon potential. A real potential gives similar results but the functions G_m^h and G_n^p show unrealistic resonance effects. The averaging procedure used to produce the factorization in Eq. (4) was checked for $W^{(2)}$ and gives errors of the order of 10% for $^{40}Ca(n,n')$.

Figure (2) shows a comparison of our microscopic[12] results for the function $\langle \mathrm{Im}\tilde{G}\rangle$ with corresponding quantities obtained from the collective model[5,13] (see Fig. 2). The two-nucleon interaction strength is normalized to give the same volume integral for $W^{(2)}$, collective and microscopic, and this same value of C_0 is then used in the determination of G. The magnitude is about the same as in the collective model, although the surface peak comes at too small a radius. The imaginary form factor at its maximum is about a factor of 3 greater than that of the collective model, and its volume integral is a factor of 1.7 greater.

In summary, we have shown that there is expected to be a substantial imaginary form factor in inelastic scattering which has the following features: (1) The hole-scattering terms subtract from the particle-scattering terms giving rise to a peak at or beyond the nuclear surface. (2) The real and imaginary form factors are related in strength. Although the collective and microscopic functions G are about the same magnitude, the geometrical differences are great and would probably give rather different effects on scattering.

The theory needs several improvements. With a finite-range interaction the position of the peak of G would presumably occur at a larger radius, giving better accord with the collective model. The local approximation is not very accurate for a nonlocality as large as that which occurs at and beyond the nuclear surface. The entire calculation should really be done nonlocally. Although the simple product particle-hole states give a complete representation of the one-particle one-hole space, the occurrence of low- and high-lying collective intermediate states well described by the random-phase approximation (RPA) may have a systematic effect on the imaginary potential. In addition, in contrast to elastic scattering, two-particle two-hole components mixed into the intermediate and final wave functions can make a nonzero contribution to the second-order form factor. It is known[14] that transitions between excited states are not accurately given by one-particle one-hole components, but that two-particle two-hole core-polarization components play an important role. With a sum over intermediate states many of these will appear with random phases and will tend to cancel each other; further work is needed to determine whether systematic effects will arise and contribute significantly to the imaginary form factor.

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