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Is there specific alpha clustering in light nuclei?*

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The role of the α-core bound state wave function in direct reaction theories of α transfer and knockout reactions is discussed. It is pointed out that for $A \leq 16$, wave functions with excessive rms radii are needed in order to reproduce predicted $1p$ shell absolute spectroscopic factors. It is argued that this may indicate considerable specific α clustering in excess of $1p$ shell estimates.

[NUCLEAR REACTIONS α-transfer and α-knockout reactions, importance of realistic α-core wave function. Possible specific α clustering.]

α clustering in $1p$ and $2s1d$ shell nuclei has been studied recently using many methods. These included the $(^6\mathrm{Li}, d)$, $(d, ^6\mathrm{Li})$, $(^3\mathrm{He}, ^7\mathrm{Be})$, $(^7\mathrm{Li}, t)$, $(\alpha, ^8\mathrm{Be})$, and $(^{16}\mathrm{O}, ^{20}\mathrm{Ne})$ transfer reactions,[1-6] as well as the $(p,p\alpha)$ and $(\alpha, 2\alpha)$ quasifree knockout processes[7-9] and the $(p, d^3\mathrm{He})$ quasifree reaction.[10] Methods of theoretical analysis include the zero range and finite range distorted wave Born approximation (DWBA) and the coupled channels Born approximation[11] (CCBA) for the transfer reactions. Analyses of the knockout reaction data have employed the distorted wave impulse approximation[12] (DWIA), either with an exact partial wave treatment of distortion effects or with an approximate parametrized treatment.[8] Frequently these calculations do not reproduce the observed absolute cross sections and discussion is limited to a comparison of extracted relative spectroscopic factors with the corresponding theoretical quantities.

In order to investigate the possibility of specific α clustering in excess of that predicted by a particular nuclear structure calculation, it is necessary to extract reliable absolute spectroscopic factors from the reaction analyses. These have been reported in some cases. However, a problem common to all three types of reaction analysis is the need to specify the α-core bound-state wave function. Specifically, if we consider a pickup or knockout experiment on a target of mass A_c+4, the wave function in question is that needed to describe the relative motion of an α particle with respect to the core A_c.

In most analyses this wave function is generated in a Woods-Saxon well, adjusted to reproduce the empirical α-core separation energy. The principal quantum number N and the angular momentum L are chosen to satisfy the oscillator shell model constraint

$$2(N-1)+L=\sum_{i=1}^{4} 2(n_i-1)+l_i\,, \qquad (1)$$

where the sum is over the corresponding quantum-numbers of the four transferred nucleons. Equation (1) results from the assumption that the α cluster is transferred in the zero quanta $(1s)^4$ configuration and that the differences in the oscillator length parameter between masses 4 and A_c+4 do not introduce significant contributions from four particle relative wave functions with nonzero oscillator quanta. As an example, for the $(1p)^4$ configuration, Eq. (1) permits NL values of $3S$, $2D$, or $1G$. In generating the α-core wave function the choice of the Woods-Saxon well radius R and diffuseness a is important. For highly surface-localized reactions the value of R strongly affects the predicted absolute cross section owing to the rapid decay of the asymptotic wave function.

Recently we have obtained data for the $(\alpha, 2\alpha)$ reaction induced by 140 MeV α particles on $^9\mathrm{Be}$, $^{12}\mathrm{C}$, $^{16}\mathrm{O}$, and $^{20}\mathrm{Ne}$. A detailed discussion will be presented elsewhere.[13] In general these data show an angular variation almost identical to the angular variation of the free α-α elastic-scattering cross section, thus supporting our interpretation of the reaction as a quasifree knockout process. Somewhat surprisingly, however, initial DWIA calculations seriously underestimated the experimental cross sections. For example, for $^{16}\mathrm{O}(\alpha, 2\alpha)$-$^{12}\mathrm{C}$(g.s.) we obtained a spectroscopic factor $S_\alpha = 40.3$, a value roughly 175 times larger than the $1p$ shell model prediction[14] of 0.23. This result was obtained using a radius of $1.09 \times 12^{1/3}$ fm and a diffuseness of $a = 0.7825$ fm. These values were obtained by folding an α-nucleon potential with a $^{12}\mathrm{C}$ density distribution.[15] The resultant wave function has an rms radius of 3.03 fm which is to be compared with the measured $^{16}\mathrm{O}$ rms radius of 2.65 fm. In Fig. 1 the resultant $3S$ wave function is plotted together with a $3S$ oscillator wave function with a length parameter $\nu = 4A_c(A_c+4)^{-4/3}$ fm^{-2} which is the expected shell model function.[16] It is

seen that the folding model potential yields a wave function which already overestimates the radial extent of the oscillator function, having the nodes and maxima at somewhat larger radii.

Detailed studies have shown that this serious discrepancy in absolute magnitude in our DWIA calculations cannot be eliminated by reasonable variations in distorting parameters.[13] However, on increasing the radius parameter to $2.52\times 12^{1/3}$ fm we obtain $S_\alpha = 0.22$ in good agreement with theory. The resultant wave function is shown in Fig. 2 along with the radial contributions to the cross section obtained by taking differences between calculations employing radial cutoffs. The surface nature of the reaction is evident and the increase in cross section is seen to arise in part from the elimination of the interference effect seen in the corresponding curve in Fig. 1. Although the value $R = 2.52\times 12^{1/3}$ fm leads to an apparently excessive rms radius of 4.37 fm, it is interesting to note that the resultant volume integral is $J/4A_c = 604$ MeV fm^3 which is consistent with fairly low energy analyses of α-nucleus scattering.

Similar behavior is found for the other targets studied, as well as for the $^{16}O(\alpha, 2\alpha)^{12}C$ reaction[8,12,17] at 90 MeV. In addition, quite similar results are obtained in analyses of transfer reactions. For example, finite range DWBA analyses of the $(\alpha, {}^8Be)$ reaction at 65 to 72.5 MeV on $1p$ shell targets[4] required $R = 2.0A_c^{1/3}$ fm with $a = 0.65$ fm in order to obtain absolute cross sections in reasonable agreement with shell model predictions. Similarly Yoshida[6] was obliged to use $R = 1.2(A_c^{1/3} + 4^{1/3})$ fm, $a = 0.65$ fm to describe $\alpha + {}^{16}O$ in his finite range DWBA analysis of $^{12}C({}^{16}O, {}^{20}Ne){}^8Be$. Finally, it seems unlikely that such anomalously large values of R serve to compensate for the omission of inelastic couplings in the entrance and exit channels. Thus Nagel and Koshel[11] in a CCBA analysis of $^{16}O(d, {}^6Li)^{12}C$ at 35 MeV found that the experimental cross section was underestimated by a factor of ~18 using $R = 1.25A_c^{1/3}$ fm, $a = 0.65$ fm, whereas good agreement with the experimental magnitude required $R = 1.27(A_c^{1/3} + 4^{1/3})$ fm which is equivalent to $2.15A_c^{1/3}$ fm. Similarly, Pisano and Parker[3] found much the same behavior in their CCBA calculations for $({}^3He, {}^7Be)$ at 25.5 MeV. These authors employed oscillator wave functions matched to correct binding energy tails to describe the α-core system. Since reasonable values are chosen for the oscillator length parameter, this procedure yields wave functions with fairly realistic rms radii. As a result the cross section for the $^{12}C({}^3He, {}^7Be){}^8Be$(g.s.) reaction is underestimated by about a factor of 11 in both DWBA and CCBA. Somewhat better results are obtained for $^{24}Mg({}^3He, {}^7Be)$-^{20}Ne(g.s.) although serious discrepancies persist

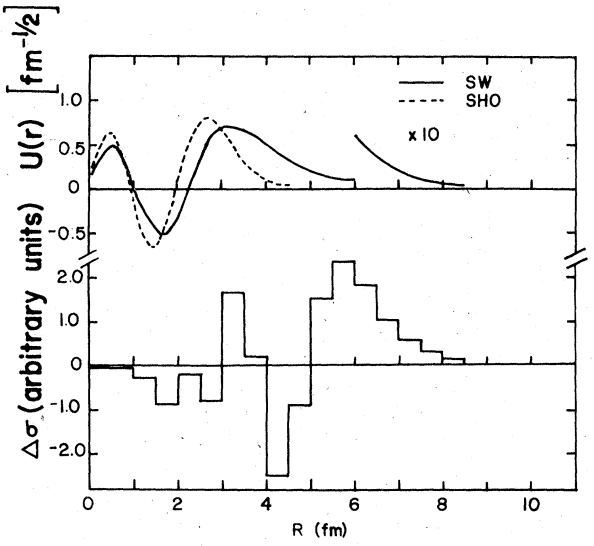

FIG. 1. $3S$ α-core wave function $U(r)$ for $^{16}O(\alpha, 2\alpha)$-^{12}C(g.s.). The broken curve is an oscillator function using $\nu = 1.19$ fm^{-2}. The continuous curve was obtained using a Woods-Saxon well with $R = 1.09\times 12^{1/3}$ fm, $a = 0.7825$ fm. Also shown is a histogram of $\Delta\sigma$, the contributions to the corresponding $(\alpha, 2\alpha)$ cross section calculated in DWIA at 140 MeV incident energy with $\theta_\alpha = \pm 43.16°$ and with equal detected energies such that the residual ^{12}C nucleus is left at rest.

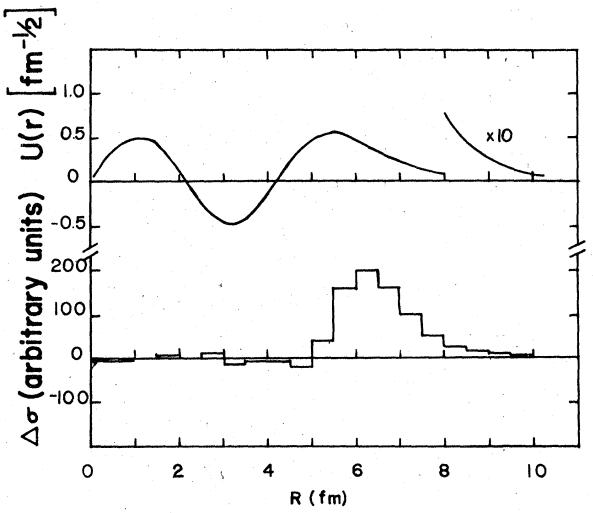

FIG. 2. $3S$ α-core wave function $U(r)$ for $^{16}O(\alpha, 2\alpha)$-^{12}C(g.s.). The continuous curve was obtained using a Woods-Saxon well with $R = 2.52\times 12^{1/3}$ fm, $a = 0.7825$ fm. Also shown is a histogram of $\Delta\sigma$, the contributions to the corresponding $(\alpha, 2\alpha)$ cross section calculated in DWIA at 140 MeV incident energy with $\theta_\alpha = \pm 43.16°$ and with equal detected energies such that the residual ^{12}C nucleus is left at rest.

for the excited state transitions.

In contrast to the above results it must be noted that some experiments on light nuclei[5,7,10] do report reasonable magnitudes for extracted absolute spectroscopic factors using α-core wave functions with realistic rms radii.[18] However, the significance of such results depends upon the degree of surface localization in the reaction studied. For example, in Refs. 7 and 10 it is found that $(p,p\alpha)$ and $(p,d^3\mathrm{He})$ reactions of $1p$ shell nuclei are less strongly surface localized than the $(\alpha,2\alpha)$ calculations shown in Figs. 1 and 2. As a result, on changing R from the folding model values originally used,[7,10] to $R\sim 2.3A_c^{1/3}$ fm, we find the extracted spectroscopic factors are reduced by no more than a factor of two. We have already seen that the corresponding change for $^{16}\mathrm{O}(\alpha,2\alpha)$-$^{12}\mathrm{C}$ (g.s.) is about two orders of magnitude.

In view of the consistent evidence from five rather dissimilar reactions and analyses favoring α-core wave functions with excessive rms radii, we feel that it is unlikely that our result $R\sim 2.5A_c^{1/3}$ merely compensates for errors in treating the $(\alpha,2\alpha)$ reaction mechanism. Thus unless it can be shown that the use of α-core wave functions such as those given in Fig. 1 with realistic rms radii somehow misinterpret the $1p$ shell model predictions for direct two-proton–two-neutron removal, the need to employ wave functions having much larger radius parameters implies considerable specific α clustering in light nuclei. Such clustering is in excess of the $1p$ shell model predictions and occurs at fairly large radii (6–7 fm).

In shell model language this result implies admixtures of four nucleon components from higher shells which would presumably enhance the wave function of Fig. 1 in the surface region. An example would be the two-particle–two-hole and four-particle–four-hole components which may be present in the $^{16}\mathrm{O}$ ground state.[19] For excitations into the $(2s1d)$ shell, such components would permit up to $5S$ terms in the α-core wave function. However, it seems likely that many higher components must also contribute. This is a consequence of the result that, using the folding model geometry which yields the $3S$ wave function shown in Fig. 1, a $5S$ admixture which has roughly 4 times the *amplitude* of the $1p$ shell prediction for the $3S$ term must be introduced in order to reproduce the observed $(\alpha,2\alpha)$ cross section. For a $4S$ function an even larger admixture is required. These results are to be compared with estimates using the wave functions of Brown and Green[16,19] which yield amplitude ratios of 0.416 and 0.135 for $4S/3S$ and $5S/3S$, respectively.[20] Thus the coherent $4S$ and $5S$ admixtures predicted in Ref. 16 are insufficient to explain the observed $(\alpha,2\alpha)$ cross section.[21]

As an alternative to large specific α clustering we must return to reaction mechanism considerations. A possible explanation for the observed effects might be some projectile induced enhancement of the α clustering which remained fairly constant from one reaction to another. In CCBA language this would imply significant inelastic couplings to excited states having large α widths. These would be entrance-channel couplings in pickup and knockout or exit-channel couplings in stripping. It is interesting that such states necessarily have lower α binding energies than the ground-state transition, or can be unbound. As a result the corresponding α-core wave functions will be enhanced in the surface region. Similar considerations suggest that, for ground-state transitions, exit-channel couplings (in pickup) will be less important owing to binding energy increases. Unfortunately suitable calculations for $(\alpha,2\alpha)$ are not yet available. However, additional CCBA calculations for transfer are currently feasible. Clearly the preceding argument is consistent with Nagel and Koshel's result[11] that, for $^{16}\mathrm{O}(d,^6\mathrm{Li})^{12}\mathrm{C}$, inelastic couplings in $^{12}\mathrm{C}$ have rather little effect. Instead it is suggested that a proper description of inelastic couplings in $^{16}\mathrm{O}$ is more important and should permit agreement with experiment using realistic rms radii. Among the levels which should be included is the 2+ state in $^{16}\mathrm{O}$ at 6.92 MeV. This level is seen strongly[5,16,22] in $^{12}\mathrm{C}(^7\mathrm{Li},t)^{16}\mathrm{O}$ and in $^{16}\mathrm{O}(d,d')^{16}\mathrm{O}$ and is close to the $^{16}\mathrm{O}$ α threshold having an α binding energy of only 0.243 MeV which is to be compared with a binding energy of 7.162 MeV for the ground state. Clearly CCBA calculations including states of this type would be of great interest.

In conclusion, we wish to emphasize the need for improved treatments of α-core wave functions and of coupled channels effects for transfer and knockout reaction studies.[23] Finally, while our discussion has been largely concerned with α cluster removal from $^{16}\mathrm{O}$, the rather weak mass dependence of the ground-state integrated cross sections of Ref. 8 suggest that similar problems persist throughout the $(2s1d)$ shell, at least for $(\alpha,2\alpha)$ at 90 MeV.

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[20]The amplitude ratios are obtained by computing the products of the coefficients $a_{NS}^{(n)}A_{NS}$ in Eq. 5.18a of Ref. 16, each divided by $a_{3S}^{(n)}A_{3S}$. Note that our principal quantum number N is one greater than the corresponding quantity in Ref. 16.
[21]This is contrary to the result reported in Ref. 16 that the α-core wave function at $r=4.8$ fm is enhanced by ~ 5.66 due to the 2p-2h and 4p-4h (4*S* and 5*S*) admixtures. Such an enhancement would lead to an encouraging increase of about $5.66^2=32$ in our $(\alpha, 2\alpha)$ calculations. Unfortunately it is a spurious result arising from the use of oscillator wave functions which have incorrect behavior in the nuclear surface.
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[23]Similar findings to our own were reported earlier by D. Mukhopadhyay and D. Robson [Bull. Am. Phys. Soc. 21, 540 (1976)]. These authors also considered possible improvements in α-core wave functions for transfer reactions.