Separable-potential three-body model of the $A = 6$ system. II. ^{6}Li ground state

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Separable-potential three-body (αnp) models of the ^{6}Li ground state are examined. Different separable-potential fits to the α-n phase shifts and the low-energy $^3S_1$-$^3D_1$ two-nucleon parameters are reviewed, after which integral equations are derived for the spectator functions of the ^{6}Li ground-state wave function assuming $S_{1/2}$, $P_{1/2}$, and $P_{3/2}$ components in the α-n interaction. These equations are solved to determine the ^{6}Li binding energy and investigate its sensitivity to the $^3D_1$ component of the two-nucleon interaction, its variation upon neglecting different components of the α-n interaction, and its dependence on different analytical forms for the P-wave α-n interactions. For two wave functions derived from the same set of α-n interactions, but with the $^3S_1$-$^3D_1$ two-nucleon interaction producing 0% and 4% d-wave components in the deuteron, respectively, the spectator functions are given and the contributions to the ground-state normalization of the various components are calculated. The latter two wave functions correspond to three-body ^{6}Li binding energies of 4.446 MeV (0%) and 4.070 MeV (4%) compared with the Coulomb-adjusted experimental value of 4.53 MeV.

[NUCLEAR STRUCTURE ^{6}Li; binding energy, wave function, wave function components, normalization. Three-body, separable-potential model.]

I. INTRODUCTION

The first paper (GL) of this series deals with the ^{6}He ground state.[1] In GL, a review of past work up to 1974 on the $A = 6$ system as a three-body problem is given with emphasis on the separable-potential approach. The types of α-n separable-potential form factors used by previous workers are delineated and the predicted α-n phase shifts displayed. Additional α-n separable potentials are generated to provide the necessary tools for a systematic study of the $A = 6$ ground states as three-body nuclei. The procedure for obtaining the ^{6}He wave function and the coupled, homogeneous integral equations for the spectator functions is outlined. The latter material is followed by a presentation of results for the ^{6}He binding energy as various properties of the underlying two-body interactions are changed. Furthermore, the structure of the ^{6}He wave function is examined through the contribution of the various components to the normalization. Two of the wave functions are tabulated for possible applications. In the present paper, we report our results for the application of the three-body model to the ^{6}Li ground state.

Since GL, two papers have appeared concerning three-body models of the $A = 6$ system, both based on separable interactions. The first, written by the present authors, concerns the α-deuteron structure of ^{6}Li from the three-body model.[2] An acceptable ^{6}Li wave function for a preliminary investigation is generated from only the n-p ($^3S_1$) and n-α ($^2P_{3/2}$) interactions. From this wave function, the $^6\mathrm{Li} \rightarrow \alpha + d$ momentum distribution, the asymptotic normalization constant of the d-α tail in the ^{6}Li wave function, and the percentage of d-α component are calculated and compared with experiment. The results are good and encourage further work with the more complete wave functions presented below. The second paper by Charnomordic, Fayard, and Lamot (CFL) pertains primarily to d-α elastic scattering.[3] CFL introduce yet another set of α-N separable-potential form factors and present some results for the ground-state ^{6}He and ^{6}Li binding energies.[4] Unfortunately, the CFL predictions for the ground-state binding energies are poor, but their results for various α-d elastic scattering quantities are on the whole quite good. One of CFL's major conclusions is the lack of sensitivity to the n-p tensor force in d-α scattering results such as polarizations. The d-α polarizations apparently arise solely from the α-N spin dependence. They conclude that the description of the α-N interaction is of primary importance in d-α scattering. Moreover, they demonstrate the sensitivity of d-α scattering quantities at 12 MeV to three different α-N separable interactions—Shanley's,[5] a set from GL, and CFL's. Though Shanley's fits to the $P_{1/2}$ and $P_{3/2}$ n-α phase shifts are poor, while those of GL and CFL fit the phase shifts well,[6-8] the results for the d-α differential

cross sections plus iT_{11} and T_{22} analyzing powers are qualitatively the same. Differences in the magnitudes of these quantities occur especially at the minima, but it is not clear that existing data are complete enough to distinguish these differences. Nevertheless, the study of low-energy d-α scattering by CFL emphasizes the validity of the three-body model as a very good first approximation in this regime. In fact, both of the above investigations give support to the viability of a three-body model of the $A=6$ system within its expected domain of validity, i.e., for excitation energies below the ^{3}He-^{3}H threshold.

The purpose of this paper is to investigate the variation of the ground-state ^{6}Li binding energy for different N-N and α-n interactions. Two overall objectives are (1) to select for future applications two wave functions which correspond to reasonable values of the binding energy and (2) to normalize the selected wave functions in order to determine the contributions of the various components to the normalization. Emphasis will be placed throughout on answering the questions raised in GL. The paper is built around these aims.

In Sec. II, the two-nucleon and α-N interactions used are discussed. The CFL α-N interactions are included and comments about amplitude singularities are made. Section III contains a derivation of the ^{6}Li integral equations from Schrödinger's equation. The binding-energy and normalization results are presented in Secs. IV and V, respectively. Finally, the major conclusions are summarized in Sec. VI.

II. INTERACTIONS

A. α-N

The α-N separable potential in momentum space is written as[1]

$$\langle \vec{k}|V_{\alpha N}|\vec{k}'\rangle = -\frac{4\pi}{2\mu}\sum_{J=1/2}^{3/2}\sum_{\substack{l=J-1/2\\ l\le 1}}^{J+1/2}\Lambda_l^J \hat{J}\,(-1)^{2J} h_l^J(k)\,h_l^J(k') \times[\mathcal{Y}^{[J]}_{l(1/2)}(\hat{k})\times\bar{\mathcal{Y}}^{[J']}_{l(1/2)}(\hat{k}')]^{[0]}, \tag{1}$$

limited to only the significant partial waves at laboratory energies $\lesssim$20 MeV.[8] The notation in Eq. (1) is as follows: μ is the α-N reduced mass which is set equal to $4M/5$ where M is the nucleon mass; Λ_l^J is the interaction strength for partial-wave l and total angular momentum J; $\hat{J}=(2J+1)^{1/2}$; $h_l^J(k)$ are form factors; and[9]

$$\mathcal{Y}^{[J]}_{l(1/2)}(\hat{k}) = \sum_{\mu\eta}\langle l\,\mu\,\tfrac{1}{2}\eta|\,JM\rangle\, Y^{[l]}_{\mu}(\hat{k})\,\chi^{[1/2]}_{\eta}\ . \tag{2}$$

Equation (1) leads to the following form for the elastic scattering amplitude with orbital and total angular momenta of l and J, respectively:

$$f^J_{l(1/2)}(k) = \frac{e^{i\delta_l^J}\sin\delta_l^J}{k} = \frac{2\pi^2[h_l^J(k)]^2}{D_l^J(k^2/2\mu+i\epsilon)}\ , \tag{3}$$

where

$$D_l^J\left(\frac{k^2}{2\mu}+i\epsilon\right) = 1-\Lambda_l^J\int d^3p\ \frac{[h_l^J(k)]^2}{p^2-k^2-i\epsilon}\ . \tag{4}$$

Thus, the relationship between the potential (parameters) and the phase shifts is through Eqs. (3) and (4).

As discussed in GL, one of the simplest one-parameter form factors which possesses both proper threshold and acceptable asymptotic behavior in momentum space is

$$h_l^J(k) = \frac{k^l}{[k^2+(\beta_l^J)^2]^{l+1}}\ , \tag{5}$$

where β_l^J acts as the inverse range of the interaction. This form has been adhered to for $l=0$ in previous work, but for $l=1$ it has not. Three forms with proper threshold behavior, but different asymptotic behavior, have been introduced:

$$\text{(A)}\quad h_1^J(k) = \frac{k}{k^2+(\beta_1^J)^2}\ , \tag{6}$$

$$\text{(B)}\quad h_1^J(k) = \frac{k}{[k^2+(\beta_1^J)^2]^2}\ , \tag{7}$$

$$\text{(C)}\quad h_1^J(k) = \frac{k}{[k^2+(\beta_1^J)^2]^{3/2}}\ . \tag{8}$$

Type (B) is preferred, since the unusual asymptotic behavior of types (A) and (C) leads to singular behavior of the coordinate-space wave function as $r\to 0$. Form (B) leads to the usual $R_1(r)$ $\to$ const $\times r$ as $r\to 0$.[10] All three forms will be considered in this work.

The α-n parameters used are presented in Table I.[11] Shanley's parameters are included for a comparative calculation even though his $P_{1/2}$ and $P_{3/2}$ interactions produce poor fits to the α-n phase shifts. His $S_{1/2}$ interaction is satisfactory. All the other interactions, except $P_{3/2}$-Pigeon, give good fits to the phase shifts. In fact, Aless. A and Aless. B are almost indistinguishable from the Arndt and Roper phase shifts below 20 MeV laboratory energy. For the details, see GL. The set of primary interest is $S_{1/2}$—Shanley, $P_{1/2}$—GL-B, and $P_{3/2}$—Aless.-B.

As mentioned in the Introduction, CFL have introduced another set of form factors. Their form factors have more parameters, but possess proper threshold and acceptable asymptotic behavior. More parameters means that it is easier to obtain good fits to the phase shifts, but with a loss of simplicity in handling quantities derived analytically from the form factors. The CFL forms are

TABLE I. Parameters for α-n potentials.

Partial wave	Name	Λ_l^J	β_l^J (fm^{-1})	Form factor	Reference
$S_{1/2}$	Shanley	−0.6373 fm^{-3}	0.7496	•••	5
	GL-1 (threshold fit)	−0.3 fm^{-3}	0.7	•••	1
	GL-2 (10–15 MeV fit)	−0.2 fm^{-3}	0.6	•••	1
$P_{1/2}$	Shanley	0.1640 fm^{-5}	0.8505	B	5
	GL-A	0.05066 fm^{-1}	0.68	A	1
	GL-B	1.104 fm^{-5}	1.177	B	1
	Pigeon [a]	0.264 fm^{-3}	1.00	C	12
$P_{3/2}$	Shanley	1.3671 fm^{-5}	1.1352	B	5
	Aless.-A [b]	0.0838 fm^{-1}	0.862	A	13
	Aless.-B [b]	4.831 fm^{-5}	1.449	B	13
	Pigeon [a]	0.724 fm^{-3}	1.25	C	12

[a] Reference 12. [b] Reference 13.

(subscripts and superscripts suppressed on the parameters)

$$h_0^{1/2}(k) = \frac{\alpha^2 + k^2}{(\beta^2 + k^2)(\gamma^2 + k^2)} \tag{9}$$

and

$$h_1^J(k) = \frac{k(\alpha^2 + k^2)}{(\beta^2 + k^2)(\gamma^2 + k^2)(\delta^2 + k^2)} \, . \tag{10}$$

The values of their parameters are given in Table II, since several comparison calculations are made with the CFL interactions. The reader is referred to Ref. 3 for a comparison of the phase shifts from the CFL and GL (preferred set: $S_{1/2}$—Shanley, $P_{1/2}$—GL-B, and $P_{3/2}$—Aless.-B) interactions.

An important aspect of the α-n interactions to be considered is the singularity structure of the amplitudes. For the $S_{1/2}$ parameters in Table I, this is of particular interest since the GL ^{6}He calculations indicate little sensitivity to the different sets, even though the strength parameters differ by as much as a factor of 3 while the range parameters remain comparable. The $S_{1/2}$ amplitude derived from Eq. (5) possesses three poles in the complex k plane[14]:

$$i\beta_0^{1/2},$$

$$\pm\pi \left(\frac{|\Lambda_0^{1/2}|}{\beta_0^{1/2}} \right)^{1/2} - i\beta_0^{1/2} .$$

The $i\beta_0^{1/2}$ pole is common to separable potentials with simple one-parameter form factors and it simulates the left-hand cut of (local) Yukawa-type potentials. The pole of interest is the one in the fourth quadrant, since it is closest to the physical region *and* varies with $\Lambda_0^{1/2}$. In Table III, we give the location of this pole for the three $S_{1/2}$ potentials of Table I. It is evident that this pole remains relatively far away from the low-energy region where the phase shifts are being fitted, even though the strength parameter is changed by a factor of 3. Thus, the fits to the *low-energy* phase shifts change very little. Of course, the higher-energy (beyond ~30 MeV) phase shifts are considerably different. This means the three-body model of ^{6}He is not sensitive to the higher-energy part of the $S_{1/2}$ interaction, but is only sensitive to the fact that the $S_{1/2}$ interaction is repulsive and has a given behavior at low energies. As we shall see, this comment will apply to ^{6}Li also.

The singularity of interest in the $P_{1/2}$ and $P_{3/2}$

TABLE II. Parameters for CFL α-n potentials.

Partial wave	Λ_l^J	α_l^J (fm^{-1})	β_l^J (fm^{-1})	γ_l^J (fm^{-1})	δ_l^J (fm^{-1})
$S_{1/2}$	8.247 fm^{-3}	0.4441	0.5785	0.5792	•••
$P_{1/2}$	0.9527 fm^{-5}	0.1781	1.136	1.112	0.2061
$P_{3/2}$	88973 fm^{-5}	0.4002	0.8071	31.62	0.4479

TABLE III. Fourth-quadrant pole positions of $S_{1/2}$ amplitudes.

Interaction	$k_R - ik_I$ (fm^{-1})	$E_R - iE_I$ (c.m.) (MeV)
Shanley	$2.897 - i\,0.7496$	$203.1 - i112.7$
GL-1	$2.057 - i\,0.7$	$97.05 - i74.70$
GL-2	$1.814 - i\,0.6$	$76.02 - i56.47$

TABLE IV. Resonance-pole positions of P-wave amplitudes.

Partial wave	Interaction	$k_R - ik_I$ (fm^{-1})	Experiment [a] $k_R - ik_I$ (fm^{-1})
$P_{1/2}$	Shanley	$0.2610 - i0.1851$	$0.3111 - i0.1404$
	GL-A	$0.3000 - i0.1800$	
	GL-B	$0.3172 - i0.1731$	
	Pigeon	$0.317 - i0.173$	
$P_{3/2}$	Shanley	$0.1633 - i0.0396$	$0.1766 - i0.03489$
	Aless.-A	$0.1700 - i0.03493$	
	Aless.-B	$0.1667 - i0.0317$	
	Pigeon	$0.208 - i0.05$	

[a] Reference 15.

amplitudes is the location of the resonance pole as emphasized by Ahmed and Shanley.[15] They have determined these quantities with high precision from the latest Arndt and Roper phase shifts.[8] Table IV lists the resonance-pole positions for the P-wave interactions of Table I. The $P_{3/2}$ interactions which give the best representation of the low-energy phase shifts also yield values for the pole location close to the Ahmed-Shanley values. The $P_{1/2}$ case is different. GL-A, GL-B, and Pigeon possess reasonable phase shifts, but have pole locations with imaginary parts roughly the same and consistently larger than the experimental values. This can be understood by turning the problem around, i.e., determining the separable-potential strength and range parameters from the Ahmed-Shanley resonance-pole positions and then predicting the phase shifts. This has been done by Lehman and Gibson.[16] They find that the $P_{3/2}$ phase shifts agree very well with experiment, but the $P_{1/2}$ ones do not. These results are interpreted as the low-energy $P_{1/2}$ amplitude not being dominated by its resonance pole, unlike the $P_{3/2}$ amplitude. There are other nearby singularities of comparable importance to the resonance pole in the $P_{1/2}$ amplitude. In the $P_{1/2}$ interactions considered here, moving the resonance-pole location away from the real axis compensates for these other singularities and permits a good fit to the low-energy phase shifts.[17]

B. *n-p*

For ^{6}Li ($J^\pi = 1^+$, $I=0$), the dominant parts of the n-p interaction which enter are the triplet-spin, s and d waves. This potential is represented by the rank-1 Yamaguchi-Yamaguchi form[18]:

$$\langle \vec{k}|V_{NN}|\vec{k}'\rangle = -\frac{4\pi\lambda_1}{2\mu}\hat{1}\sum_{\substack{l,l'=0\\ l,l'\neq 1}}^{2} g_l^1(k)\,g_{l'}^1(k')[[Y^{[l]}(\hat{k})\times\chi^{[1]}(12)]^{[1]}\times[Y^{[l']}(\hat{k})\times\bar{\chi}^{[1]}(12)]^{[1]}]^{[0]}, \tag{11}$$

where $\mu = \frac{1}{2}M$, λ_1 is the triplet coupling strength, and the form factors $g_l^1(k)$ are

$$g_0^1(k) = \frac{1}{k^2 + (\beta_0^1)^2} \tag{12}$$

and

$$g_2^1(k) = \frac{tk^2}{[k^2 + (\beta_2^1)^2]^2}, \tag{13}$$

with t as a parameter. We follow Shanley[5] and use Phillips's parameters[19] when the tensor component of the interaction is nonzero. These parameters

TABLE V. Parameters for n-p potentials (the binding energy of the deuteron is 2.225 MeV).

P_D (%)	a_t (fm)	r_{0_t} (fm)	Q (fm^2)	P_t	λ_1 (fm^{-3})	t	β_0^1 (fm^{-1})	β_2^1 (fm^{-1})
0	5.400	1.747	•••	−0.0004	0.3910	•••	1.418	•••
4	5.397	1.727	0.282	−0.0002	0.2432	1.689	1.3134	1.5283
5.5	5.397	1.725	0.282	0.0049	0.1876	2.950	1.2766	1.7610
7	5.397	1.722	0.283	0.0105	0.1430	4.495	1.2412	1.9476
?	5.425 ± 0.004 [a]	1.749 ± 0.008 [a]	0.286 [b]	≈ 0.025 [c]	← Experimental values			

[a] Reference 20.
[b] Reference 21.
[c] Quoted by Phillips, Ref. 19.

are given in Table V along with the quantities fitted. One can criticize the use of a rank-1 separable potential to describe the $^3S_1$-$^3D_1$ n-p interaction, since it does not describe well the $^3D_1$ phase shift and mixing parameter ϵ_1. Yet, such an interaction can represent the low-energy parameters quite well as seen in Table V. Moreover, it can give an indication of the role played by the n-p tensor force in the binding of ^{6}Li. We shall see below that, with identical α-N interactions, the results with the rank-1 interaction are essentially the same as those of CFL with a rank-2 n-p interaction.

III. DERIVATION OF ^{6}Li EQUATIONS

We derive the ^{6}Li three-body bound-state equations from Schrödinger's equation in the momentum representation with particle 3 designated as the α particle. Equations (1) and (11) are substituted into Schrödinger's equation to obtain

$$
\begin{aligned}
&M^{-1}(K^2+k_{12}^{\,2}+\tfrac{3}{8}p_3^{\,2})\,\Psi_M^{[1]}(\vec{k}_{12},\vec{p}_3)\\
&=\frac{4\pi}{2\mu_{23}}\sum_{J=1/2}^{3/2}(-1)^{2J}\hat{J}\sum_{\substack{l=J-1/2\\ l\le 1}}^{J+1/2}\Lambda_l^J h_l^J(k_{23})\int d^3k'_{23}\,h_l^J(k'_{23})[\mathcal{Y}^{[J]}_{l(1/2)}(\hat{k}_{23},2)\chi\tilde{\mathcal{Y}}^{[J]}_{l(1/2)}(\hat{k}'_{23},2)]^{[0]}\Psi_M^{[1]}(\vec{k}'_{23},\vec{p}_1)\\
&+\frac{4\pi}{2\mu_{31}}\sum_{J=1/2}^{3/2}(-1)^{2J}\hat{J}\sum_{\substack{l=J-1/2\\ l\le 1}}^{J+1/2}\Lambda_l^J h_l^J(k_{31})\int d^3k'_{31}h_l^J(k'_{31})[\mathcal{Y}^{[J]}_{l(1/2)}(\hat{k}_{31},1)\times\tilde{\mathcal{Y}}^{[J]}_{l(1/2)}(\hat{k}'_{31},1)]^{[0]}\Psi_M^{[1]}(\vec{k}'_{31},\vec{p}_2)\\
&+\frac{4\pi}{2\mu_{12}}\lambda_1\hat{1}\sum_{\substack{l,l'=0\\ l,l'\neq 1}}^{2}g_l^1(k_{12})\int d^3k'_{12}\,g_{l'}^1(k'_{12})[[Y^{[l]}(\hat{k}_{12})\times\chi^{[1]}(12)]^{[1]}\times[Y^{[l']}(\hat{k}'_{12})\times\tilde{\chi}^{[1]}(12)]^{[1]}]^{[0]}\Psi_M^{[1]}(\vec{k}'_{12},\vec{p}_3)\,,
\end{aligned}
\tag{14}
$$

where $-K^2/M$ is the three-body binding energy, the extra argument on the vector spherical harmonic denotes the spin-$\frac{1}{2}$ particle, $\vec{k}_{ij}$ is the relative momentum between particles i and j, and $\vec{p}_k$ is the relative momentum of particle k with respect to the center-of-mass of particles i and j (i, j, and k permuted cyclically). The ^{6}Li(1^+), isospin singlet, wave function can be written by observing the structure of Eq. (14) and remembering that its spatial-spin part must be symmetric under exchange of particles 1 and 2, the neutron and proton:

$$
\begin{aligned}
\Psi_M^{[1]}(\vec{k}_{12},\vec{p}_3)=&\frac{4\pi}{K^2+k_{12}^{\,2}+\frac{3}{8}p_3^{\,2}}\\
&\times\Bigg(\lambda_1\sum_{\substack{l,l'=0\\ l,l'\neq 1}}^{2}g_l^1(k_{12})[[Y^{[l]}(\hat{k}_{12})\times\chi^{[1]}(12)]^{[1]}\times Y^{[l']}(\hat{p}_3)]^{[1]}_M\,G^{l'}(p_3)\\
&+\frac{5}{8}\sum_{J=1/2}^{3/2}\sum_{J'=|1-J|}^{1+J}\sum_{\substack{l=J-1/2\\ l\le 1}}^{J+1/2}\sum_{l'=J'-1/2}^{J'+1/2}P^{l+l'}\Lambda_l^J\{h_l^J(k_{23})[\mathcal{Y}^{[J]}_{l(1/2)}(\hat{k}_{23},2)\times\mathcal{Y}^{[J']}_{l'(1/2)}(\hat{p}_1,1)]^{[1]}_M F^{J'}_{l'(Jl)}(p_1)\\
&\qquad+(-1)^l h_l^J(k_{31})[\mathcal{Y}^{[J]}_{l(1/2)}(\hat{k}_{31},1)\times\mathcal{Y}^{[J']}_{l'(1/2)}(\hat{p}_2,2)]^{[1]}_M\\
&\qquad\times F^{J'}_{l'(Jl)}(p_2)\}\Bigg),
\end{aligned}
\tag{15}
$$

where $P^L=\frac{1}{2}[1+(-1)^L]$ and the isospin function $\zeta_0^{[0]}(12)$ is suppressed. The spectator function $G^l(p)$ gives the l-wave momentum distribution of the α particle relative to the two-nucleon center of mass, while $F^{J'}_{l'(Jl)}(p)$ gives the total-angular-momentum J'–orbital-angular-momentum-l' momentum distribution of a nucleon relative to the center of mass of an α-N pair interacting in the state l_J. There are nine spectator functions: two G's and seven F's. By comparing Eqs. (14) and (15), we can write integral expressions for the spectator functions. For the $G^l(p)$, we obtain initially

$$
\begin{aligned}
&\sum_{\substack{l'=0\\ l'\neq 1}}^{2}[[Y^{[l]}(\hat{k})\times\chi^{[1]}(12)]^{[1]}\times Y^{[l']}(\hat{p})]^{[1]}_M G^{l'}(p)\\
&\qquad=\hat{1}\sum_{\substack{l''=0\\ l''\neq 1}}^{2}\int d^3k'\,g_l^1(k')[[Y^{[l]}(\hat{k})\times\chi^{[1]}(12)]^{[1]}\times[Y^{[l'']}(\hat{k}')\times\tilde{\chi}^{[1]}(12)]^{[1]}]^{[0]}\Psi_M^{[1]}(\vec{k}',\vec{p})\,,
\end{aligned}
\tag{16}
$$

which can be recoupled to

$$
G^l(p)=4\pi\sum_{\substack{l'=0\\ l'\neq 1}}^{2}\sum_J\hat{J}\begin{Bmatrix}1&1&J\\ l&l'&1\end{Bmatrix}\int d^3k'g_{l'}^1(k')[[Y^{[l]}(\hat{p})\times Y^{[l']}(\hat{k}')]^{[J]}\times[\tilde{\chi}^{[1]}(12)\times\Psi^{[1]}(\vec{k}',\vec{p})]^{[J]}]^{[0]}\,.
\tag{17}
$$

Similarly for the $F^{J'}_{l'(Jl)}(p)$, we get

$$\sum_{J'=|1-J|}^{1+J}\ \sum_{l'=J-1/2}^{J'+1/2} P^{l+l'}[\mathcal{Y}^{[J]}_{l(1/2)}(\hat{k},2)\times\mathcal{Y}^{[J']}_{l'(1/2)}(\hat{p},1)]^{[1]}_{M}F^{J'}_{l'(Jl)}(p)$$
$$=(-1)^{2J}\hat{J}\int d^3k' h^J_l(k')[\mathcal{Y}^{[J]}_{l(1/2)}(\hat{k},2)\times\tilde{\mathcal{Y}}^{[J]}_{l(1/2)}(\hat{k}',2)]^{[0]}\Psi^{[1]}_{M}(\vec{k}',\vec{p}), \quad (18)$$

which after some Racah algebra becomes

$$F^{J'}_{l'(Jl)}(p)=\frac{4\pi}{\hat{1}}(-1)^{1-J-J'}\int d^3k' h^J_l(k')[[\tilde{\mathcal{Y}}^{[J']}_{l'(1/2)}(\hat{p},1)\times\tilde{\mathcal{Y}}^{[J]}_{l(1/2)}(\hat{k}',2)]^{[1]}\times\Psi^{[1]}(\vec{k}',\vec{p})]^{[0]}. \quad (19)$$

Equations (17) and (19) are the key elements in deriving the coupled equations which must be solved to find the ^{6}Li binding energy and spectator functions.

The detailed form of the nine coupled, homogeneous, integral equations for the spectator functions is obtained by substituting Eq. (15) into Eqs. (17) and (19), respectively. Again, considerable angular-momentum recoupling must be carried out. The results are

$$D_g(K^2,p)G^l(p)=\tfrac{5}{4}(4\pi\hat{1})^2\sum_{\substack{l'=0\\ l'\neq 1}}^{2}\sum_{L}\hat{L}\begin{Bmatrix}1&1&L\\ l&l'&1\end{Bmatrix}$$
$$\times\sum_{J=1/2}^{3/2}\sum_{J'=|1-J|}^{1+J}\sum_{\substack{l_1=J-1/2\\ l_1\leq 1}}^{J+1/2}\sum_{l_2=J'-1/2}^{J'+1/2}P^{l_1+l_2}\Lambda^J_{l_1}\hat{J}\hat{J}'\begin{Bmatrix}l_1&\frac{1}{2}&J\\ l_2&\frac{1}{2}&J'\\ L&1&1\end{Bmatrix}$$
$$\times\int d^3k_{12}\frac{g^1_{l'}(k_{12})h^J_{l_1}(k_{23})}{K^2+k_{12}^2+\frac{3}{8}p^2}[[Y^{[l]}(\hat{p})\times Y^{[l']}(\hat{k}_{12})]^{[L]}$$
$$\times[Y^{[l_1]}(\hat{k}_{23})\times Y^{[l_2]}(\hat{p}_1)]^{[L]}]^{[0]}F^{J'}_{l_2(Jl_1)}(p_1) \quad (20)$$

and

$$D^{lJ}_h(K^2,p)F^{J'}_{l'(Jl)}(p)=(4\pi)^2\hat{J}\hat{J}'(-1)^{3J'}\Bigg((-1)^{1-J}\hat{1}^2\lambda_1\sum_{\substack{l_1,l_2=0\\ l_1,l_2\neq 1}}^{2}\sum_{L}(-1)^L\hat{L}\begin{Bmatrix}l'&\frac{1}{2}&J'\\ l&\frac{1}{2}&J\\ L&1&1\end{Bmatrix}\begin{Bmatrix}l_1&l_2&L\\ 1&1&1\end{Bmatrix}$$
$$\times\int d^3k_{23}\frac{h^J_l(k_{23})g^1_{l_1}(k_{12})}{K^2+\frac{5}{8}k_{23}^2+\frac{3}{5}p^2}[[Y^{[l']}(\hat{p})\times Y^{[l]}(\hat{k}_{23})]^{[L]}$$
$$\times[Y^{[l_1]}(\hat{k}_{12})\times Y^{[l_2]}(\hat{p}_3)]^{[L]}]^{[0]}G^{l_2}(p_3)$$
$$-\frac{5}{8}\sum_{J_1=1/2}^{3/2}\sum_{J_2=|1-J_1|}^{1+J_1}\sum_{\substack{l_1=J_1-1/2\\ l_1\neq 1}}^{J_1+1/2}\sum_{l_2=J_2-1/2}^{J_2+1/2}P^{l_1+l_2}\Lambda^{J_1}_{l_1}\hat{J}_1\hat{J}_2\sum_{L}(-1)^L\hat{L}$$
$$\times\begin{Bmatrix}J'&J&1\\ J_2&J_1&L\end{Bmatrix}\begin{Bmatrix}J'&J_1&L\\ l_1&l'&\frac{1}{2}\end{Bmatrix}\begin{Bmatrix}J&J_2&L\\ l_2&l&\frac{1}{2}\end{Bmatrix}$$
$$\times\int d^3k_{23}\frac{h^J_l(k_{23})h^{J_1}_{l_1}(k_{31})}{K^2+\frac{5}{8}k_{23}^2+\frac{3}{5}p^2}[[Y^{[l']}(\hat{p})\times Y^{[l_1]}(\hat{k}_{31})]^{[L]}$$
$$\times[Y^{[l]}(\hat{k}_{23})\times Y^{[l_2]}(\hat{p}_2)]^{[L]}]^{[0]}F^{J_2}_{l_2(J_1l_1)}(p_2)\Bigg), \quad (21)$$

where

$$D_g(K^2,p)=1-\lambda_1\int d^3k\frac{[g^1_0(k)]^2+[g^1_2(k)]^2}{K^2+k^2+\frac{3}{8}p^2}, \quad (22)$$

$$D^{lJ}_h(K^2,p)=1-\tfrac{5}{8}\Lambda^J_l\int d^3k\ \frac{[h^J_l(k)]^2}{K^2+\frac{5}{8}k^2+\frac{3}{5}p^2}, \quad (23)$$

$$\vec{k}_{23}=-\tfrac{4}{5}\vec{k}_{12}+\tfrac{3}{5}\vec{p}_3,\quad \vec{p}_1=-\vec{k}_{12}-\tfrac{1}{2}\vec{p}_3, \quad (24)$$

and

$$\vec{k}_{31}=-\tfrac{4}{5}\vec{k}_{12}-\tfrac{3}{5}\vec{p}_3,\quad \vec{p}_2=\vec{k}_{12}-\tfrac{1}{2}\vec{p}_3. \quad (25)$$

Observe that the $G^l(p)$ do not couple to themselves, so in principle the nine equations can be reduced to seven. In practice, we simplify Eqs. (20) and (21) further for coding and then iterate all nine equations simultaneously. The details are given in Rai's dissertation.[22]

After the spectator functions are obtained numerically, the unnormalized three-body ^{6}Li wave

function can be constructed by means of Eq. (15). The wave function is normalized in the standard manner, i.e., the normalization constant, N, which multiplies the right-hand side of Eq. (15) is defined as

$$N^{-2} = \frac{1}{3} \sum_{M=-1}^{1} \int d^3k d^3p \Psi_M^{[1]\dagger}(\vec{k}, \vec{p}) \Psi_M^{[1]}(\vec{k}, \vec{p}) \quad (26)$$

$$= \frac{1}{\hat{1}} \int d^3k d^3p [\tilde{\Psi}^{[1]}(\vec{k}, \vec{p}) \times \Psi^{[1]}(\vec{k}, \vec{p})]^{[0]} . \quad (27)$$

IV. BINDING ENERGY RESULTS

Equations (20) and (21) are solved numerically by iteration. The advantage of this method is that it yields the spectator functions as well as the binding energy, so that the ^{6}Li three-body wave function is easily constructed. In the calculations, we use Gegenbauer quadrature for the infinite-range momentum integrals and Gaussian quadrature for the angular integrals. Ten- and six-point quadrature formulas are used, respectively, in the binding-energy results. Increasing the number of points to 16 and 10, respectively, gives an estimate of the numerical accuracy. The binding-energy results are stable to within 1% under such changes. The wave functions used in the next section are obtained with 16- and 10-point quadrature, respectively.

To assure consistency with previous work, we repeat Shanley's binding-energy calculations, i.e., with his α-n interactions and Phillips's parameters for the n-p tensor force. Our results are 3.34 MeV ($P_D = 0\%$), 3.15 MeV (4%), and 3.04 MeV (7%), compared with Shanley's values of 3.35, 3.25, and 3.16 MeV, respectively. These results are compatible with Shanley's estimated errors. Furthermore, CFL also repeated Shanley's calculation for the 7% case and they obtained 3.02 MeV, in excellent agreement with our 3.04 MeV. Thus, our starting points are equivalent.

With confidence in our equations and computer codes, we can now seek answers to the following questions about the three-body model of ^{6}Li: (1) What differences in the ^{6}Li binding energy occur when the $S_{1/2}$ and $P_{1/2}$ α-n interactions are neglected? (2) What effects do slight changes in the fit to the low-energy α-n $S_{1/2}$ phase shift, brought about by a factor of 2 or 3 change in the interaction strength with the range essentially unchanged, have on the binding energy? (3) How dependent is the binding energy on the analytical form of the α-n P-wave form factors, when the form factors give essentially equivalent fits to the low-energy phase shifts? (4) How sensitive is the binding energy to P_D and the analytical form of the n-p $^3S_1$-$^3D_1$ interaction? These questions are answered on the basis of the results in Tables VI and VII.

The significance of the various components of the α-N interaction to the three-body binding energy of ^{6}Li is apparent from Table VI by comparing the top row of the first three columns for any P-wave form factor. The dominant interaction is $P_{3/2}$, followed by the repulsive $S_{1/2}$ and then the $P_{1/2}$. Including the attractive $P_{1/2}$ component with the strongly resonant $P_{3/2}$ increases the binding by ~35%. The binding energy produced by the $P_{1/2}$ and $P_{3/2}$ interactions is reduced by ~40% by introducing the repulsive $S_{1/2}$ interaction. These percentages are somewhat different than the ^{6}He values[1] of ~25% and ≳50%, respectively. This difference is due to the different two-nucleon interactions involved: ^{6}He-$^1S_0$ and ^{6}Li-$^3S_1$. The two-nucleon $^3S_1$ interaction is stronger than the $^1S_0$ in the sense that it supports a bound state. Therefore, the two nucleons in ^{6}Li are expected to be closer together, on the average, than those in ^{6}He. Thus, the $P_{1/2}$ α-n interaction is somewhat more effec-

TABLE VI. ^{6}Li binding energies (MeV).

α-n P-wave form factor	$P_{3/2}$ only	$P_{3/2}, P_{1/2}$ only	$P_{3/2}, P_{1/2}$ $S_{1/2}$-Shanley	$P_{3/2}, P_{1/2}$ $S_{1/2}$-GL-1	$P_{3/2}, P_{1/2}$ $S_{1/2}$-GL-2
A	4.835	6.624	3.727 0%[a] 3.413 4% 3.324 5.5% 3.245 7%	3.812	3.585
B	5.222	6.842	4.446 0% 4.070 4% 3.955 5.5% 3.847 7%	4.497	4.282
C	4.590	6.418	3.692 0% 3.373 4% 3.274 5.5% 3.187 7%	3.750	3.523

[a] Percentages refer to P_D. When unspecified, $P_D = 0\%$.

TABLE VII. ^{6}Li binding energies with CFL α-n interactions.

S-wave α-n interaction [a]	n-p interaction (P_D)	Binding energy (MeV)	Reference
CFL	YY [b] (0%)	3.577	•••
CFL	YY (4%)	3.288	•••
Shanley	YY (4%)	3.623	•••
CFL	ACS-4 (4%)	3.188	3

[a] The P-wave interactions are always CFL.
[b] Reference 18 and the parameters of Table V.

tive in ^{6}Li than ^{6}He and the opposite is true for the $S_{1/2}$.

An illuminating comparison in Table VI involves the top row of the last three columns for each P-wave form factor. All interactions remain unchanged, except the α-n $S_{1/2}$. As pointed out above, the predictions for the low-energy phase shifts for these three $S_{1/2}$ interactions are not very different, but their higher-energy phase shifts are markedly different. Yet, the predicted differences in the ^{6}Li binding energy are only a few percent. This is similar to what GL found for ^{6}He and it is explained by our remarks in Sec. II above.

By scanning down the columns of Table VI, it is apparent that the ^{6}Li binding energy is sensitive to the analytical form of the P-wave α-n form factors. Since the different P-wave interactions fit the low-energy phase shifts equally well (except $P_{3/2}$-Pigeon), the higher-energy behavior of the form factors and off-shell effects are playing a role. There is sensitivity to more than just the asymptotic behavior of the coordinate-space, P-wave, α-n function.

The general effects of the tensor force as P_D is increased can be seen in column three of Table VI. Firstly, the ^{6}Li binding energy decreases as P_D is increased—as expected.[23] Secondly, the reduction of the binding energy is smaller as P_D becomes larger. Thirdly, the maximum decrease in binding energy is ~14% for $P_D = 7\%$. Finally, in Table VII, it is apparent that the ^{6}Li binding energy is not very sensitive to the analytical form chosen to represent the n-p $^3S_1$-$^3D_1$ interaction as long as the low-energy parameters as well represented. This is made clear by comparing the second and fourth lines. ACS-4 is a rank-2 $^3S_1$-$^3D_1$ interaction. The difference in binding is only 3%.

The results in Table VII are useful for contrasting the CFL interactions with the form B interactions of this work. If the first two lines of Table VII are compared with the first two entries, column 3, form B of Table VI, we see that the CFL interactions produce ~20% less binding than our preferred set of α-N interactions. When the CFL-$S_{1/2}$ interaction is replaced by Shanley's, the binding is 11% smaller. This implies that it is not due to just a difference in one interaction, say the $S_{1/2}$, but is probably traceable to all of them. As indicated in Sec. II, the more sophisticated CFL form factors lead to different singularity structure of the α-N amplitudes than that obtained from the simpler choices. It would be of interest to contrast them.[24]

The last point to be made in this section concerns a comparison of the predicted ^{6}Li binding energy with the experimental value of 4.53 MeV [3.697 + 0.834 (Coulomb)]. The preferred set of α-N interactions are form B. The results to be compared with experiment are in column 3 of Table VI. The agreement with experiment becomes progressively worse as P_D increases, beginning with a 2% discrepancy (4.446 MeV) and ending with a 15% discrepancy (3.847 MeV). Compared to ^{6}He where the discrepancy is $\gtrsim$50%, the ^{6}Li results are quite good. In fact, we are encouraged enough to construct the wave functions for $P_D = 0\%$ and 4% in order that they can be examined and tested in processes involving ^{6}Li. Thus, the next section concerns the normalization of these two wave functions.

V. NORMALIZATION

The two wave functions normalized correspond to the 4.446 MeV (0%) and 4.070 MeV (4%) binding energies (P_D), respectively. By contrasting the contributions to the normalization for these two cases, it will be possible to examine the effect of the tensor force on the components of the wave function. Besides this, the normalization gives a hint as to the nature of the three-body ^{6}Li wave functions, e.g., as to what components are most significant.

In order to make the normalization calculation easier and for later applications, we fitted the spectator functions to the forms

$$\varphi_{\text{spec}}(p) = \frac{A + Bp^2 + Cp^4}{1 + ap^2 + bp^4 + cp^6 + dp^8 + ep^{10}} \,. \qquad (28)$$

This allows for threshold and asymptotic behavior as extracted from Eqs. (20) and (21). The parameters are given in Table VIII. The quality of the fits are excellent, especially for $0 \leq p \lesssim 8$ fm^{-1} where the main contributions to most integrals in applications occur.[22]

When the expression for the normalization integral, Eqs. (26) and (27), is worked out, there are 49 different integrals—16 of which have two pieces when $P_D \neq 0$. There are integrals containing $(G^0)^2$,

TABLE VIII. Spectator-function parameters. [The first and second entries for each parameter correspond to the 4.446 MeV (0%) and 4.070 MeV (4%) wave functions, respectively. The notation is $1.883(-8) = 1.883 \times 10^{-8}$.]

	$G^0(p)$	$G^2(p)$	$F^{1/2}_{0[(1/2)0]}$	$F^{3/2}_{2[(1/2)0]}$	$F^{1/2}_{1[(1/2)1]}$	$F^{3/2}_{1[(1/2)1]}$	$F^{1/2}_{1[(3/2)1]}$	$F^{3/2}_{1[(3/2)1]}$	$F^{5/2}_{3[(3/2)1]}$
A	1.003	1.882(−8)	3.551(−2)	6.451(−10)	−9.087(−4)	2.676(−3)	−2.761(−3)	4.535(−3)	1.159(−7)
	1.004	−4.198(−8)	2.445(−2)	−4.000(−9)	−4.883(−4)	1.911(−3)	−2.033(−3)	3.028(−3)	2.193(−6)
B	2.107(−1)	8.861(−2)	−9.002(−3)	−2.198(−4)	−7.407(−1)	2.179	−2.310	3.782	−3.591(−4)
	−1.132(−1)	8.612(−2)	−3.740(−3)	−1.663(−2)	−4.258(−1)	1.564	−1.764	2.832	−6.645(−3)
C	−8.302(−1)	−1.385(−5)	9.259(−4)	1.254(−3)	7.142(−2)	9.200(−2)	1.791(−1)	2.529(−2)	1.973(−2)
	−6.839(−1)	−3.816(−3)	4.377(−4)	3.469(−3)	1.047(−2)	−6.016(−2)	1.702(−1)	1.403(+1)	3.353(−4)
a	8.395	7.777	3.409	6.113	4.644(+1)	4.616(+1)	5.088(+1)	5.019(+1)	−4.450(+1)
	9.631	8.305	4.080	2.902	5.499(+1)	4.714(+1)	5.526(+1)	6.843(+1)	−2.027
b	1.095(+1)	5.538	−1.336	6.756	1.661(+1)	1.229(+1)	5.651(+1)	4.697(+1)	−1.281(+1)
	1.104(+1)	1.874	−2.696(−1)	3.581(−1)	8.350	1.785(+1)	4.799(+1)	2.465(+2)	5.634
c	6.661	1.032	1.001	8.169	4.421(+1)	4.416(+1)	5.896(+1)	5.095(+1)	−6.781(+1)
	7.464	1.777	1.157	9.308(−1)	2.346(+2)	4.584(+1)	8.022(+1)	6.868(+2)	−1.892
d	3.967(−1)	1.092(−1)	−2.485(−2)	−4.113(−1)	2.057(−1)	−9.848(−1)	1.956	2.680	2.521
	1.086(−1)	−6.980(−2)	−9.384(−2)	−8.514(−2)	−1.901(+1)	−9.088(−1)	6.513(−2)	1.088(+2)	2.949(−1)
e	2.195(−1)	2.002(−3)	8.913(−4)	1.397(−2)	2.162(−1)	2.861(−1)	5.149(−1)	2.905(−1)	−4.020(−1)
	2.835(−1)	4.140(−3)	2.518(−3)	7.168(−3)	5.254(−1)	4.321(−1)	7.912(−1)	5.490(+1)	−5.937(−3)

$G^2 F^{J'}_{l'(Jl)}$, $(F^{J'}_{l'(Jl)})^2$, plus others. From these, we list the contributions $\gtrsim 1\%$ to the normalization and the normalization constants in Table IX. There are several features of the results which should be noted. A quick glance indicates that the dominate components in the $P_D = 0\%$ wave function remain dominant in the 4% one and with essentially the same percentages. The dominant components are the ones with the spectator functions G^0, $F^{3/2}_{1[(3/2)1]}$, and $F^{1/2}_{0[(1/2)0]}$. The G^0 and $F^{3/2}_{1[(3/2)1]}$ components contribute ~50% and 26–30% alone, respectively, while their interference contributes another 16–

TABLE IX. Normalization contributions (contributions $\gtrsim 1\%$ only).

Component	4.446 MeV (0%) $N = 2.714$ fm^2 (%)	4.071 MeV (4%) $N = 4.071$ fm^2 (%)
$G^0 \times G^0$	48.7	47.8 + 3.7 [a] = 51.5
$G^0 \times F^{1/2}_{0[(1/2)0]}$	−24.2	−23.0
$G^0 \times F^{3/2}_{1[(1/2)1]}$	5.5	5.5 + 0.5 = 6.0
$G^0 \times F^{1/2}_{1[(3/2)1]}$	9.2	9.6 + 0.8 = 10.4
$G^0 \times F^{3/2}_{1[(3/2)1]}$	19.0	17.2 − 1.2 = 16.0
$F^{1/2}_{0[(1/2)0]} \times F^{1/2}_{0[(1/2)0]}$	3.5	3.2
$F^{3/2}_{1[(1/2)1]} \times F^{3/2}_{1[(1/2)1]}$	1.5	1.6
$F^{1/2}_{1[(3/2)1]} \times F^{1/2}_{1[(3/2)1]}$	5.2	6.1
$F^{3/2}_{1[(3/2)1]} \times F^{3/2}_{1[(3/2)1]}$	17.4	15.5
$F^{1/2}_{0[(1/2)0]} \times F^{1/2}_{0[(1/2)0]}$ (cr) [b]	−3.3	−3.0
$F^{3/2}_{1[(3/2)1]} \times F^{3/2}_{1[(3/2)1]}$ (cr)	12.1	10.4
$F^{1/2}_{1[(3/2)1]} \times F^{3/2}_{1[(1/2)1]}$	4.1	4.5
	98.7	99.2

[a] Due to presence of tensor force.
[b] (cr) means cross term, i.e., the (2,31) term with the (1,23) term.

19%. The interference of G^0 with $F^{1/2}_{0[(1/2)0]}$ subtracts from the normalization due to the fact that the $S_{1/2}$ α-n interaction is repulsive. As pointed out by GL, the same three terms dominate for ^{6}He,[25] but the percentages are interestingly different except for the $G^0 \times F^{1/2}$ interference. In ^{6}He, the G^0 and $F^{3/2}_1$ components contribute ~23% and ~62% alone, respectively, and ~21% through their interference. The shift from $F^{3/2}_1$ dominating over G^0 in ^{6}He to G^0 dominating over $F^{3/2}_{1[(3/2)1]}$ in ^{6}Li is due to the fact that the $^1S_0$ two-nucleon interaction has no bound state and the $^3S_1$ does. For ^{6}Li, the configuration of the pair of nucleons interacting in the $^3S_1$ state while the α particle interacts relative to their center of mass in an s wave is favored over the configuration of an α-N pair interacting in the $P_{3/2}$ state while the remaining N couples relative to their center of mass in a $P_{3/2}$ wave. The opposite is true for ^{6}He with $^3S_1$ replaced by $^1S_0$. These two configurations are not mutually exclusive, however, due to their interference. Finally, we note the contribution to the normalization in the $P_D = 4\%$ case from the terms due directly to the tensor force is only 3.8%.

VI. CONCLUSIONS

The following list summarizes our major conclusions about the three-body separable-potential model of the ^{6}Li ground state:

(1) The ^{6}Li binding energy decreases as P_D is increased, but more slowly the larger P_D, with an ~14% decrease in binding energy for $P_D = 7\%$—independent of the α-N interaction set. Furthermore, the ^{6}Li binding energy is not very sensitive to the analytical form chosen to represent the $^3S_1$-$^3D_1$ interaction as long as the low-energy n-p parameters are well represented.

(2) The $P_{3/2}$ component of the α-N interaction is primarily responsible for the binding. When the $P_{1/2}$ component is included with the $P_{3/2}$, the binding energy increases by ~35%. The repulsive $S_{1/2}$ component reduces the latter binding energy by ~40%.

(3) The three-body model of ^{6}Li is not sensitive to the higher-energy ($\gtrsim$30 MeV) part of the $S_{1/2}$ α-N interaction, but only to the fact that the $S_{1/2}$ interaction is repulsive and has a given behavior at low energies.

(4) The ^{6}Li binding energy is sensitive to the analytical form of the P-wave α-n form factors which produce comparable fits to the low-energy phase shifts.

(5) The dominant wave function components, in decreasing significance, are those with the spectator functions G^0, $F^{3/2}_{1[(3/2)1]}$, and $F^{1/2}_{0[(1/2)0]}$, respectively (see the previous section.)

(6) Two calculated values of the ^{6}Li binding energy, from the preferred set of α-N interactions with $P_D = 0\%$ and 4% for the n-p interaction, are 4.446 MeV (0%) and 4.070 MeV (4%) compared with the Coulomb-adjusted experimental value of 4.53 MeV.

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[14]Only *repulsive* s-wave separable potentials of the type considered yield amplitudes with poles in the third and fourth quadrants. Attractive potentials of this type give amplitudes with purely imaginary poles. In fact, one can fit the n-α $S_{1/2}$ phase shift with an attractive potential, but it will possess a bound state, whereas the n-α system is unbound. Nevertheless, one can project this bound state out of the two-body amplitude in three-body calculations, but the *projected* amplitude will not reproduce the experimental phase shift. This is not in the spirit of our approach. Furthermore, the $S_{1/2}$ N-α interaction taken as repulsive has the physical interpretation of being due to the Pauli exclusion principle.

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[24]Two calculations of the ^{6}He binding energy were made with the CFL interactions and the $^1S_0$ n-p best fit interaction of GL. The first used the complete set of CFL α-N interactions and *no* bound state was found. The second replaced their $S_{1/2}$ with Shanley's and the binding energy is 0.0569 MeV. Our result with the complete set is interesting, since CFL quote a result of 0.008 MeV with the complete set and a $^1S_0$ interaction which has the same low-energy parameters as ours, but also includes repulsion, i.e., it is rank 2.

[25]Since ^{6}He is 0^+, there is only one G^l, i.e., $l=0$, and three $F^{J'}_{l'(Jl)}$ with $J=J'$ and $l=l'$.