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$^{12}C(\gamma,\pi^-)^{12}N_{g.s.}$ near threshold*

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We calculate the total cross section for the reaction $^{12}C(\gamma,\pi^-)^{12}N_{g.s.}$ and study its energy dependence in the threshold region. Contrary to previous theoretical analyses, we find agreement between theory and experiment using a $\vec{\sigma}\cdot\vec{\epsilon}$ interaction. Our results depend strongly on the inclusion of all pion partial waves and a correct treatment of the isospin dependent terms in the pion-nucleus optical potential.

[NUCLEAR REACTIONS $^{12}C(\gamma,\pi^-)^{12}N_{g.s.}$; photon lab. energy 0–10 MeV above threshold; realistic pion optical potential; shell model wave functions; $\vec{\sigma}\cdot\vec{\epsilon}$ interaction. Calculated total cross section.]

I. INTRODUCTION

Recently, the total cross section σ_T for the reaction $^{12}C(\gamma,\pi^-)^{12}N_{g.s.}$ has been measured in the threshold region, i.e., for photon laboratory energy ranging from 0 to 12 MeV above threshold.[1] A comparison with previous theory shows a marked discrepancy: σ_T rises significantly faster with photon energy than calculations predict. The discrepancy is illustrated in Fig. 1. The theoretical results of Koch (K)[2] and Nagl and Uberall (1, 2)[3] are shown along with the experimental fit of Bernstein *et al.* (B)[1] and its associated uncertainty (shaded area).

To analyze this discrepancy, it is useful to observe that there are three principal ingredients which enter the theoretical analysis:

(1) the pion-nucleus optical potential $V_{\pi\eta}$;
(2) the nuclear wave functions ψ_η;
(3) the elementary interaction operator $H_{\gamma\pi}$ which describes absorption of the photon and emission of the pion.

All of these ingredients appear to be reliably determined from independent analyses: since we are in the threshold region it is reasonable to use $V_{\pi\eta}$ fitted to pionic atom data[4]; the ^{12}C, $^{12}N_{g.s.}$ wave functions are constrained by electron scattering data[5]; $H_{\gamma\pi}$ is rather well fixed by data from photoproduction of pions from nucleons and the associated multipole analyses.[6] In view of the well determined nature of this input information, it is rather surprising that any discrepancy exists in the threshold region. For this reason we have carried out a straightforward, independent theoretical analysis to determine the origin of the discrepancy. We use a shell model framework so that our work may be viewed as a checkout and extension of the Koch-Donnelly (KD) calculation.[7] We build our analysis in the following manner:

(1) We check the original KD calculation.
(2) We ensure that the isospin dependent part of $V_{\pi\eta}$ is correctly treated.
(3) We put in nuclear recoil, omit an unjustified form factor, and use a more accurate value for the πNN coupling constant.
(4) We include all pion partial waves.

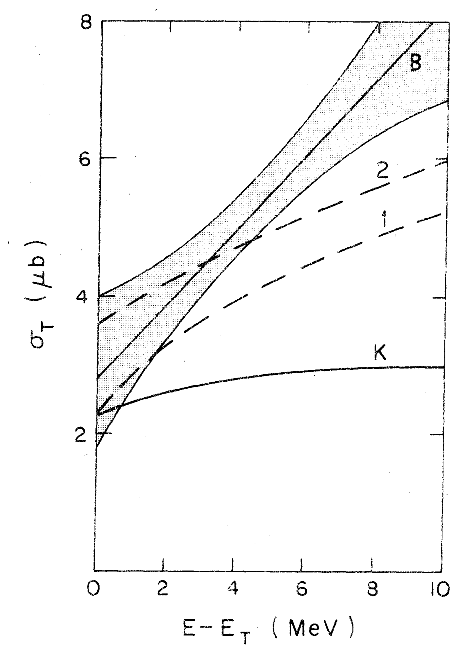

FIG. 1. The total cross section σ_T for $^{12}C(\gamma,\pi^-)^{12}N_{g.s.}$. E is the photon lab energy and E_T the threshold photon lab energy. The labeling is: B, experimental results of Berstein *et al.* (Ref. 1); 1, 2, theoretical results of Nagl and Uberall (Ref. 3); K theoretical results of Koch (Ref. 2).

(5) We improve the soft pion $\vec{\sigma}\cdot\vec{\epsilon}\ H_{\gamma\pi}$ interaction by altering its strength so that it gives a better fit to the $\gamma n \to \pi^- p$ data.

We find that with each improvement in the analysis, our results move closer to the experimental data. Our final results are consistent with experiment. Thus we find that if one uses $V_{\pi\eta}$ fitted to pionic atom data, nuclear shell model wave functions consistent with electron scattering data, and the simplest $H_{\gamma\pi}$ interaction consistent with the low energy $\gamma n \to \pi^- p$ data, then theory and experiment for $^{12}C(\gamma,\pi^-)^{12}N_{g.s.}$ are in accord.

This paper is organized as follows. In Sec. II we give a brief summary of the formulas we use to compute σ_T. Section III discusses the calculation of the pion distorted waves. Section IV details and discusses the results of our first set of calculations. Section V contains an analysis of $H_{\gamma\pi}$ and shows how improving $H_{\gamma\pi}$ affects σ_T. Section VI provides a summary and conclusions.

II. CROSS-SECTION FORMULAS

The derivation proceeds in a straightforward fashion. Suppose we know the Hamiltonian which induces the process $\gamma N \to \pi N$ and we call it $H_{\gamma\pi}(j)$ where j is the nucleon label. Then, if we neglect exchange current effects, the Hamiltonian which induces pion photoproduction in a nucleus will be

$$H_{\gamma\pi}=\sum_{j=1}^{A} H_{\gamma\pi}(j), \tag{2.1}$$

where the sum extends over the A nucleons in the nucleus. Choosing the center-of-mass system we can write down the expression for the cross section for $\eta(\gamma,\pi^-)\eta'$, where η,η' are the initial and final nuclear states:

$$\left(\frac{d\sigma}{d\Omega}\right)_{\text{c.m.}}=\frac{(2\pi)^4 qE_\pi E_\eta E_{\eta'}}{(k+E_\eta)(E_\pi+E_{\eta'})}\,|\langle\eta',\pi|H_{\gamma\pi}|\eta,\gamma\rangle|^2, \tag{2.2}$$

where q is the pion momentum; E_π the pion energy; k the photon momentum; $E_\eta, E_{\eta'}$ the initial and final nucleus energy, respectively. To obtain σ_T, the total cross section, we integrate over angles, sum over the final nuclear spin states, and average over the initial nuclear spin and photon polarization states:

$$\sigma_T=\frac{1}{2J_i+1}\sum_{M_i,M_f}\frac{1}{2}\sum_{\lambda}\frac{(2\pi)^4 qE_\pi E_\eta E_{\eta'}}{(k+E_\eta)(E_\pi+E_{\eta'})}\times\int d\Omega_{\hat{q}}\,|\langle\eta',\pi|H_{\gamma\pi}|\eta,\gamma\rangle|^2, \tag{2.3}$$

where J_i is the initial nucleus spin; M_i, M_f are the initial and final nuclear spin z components, respectively; λ denotes the photon polarization state.

Now, we take the photoproduction operator to be of the following form:

$$H_{\gamma\pi}(j)=\frac{iC_0}{2(E_\pi k)^{1/2}}\vec{\sigma}_j\cdot\vec{\epsilon}_\lambda\tau_j^+\,\delta(\vec{x}-\vec{x}_j), \tag{2.4}$$

where C_0 is a constant; $\vec{\sigma}_j$ is the spin operator of the jth nucleon; $\vec{\epsilon}_\lambda$ is the polarization vector of the photon; τ_j^+ is the isospin raising operator acting on the jth nucleon; $\vec{x}_j$ is the position coordinate for the jth nucleon.

With this choice we now proceed to evaluate the matrix element $\langle\eta',\pi|H_{\gamma\pi}|\eta,\gamma\rangle$. We write

$$\langle\eta',\pi|H_{\gamma\pi}|\eta,\gamma\rangle=\frac{iC_0\xi}{2[E_\pi k(2\pi)^3]^{1/2}}, \tag{2.5}$$

where

$$\xi=\langle\eta'|\int d\vec{x}[\Phi^{(-)}(\vec{x},\vec{q})]^*\sum_j \vec{\sigma}_j\cdot\vec{\epsilon}_\lambda\tau_j^+\,\delta(\vec{x}-\vec{x}_j)e^{i\vec{k}\cdot\vec{x}}|\eta\rangle$$

and we use Goldberger and Watson conventions.[8] $\Phi^{(-)}(\vec{x},\vec{q})$ is the pion wave function with the appropriate boundary conditions. Now we make use of the relation $[\Phi^{(-)}(\vec{x},\vec{q})]^*=\Phi^{(+)}(\vec{x},-\vec{q})$ and employ the following partial wave decompositions[3]:

$$\Phi^{(+)}(\vec{x},\vec{q})=\frac{4\pi}{(2\pi)^{3/2}}\sum_{l=0}^{\infty}\sum_{m=-l}^{l} i^l\phi_l(qr)Y_{lm}(\hat{x})Y^*_{lm}(\hat{q}),$$
$$e^{ikz}=\sum_{l'=0}^{\infty} i^{l'}[4\pi(2l'+1)]^{1/2}j_{l'}(kr)Y_{l'0}(\theta), \tag{2.6}$$

where we choose the z axis in the direction of the incoming photon beam. Thus, we obtain

$$e^{ikz}\Phi^{(+)}(\vec{x},-\vec{q})=\frac{4\pi}{(2\pi)^{3/2}}\sum_{ll'Lm} i^{l+l'}(2l'+1)(2l+1)^{1/2}(2L+1)^{1/2}j_{l'}(kr)\phi_l(qr)Y^*_{lm}(-\hat{q})Y_{Lm}(\hat{x})(-1)^m \times\begin{pmatrix} l & l' & L\\ m & 0 & -m\end{pmatrix}\begin{pmatrix} l & l' & L\\ 0 & 0 & 0\end{pmatrix}, \tag{2.7}$$

where we have used the addition theorem for spherical harmonics.[9] Next, we note that[9]

$$\vec{e}_\lambda Y_{Lm} = \sum_{JM_J} \langle JM_J \,|\, L1m\lambda\rangle \vec{Y}^{M_J}_{JL1} \tag{2.8}$$

so that

$$\xi = \sum_{lm} C_{lm}\,\chi_{lm}(-\vec{q}), \tag{2.9}$$

where

$$C_{lm} = \frac{4\pi}{(2\pi)^3} \sum_{l'LJM_J} i^{l+l'}(2l'+1)(2l+1)^{1/2}(2L+1)^{1/2}(-1)^m \begin{pmatrix} l & l' & L \\ m & 0 & -m \end{pmatrix} \begin{pmatrix} l & l' & L \\ 0 & 0 & 0 \end{pmatrix}$$

$$\times \langle JM_J \,|\, L1m\lambda\rangle \langle J_f M_f \,|\, T^{Ll l'}_{JM_J} \,|\, J_i M_i\rangle \tag{2.10}$$

and $T^{Lll'}_{JM_J}$ is a one body operator given by

$$T^{Lll'}_{JM_J} = \sum_j \int d\vec{x}\,\vec{\sigma}_j \cdot \tau_j^+ \delta(\vec{x} - \vec{x}_j)\, j_{l'}(kr)\phi_l(qr) \vec{Y}^{M_J}_{JL1}(\hat{x}). \tag{2.11}$$

Note that J_f is the spin of the final nucleus.

At this point, we can make contact with the KD formulation[7] by making the following approximations: (i) neglect nuclear recoil; (ii) include only s-wave pions; (iii) choose $C_0 = \sqrt{2}\, ef/m_\pi$ where e is the electron charge,[10] f the πNN coupling constant, m_π the pion mass.

With the above approximations and writing the s-wave pion wave function as $(1/2\pi)^{3/2}\phi_\pi(\vec{x})$, Eq. (2.3) may be written as

$$\sigma_T^{\text{KD}} = \frac{1}{4\pi}\frac{q}{k}\,|M|^2,$$

where

$$|M|^2 = \frac{1}{2J_i+1} \sum_{M_i, M_f} \frac{1}{2}\sum_\lambda \left(\frac{\sqrt{2}\,ef}{m_\pi}\right)^2 \left| \langle \eta' | \int d\vec{x}\,\vec{\epsilon}_\lambda \cdot \vec{\mathcal{G}}^{\,5}_{+}(\vec{x}) \phi_\pi^*(\vec{x}) e^{i\vec{k}\cdot\vec{x}} | \eta \rangle \right|^2 \tag{2.12}$$

and

$$\vec{\mathcal{G}}^{\,5}_{+}(\vec{x}) = \sum_j \vec{\sigma}_j \tau_j^+ \delta(\vec{x} - \vec{x}_j).$$

These are precisely the KD expressions.[7]

Having made this connection, we can return to the relations (2.9)–(2.11) and observe that to calculate σ_T, we need both nuclear and pion wave functions. The pion wave functions are discussed in the next section and since we are using *exactly* the same shell model wave functions as KD, we refer the reader to their paper for details.[7] Suffice it to say that the harmonic oscillator parameter used is $b = 1.77$ fm and the amplitude for the $^{12}\text{N}_{\text{g.s.}}$ $(J^\pi T = 1^+1)$ $(1p_{1/2})(1p_{3/2})^{-1}$ state was reduced by the customary factor of 2.25 as determined by O'Connell, Donnelly, and Walecka[5] to ensure consistency with electron scattering to the 1^+1 15.11 MeV state in ^{12}C.

III. PION DISTORTED WAVES

The photoproduction cross section depends on the wave function of the pion. The pion experiences multiple scattering from the constituent nucleons and is also subject to direct absorption. To include these effects, the pion distorted waves are calculated using an optical potential $V_{\pi\eta}$. We follow standard practice[11] and solve an approximate Klein-Gordon equation for the pion wave function Φ:

$$(E - V_C - V_{\pi\eta})^2\Phi \simeq [E^2 - 2E(V_C + V_{\pi\eta}) + V_C{}^2]\Phi$$

$$= (-\vec{\nabla}^2 + m_\pi{}^2)\Phi, \tag{3.1}$$

$$\text{i.e., } (\nabla^2 + q^2)\Phi = [2E(V_C + V_{\pi\eta}) - V_C{}^2]\Phi,$$

where V_C is the pion-nucleus Coulomb potential which includes a finite nucleus size effect for a uniform charge distribution of radius $R_C = 3.125$ fm. The terms $2V_C V_{\pi\eta}$ and $V_{\pi\eta}{}^2$ have been dropped as is customary[11] and we have set $E^2 - m_\pi{}^2 = q^2$ with $\hbar = c = 1$.

The pion-nucleus optical potential $V_{\pi\eta}$ is taken to be of the form[4,12]

$$V_{\pi\eta}=-\frac{2\pi}{E_\pi}[p_1 b_0\rho+p_1 b_1(\rho_n-\rho_p)+ip_2\,\mathrm{Im}B_0\rho^2-\overleftarrow{\nabla}\cdot\alpha\overrightarrow{\nabla}],$$

$$\alpha=\beta\left(1+\frac{4\pi\xi'}{3}\beta\right)^{-1}, \qquad (3.2)$$

$$\beta=p_1^{-1}c_0\rho+p_1^{-1}c_1(\rho_n-\rho_p)+ip_2^{-1}\,\mathrm{Im}C_0\rho^2,$$

$$p_1=1+\frac{m_\pi}{m};\quad p_2=1+\frac{m_\pi}{2m};\quad m=\text{nucleon mass},$$

where $\rho=\rho_n+\rho_p$ and ρ_n, ρ_p denote the neutron and proton densities normalized to N and Z, respectively. This form is for π^- mesons; for π^+ one switches $\rho_n \leftrightarrow \rho_p$. The Ericson-Ericson effect[13] is included in the α term.

To solve (3.1) with the above potential, we have used the code PIRK[11] (kindly provided by R. A. Eisenstein). Our only modifications have been to include the Ericson-Ericson effect, to use an improved Coulomb routine (also provided by R. A. Eisenstein), and to correct ρ for finite nucleon size effects. The improved Coulomb routine permits us to reliably calculate very low energy pion photoproduction, corresponding to a large Sommerfeld parameter.

The parameters in (3.2) are related to the basic pion-nucleon amplitudes and the amplitudes for the process $\pi NN\to NN$. However, it is known that the theoretical values for these parameters do not agree completely with the values extracted from fits to pionic atom data. Here we choose to use the values[4]:

$$b_0=-0.03\lambda\!\!\!^-_\pi,\qquad c_0=0.22(\lambda\!\!\!^-_\pi)^3,$$

$$b_1=-0.08\lambda\!\!\!^-_\pi,\qquad c_1=0.18(\lambda\!\!\!^-_\pi)^3, \qquad (3.3)$$

$$\mathrm{Im}B_0=0.04(\lambda\!\!\!^-_\pi)^4,\qquad \mathrm{Im}C_0=0.08(\lambda\!\!\!^-_\pi)^6,$$

where $\lambda\!\!\!^-_\pi$ is the pion Compton wavelength. It is reasonable to expect that these values are close to those at slightly positive energies.

Several aspects of $V_{\pi\eta}$ are noteworthy. Firstly, the $b_0\rho$ term is repulsive. This term will, therefore, reduce the pion wave function near the nucleus and significantly decrease the photoproduction cross section relative to the result obtained with only Coulomb distortions. Secondly, the isospin term $b_1(\rho_n-\rho_p)$ which arises from the isospin dependence of the basic pion-nucleon interaction, is sizable and tends to cancel the $b_0\rho$ term for the reaction $^{12}C(\gamma,\pi^-)^{12}N_{g.s.}$. [Note that isospin symmetry implies an identical cancellation for the reaction $^{12}C(\gamma,\pi^+)^{12}B_{g.s.}$.] To a lesser degree a similar effect occurs for the $c_0\rho$ and $c_1(\rho_n-\rho_p)$ terms.

Another interesting aspect of $V_{\pi\eta}$ is the simple quadratic absorption terms, which also tend to reduce the photoproduction cross section. We shall come back to the sensitivity of the photoproduction cross section to the various parameters in $V_{\pi\eta}$ later on. Finally, we note that we use a Gaussian matter distribution with range 1.64 fm (i.e., $c=1.64$ fm in NDEN=1 option of PIRK, see Ref. 11).

IV. RESULTS OF CALCULATIONS WITH SIMPLE $H_{\gamma\pi}$

Having specified the pion waves, the nuclear wave functions, and the form of the photoproduction operator, we are now in a position to carry out the calculations indicated in the Introduction. We set $C_0=\sqrt{2}\,ef/m_\pi$ here and discuss this choice in the next section.

Our first task is to check the KD calculation and it is important to note that KD used a rather large πNN coupling: $f^2/4\pi=0.088$. So, to check KD, we use this value, neglect nuclear recoil, include only s-wave pions, turn off the Ericson-Ericson effect (i.e., set $\xi'=0$), and put in the KD "finite nucleon size form factor" F_{SN} where[7]

$$F_{SN}=\left(\frac{\Lambda^2}{\Lambda^2+k^2}\right)^2,\qquad \Lambda=855\ \text{MeV}. \qquad (4.1)$$

Then, we find that

$$\sigma_T^{KD}=\frac{q}{4\pi k}\left(\frac{ef}{m_\pi}\right)^2 F_{c.m.}{}^2(k)F_{SN}{}^2(k)\sum_{M_f,\lambda}|C_{00}(\lambda)|^2, \qquad (4.2)$$

where $F_{c.m.}(k)=\exp(k^2b^2/4A)$ is the usual correction for the shell model wave functions' lack of translational invariance.[14] The results for σ_T are shown in Fig. 2 (curve 1) along with the original KD result sults (labeled KD).[7] It is clear that we disagree. (We note that when $V_{\pi\eta}$ is turned off, we agree exactly with KD for both plane waves and Coulomb distorted pions.) The disagreement proves to be of simple origin. It appears that the KD calculation employed a pion optical potential which had the wrong sign for the isospin dependent part of the potential, i.e., it appears that KD actually used

$$b_1=+0.08\lambda\!\!\!^-_\pi,\qquad c_1=-0.18(\lambda\!\!\!^-_\pi)^3. \qquad (4.3)$$

When we do the calculation with the above incorrect sign for the b_1, c_1 terms, we obtain exact agreement with KD (curve 1). We have extensively checked our code and also confirmed that this is not a chance result by using our code with the correct signs for b_1, c_1 to calculate $^{12}C(\gamma,\pi^+)^{12}B_{g.s.}$, where we obtain exact agreement with KD. Thus, apparently, KD have the isospin terms in with the wrong sign for $^{12}C(\gamma,\pi^-)^{12}N_{g.s.}$, and the correct sign for $^{12}C(\gamma,\pi^+)^{12}B_{g.s.}$. Although algebraically

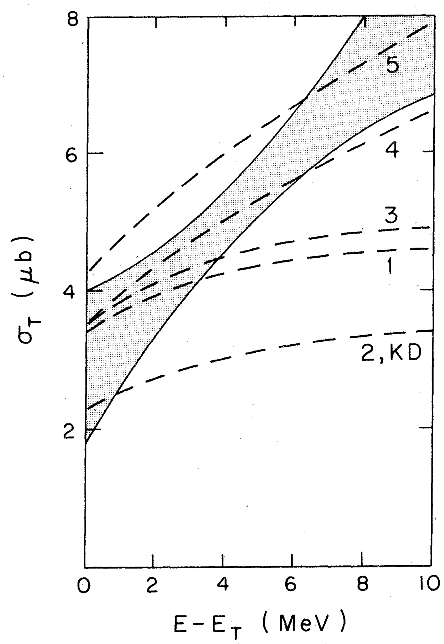

FIG. 2. The total cross section σ_T for $^{12}C(\gamma,\pi^-)^{12}N_{g.s.}$. E is the photon lab energy and E_T the threshold photon lab energy. The shaded area indicates the experimental results of Berstein *et al.* (Ref. 1). The dashed curves are our theoretical results. See text for explanation of labeling.

trivial, this error does have a significant effect on σ_T for $^{12}C(\gamma,\pi^-)^{12}N_{g.s.}$, as our results show. Already it is clear that the theoretical result has moved much closer to the experimental numbers. [Note that the curve K in Fig. 1 is not the same as the curve KD in Fig. 2. Curve K corresponds to some subsequent modifications made by Koch to the original KD calculations.[1,2]]

Next, we put in nuclear recoil, i.e., we calculate the kinematics using

$$k+(k^2+m_C{}^2)^{1/2}=(q^2+m_\pi{}^2)^{1/2}+(q^2+m_N{}^2)^{1/2}, \quad (4.4)$$

where m_C is the mass of $^{12}C_{g.s.}$, m_N is the mass of $^{12}N_{g.s.}$, and we obtain for the total cross section

$$\sigma_T=\frac{q}{4\pi k}\left(\frac{ef}{m_\pi}\right)^2 F_{c.m.}{}^2(k)\frac{E_N E_C}{(E_\pi+E_N)(E_C+k)} \times \sum_{M_f,\lambda} |C_{00}(\lambda)|^2 \quad (4.5)$$

$$E_N=(q^2+m_N{}^2)^{1/2}, \quad E_C=(k^2+m_C{}^2)^{1/2}.$$

Note that we have dropped the form factor $F_{SN}(k)$. We do this for the following reasons. First of all, this form factor is appropriate to electron scattering, not to the process γ,π. The γNN vertex for on-shell nucleons has a Lorentz invariant form factor F, commonly represented by the form[15]

$$F=\left(\frac{\Lambda_1{}^2}{\Lambda_1{}^2+k_\mu k_\mu}\right)^2, \quad (4.6)$$

where k_μ is the photon four-momentum and Λ_1 is a mass. From elastic electron-nucleon scattering experiments, where the photon is virtual and carries no energy, $\Lambda_1=855$ MeV$/c^2$ gives a good fit to the data[15] so that

$$F=\left(\frac{\Lambda_1{}^2}{\Lambda_1{}^2+\vec{k}^2}\right)^2 \quad (4.7)$$

with $\Lambda_1=855$ MeV$/c^2$. However, for the present case of real photons $k_\mu k_\mu=0$ and hence $F=1$. Thus the F_{SN} form factor used by KD is inappropriate. In addition, although it is clear that in a consistent approach, form factor effects will be present in $H_{\gamma\pi}$, such form factors should be very slowly varying functions of the kinematic variables for our region of interest and so are encompassed by a constant strength $\vec{\sigma}\cdot\vec{\epsilon}\, H_{\gamma\pi}$ model. In other words, in our model such effects are included in the constant C_0. (We consider the strength for C_0 in the next section.)

It is useful to note that omitting F_{SN} increases σ_T by about 14%. At this point, we also replace the KD value for the πNN coupling by the more realistic value[16] given by $f^2/4\pi=0.08$. This reduces σ_T by about 9%. The results obtained when the F_{SN} form factor is omitted, nuclear recoil is included, and the correct value for f is used, are shown in Fig. 2 (curve 3). Again the curve has moved closer to the experimental data. When we turn on all pion partial waves (curve 4), it is clear that we already have almost complete agreement between theory and experiment.

It is useful at this point to consider the sensitivity of σ_T to the various pieces of the pion optical potential $V_{\pi\eta}$. We find that if we neglect all terms in $V_{\pi\eta}$ except b_0, b_1 then the results lie within 10% of curve 4. It is clear then that the b_0, b_1 terms dominate in the energy range considered here. We note in passing that the Ericson-Ericson effect (with $\xi'=1$) produces less than a 1% increase in σ_T.

To complete this section, we note that we have used reasonable nuclear wave functions and a realistic pion optical potential; however, it is possible to improve the $H_{\gamma\pi}$ interaction in a simple fashion. We examine this in the next section.

V. IMPROVING $H_{\gamma\pi}$ AND ITS EFFECT ON σ_T

So far we have employed the soft pion $H_{\gamma\pi}$, as did KD. It is natural to ask how accurate this is, i.e., do we have to use a more complicated form to obtain reliable results for $^{12}C(\gamma,\pi^-)^{12}N_{g.s.}$? [For example, Nagl and Uberall[3] (NU) put in the complete pion photoproduction operator.] To examine the

TABLE I. Experimental values for $|M|^2$ at threshold [see Eq. (5.1) of text].

$\|M\|^2$	Year	Reference
20.6 ± 0.8	1969, 1975	17
21.30 ± 0.85	1973	18
19.73 ± 1.74	1965	19

need for terms beyond the $\vec{\sigma}\cdot\vec{\epsilon}$ form, we have compared the prediction of the $\vec{\sigma}\cdot\vec{\epsilon}$ $H_{\gamma\pi}$ with data for the reaction $\gamma n\to\pi^-p$. It is useful to write for the $\gamma n\to\pi^-p$ reaction

$$\frac{d\sigma}{d\Omega}=\frac{q}{k}\,|M|^2 \tag{5.1}$$

so that at threshold the $\vec{\sigma}\cdot\vec{\epsilon}$ result is

$$|M|^2=\left[\frac{mC_0}{4\pi(m+m_\pi)}\right]^2. \tag{5.2}$$

In the soft pion limit used by KD, $C_0=\sqrt{2}\,ef/m_\pi$ (with $f^2/4\pi=0.08$), we find that $|M|^2=17.7$ μb/s. This is to be compared with the experimental results[17-19] shown in Table I. It is clear that this choice of C_0 is too small at threshold. Thus a simple way to improve $H_{\gamma\pi}$ is to adjust C_0 to reproduce the experimental value of $|M|^2$, which we take to be the average of the results in Refs. 17 and 18 (note that these are the most recent results and that the experimental error is of order 5%). Then we find that

$$C_0=\frac{\sqrt{2}\,ef}{m_\pi}(1.09). \tag{5.3}$$

With this value we are assured of a fit to the $\gamma n\to\pi^-p$ data at threshold. However, now we must check how well this improved $H_{\gamma\pi}$ model fits the $\gamma n\to\pi^-p$ data above threshold. At once we run into the problem of scarce $\gamma n\to\pi^-p$ data in the region from threshold up to about 200 MeV photon lab energy. However, we can take advantage of the available multipole fits to $\gamma N\to\pi N$ reactions. These are most conveniently (and accurately) represented by the recently published Blomqvist-Laget (BL) model.[20] The BL model can then be "turned around" and used to generate both total $\sigma_T^{\gamma n}$ and differential cross sections $d\sigma^{\gamma n}/d\Omega$ for the near threshold region of $\gamma n\to\pi^-p$. Thus we use the BL model to conveniently represent the data for $\gamma n\to\pi^-p$. (We observe that most of the $\gamma n\to\pi^-p$ information comes from experiments using deuterium targets and hence some theoretical considerations are necessary in the extraction of this information.) We then compare the results for $\sigma_T^{\gamma n}, d\sigma^{\gamma n}/d\Omega$ from our improved $\vec{\sigma}\cdot\vec{\epsilon}$ model with the BL model results. We find that in the region from 0 up to 10 MeV above threshold, our improved $\vec{\sigma}\cdot\vec{\epsilon}$ model fits the BL $\sigma_T^{\gamma n}, d\sigma^{\gamma n}/d\Omega$ to within 5% and 10%, respectively. This is quite good in view of the fact that typical experimental errors are 5% for σ_T[17-19] and 5–10% for $d\sigma^{\gamma n}/d\Omega$.[21] If we now use this improved model for $H_{\gamma\pi}$ in our $^{12}C(\gamma,\pi^-)^{12}N_{g.s.}$ calculation, we obtain the results shown in Fig. 2 (curve 5). These final results have a typical accuracy of ±10% corresponding to the accuracy of the improved model fit to the $\gamma n\to\pi^-p$ data. With this error allowance the final results agree with the experimental data.

Thus, we see that with a simple $H_{\gamma\pi}$ operator it is possible to fit the $\gamma n\to\pi^-p$ data rather well and so predict σ_T for $^{12}C(\gamma,\pi^-)^{12}N_{g.s.}$ with reasonable confidence in the energy range we have considered. To go to higher energies it is necessary to put in the complete $H_{\gamma\pi}$ operator, not just its leading $\vec{\sigma}\cdot\vec{\epsilon}$ term. We shall present details of this and results for $^{12}C(\gamma,\pi^-)^{12}N_{g.s.}$ in a subsequent paper. However, it is useful to make some comments at this stage. In the region 0–10 MeV above threshold the additional momentum dependent terms in $H_{\gamma\pi}$ contribute only about 5% to the $\gamma n\to\pi^-p$ cross sections $\sigma_T^{\gamma n}, d\sigma^{\gamma n}/d\Omega$ (they tend to reduce $\sigma_T^{\gamma n}$). Their neglect in the near threshold region is supported by this fact, the current experimental errors in $\sigma_T^{\gamma n}, d\sigma^{\gamma n}/d\Omega$, and the observation that pion distortions for low pion energies are not strong enough to significantly magnify the momentum dependent terms when operating in $^{12}C(\gamma,\pi^-)^{12}N_{g.s.}$. At higher energies [especially as the $\Delta(1232)$ resonance is approached] the momentum dependent terms increase in importance in $\gamma n\to\pi^-p$ (and pion distortions become stronger) so that their omission in $^{12}C(\gamma,\pi^-)^{12}N_{g.s.}$ would cause large errors.

We end this section by observing that once we have the photoproduction calculation under complete control, with all terms in $H_{\gamma\pi}$ included, then the (γ,π) process may yield useful information on the pion distortions, i.e., we hope to use the (γ,π) process to learn about pion-nucleus interactions and the associated absorption mechanism.

VI. DISCUSSION AND CONCLUSIONS

We have completed a simple direct analysis of the reaction $^{12}C(\gamma,\pi^-)^{12}N_{g.s.}$ which employs realistic nuclear wave functions, takes full account of pion distortions, and uses a photoproduction operator $H_{\gamma\pi}$ which gives a good fit to the $\gamma n\to\pi^-p$ data near threshold. Our theoretical predictions are expected to be reliable to about 10% in the region from 0–10 MeV above threshold for the photon lab energy. This accuracy is comparable to the current experimental uncertainty in the elementary process $\gamma n\to\pi^-p$. To obtain more accurate predictions for $^{12}C(\gamma, \pi^-)^{12}N_{g.s.}$ in the above energy range it will be necessary to have more precise data on $\gamma n \to \pi^- p$ in the threshold region, and to include the full $H_{\gamma\pi}$ operator. It is clear from our analysis that at the current level of accuracy, there is agreement between theory and experiment for $^{12}C(\gamma, \pi^-)^{12}N_{g.s.}$ in the near threshold region. Vital factors in obtaining this agreement are inclusion of all pion partial waves and a correct treatment of the isospin dependent part of the pion-nucleus optical potential.

At this point, it is important to make some comparison of our results with those of Nagl and Uberall.[3] NU employed a nuclear model consistent with electron scattering, included the complete $H_{\gamma\pi}$ operator, and put in distorted pion waves. Yet their results do not agree well with experiment. In fact, their closest approach to the data is given by curve 2 in Fig. 1, which corresponds to, what appears to us to be, an unjustified recipe for treating the photoproduction operator. Let us elaborate on this point. NU use Behrend's elementary amplitude[6] for $\gamma n \to \pi^- p$ which may be written as

$$\mathfrak{F} = i\vec{\sigma}\cdot\vec{\epsilon}_\lambda \mathfrak{F}_1 + \vec{\sigma}\cdot\vec{q}\vec{\sigma}\cdot(\vec{k}\times\vec{\epsilon}_\lambda)(\mathfrak{F}_2/qk) + i\vec{\sigma}\cdot\vec{k}\vec{q}\cdot\vec{\epsilon}_\lambda(\mathfrak{F}_3/qk) + i\vec{\sigma}\cdot\vec{q}\vec{q}\cdot\vec{\epsilon}_\lambda(\mathfrak{F}_4/q^2).$$

Now, NU recipe 1 simply replaces the explicit pion momentum $\vec{q}$ by an operator $-i\vec{\nabla}$ which acts on the pion distorted wave. Recipe 2 follows recipe 1 and in addition, uses local pion momenta ($-i\vec{\nabla}$!) for the expressions $q = |\vec{q}|$ in the denominators. This appears to us to be inconsistent with basic theory. For example in a Feynman diagram model of $\gamma n \to \pi^- p$ (such as the Blomqvist-Laget model[20]) there is no evidence for such factors of q in the denominator—they are always canceled out by factors of q present in $\mathfrak{F}_2$, $\mathfrak{F}_3$, and $\mathfrak{F}_4$. On this basis we take curve 1 in Fig. 1 to be the appropriate curve to consider in our comparison with NU. Having made this point we now proceed.

Both the NU and our nuclear models are consistent with electron scattering to the 15.11 MeV 1^+1 level in ^{12}C so we can find no obvious difference there. However, from their description, it appears that NU may not have included the isospin terms in $V_{\pi\eta}$. Certainly in the references[22] quoted by NU the pion optical potentials do not include these effects. It is possible to make a direct comparison with NU if we assume that there is little quantitative difference in our nuclear models. We proceed as follows. NU[3] show results for the case where $\mathfrak{F}_2 = \mathfrak{F}_3 = \mathfrak{F}_4 = 0$, i.e., where they keep only the $\vec{\sigma}\cdot\vec{\epsilon}_\lambda$ term in $\mathfrak{F}$.

Now numerical studies we have carried out show that $\mathfrak{F}_1$ is approximately constant for photon lab energy ranging from threshold to 200 MeV above threshold. In addition, $\mathfrak{F}_1$ has such a magnitude that keeping only the $\vec{\sigma}\cdot\vec{\epsilon}_\lambda$ term in $\mathfrak{F}$ is the same as using our improved model for $H_{\gamma\pi}$. In other words, the NU $\vec{\sigma}\cdot\vec{\epsilon}_\lambda$ photoproduction operator and our improved model for $H_{\gamma\pi}$ are quantitatively the same. However, the cross sections obtained by NU for the $\vec{\sigma}\cdot\vec{\epsilon}_\lambda$ case differ markedly from ours. It appears therefore that our difference with the NU results most likely is due to different pion distorted waves.

In conclusion we can simply say that our calculation of σ_T for $^{12}C(\gamma, \pi^-)^{12}N_{g.s.}$ agrees with experiment in the near threshold region. We differ with the results of Nagl and Uberall[3] possibly because we use more realistic pion wave functions. It is clear that this photoproduction process is sensitive to to details of the pion-nucleus interaction and it would be very useful if the experiments could be extended to higher energies.

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