PHYSICAL REVIEW C VOLUME 17, NUMBER 2 FEBRUARY 1978

Modified one-body nuclear dissipation*

Arnold J. Sierk,† Steven E. Koonin, and J. Rayford Nix

Theoretical Division, Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico 87545 and W. K. Kellogg Radiation Laboratory, California Institute of Technology, Pasadena, California 91125

(Received 6 October 1977)

We study a modification of the one-body dissipation mechanism for the conversion of energy of collective nuclear motion into internal single-particle excitation energy. One-body nuclear dissipation is a consequence of the long mean free path of nucleons inside a nucleus, and arises from nucleons colliding with the moving boundary of the nucleus rather than with other individual nucleons. In our modification, which attempts to incorporate self-consistency, the dissipation rate is proportional to an integral over the nuclear surface of the square of the normal component of the normal derivative of the velocity. The resulting properties of this dissipation are qualitatively similar to those of ordinary two-body viscosity rather than to those of the original one-body dissipation. In particular, for small oscillations about a sphere the dissipation rate increases with increasing multipole degree, and in fission this dissipation leads to more elongated scission shapes and to decreased fission-fragment kinetic energies. By adjusting the parameter that specifies the magnitude of this dissipation, we are able to reproduce adequately the experimental most probable fission-fragment kinetic energies for the fission of nuclei throughout the Periodic Table.

NUCLEAR REACTIONS, FISSION Calculated dependence of fission-fragment kinetic energies upon modified one-body dissipation. Dynamics of large-scale nuclear collective motion, nuclear dissipation, hydrodynamical model, Werner-Wheeler method.

I. INTRODUCTION

Nuclear dissipation—the conversion of energy of collective nuclear motion into internal single-particle excitation energy—is now receiving much attention. Following the recognition by Swiatecki in 1969 that nuclear dissipation is possibly very large,[1,2] several studies were made of ordinary two-body viscosity, in which the dissipation proceeds from collisions between individual nucleons.[3-11] When applied to nuclear fission, two-body viscosity leads to more elongated scission shapes and to decreased fission-fragment kinetic energies.[8-11] Experimental most probable fission-fragment kinetic energies for the fission of nuclei throughout the Periodic Table are reproduced adequately by a two-body viscosity coefficient of

$$\mu = 0.03 \pm 0.01 \text{ TP} = 19 \pm 6 \times 10^{-24} \text{ MeV s/fm}^3 ,$$

when account is taken of the rupture of the neck at a finite radius.[11]

However, at low excitation energies of the nucleus, the Pauli exclusion principle strongly inhibits two-nucleon scattering by limiting the phase space into which the nucleons can scatter. The mean free path for a nucleon near the Fermi surface is therefore large, possibly many times as large as nuclear dimensions. As a consequence, the short-mean-free-path assumption implicit in ordinary two-body viscosity is questionable, and new dissipative mechanisms may be important.

In particular, it has been suggested that in the long-mean-free-path regime, the dominant dissipative mechanism is the elastic collision of nucleons with the time-dependent nuclear single-particle potential, which is referred to as one-body dissipation.[12-23] The traditional microscopic interpretation of nuclear dissipation—the excitation of higher levels, primarily in the vicinity of level crossings—involves features of both one-body and two-body dissipation.[24-33]

By considering classically a rigid potential wall driven through cold nuclear matter, Swiatecki derived an expression for the rate of one-body dissipation associated with arbitrary changes in the nuclear shape.[16,17] His result is

$$\frac{dE_{\text{dis}}}{dt} = \tfrac{3}{4}\rho_m v_F \oint v_n^2 dS , \tag{1}$$

where ρ_m is the mass density, v_F is the Fermi velocity, and v_n is the normal velocity of the surface. The integral is over the nuclear surface. In the derivation of Eq. (1) it is assumed that $|v_n| \ll v_F$ and that there is no overall motion of the matter inside the nucleus.

Equation (1) predicts that nuclear dissipation should be extremely large and—unlike the case for ordinary two-body viscosity—that the low-multipole oscillations should be damped more strongly than high-multipole oscillations. In fission this leads to a slow descent from saddle to scission and to a compact scission configuration.[10,11,17,19] Without the use of any adjustable parameters, the resulting calculated most probable fission-fragment kinetic energies for the fission of nuclei throughout the Periodic Table are in approximate agreement with experimental values. In particular, the calculated energies are about 8% larger than the experimental values, when account is taken of the rupture of the neck at a finite radius.[11]

In dynamical calculations of fission, Eq. (1) cannot be used past the scission point because for the uniform translation of the fission fragments it yields a finite dissipation rather than zero. For a uniform translation or a uniform rotation, Eq. (1) can be modified to remove this spurious dissipation,[17] but there is no satisfactory way to interpolate between the one-body dissipation formulas that are meant to apply before scission and after scission.

Later work within the linear-response formalism has shown that even with the simplifying assumptions of rigid potential walls and cold nuclear matter, one-body dissipation is a more complicated phenomenon[20,22,23] than is suggested by Eq. (1). Because of the long mean free path for nucleons, this type of dissipation is highly nonlocal, coupling the velocities of points widely separated on the nuclear surface. Equation (1) is only the local contribution to the dissipation rate. Important quantal corrections for the effects of curvature and diffuseness of the nuclear surface also exist.

Here we explore an aspect of one-body dissipation that has been overlooked by the previous perturbative treatments, namely, self-consistency. In Sec. II we attempt to incorporate self-consistency by means of a heuristic argument, which leads to a modification of Eq. (1). The properties of the resulting dissipation formula are compared in Sec. III with those for ordinary two-body viscosity and for the original one-body dissipation, for small oscillations about a sphere. The large distortions encountered in fission are considered in Sec. IV, where we solve the dynamical equations of motion for the descent from the fission saddle point and calculate most probable fission-fragment kinetic energies. Our summary and conclusion are presented in Sec. V.

II. MODIFICATION OF DISSIPATION FORMULA

One-body dissipation is an energy exchange between collective and single-particle degrees of freedom due to an incoherence arising between these coordinates during the collective motion. But the motion of the nucleons themselves is the ultimate source of the time dependence of the nuclear mean field, such as occurs, for example, in the time-dependent Hartree-Fock approach. Therefore, the single-particle degrees of freedom are not totally independent of the collective coordinates. In particular, the mean field at any point is sensitive to the presence of nucleons within a distance approximately that of the range of the internucleon force. Thus, any motion of the nuclear surface (mean-field potential wall) inescapably implies a corresponding collective motion of the nucleons near the surface region.

Equation (1), which assumed the nuclear matter to be at rest with respect to the moving surface, must therefore be modified to take into account the overall motion of the matter inside the wall. In particular, v_n must be replaced by the *relative* normal velocity Δv_n between the wall and the nuclear matter colliding with it. If the scale for the variation of the collective velocity field $\vec{v}(\vec{r})$ is large compared to the range of the internucleon force, this relative normal velocity can be approximated by expanding $\vec{v}$ in a Taylor series about the surface. This leads to

$$\Delta v_n = \lambda \hat{n} \cdot \frac{\partial \vec{v}}{\partial n} + \cdots , \qquad (2)$$

where λ is the effective distance between the nuclear surface and the nuclear matter colliding with it and where $\hat{n}$ is the outward-directed normal unit vector. The normal derivative $\partial/\partial n = \hat{n}\cdot\nabla$ is evaluated on the surface. Alternatively, Eq. (2) may be regarded as the first term in an expansion that expresses the relative normal matter-wall velocity as a folding of the collective velocity field with the internucleon potential.

With this replacement, Eq. (1) becomes

$$\frac{dE_{\rm dis}}{dt} = \tfrac{3}{4}\rho_m v_F \lambda^2 \oint \left(\hat{n}\cdot\frac{\partial \vec{v}}{\partial n}\right)^2 dS . \qquad (3)$$

In this modified formula, the dissipation rate is proportional to an integral over the nuclear surface of the square of the normal component of the normal derivative of the velocity. This leads automatically to zero dissipation for a uniform translation or a uniform rotation, as should be the case.

Because of the uncertainties associated with the derivation of Eq. (3), and because we approximate the velocity field in terms of incompressible, nearly irrotational flow, we treat the effective distance λ as an adjustable parameter that specifies the magnitude of the dissipation. Its value, which is determined in Sec. IV from a comparison of calculated and experimental fission-fragment kinetic energies for the fission of nuclei throughout the Periodic Table, turns out to be slightly larger than the range of the internucleon force.

III. SMALL OSCILLATIONS ABOUT A SPHERE

We now compare the damping of small oscillations about a sphere for three types of dissipation: (1) ordinary two-body viscosity, (2) the original one-body dissipation, and (3) our modified one-body dissipation. For this purpose we describe the small axially symmetric deformations of the nucleus by expanding its radius vector as a function of polar angle θ and time t in a series of Legendre polynomials,

$$R(\theta, t) = R_0\left[1 + \sum_{n=1}^{\infty} \alpha_n(t)P_n(\cos\theta)\right],$$

where R_0 is the radius of the spherical nucleus.

The equations of motion are then those of a damped harmonic oscillator,

$$M_n\ddot{\alpha}_n + \eta_n\dot{\alpha}_n + C_n\alpha_n = 0,$$

where M_n, η_n, and C_n are the inertia coefficient, damping coefficient, and stiffness coefficient, respectively, for the nth multipole oscillation. The qualitative behavior of the solutions of this equation depends upon the relative magnitude of η_n compared to the critical value

$$\eta_n^{\rm crit} = 2(M_nC_n)^{1/2}.$$

For $\eta_n < \eta_n^{\rm crit}$ the nth mode corresponds to exponentially damped sinusoidal oscillations, for $\eta_n = \eta_n^{\rm crit}$ the nth mode is critically damped, and for $\eta_n > \eta_n^{\rm crit}$ the nth mode is overdamped.

On the basis of the uniformly charged liquid-drop model, the stiffness coefficient C_n is given by[34]

$$C_n = \frac{(n-1)(n+2)}{(2n+1)} E_s^{(0)} - \frac{10(n-1)}{(2n+1)^2} E_C^{(0)},$$

where $E_s^{(0)}$ is the surface energy of the spherical nucleus and $E_C^{(0)}$ is the Coulomb energy of the spherical nucleus. When account is taken of the finite range of the nuclear force, the stiffness coefficient C_n becomes,[35] for $n \geq 2$,

$$C_n = \frac{2}{(2n+1)}\left\{\left(\frac{R_0}{a}+1\right)\left[\frac{R_0}{a} - 1 + \left(\frac{R_0}{a}+1\right)\exp\left(-\frac{2R_0}{a}\right)\right] - 2\left(\frac{R_0}{a}\right)^3 I_{n+1/2}\left(\frac{R_0}{a}\right)K_{n+1/2}\left(\frac{R_0}{a}\right)\right\}E_s^{(0)} - \frac{10(n-1)}{(2n+1)^2} E_C^{(0)},$$

where a is the range of the Yukawa effective two-nucleon interaction and where $I_{n+1/2}$ and $K_{n+1/2}$ are modified Bessel and Hankel functions, respectively. For a uniform translation, described by $n=1$, the stiffness coefficient is zero. As the degree n of the multipole oscillation increases, the stiffness coefficient increases. This rate of increase is more rapid in the liquid-drop model than when account is taken of the finite range of the nuclear force.

In calculating the inertia and damping coefficients, we approximate the velocity field inside the nucleus in terms of incompressible, irrotational flow. The inertia coefficient M_n is then given by[9,34]

$$M_n = \frac{3}{n(2n+1)} M_0R_0^2,$$

where M_0 is the total mass of the nucleus. For a uniform translation, the inertia coefficient is simply $M_0R_0^2$. As the degree n of the multipole oscillation increases, the inertia coefficient decreases.

For ordinary two-body viscosity and the assumed irrotational flow, the damping coefficient η_n is given by[3,9]

$$\eta_n^{\rm 2\text{-}body} = \frac{8\pi(n-1)}{n} R_0^3\mu,$$

where μ is the ordinary two-body viscosity coefficient. The damping coefficient for this type of dissipation is zero for a uniform translation and increases slowly as n increases.

For the original one-body dissipation, the damping coefficient η_n is given by[17]

$$\eta_n^{\rm 1\text{-}body,orig} = \frac{3\pi}{(2n+1)} R_0^4\rho_m v_F,$$

where ρ_m is the mass density and v_F is the Fermi velocity. (Unlike the case for the other two types of dissipation, the original one-body dissipation is independent of the nature of the internal velocity field.) The damping coefficient for the original one-body dissipation is spuriously finite for a uniform translation and decreases as n increases. However, with the modification discussed in Ref. 17, the result for a uniform translation or a uniform rotation is zero.

For our modified one-body dissipation and irrotational flow, we find that the damping coefficient η_n is given by

$$\eta_n^{\rm 1\text{-}body,mod} = \frac{3\pi(n-1)^2}{(2n+1)} R_0^2\rho_m v_F\lambda^2,$$

where the effective distance λ specifies the magnitude of the dissipation. The damping coefficient for this type of dissipation is zero for a uniform translation and increases as n increases.

For the $n=2$, 3, and 4 multipole oscillations, we show in Table I the damping coefficients for a ^{236}U nucleus corresponding to each of these three types of dissipation. For ordinary two-body viscosity, the oscillations are somewhat under-

TABLE I. Comparison of damping coefficients for the small oscillations of ^{236}U about a sphere. The inertia and damping coefficients are calculated for incompressible, irrotational hydrodynamical flow, and the stiffness coefficients are calculated by taking into account the finite range of the nuclear force, using the constants of Ref. 35. The value of the two-body viscosity coefficient μ is taken to be 0.03 TP (Ref. 11), and the value of the modified one-body dissipation coefficient λ^2 is taken to be 3 fm^2 (see Sec. IV). The results are presented in units of the critical damping coefficient $\eta_n^{\rm crit} = 2(M_n C_n)^{1/2}$, which is different for each multipole oscillation.

n	$\eta_n^{\text{2-body}}/\eta_n^{\rm crit}$	$\eta_n^{\text{1-body, orig}}/\eta_n^{\rm crit}$	$\eta_n^{\text{1-body, mod}}/\eta_n^{\rm crit}$
2	0.72	5.44	0.32
3	0.71	2.90	0.68
4	0.85	2.38	1.25

damped. The $n = 3$ oscillation is damped slightly less rapidly than the $n = 2$ oscillation, whereas the $n = 4$ oscillation is damped somewhat more rapidly. For the original one-body dissipation, the oscillations are highly overdamped, with the amount of overdamping decreasing as n increases. For our modified one-body dissipation, the $n = 2$ oscillation is highly underdamped, the $n = 3$ oscillation is somewhat underdamped, and the $n = 4$ oscillation is somewhat overdamped. This strong dependence of the damping upon n means that the quadrupole oscillations of nuclei in their ground states are damped only partially, while fission, which involves also higher multipole oscillations, is damped somewhat more highly. (Because for ground-state nuclei the true nuclear inertia is substantially larger than the value for incompressible, irrotational flow, the actual damping of the quadrupole oscillations is even less than that given in Table I.)

For each of the three types of dissipation that we are considering, the damping can also be calculated exactly for pure spheroidal distortions of arbitrarily large eccentricity, provided that we approximate the velocity field inside the nucleus in terms of incompressible, irrotational flow. The relevant formulas are given in the Appendix.

IV. LARGE DISTORTIONS IN FISSION

To study the effect of our modified one-body dissipation on the dynamics of fission, we solve numerically the classical equations of motion for a fissioning nucleus,[9,11] with the dissipation rate specified by Eq. (3). Prior to scission, where the nuclear shape is parametrized in terms of smoothly joined portions of three quadratic surfaces of revolution,[36] Eq. (3) is integrated numerically. After scission, where the fission fragments are described in general by two spheroids,[37] Eq. (3) is integrated analytically, with the result given in the Appendix. In calculating the nuclear macroscopic energy, we take into account the finite range of the nuclear force.[35] This permits us to determine scission as the point where stability against neck rupture is lost.[11] The Werner-Wheeler method is used for determining the incompressible, nearly irrotational hydrodynamical flow pattern.[9,36] The Rayleigh dissipation function F that enters into the modified Lagrange equations of motion is given by $F = \frac{1}{2} dE_{\rm dis}/dt$.

Figure 1 displays the dependence of the calculated dynamical path for the fission of ^{236}U upon the magnitude of our modified one-body dissipation. Relative to the path for zero dissipation, modified one-body dissipation leads to more elongated shapes during the descent from the fission saddle point. This is similar to ordinary two-body viscosity and opposite to the original one-body dissipation, which leads to compact shapes. After scission, the path for zero dissipation corresponds to an oscillation of the fission fragments about their own centers of mass as they separate, whereas the path for infinite dissipation corresponds to a rigid separation of the fission fragments without change in their shape.

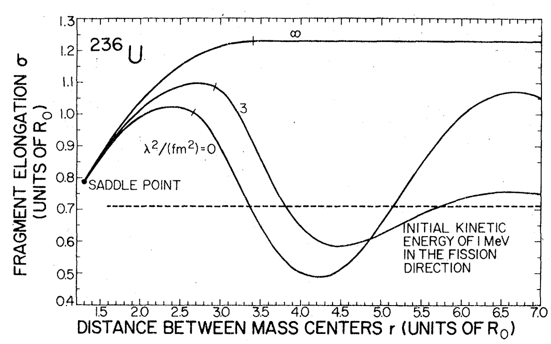

FIG. 1. Calculated dynamical paths in the r-σ plane for the fission of ^{236}U. The moment r gives the distance between the centers of mass of the two halves of the system, and the moment σ gives the sum of the root-mean-square extensions of the matter distribution of each half about its own center of mass. The radius R_0 of the spherical nucleus is given by $R_0 = r_0 A^{1/3} = 1.16$ $(236)^{1/3}$ fm $= 7.17$ fm. The oscillating curve is calculated for zero dissipation, the upper curve is calculated for infinite modified one-body dissipation, and the intermediate curve is calculated for a value of $\lambda^2 = 3$ fm^2. In each case the initial conditions correspond to starting from the fission saddle point with 1 MeV of kinetic energy in the fission direction. The short line perpendicular to each path gives the location where stability against neck rupture is lost. The dashed horizontal line corresponds to configurations of two separated spherical nuclei.

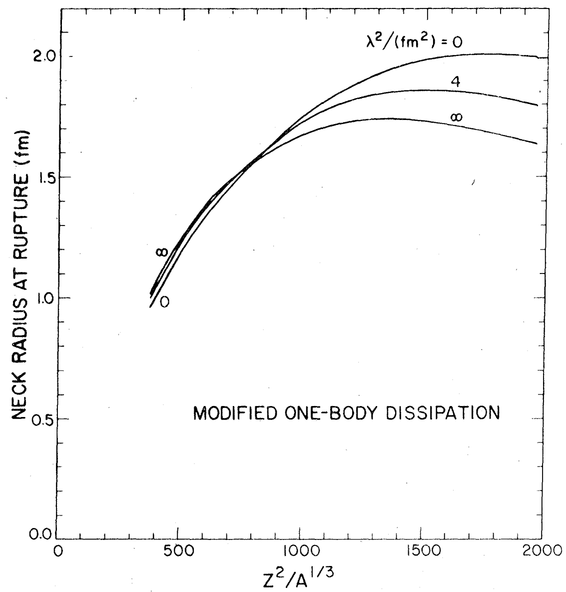

FIG. 2. Dependence of the neck radius at which instability occurs upon $Z^2/A^{1/3}$ of the fissioning nucleus, for three values of the magnitude of our modified one-body dissipation. The initial conditions correspond to starting from rest an infinitesimal distance beyond the fission saddle point in the fission direction.

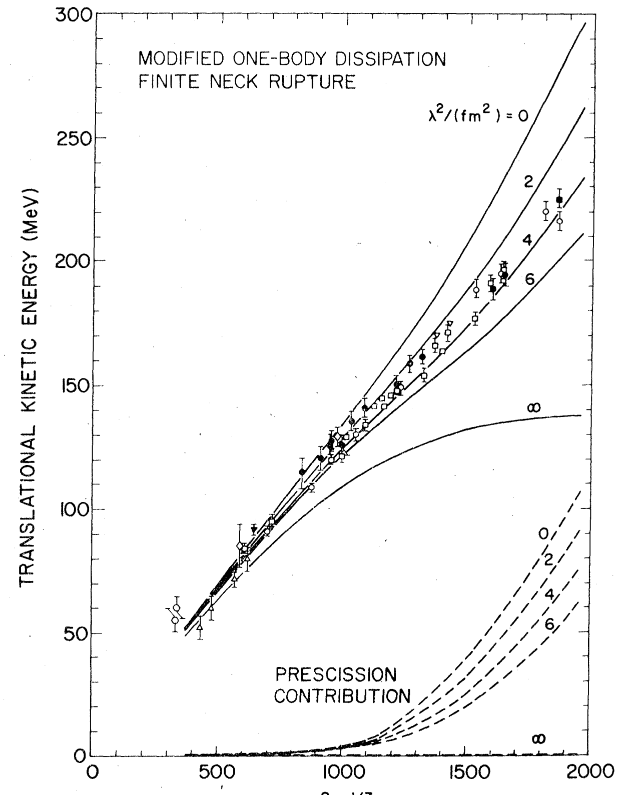

FIG. 3. Dependence of the most probable fission-fragment kinetic energy upon $Z^2/A^{1/3}$ of the fissioning nucleus, for five values of the magnitude of our modified one-body dissipation. The initial conditions correspond to starting from rest an infinitesimal distance beyond the fission saddle point in the fission direction. The dashed curves give the calculated contribution to the translational kinetic energy acquired prior to the rupture of the neck, and the solid curves give the calculated total translational kinetic energy of the fragments at infinity. The open and solid points give the experimental data, whose sources can be found in Ref. 9.

The intermediate path calculated for $\lambda^2 = 3$ fm^2 corresponds to underdamped fission fragments whose oscillations decrease less rapidly than those for fragments with a two-body viscosity coefficient[11] of $\mu = 0.03$ TP.

The short line perpendicular to each path in Fig. 1 gives the location where stability against neck rupture is lost. This occurs because the attractive nuclear force can no longer withstand the repulsive Coulomb force when the neck's size is reduced below a critical value.[11] As seen in Fig. 2, the neck radius at which instability occurs for heavy actinide nuclei is roughly 2 fm for zero dissipation and decreases somewhat with increasing modified one-body dissipation. This decrease occurs because of the reduced importance of the disruptive Coulomb force for more elongated shapes. Also, the neck radius at instability decreases for lighter nuclei, again because of the reduction in the Coulomb force.

In our calculations, we assume that the neck ruptures instantaneously at the point of instability and, with the exception of the case for infinite dissipation, treat the subsequent motion of the system in terms of spheroidal fission fragments. The transition from the three-quadratic-surface shape parametrization to spheroidal fission fragments is accomplished by making continuous the values of the moments r and σ defined in Fig. 1 and their time derivatives.[11] The equations of motion are then integrated until the spheroids have separated a distance of about $15R_0$. The translational fission-fragment kinetic energy at infinity is then calculated as the sum of the translational kinetic energy at this point plus the Coulomb interaction energy of two spherical fragments located at this point. For the case of infinite dissipation, where the framents separate rigidly to infinity without changing their shape, the fission-fragment kinetic energy at infinity is calculated exactly by taking the sum of the Coulomb and nuclear interaction energies at the point of instability.

The most probable fission-fragment kinetic energies calculated in this way are shown in Fig. 3 for the fission of nuclei throughout the Periodic

Table. As the magnitude of our modified one-body dissipation increases, the calculated kinetic energy decreases, as a result of two different effects. First, as shown by the dashed curves, increased dissipation causes the system to acquire less translational kinetic energy prior to neck rupture. Second, increased modified one-body dissipation leads to more elongated shapes at neck rupture and hence to less kinetic energy being acquired after neck rupture.

It is seen from Fig. 3 that the value

$$\lambda^2 = 3 \pm 1 \text{ fm}^2$$

for the magnitude of our modified one-body dissipation adequately reproduces the experimental fission-fragment kinetic energies for the fission of nuclei throughout the Periodic Table. This means that the effective distance between the nuclear surface and the nuclear matter colliding with it is

$$\lambda = 1.7 \pm 0.3 \text{ fm},$$

which is slightly larger than the range of the internucleon force.

V. SUMMARY AND CONCLUSION

In a study of the mechanism of nuclear dissipation, we have attempted to incorporate self-consistency into the original formula for one-body dissipation. We argued heuristically that the motion of the nuclear surface is caused by the motion of the matter inside the nucleus, and that therefore the normal surface velocity appearing in the original formula must be replaced by the relative normal velocity between the matter at the surface and the matter inside. This led to a modified formula for one-body dissipation in which the dissipation rate is proportional to an integral over the nuclear surface of the square of the normal component of the normal derivative of the velocity.

We studied this modified formula for small oscillations about a sphere and for the large distortions encountered in fission. In both cases the properties of our modified one-body dissipation are qualitatively similar to those of ordinary two-body viscosity rather than to those of the original one-body dissipation. In particular, the dissipation rate increases with increasing multipole order, which leads to more elongated scission shapes compared to those for zero dissipation. Experimental most probable fission-fragment kinetic energies for the fission of nuclei throughout the Periodic Table are reproduced adequately when the parameter that specifies the magnitude of our modified one-body dissipation has the value

$$\lambda^2 = 3 \pm 1 \text{ fm}^2.$$

There are some uncertainties associated with our heuristic derivation, and we do not expect our modified formula to be the ultimate word on nuclear dissipation. Nevertheless, we hope that our present considerations will prove useful in stimulating further progress on this important question.

We are grateful to W. J. Swiatecki for his strong criticism of this work and to R. A. Broglia and J. W. Negele for their encouragement concerning it. J. R. Nix thanks the W. K. Kellogg Radiation Laboratory for its hospitality during the early phases of this work.

APPENDIX: DAMPING FOR PURE SPHEROIDAL DISTORTIONS

For pure spheroidal distortions, the rate of energy dissipation corresponding to each of the three types of dissipation that we are considering can be written in the form

$$\frac{dE_{\text{dis}}}{dt} = \eta_c(c)\dot{c}^2,$$

provided that we approximate the velocity field inside the nucleus in terms of incompressible, irrotational flow. The coordinate c is the symmetry semiaxis of the spheroid, $\dot{c}$ is its time derivative, and $\eta_c(c)$ is the damping term, which is a function of the deformation of the spheroid.

Because of volume conservation, the transverse semiaxis a of the spheroid is related to c by

$$a^2 c = R_0{}^3,$$

where R_0 is the radius of the sphere. In terms of cylindrical coordinates ρ and z, the equation for the surface of the spheroid is then

$$\rho^2 = a^2 - \frac{a^2}{c^2} z^2 = \frac{R_0{}^3}{c}\left(1 - \frac{z^2}{c^2}\right).$$

For incompressible, irrotational flow the velocity $\vec{v}$ at an interior point is

$$\vec{v} = \frac{\rho}{a}\dot{a}\hat{e}_\rho + \frac{z}{c}\dot{c}\hat{e}_z = (-\tfrac{1}{2}\rho\hat{e}_\rho + z\hat{e}_z)\frac{\dot{c}}{c},$$

where $\hat{e}_\rho$ and $\hat{e}_z$ are unit vectors in the ρ and z directions, respectively.

For ordinary two-body viscosity, the damping term $\eta_c(c)$ is given by[3,9]

$$\eta_c^{\text{2-body}} = 4\pi\frac{R_0{}^3}{c^2}\mu,$$

where μ is the ordinary two-body viscosity coefficient.

For the original one-body dissipation, we find from Eq. (1) that the damping term $\eta_c(c)$ is given by

$$\eta_c^{\text{1-body,orig}} = \tfrac{3}{5}\pi f(\alpha)\frac{R_0^3}{c}\rho_m v_F\,,$$

where ρ_m is the mass density, v_F is the Fermi velocity, and $f(\alpha)$ is a shape-dependent function of

$$\alpha = 1 - \frac{a^2}{c^2} = 1 - \frac{R_0^3}{c^3}\,.$$

For a prolate spheroid, where α is positive, the quantity α is simply the square of the eccentricity. However, this is not true for an oblate spheroid, where α is negative. The function $f(\alpha)$ is given by

$$f(\alpha) = \tfrac{5}{32}[(27 - 24\alpha + 8\alpha^2)(1-\alpha)^{1/2}h(\alpha) - 27 + 33\alpha - 6\alpha^2]/\alpha^2\,,$$

where

$$h(\alpha) = \begin{cases} (\sin^{-1}\alpha^{1/2})/\alpha^{1/2}\,, & \alpha > 0 \\ \{\ln[(1-\alpha)^{1/2} + (-\alpha)^{1/2}]\}/(-\alpha)^{1/2}\,, & \alpha < 0\,. \end{cases}$$

For nearly spherical shapes, where $|\alpha| \ll 1$, $f(\alpha)$ can be calculated conveniently by means of the series expansion

$$f(\alpha) = 1 - \tfrac{5}{21}\alpha - \tfrac{2}{21}\alpha^2 - \tfrac{2}{33}\alpha^3 - \tfrac{404}{9009}\alpha^4 + \cdots\,.$$

For our modified one-body dissipation, we find from Eq. (3) that the damping term $\eta_c(c)$ is given by

$$\eta_c^{\text{1-body,mod}} = \tfrac{3}{5}\pi g(\alpha)\rho_m v_F \lambda^2\,,$$

where λ is a parameter that specifies the magnitude of the dissipation. The shape-dependent function $g(\alpha)$ is given by

$$g(\alpha) = \tfrac{5}{8}[(-27 + 48\alpha - 20\alpha^2)(1-\alpha)^{1/2}h(\alpha) + 27 - 57\alpha + 34\alpha^2 - 4\alpha^3]/\alpha^2\,.$$

For nearly spherical shapes, where $|\alpha| \ll 1$, $g(\alpha)$ can be calculated conveniently by means of the series expansion

$$g(\alpha) = 1 - \tfrac{22}{21}\alpha + \tfrac{5}{21}\alpha^2 + \tfrac{4}{77}\alpha^3 + \tfrac{8}{1287}\alpha^4 + \cdots\,.$$

---

*This work was supported by the U. S. Energy Research and Development Administration under Contract No. W-7405-ENG-36 and by the U. S. National Science Foundation under Contract No. PHY76-02724.

†Alfred P. Sloan Foundation Research Fellow, 1975–1977.

[1]W. J. Swiatecki, in *Proceedings of the International Conference on Nuclear Reactions Induced by Heavy Ions, Heidelberg, Germany, 1969*, edited by R. Bock and W. R. Hering (North-Holland, Amsterdam/American Elsevier, New York, 1970), p. 729.

[2]W. J. Swiatecki, J. Phys. Suppl. 33, C5-45 (1972).

[3]J. Schirmer, S. Knaak, and G. Süssman, Nucl. Phys. A199, 31 (1973).

[4]R. Wieczorek, R. W. Hasse, and G. Süssmann, in *Proceedings of the Third International Atomic Energy Agency Symposium on the Physics and Chemistry of Fission, Rochester, New York, 1973* (International Atomic Energy Agency, Vienna, 1974), Vol. I, p. 523.

[5]R. W. Hasse, in Proceedings of the International Workshop III on Gross Properties of Nuclei and Nuclear Excitations, Hirschegg, Kleinwalsertal, Austria, 1975, edited by W. D. Myers [Technische Hochschule Darmstadt Report No. AED-Conf-75-009-000, 1975 (unpublished)], p. 22.

[6]R. W. Hasse, University of Munich Habilitationsschrift, 1977 (unpublished).

[7]A. J. Sierk and J. R. Nix, in *Proceedings of the Third International Atomic Energy Agency Symposium on the Physics and Chemistry of Fission, Rochester, New York, 1973* (see Ref. 4), Vol. II, p. 273.

[8]K. T. R. Davies, S. E. Koonin, J. R. Nix, and A. J. Sierk, in Proceedings of the International Workshop III on Gross Properties of Nuclei and Nuclear Excitations, Hirschegg, Kleinwalsertal, Austria, 1975 (see Ref. 5), p. 8.

[9]K. T. R. Davies, A. J. Sierk, and J. R. Nix, Phys. Rev. C 13, 2385 (1976).

[10]A. J. Sierk and J. R. Nix, in Proceedings of the Symposium on Macroscopic Features of Heavy-Ion Collisions, Argonne, Illinois, 1976 [Argonne National Laboratory Report No. ANL/PHY-76-2, 1976 (unpublished)], Vol. I, p. 407.

[11]K. T. R. Davies, R. A. Managan, J. R. Nix, and A. J. Sierk, Phys. Rev. C 16, 1890 (1977).

[12]C. Toepffer, in *Proceedings of the International Conference on Reactions between Complex Nuclei, Nashville, Tennessee, 1974*, edited by R. L. Robinson, F. K. McGowan, J. B. Ball, and J. H. Hamilton (North-Holland, Amsterdam/American Elsevier, New York, 1974), Vol. 1, p. 129.

[13]G. Wegmann, Phys. Lett. 50B, 327 (1974).

[14]G. Wegmann, in Proceedings of the International Workshop III on Gross Properties of Nuclei and Nuclear Excitations, Hirschegg, Kleinwalsertal, Austria, 1975 (see Ref. 5), p. 28.

[15]D. H. E. Gross, Nucl. Phys. A240, 472 (1975).

[16]W. J. Swiatecki, in Proceedings of the International School-Seminar on Reactions of Heavy Ions with Nuclei and Synthesis of New Elements, Dubna, U.S.S.R., 1975 [Joint Institute for Nuclear Research Report No. JINR-D7-9734, 1976 (unpublished)], p. 89.

[17]J. Blocki, Y. Boneh, J. R. Nix, J. Randrup, M. Robel, A. J. Sierk, and W. J. Swiatecki, Lawrence Berkeley Laboratory Report No. LBL-6536, 1977 (unpublished).

[18]J. Randrup, in Proceedings of the International Workshop IV on Gross Properties of Nuclei and Nuclear Excitations, Hirschegg, Kleinwalsertal, Austria, 1976 [Technische Hochschule Darmstadt Report No. AED-Conf-76-015-000, 1976 (unpublished)], p. 69.

[19]W. D. Myers, in Proceedings of the International Workshop IV on Gross Properties of Nuclei and Nuclear Excitations, Hirschegg, Kleinwalsertal, Austria, 1976 (see Ref. 18), p. 86.

[20]S. E. Koonin, J. Randrup, and R. L. Hatch, in Proceedings of the International Workshop V on Gross Properties of Nuclei and Nuclear Excitations, Hirschegg, Kleinwalsertal, Austria, 1977 [Technische Hochschule Darmstadt Report No. AED-Conf-77-017-001-AED-Conf-77-017-043, 1977 (unpublished)], p. 107.

[21]J. Randrup, in Proceedings of the International Workshop V on Gross Properties of Nuclei and Nuclear Excitations, Hirschegg, Kleinwalsertal, Austria, 1977 (see Ref. 20), p. 115.

[22]S. E. Koonin, R. L. Hatch, and J. Randrup, Nucl. Phys. A283, 87 (1977).

[23]S. E. Koonin and J. Randrup, Nucl. Phys. A289, 475 (1977).

[24]G. Schütte and L. Wilets, in *Proceedings of the Third International Atomic Energy Agency Symposium on the Physics and Chemistry of Fission, Rochester, New York, 1973* (see Ref. 4), Vol. I, p. 503.

[25]G. Schütte and L. Wilets, Nucl. Phys. A252, 21 (1975).

[26]Y. Boneh, Z. Fraenkel, and Z. Paltiel, in *Proceedings of the Third International Atomic Energy Agency Symposium on the Physics and Chemistry of Fission, Rochester, New York, 1973* (see Ref. 4), Vol. I, p. 515.

[27]Y. Boneh and Z. Fraenkel, Phys. Rev. C 10, 893 (1974).

[28]Y. Boneh, J. P. Blocki, and W. D. Myers, Phys. Lett. 63B, 265 (1976).

[29]T. Ledergerber, Z. Paltiel, H. C. Pauli, G. Schütte, Y. Yariv, and Z. Fraenkel, Phys. Lett. 56B, 417 (1965).

[30]T. Ledergerber, Z. Paltiel, Z. Fraenkel, and H. C. Pauli, Nucl. Phys. A275, 280 (1977).

[31]D. Glas and U. Mosel, Phys. Lett. 49B, 301 (1974).

[32]D. Glas and U. Mosel, Nucl. Phys. A264, 268 (1976).

[33]S. E. Koonin and J. R. Nix, Phys. Rev. C 13, 209 (1976).

[34]J. R. Nix, Ann. Phys. (N.Y.) 41, 52 (1967).

[35]H. J. Krappe and J. R. Nix, in *Proceedings of the Third International Atomic Energy Agency Symposium on the Physics and Chemistry of Fission, Rochester, New York, 1973* (see Ref. 4), Vol. I, p. 159.

[36]J. R. Nix, Nucl. Phys. A130, 241 (1969).

[37]J. R. Nix and W. J. Swiatecki, Nucl. Phys. 71, 1 (1965).