Levels in ^{23}Na populated by the ^{12}C(^{15}N,α) reaction

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Forty-nine angular distributions for states in ^{23}Na up to 16.6 MeV in excitation energy populated by the ^{12}C(^{15}N,α)^{23}Na reaction have been measured in 400 keV intervals for bombarding energies from 36.0 to 39.2 MeV. Hauser-Feshbach calculations have been made to suggest spins for most of the states observed. Shell-model predictions were used to guide the suggestions for positive-parity spin state locations. High-spin members of three rotational bands, $K^\pi = 3/2^+$, $1/2^+$, and $1/2^-$, were suggested and average moment-of-inertia parameters were extracted from the band systematics.

[NUCLEAR REACTIONS ^{12}C(^{15}N,α), $E_{^{15}\text{N}}$=36.0 to 39.2 MeV, measured $\sigma(E)$ for $\theta_{\text{lab}}=7°$; $E_{^{15}\text{N}}$=36.4, 37.2, 38.0, 38.8 MeV, measured $\sigma(\theta)$; ^{23}Na deduced levels, classification into rotational bands. Hauser-Feshbach calculations, suggested J values, moment of inertia calculations. Natural target.]

I. INTRODUCTION

In recent years the nucleus ^{23}Na has been widely studied because it reflects the typical rotational band structure of *sd*-shell nuclei. Speculation concerning probable members of the $K^\pi=\frac{3}{2}^+$ band such as the 9.04 and 9.81 MeV states has resulted in extensive experimental work.[1-7] There is now general agreement[6,7] that the spin of the 9.04 MeV state is $\frac{15}{2}^+$. KeKelis *et al.*[6] have also assigned this same spin to the 9.81 MeV state as opposed to the $\frac{17}{2}^+$ value suggested by previous authors.[2,3,5] More recently, the work of Evers *et al.*[7] tends to refute the $\frac{15}{2}^+$ assignment for the 9.81 MeV state. However, the present results are more consistent with a $\frac{15}{2}^+$ spin value.

The $K^\pi=\frac{1}{2}^-$ band in ^{23}Na is interesting because it can be described[8] reasonably well by both the Nilsson model and a weak coupling picture in which a $p_{1/2}^{-1}$ proton is coupled to the ground state rotational band of ^{24}Mg. The close relationship between the nuclear structure of the $\frac{1}{2}^-$ band in ^{23}Na and the ground state band of ^{24}Mg is discussed in the present work.

In a previous publication[9] we reported on the $K^\pi=\frac{3}{2}^+$ and $\frac{1}{2}^+$ bands of ^{23}Na populated in the ^{12}C(^{15}N, α)^{23}Na reaction. We have repeated and extended the measurements with an improved energy resolution of 60 keV in order to obtain angular distributions for members of previously unseparated doublets.

In this paper we discuss our results for the $K^\pi=\frac{3}{2}^+$, $\frac{1}{2}^+$, and $\frac{1}{2}^-$ band members, as well as for other detected states. A total of 49 angular distributions were measured for transitions to final states in ^{23}Na. Our suggestions for high-spin states are remarkably consistent with the shell-model predictions of Wildenthal[10] for the positive-parity states. The negative-parity states are in good agreement with rotational band systematics for the $K^\pi=\frac{1}{2}^-$ band.

II. EXPERIMENTAL PROCEDURE AND RESULTS

A beam of ^{15}N particles, accelerated by the ORNL EN tandem accelerator, bombarded thin ^{12}C targets (~8 μg/cm^2), and the emitted α particles were momentum analyzed in the Enge split-pole magnetic spectrograph and detected by a position sensitive proportional counter. The improved resolution of 60 keV was obtained primarily by using thinner targets and by optimizing the parameters of the main shaping amplifiers for the wire signals. In addition to the previous measurements, additional data were taken at lab energies (lab angles) of 36.4 MeV (15°, 22°), 37.2 MeV (7°, 15°, 22°), 38.0 MeV (7°, 15°, 22°), and 38.8 MeV (7°, 22°). Doublets at 2.64–2.70 MeV, 3.85–3.91 MeV, and 6.04–6.12 MeV were usually resolved in the new data as can be seen in a typical spectrum at 38.0 MeV (7°) shown in Fig. 1. The states identified are listed in Table I along with known spins and parities. Excitation energies are generally those of Endt and van der Leun[11] or Moss,[12] who recently made accurate measurements of excitation energies for 150 levels in ^{23}Na by means of the ^{23}Na(p,p')^{23}Na reaction. For the higher-lying states where it is uncertain as to which of the many states is being populated, our own excitation energy assignments are quoted to ±30 keV. Some

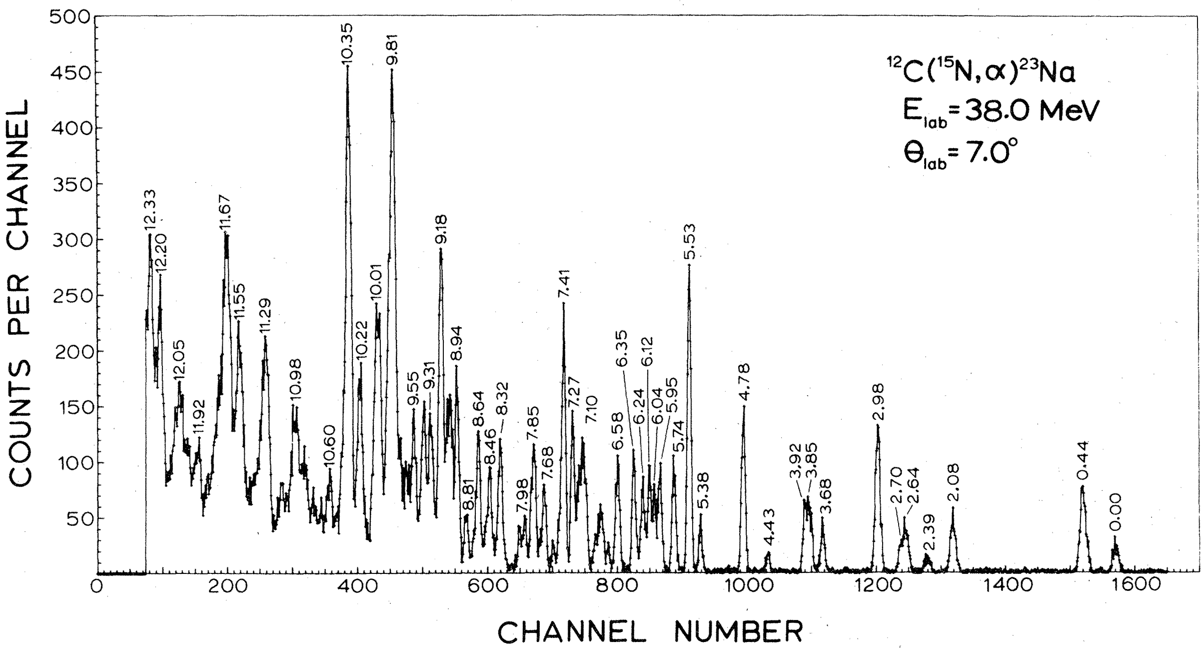

FIG. 1. Typical high resolution spectrum for the ^{12}C(^{15}N,α) ^{23}Na reaction. The excitation energies are noted.

differences in excitation energies exist between the present work and the ^{11}B(^{16}O, α)^{23}Na work of Gomez del Campo *et al.*[13] which can be explained by the improved resolution in the present ^{12}C(^{15}N, α)^{23}Na measurements and by the elimination of contaminant peaks such as from the ^{12}C(^{16}O, α) reaction which had high yields in their work.

In order to remove the effects of statistical fluctuations in the angular distributions, measurements of $\sigma(\theta)$ were made at bombarding energies in 400 keV steps between 36.0 and 39.2 MeV for much of the data. The energy averaged differential cross sections are presented in Figs. 2–6.

III. HAUSER-FESHBACH CALCULATIONS

Hauser-Feshbach calculations were made for comparison with the averaged cross sections of the ^{12}C(^{15}N, α)^{23}Na reaction. The calculations were as previously described[9] with identical optical model parameters; however, the effect of the level density parameters and of the critical angular momentum J_c on the cross sections were further investigated.

Recently it has been shown that at energies well above the Coulomb barrier cross sections may be limited[14,15] by a critical angular momentum. The origin of the effect is not clear. We have determined J_c in the present analysis from several considerations. For the optical model potential used for the entrance channel (see Ref. 9), a grazing angular momentum of $\frac{27}{2}$ corresponds to a transmission coefficient of 0.5. From the rigid rotator model with r_0 = 1.2 fm, we calculate a value of $J_c = \frac{29}{2}$ (if r_0 = 1.4 fm, then $J_c = \frac{35}{2}$). We have also determined J_c by considering relative cross sections as suggested by Klapdor *et al.*[15] Since relative cross sections (in our case relative to the ground state) are less sensitive to the level density parameters than are the absolute cross sections, the experimental data may be compared to statistical compound nucleus calculations with the computer program HELGA[16] by using different J_{max} values. The relative cross sections tend to be fairly level, rise dramatically, and then level off again as J_{max} is raised. A critical angular momentum of $\frac{27}{2}$ or $\frac{29}{2}$ was indicated from the values at which the computed relative cross sections equalled the experimental cross section ratios.

Other limitations (see, e.g., Ref. 17), as from fission or applicability of the statistical model, indicated limits of J_c much larger than $\frac{27}{2}$. For the present analysis a J_c value of $\frac{27}{2}$ was used in good agreement with the grazing angular momentum in the entrance channel. No change in J_c was expected over the range of bombarding energies used for the angular distributions.

The absolute cross sections depend strongly[13] on the level density parameter a. We calculated the cross sections for nine levels below an excitation energy of 7 MeV in ^{23}Na using $J_{max} = \frac{27}{2}$ in order to

TABLE I. Excitation energies, rotational bands, and spin values for ^{23}Na states.

E_c [a] (MeV)	Known [b] J^π	Band [c] K^π	Present [d] J^π	E_x [a] (MeV)	Known [b] J^π	Band [c] K^π	Present [d] J^π
0.0	$\frac{3}{2}^+$	$\frac{3}{2}^+$	$\frac{3}{2}^+$	9.81 [e]	$\frac{15}{2}^+$		$\frac{15}{2}^+$
0.439	$\frac{5}{2}^+$	$\frac{3}{2}^+$	$\frac{5}{2}^+$	10.01 [e] (multiplet)		$(\frac{1}{2}^-)$	$\begin{pmatrix}\frac{11}{2}^- \\ \frac{13}{2}^+ \\ \frac{13}{2}^+\end{pmatrix}$
2.076	$\frac{7}{2}^+$	$\frac{3}{2}^+$	$\frac{7}{2}^+$				
2.390	$\frac{1}{2}^+$	$\frac{1}{2}^+$	$\frac{1}{2}^+$				
2.639	$\frac{1}{2}^-$	$\frac{1}{2}^-$	$\frac{1}{2}^-$	10.22 [e]			$(\frac{11}{2}^+)$
2.703	$\frac{9}{2}^+$	$\frac{3}{2}^+$	$\frac{9}{2}^+$	10.35 [e]		$(\frac{1}{2}^-)$	$\begin{pmatrix}\frac{13}{2}^- \\ \frac{11}{2}^+ \\ \frac{11}{2}^+\end{pmatrix}$
2.982	$\frac{3}{2}^+$	$\frac{1}{2}^+$	$\frac{3}{2}^+$				
3.678	$\frac{3}{2}^-$	$\frac{1}{2}^-$	$\frac{3}{2}^-$				
3.848	$\frac{5}{2}^-$	$\frac{1}{2}^-$	$\frac{5}{2}^-$	10.60 [e]			$(\frac{9}{2}^+)$
3.914	$\frac{5}{2}^+$	$\frac{1}{2}^+$	$\frac{5}{2}^+$	10.98 [e]			$(\frac{9}{2}^+)$
4.432	$\frac{1}{2}^+$		$\frac{1}{2}^+$	11.29 [e]		$(\frac{3}{2}^+)$	$\begin{pmatrix}\frac{17}{2}^+ \\ \frac{13}{2}^+ \\ \frac{9}{2}, \frac{11}{2}\end{pmatrix}$
4.775	$\frac{7}{2}^+$	$\frac{1}{2}^+$	$\frac{7}{2}^+$				
5.380	$\frac{5}{2}^+$		$\frac{5}{2}^+$				
5.536	$\frac{11}{2}^+$	$\frac{3}{2}^+$	$\frac{11}{2}^+$	11.55 [e]			$\begin{pmatrix}\frac{13}{2}^+ \\ \frac{17}{2}^+, \frac{15}{2}^+\end{pmatrix}$
5.741	$(\frac{3}{2}, \frac{5}{2})^+$		$\begin{pmatrix}\frac{3}{2}^+ \\ \frac{1}{2}^+ \\ \frac{1}{2}^+\end{pmatrix}$				
5.766				11.67 [e]			$\begin{pmatrix}\frac{9}{2}^+ \\ \frac{15}{2}^+\end{pmatrix}$
5.781							
5.931			$(\frac{3}{2}^+)$	12.05 [e]			$(\frac{9}{2}^+)$
5.967	$\frac{3}{2}^-$		$\frac{3}{2}^-$	12.33 [e]			$\begin{pmatrix}\frac{9}{2}^+ \\ \frac{15}{2}^+\end{pmatrix}$
6.043	$\frac{7}{2}^-$	$\frac{1}{2}^-$	$\frac{7}{2}^-$				
6.117	$\frac{11}{2}^+$		$\frac{11}{2}^+$	12.54 [e]			$(\frac{15}{2}^+)$
6.191				12.82 [e]			$(\frac{13}{2}^+)$
6.236	$\frac{13}{2}^+$	$\frac{3}{2}^+$	$\frac{13}{2}^+$	12.92 [e]			$(\frac{11}{2})$
6.263				13.05 [e]			$\begin{pmatrix}\frac{13}{2}^+ \\ \frac{17}{2}^+\end{pmatrix}$
6.350	$\frac{9}{2}^-$	$\frac{1}{2}^-$	$\frac{9}{2}^-$				
6.577		$(\frac{1}{2}^+)$	$(\frac{9}{2}^+)$	13.21 [e]			$(\frac{13}{2}^+)$ or $(\frac{11}{2}, \frac{9}{2})$
7.1 (multiplet)				13.72 [e]			$\begin{pmatrix}\frac{17}{2}^+ \\ \frac{15}{2}^+\end{pmatrix}$
7.267	$\frac{13}{2}^+$		$\frac{13}{2}^+$				
7.386	$(\frac{1}{2}, \frac{3}{2})^-$		$(\frac{1}{2}, \frac{3}{2})^-$	14.08 [e]			
7.410			$(\frac{9}{2}, \frac{11}{2})$	14.38 [e]			$(\frac{11}{2}, \frac{13}{2})$
7.446	$(\frac{3}{2})^+$		$(\frac{3}{2})^+$	14.44 [e]			$\begin{pmatrix}\frac{15}{2}^+ \\ \frac{11}{2}^+\end{pmatrix}$
7.68							
7.84 (multiplet)				14.98 [e]		$(\frac{3}{2}^+)$	$(\frac{19}{2}^+)$
7.966			$\begin{pmatrix}\frac{7}{2}, \frac{9}{2} \\ \frac{1}{2} \\ \frac{1}{2}\end{pmatrix}$	15.45 [e]		$(\frac{1}{2}^-)$	$\begin{pmatrix}\frac{15}{2}^- \\ \frac{15}{2}^+\end{pmatrix}$
7.983							
8.060				15.90 [e]		$(\frac{1}{2}^-)$	$\begin{pmatrix}\frac{17}{2}^- \\ \frac{15}{2}^+\end{pmatrix}$
8.32			$(\frac{11}{2}, \frac{13}{2})$ or multiplet	15.98 [e]			$(\frac{19}{2}^+)$

TABLE I. (*Continued*)

E_c [a] (MeV)	Known [b] J^π	Band [c] K^π	Present [d] J^π	E_x [a] (MeV)	Known [b] J^π	Band [c] K^π	Present [d] J^π
8.48 (multiplet)				16.32 [e]			$\left(\begin{array}{c}\frac{15}{2}^+\\ \frac{13}{2}^-\end{array}\right)$
8.64 (multiplet)							
8.799 } 8.822			$(\frac{5}{2},\frac{7}{2})$	16.60			$(\frac{19}{2}^+,\frac{17}{2}^-)$
9.038	$\frac{15}{2}^+$	$(\frac{3}{2}^+)$	$\frac{15}{2}^+$				
9.18 (multiplet)							

[a] Excitation energies from Refs. 11 and 12 except where noted.
[b] See Ref. 11.
[c] See Ref. 24.
[d] Spins suggested in present work.
[e] Excitation energies from present experiment.

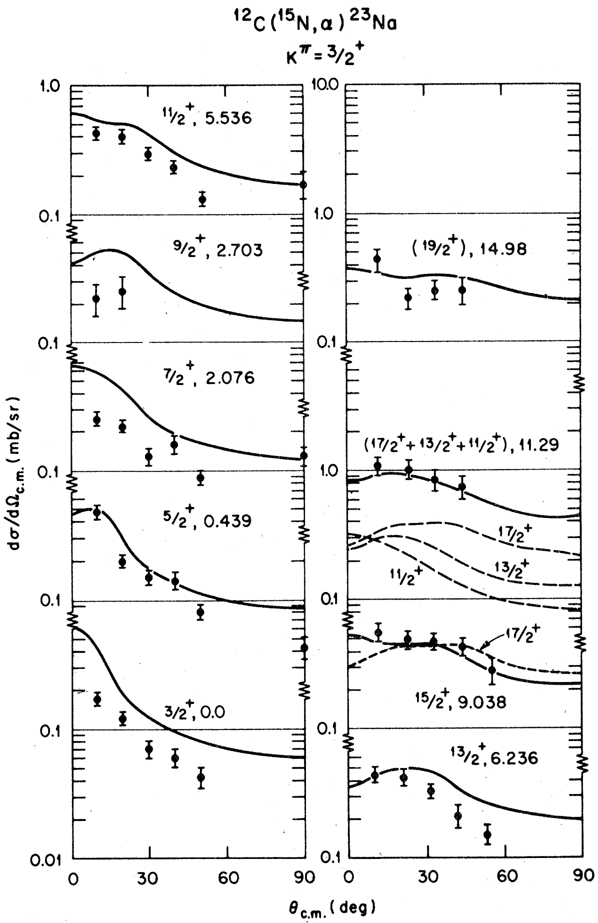

FIG. 2. Energy averaged experimental (36–39.2 MeV) and HF calculated angular distributions for the known and proposed members of the $K^\pi = \frac{3}{2}^+$ rotational band of ^{23}Na. The lines indicate the HF predicted $\sigma(\theta)$ with the solid line indicating that obtained with the preferred spin.

determine the value of a by A/x where A is the mass value of the residual nucleus for a given channel and x is a variable. Comparing the predicted with the experimental cross sections gave a least squares best fit value of $a = A/8.2$ when using this parameter for the n, p, d, and α channels. Similar calculations including other channels such as for t, ^{3}He, Li, and Be emission indicated that 99% of the cross section is contained in the n, p, d, and α channels. The level density parame-

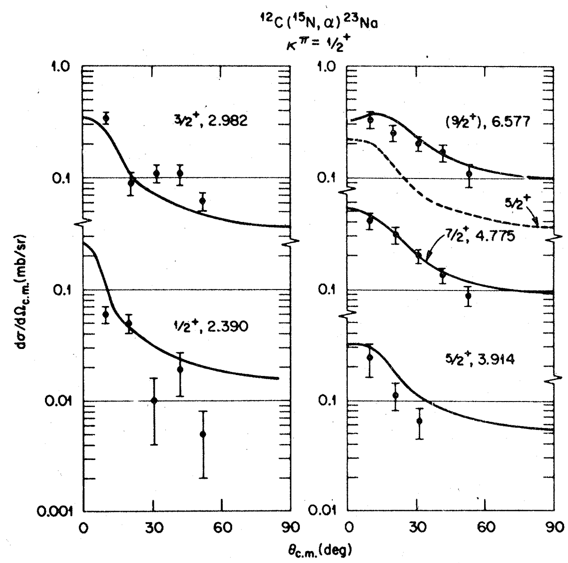

FIG. 3. Energy averaged experimental (36–39.2 MeV) and HF calculated angular distributions for the known and proposed members of the $K^\pi = \frac{1}{2}^+$ rotational band of ^{23}Na. The lines indicate the HF predicted $\sigma(\theta)$ with the solid line indicating that obtained with the preferred spin.

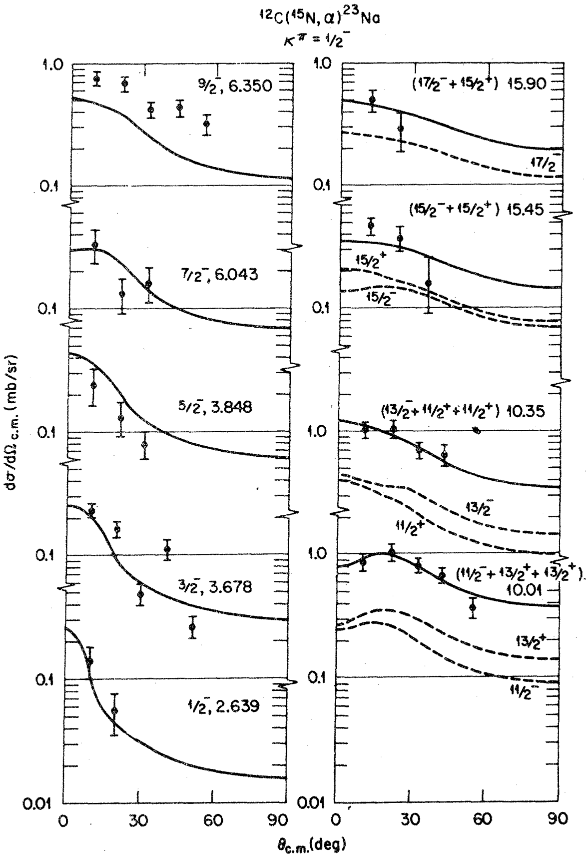

FIG. 4. Energy averaged experimental (36–39.2 MeV) and HF calculated angular distributions for the known and proposed member of the $K^\pi = \frac{1}{2}^-$ rotational band of ^{23}Na. The lines indicate the HF predicted $\sigma(\theta)$ with the solid line indicating that obtained with the preferred spin.

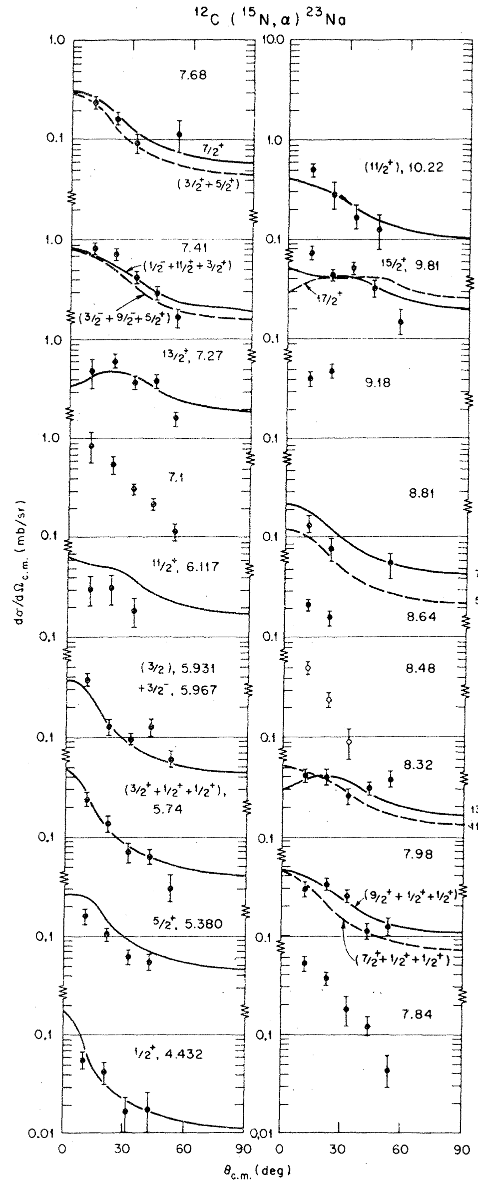

FIG. 5. Energy averaged experimental (36–39.2 MeV) and HF calculated angular distributions for several excited states in ^{23}Na with excitation energies from 4.432 to 10.22 MeV. The lines indicate the HF predicted $\sigma(\theta)$ with the solid line indicating that obtained with the preferred spin.

ters used for the various channels are shown in Table II.

IV. DISCUSSION

Hauser-Feshbach (HF) calculations were performed using the parameters previously discussed for comparison with the energy averaged experimental cross sections. This comparison suggests higher-spin members of the extensively stud-

TABLE II. Level density parameters for the ^{12}C(^{15}N, α)^{23}Na reaction.

	^{15}N + ^{12}C	^{26}Al + n	^{26}Mg + p	^{23}Na + α	^{25}Mg + d
a	3.41	3.17 [a]	3.17 [a]	2.80 [a]	3.05 [a]
Δ [b]	2.25	0.0	4.26	2.67	2.46
E_c (MeV)	10.40	3.75	6.90	16.92	7.00
No. of discrete levels	5	23	20	310	41

[a] Best fit $a = A/8.2$.

[b] Values from Ref. 30.

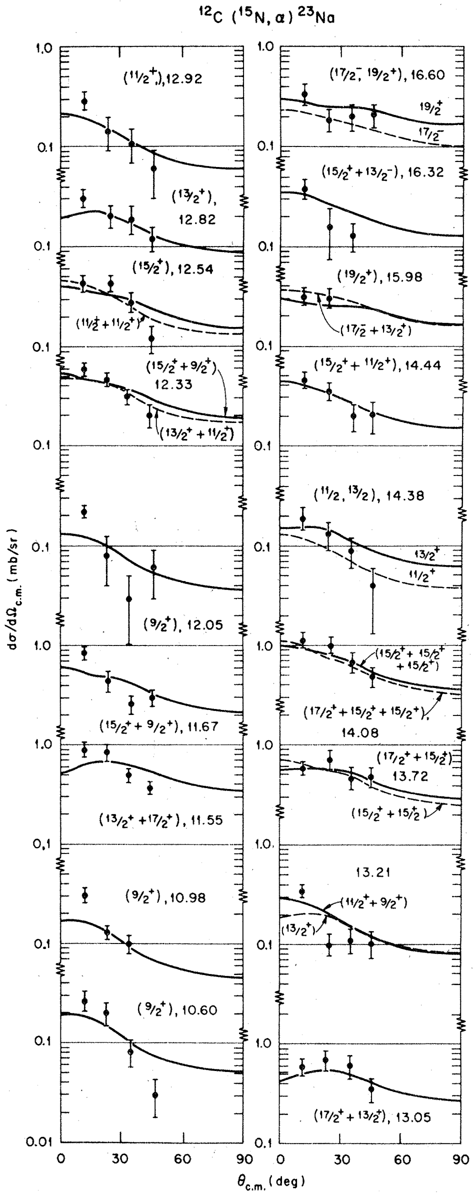

FIG. 6. Energy averaged experimental (36–39.2 MeV) and HF calculated angular distributions for several excited states in ^{23}Na with excitation energies from 10.60 to 16.60 MeV. The lines indicate the HF predicted $\sigma(\theta)$ with the solid line indicating that obtained with the preferred spin.

ied[18–24] $K^{\pi}=\frac{3}{2}^{+}$, $\frac{1}{2}^{+}$, and $\frac{1}{2}^{-}$ rotational bands.

Recent measurements by Gomez del Campo *et al.*[25] indicate resonant structure in the excitation functions of the ^{12}C(^{15}N, α)^{23}Na reaction. However, in the energy region studied here, $E_{\rm lab}=36$–39.2 MeV, the resonantlike structure was not pronounced,[25] and furthermore, averaging the cross sections with energy should minimize any effects due to nonstatistical processes. Although some of the angular distributions show structure which may be due to insufficient averaging of the data (see, e.g., those for the 2.390 and 2.982 MeV states in Fig. 3), in general the shapes and relative magnitudes of the angular distributions for different states are compatible with the statistical model. Thus comparison between the present data and calculations should yield spin values to within the usual 1–2$\hbar$ for high-spin states.

A. $K^{\pi}=\frac{3}{2}^{+}$ rotational band members

The $\sigma(\theta)$ for the known and proposed members of the $K^{\pi}=\frac{3}{2}^{+}$ band are displayed in Fig. 2. With the help of shell-model predictions,[10,26–28] we are able to suggest members of this band through the $\frac{19}{2}^{+}$ state. The spins of all band members through the $\frac{13}{2}^{+}$ member at 6.236 MeV are known.

The recent work of KeKelis *et al.*[6] establishes a $\frac{15}{2}^{+}$ spin for both the 9.04 and 9.81 MeV states which is consistent with our earlier work.[9] Our present calculations also favor a $\frac{15}{2}^{+}$ spin for the 9.81 MeV state rather than $\frac{17}{2}^{+}$ as suggested by several other groups,[2,3,5] since the shape of the curve for a spin of $\frac{15}{2}^{+}$ more closely resembles the data as seen in Fig. 5. It is not obvious, however, which $\frac{15}{2}^{+}$ state, 9.04 or 9.81 MeV, is a member of the ground state band. In fact, the recent shell-model calculations of Cole *et al.*[28] indicate that the $\frac{15}{2}^{+}$ strength of the $K^{\pi}=\frac{3}{2}^{+}$ band is divided between the two levels. This group has shown[27] that members of rotational bands have similar subshell occupancies, thus allowing the possibility of determining the rotational band assignment. Both $\frac{15}{2}^{+}$ states are predicted by Wildenthal[10] (at 9.06 and 9.81 MeV), and there are few differences between the theoretical predictions.[10,28] Cole *et al.*[28] point out that their calculations indicate considerable fragmentation above the $\frac{13}{2}^{+}$ member for the ground state band. Thus the $\frac{3}{2}^{+}$ band members with spins above $\frac{13}{2}^{+}$ shown in Figs. 2 and 7 must be considered as tentative. Further experimental and theoretical work needs to be performed in order to clarify the fragmentation of high-spin states among the rotational bands. For purposes of displaying possible band members in Fig. 2, we have chosen the lowest excitation energy of a given J^{π} to be the ground state rotational band member.

Evers *et al.*[7] have examined high-spin states in the mirror nuclei ^{23}Mg and ^{23}Na. Their data favor a $\frac{17}{2}^{+}$ assignment for both the 9.61 MeV state in ^{23}Mg and its mirror state at 9.81 MeV in ^{23}Na. They question whether the 4.27 MeV γ ray observed by KeKelis *et al.*[6] really occurs between the 9.81 MeV→ 5.54 MeV ($\frac{11}{2}^{+}$) states. If such a transition does occur, a $\frac{17}{2}^{+}$ assignment would be unlike-

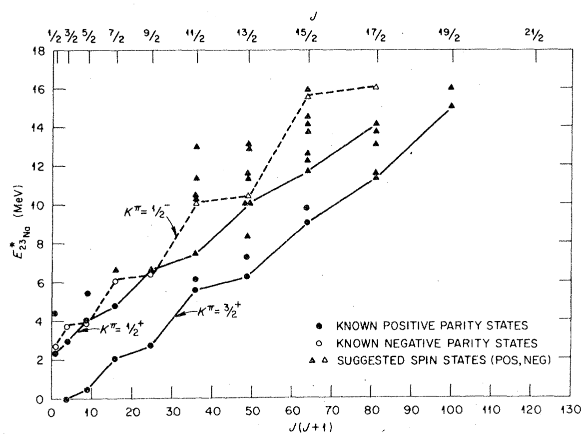

FIG. 7. A plot of the excitation energies for the known and proposed members of the $K^\pi = \frac{3}{2}^+$, $\frac{1}{2}^+$, and $\frac{1}{2}^-$ band in ^{23}Na versus $J(J+1)$.

ly.

We suggest that the lowest $\frac{17}{2}^+$ and $\frac{19}{2}^+$ states may be at 11.29 and 14.98 MeV, respectively. The peak at 11.29 MeV clearly appears in our high resolution data as a multiplet consisting of at least two states and possibly three. It was also necessary to include three levels in the calculations in order to fit the experimental $\sigma(\theta)$, and very good agreement was obtained with a triplet with spins $\frac{17}{2}^+$, $\frac{13}{2}^+$, and $\frac{11}{2}^+$. Undoubtedly, other spin combinations would work, but a $\frac{17}{2}^+$ state is needed to obtain the correct shape of the angular distribution. The individual contributions for the $\frac{17}{2}^+$, $\frac{13}{2}^+$, and $\frac{11}{2}^+$ spins are given in Fig. 2 to show the possible combinations that could fit the data. The $\frac{17}{2}^+$ suggestion for this level is consistent with our earlier work[9] and with shell-model calculations[10] which predict that the lowest $\frac{17}{2}^+$ state is at 10.92 MeV.

The lowest $\frac{19}{2}^+$ state predicted[10] by the shell model is at 14.78 MeV. The shape of $\sigma(\theta)$ for a $\frac{19}{2}^+$ state in this energy region is quite characteristic with a secondary maximum near 35°. The $\frac{19}{2}^+$ state is possibly located at 14.44 or 14.98 MeV. For the latter state there is better agreement between the HELGA predictions and the data for spin $\frac{19}{2}$, whereas the peak at 14.44 MeV seems better fitted with a $\frac{15}{2}^+ + \frac{11}{2}^+$ doublet (see Fig. 6).

A graph of excitation energy versus $J(J+1)$ for the ground state band is shown in Fig. 7. The average behavior of the ground state band can be described by a moment-of-inertia parameter $\hbar^2/2\mathscr{G}$ of 150 keV. This moment-of-inertia parameter is lower than that of 240 keV used by Frank *et al.*[24] in their Nilsson model calculation. However, in that work the low spins were predicted at too low energies and the high spins at slightly too high energies. Since their calculations stopped with the $\frac{15}{2}^+$ state, it would be interesting to extend the calculations to higher energies. Our value of $\hbar^2/2\mathscr{G}$ is consistent with the value of 156 keV obtained for ^{23}Mg by Evers *et al.*[7] for $J^\pi \leq \frac{13}{2}^+$ and with the value of 180 keV determined by KeKelis *et al.*[6] for ^{23}Na. For excitation energies above the $J^\pi = \frac{13}{2}^+$ member, Evers *et al.* find a value of 133 keV, but this results from a difference in location of the higher spins because of their $\frac{17}{2}^+$ spin state assignment.

The Nilsson and shell models adequately predict the location of the yrast levels through the $\frac{13}{2}^+$ state at 6.24 MeV. Frank *et al.*[24] conclude that their Nilsson model calculations adequately explain their data including γ-ray transitions. Transition strengths, branching ratios, lifetimes, and mixing ratios measured in the ^{12}C(^{12}C,$p\gamma$)^{23}Na reaction were compared with the Nilsson and shell-model predictions by KeKelis *et al.*[6] The highly sophisticated shell-model calculation of Wildenthal[10] compared more favorably with the data. The attractiveness of the Nilsson model is in its simplicity, but in order to compete with the shell model, more drastic changes in the Nilsson model, such as changing the deformation and moment-of-inertia parameters within bands, would be needed.

B. $K^\pi = \frac{1}{2}^+$ rotational band members

The known members of this band are displayed in Fig. 3 and include states at 2.390 ($\frac{1}{2}^+$), 2.982 ($\frac{3}{2}^+$), 3.914 ($\frac{5}{2}^+$), and 4.775 MeV ($\frac{7}{2}^+$). Lindgren *et al.*[22] limit the spin of the 6.577 MeV state to ($\frac{5}{2}$, $\frac{9}{2}$). A spin of $\frac{9}{2}^+$ clearly fits our data (see Fig. 3) and is consistent with the expected location of the $\frac{9}{2}^+$ member of the $K^\pi = \frac{1}{2}^+$ band.

The locations of the higher members of the $K^\pi = \frac{1}{2}^+$ band are less certain. From rotational band systematics, one would expect the $\frac{11}{2}^+$ member to be near 7.6 MeV. Besides the ground state rotational band member, Wildenthal[10] predicts $\frac{11}{2}^+$ states at 6.33, 7.20, 7.70, and 9.04 MeV. An $\frac{11}{2}^+$ state has been identified by KeKelis *et al.*[6] at 6.114 MeV, but this state's energy is too low to be a member of the $\frac{1}{2}^+$ band. We identify possible $\frac{11}{2}$ states in a broad multiplet near 7.41 MeV, and at 7.68, 7.84, and 8.32 MeV (see Fig. 5). Systematics and comparison with Hauser-Feshbach calculations suggest the state near 7.41 MeV as the most likely candidate for the $\frac{11}{2}^+$ member.

Two peaks observed near an excitation energy of 10 MeV may contain the $\frac{13}{2}^+$ member of the $K^\pi = \frac{1}{2}^+$ band. In our better resolution data, we see three and possibly four large peaks near 10 MeV. We are unable to extract $\sigma(\theta)$ for each because of large experimental uncertainties, but Fig. 4 displays the combined cross sections fitted with states of spin $\frac{11}{2}^-$, $\frac{13}{2}^+$, and $\frac{13}{2}^+$. Although we cannot be certain how many levels actually comprise peaks such as that observed near 10 MeV and the choice of spins chosen to fit the experimental angular distribution is certainly not unique, the fits given as multiplets do illustrate the number of levels and magnitudes of the spins required in order to obtain satisfactory fits to the data. The shell model does predict[10] $\frac{13}{2}^+$ states at 9.70 and 10.03 MeV as well as at lower energies (see Table I in Ref. 9).

The $\frac{15}{2}^+$ member is expected to be near 12 MeV from the $\frac{1}{2}^+$ band systematics, and the shell model predicts three $\frac{15}{2}^+$ states between 11.5 and 12.5 MeV. Possible $\frac{15}{2}^+$ states lie at 11.55, 11.67, 12.33, and 12.54 MeV (Fig. 6), although we cannot determine which state might be the $\frac{15}{2}^+$ member of the $K^\pi = \frac{1}{2}^+$ band.

The shell model predicts[10] $\frac{17}{2}^+$ states at 11.58, 12.84, 14.51, and 14.84 MeV. Peaks corresponding to excitation energies of 11.55, 13.05, 13.72, and 14.08 MeV (see Fig. 6) may contain $\frac{17}{2}^+$ states. Although with increasing excitation energy it becomes increasingly difficult to suggest high-spin values, states at 13.72 or 14.08 MeV appear most likely to be the $\frac{17}{2}^+$ member of the $K^\pi = \frac{1}{2}^+$ band.

The moment-of-inertia parameter $\hbar^2/2\mathscr{I}$ for the $\frac{1}{2}^+$ band is 150 keV in agreement with the ground state band. The moment of inertia $\mathscr{I}$ seems to be increasing slightly at higher excitation energies if our $\frac{15}{2}^+$ and $\frac{17}{2}^+$ spin suggestions are correct. This $K^\pi = \frac{1}{2}^+$ band is based on the Nilsson $\frac{1}{2}^+$ [211] orbital No. 9. The Nilsson model calculations of Frank *et al.*[24] predict increasingly higher excitation energies for the band than the data indicate. This is undoubtedly the result of the large value of $\hbar^2/2\mathscr{I}$ (240 keV) used for their band-mixing calculations.

C. $K^\pi = \frac{1}{2}^-$ rotational band

The $K^\pi = \frac{1}{2}^-$ rotational band is extremely interesting as suggestions have been made that it is due to strong coupling in the Nilsson model by raising a proton from the $\frac{1}{2}^-$ [101] orbital No. 4 to the $\frac{3}{2}^+$ [211] orbital No. 7. Alternately in terms of the weak coupling model, one can imagine a $1p_{1/2}^{-1}$ proton hole coupled to the ^{24}Mg ground state band.

From early work[18] it appeared that states at 2.64 ($\frac{1}{2}^-$), 3.68 ($\frac{3}{2}^-$), and 3.85 MeV ($\frac{5}{2}^-$) might form a rotational band that could be explained by the Nilsson model. Powers *et al.*[20] performed a Nilsson model calculation and concluded that it could not describe the low-lying negative-parity states of ^{23}Na. Their calculations indicated the orbital No. 4 hole states were too high in energy, and that one should expect to form lower-lying states from raising a proton to orbital No. 14.

They suggested fitting the experimental data by using different deformations and *ad hoc* mixing between bands. The latter result may not be unreasonable. Pilt[8] showed that the Nilsson model correctly predicts the location of the low-lying states when isospin dependence is considered.

Middleton *et al.*,[29] De Meijer,[31] and Pilt[8] point out that the low-lying states in ^{23}Na appear to be 8p-1h states based on removing a $1p_{1/2}$ proton from the ^{16}O core and placing it in the higher shells. Since the ^{24}Mg ground state band appears to be 8p-0h states, the corresponding states in ^{23}Na are a hole coupled to ^{24}Mg. In Fig. 8 we show the similarity between the ^{24}Mg ground state band and the $K^\pi = \frac{1}{2}^-$ band of ^{23}Na. We have performed a fit of the known $\frac{1}{2}^-$ band members ($\frac{1}{2}^-, \frac{3}{2}^-, \frac{5}{2}^-, \frac{9}{2}^-$) with the relation

$$E = \epsilon + \frac{\hbar^2}{2\mathscr{I}}\left[J(J+1) + (-1)^{J+1/2}(J+1/2)a'\right], \quad (1)$$

where $\hbar^2/2\mathscr{I}$ is the moment-of-inertia parameter and a' is the decoupling parameter. The best fit was obtained for $\hbar^2/2\mathscr{I} = 179$ keV and $a' = 0.85$. The predictions for the unknown states were then: $\frac{7}{2}^-$ (6.12 MeV), $\frac{11}{2}^-$ (10.01 MeV), $\frac{13}{2}^-$ (10.36 MeV), $\frac{15}{2}^-$ (15.33 MeV), and $\frac{17}{2}^-$ (15.78 MeV). The energy

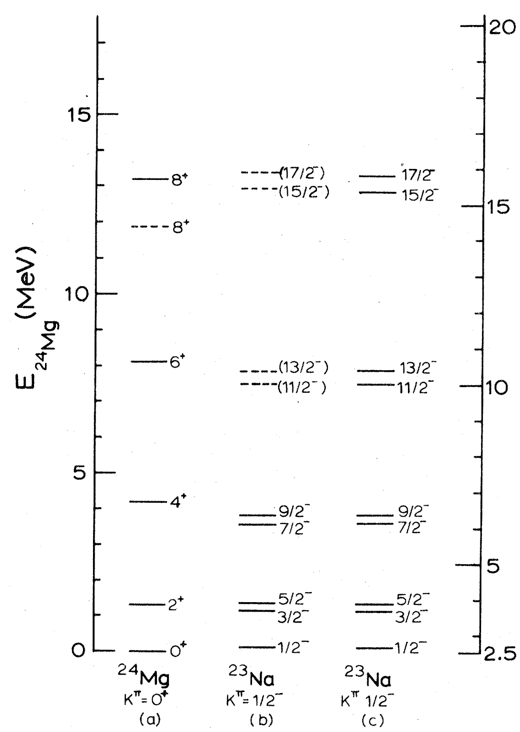

FIG. 8. Comparison of the normalized excitation energies for (a) the ground state members of ^{24}Mg, (b) known and proposed members of the $K^\pi = \frac{1}{2}^-$ band of ^{23}Na, and (c) predicted energies from Eq. (1) in the text with $\hbar^2/2\mathscr{I} = 179$ keV and $a' = 0.85$.

of the $\frac{19}{2}^-$ state would be above 22 MeV. These predictions are shown in Fig. 8 in column c. Interestingly enough, the $\frac{7}{2}^-$ state was previously believed[24] to be located at 6.04 MeV and the $\frac{13}{2}^-$ state at 10.35 MeV, remarkably close to the values obtained with our best fit. Notice that the $\frac{15}{2}^-$ and $\frac{17}{2}^-$ states appear to be coupled to the second 8^+ state in ^{24}Mg rather than to the first. Recently, Watt, Kelvin, and Whitehead[26] pointed out that from the subshell populations in their shell-model calculation the second 8^+ state is the ground state member and that the band probably terminates with that state. If the weak coupling picture is correct, the $K^\pi=\frac{1}{2}^-$ band would then terminate with the $\frac{17}{2}^-$ member.

Our angular distributions for the $K^\pi=\frac{1}{2}^-$ band members are shown in Fig. 4. The $\frac{7}{2}^-$ state at 6.043 MeV is so close to the expected energy of 6.12 MeV that it probably is a member of the $K^\pi=\frac{1}{2}^-$ band. No other nearby states are likely candidates, although there are unassigned spin states near 5.74 and 6.24 MeV. The peak at 10.01 MeV in our data is quite large and appears to contain at least three peaks in our higher resolution data. The ^{12}C(^{12}C,p)^{23}Na reaction data of Frank *et al.*[24] also show strong peaks near 10 MeV. Since no other nearby states have angular distributions with the correct shape, and since we predict 10.01 MeV to be the correct location for an $\frac{11}{2}^-$ state, we believe one of the states near 10 MeV is the $\frac{11}{2}^-$ band member.

Candidates for the $\frac{15}{2}^-$ and $\frac{17}{2}^-$ states are difficult to suggest because of the high density of $\frac{13}{2}^+, \frac{15}{2}^+, \frac{17}{2}^+$ states above 15 MeV. Locations predicted by the shell model for the positive-parity states and by the rotational model predictions for the $\frac{15}{2}^-$ and $\frac{17}{2}^-$ states have guided our attempts to account for all the predicted high-spin states in making our suggestions for spin values. The most likely candidates for the $\frac{15}{2}^-$ and $\frac{17}{2}^-$ members of the $K^\pi=\frac{1}{2}^-$ band are at 15.45 and 15.90 MeV, respectively. The data in Fig. 4 are fitted as doublets. The known and proposed members of the $K^\pi=\frac{1}{2}^-$ band are shown in solid and dashed lines, respectively, in column b of Fig. 8.

D. Other states in ^{23}Na

Angular distributions for the states that have not been proposed as members of one of the three rotational bands ($K^\pi=\frac{3}{2}^+, \frac{1}{2}^+, \frac{1}{2}^-$) are shown in Figs. 5 and 6. Several of these states have been mentioned previously, especially as possible members of the $K^\pi=\frac{1}{2}^+$ band.

The method of suggesting possible spin values was similar for all the states. Attempts were made to fit the experimental shape using first a single level, and if completely unsuccessful, then a multiplet. For several states one particular spin produced an obviously better fit. For some states, especially the multiplets, one or more spins (or combinations) produced about equal fits. The fits given as multiplets illustrate only the number of states and magnitude of the spin values required to obtain fits for some of the large peaks observed in the data. The energy resolution of the present experiment was sufficient to definitely indicate that some of these peaks, such as those at 10.01 and 11.67 MeV, were in reality multiplets. However, others such as that observed at 10.35 MeV appeared to be largely due to a single level at any given bombarding energy. The fact that angular distributions needed to be fitted as multiplets may indicate either that the present experimental resolution was indeed limited, or that while a single level may be dominant at any particular incident energy, the energy average may nevertheless include the effects of several levels since their strengths fluctuate rapidly, or lastly that resonant components have not been completely eliminated by the averaging interval.

Despite the problems associated with fitting angular distributions at high excitation energies, many of the fits are quite informative. For example, a good fit was obtained for the 7.27 MeV state to which KeKelis *et al.*[6] have assigned a spin of $\frac{13}{2}^+$. Similarly, the multiplet at 5.74 MeV was fitted well by assuming three states of $\frac{3}{2}^+$, $\frac{1}{2}^+$, and $\frac{1}{2}^+$ spins. Moss[12] has found three states near this energy. Krämer *et al.*[19] assigned a $\frac{3}{2}^-$ spin to the 5.967 MeV state. We fit the 5.931-5.967 MeV doublet by using a spin of $\frac{3}{2}$ for the 5.931 MeV state. The peak at 7.41 MeV has already been discussed in connection with the $K^\pi=\frac{1}{2}^+$ band. Two of the states are believed to have low spin,[11] $(\frac{1}{2},\frac{3}{2})^-$ for the 7.39 MeV level and $(\frac{3}{2},\frac{5}{2})^+$ for the 7.45 MeV level. Assuming a spin of $\frac{9}{2}$ or $\frac{11}{2}$ for the third member of a triplet provides an adequate fit to our data.

The best fits obtained for the remaining angular distributions (and the spin values assumed) are shown in Figs. 5 and 6. We have suggested many more high-spin positive-parity states than negative ones. This is because the structure of the $2s$-$1d$ shells allows only positive-parity states. Negative-parity states result from hole states or highly excited states in, for example, the $f_{7/2}$ shell. The lowest-lying $\frac{15}{2}^+$ state is believed to be at 9.04 MeV, whereas the lowest $\frac{15}{2}^-$ state is not suggested until 15.45 MeV from our $K^\pi=\frac{1}{2}^-$ band systematics.

V. CONCLUSIONS

In this investigation three rotational bands of ^{23}Na have been examined with emphasis on their high-spin members. Suggestions have been made for members of the $K^{\pi}=\frac{3}{2}^{+}$ ground state band through the $\frac{19}{2}^{+}$ state although theoretical calculations[28] indicate considerable fragmentation above the $\frac{13}{2}^{+}$ member. The members of the $\frac{1}{2}^{+}$ band are less certain, but suggestions through the $\frac{9}{2}^{+}$ member appear reasonable.

The interesting $K^{\pi}=\frac{1}{2}^{-}$ band shows significant coupling effects, and suggestions are made for the members of this band up through the $\frac{17}{2}^{-}$ member, which may be the end of the band if indeed an 8^{+} state is the end of the ^{24}Mg ground state band. The states in both ^{24}Mg and ^{23}Na need further study in order to test the weak coupling picture.

Many other spin suggestions were proposed for excited states in ^{23}Na which should be useful in studying other rotational bands in ^{23}Na. The shell model has been quite successful in predicting spins for positive-parity spin levels. The simple Nilsson model is useful, but would require complex calculations including different deformations and moment-of-inertia parameters for some bands in order to be as successful as the shell model. The moment of inertia parameters $\hbar^2/2\mathscr{I}$ determined in the present work for the $K^{\pi}=\frac{3}{2}^{+}$, $\frac{1}{2}^{+}$, and $\frac{1}{2}^{-}$ bands are 150, 150, and 179 keV, respectively. These values are generally lower than those previously determined and reflect the influence of the location of the higher-spin band members.

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