Asymptotic normalization of the deuteron D state

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(Received 7 September 1977)

The ratio of the D-state to S-state asymptotic normalization of the deuteron wave function provides a model independent measure of the deuteron D state. We show how that ratio can be determined directly from experimental data by extrapolating the tensor polarization $T_{2\nu}$ in p-d scattering to the neutron exchange pole.

[NUCLEAR STRUCTURE Method proposed for determining asymptotic strength of deuteron D state.]

It is well known that the deuteron is not pure S wave, but an unambiguous model independent description of the D-wave part does not exist. Attempts to quantify the D state in terms of a single number usually use P_D, the D-state probability.[1] However, the wide range (factor of 2) of values determined for this quantity[2] show that P_D is not in fact an observable of the deuteron. In this note we advocate the use of a different single quantity to "measure" the D state—its asymptotic normalization. We show how that quantity can be extracted from experiment in a model independent way, we give a preliminary determination of it based on presently available data, and we comment on experimental improvements needed to obtain a precise value.

The quantity we study is the ratio ρ_D of the asymptotic (large r) part of the D-wave deuteron wave function to the S-wave part. This is equivalent to the D-wave fraction of the $d \rightleftarrows n+p$ vertex at the (unphysical) on-shell point, or to the D-wave part of the $d \rightleftarrows n+p$ "coupling constant." As with all such "coupling constants," its value can be found from the residue at the pole of some amplitude outside the physical region, but quite close to it, for problems involving the weakly bound deuteron. The nearness of poles for other few-body systems such as ^{3}H or ^{3}He can similarly be used to extract the asymptotic normalization of wave function components.

Most two-body models of the deuteron give $\rho_D = 0.026 \pm 0.002$ with little variation[3] and Wong[4] has shown that a similar value follows from dispersion theory using only one-pion exchange and the value of the deuteron binding energy, but a direct measurement of ρ_D has not been made. Knutson and Häberli[5] have tried to extract a related quantity from polarization measurements in sub-Coulomb (d,p) reactions. Their result can be interpreted as giving a slightly smaller value of ρ_D but it is not a direct determination of ρ_D.

Reactions sensitive to ρ_D in leading order must involve tensor polarization. As an example, we study here the tensor polarization $T_{2\nu}$ in elastic p-d scattering. The pole is the well known neutron exchange pole. To lowest order (in ρ_D) the pole contribution to $T_{2\nu}$ is linear in ρ_D. That is, the leading contribution comes from interferences between one D-wave and three S-wave vertices. We define the vertex function of the deuteron in its rest frame as

$$\begin{aligned}\langle n\vec{k}m, p-\vec{k}m' | v_{np} | dM_d\rangle &= \gamma_S\Big[g_S(k^2)Y_{0,0}(\hat{k})\langle \tfrac{1}{2}m, \tfrac{1}{2}m' | 1M_d\rangle \\ &\quad - \rho_D \bar{k}^2 g_D(k^2) \sum_{\mu, M_S} Y_{2,\mu}(\hat{k})\langle \tfrac{1}{2}m, \tfrac{1}{2}m' | 1M_S\rangle \\ &\qquad \times \langle 1M_S, 2\mu | 1M_d\rangle\Big], \qquad (1)\end{aligned}$$

where $\hat{k}$ is a unit vector in the direction of the relative momentum $\vec{k}$, the form factors g_S and g_D are normalized to 1 at the on-shell point $k^2 = -MB$ (B the deuteron binding energy, M the nucleon mass), $\bar{k}^2$ is a normalized threshold factor $\bar{k}^2 = k^2/MB$, and m, m', and M_d are z components of spin. The definition in (1) agrees with the usual definition of ρ_D as the ratio of the asymptotic normalization of the D- and S-wave parts of the deuteron.[6] ρ_D has the same sign as the quadrupole moment and is therefore positive. The ratio of the residues from (1) is also positive since $\bar{k}^2 = -1$ at the on-shell point.

In terms of (1) and the definition of $T_{2\nu}$ (in the Madison convention[7]), we can calculate the contribution to $T_{2\nu}$ from the neutron exchange pole only. For center of mass scattering (from momentum $\vec{p}$ to $\vec{p}\,'$) we obtain

$$T^{(\text{pole})}_{2\nu} = -\frac{\rho_D\bar{k}^2(2+\rho_D\bar{k}^2/\sqrt{2}\,)}{1+\rho_D{}^2(\bar{k}^2)^2}\left(\frac{4\pi}{5}\right)^{1/2} Y^*_{2,\nu}(\hat{k}) , \qquad (2)$$

where $\vec{k}=\vec{p}'+\frac{1}{2}\vec{p}$. In all places we have evaluated the form factors g_S and g_D at the on-shell point where they are 1. The expression (2) is exact at the pole ($\bar{k}^2=-1$). In terms of the deuteron laboratory kinetic energy E, this pole comes at a cosine of the center of mass scattering angle, $z = z_p$, given by

$$z_p = -(\tfrac{5}{4}+\tfrac{9}{4}B/E) . \qquad (3)$$

Thus, to find ρ_D we must fit some angular distribution for $T_{2\nu}$, for example in Legendre polynomials of $\cos\theta_{\text{c.m.}}$, and extrapolate to $\cos\theta_{\text{c.m.}}=z_p$. The expression (2) for $T^{(\text{pole})}_{2\nu}$ has kinematic zeros arising from the $Y^*_{2\nu}$ factor for $\nu=0,1$ that are not also kinematic zeros of the full $T_{2\nu}$. These zeros are near z_p and make straightforward extrapolation difficult. $T^{(\text{pole})}_{22}$ vanishes (like $\sin^2\theta$) at $\theta_{\text{c.m.}}=0$ and 180°, but so does the full T_{22}. Hence, $T_{22}/\sin^2\theta_{\text{c.m.}}$ can be rather easily and smoothly extrapolated.

In Table I we show values of ρ_D obtained by fitting a sample of existing data for $T_{22}/\sin^2\theta_{\text{c.m.}}$ with Legendre polynomials in $\cos\theta_{\text{c.m.}}$ and extrapolating. Also shown is the order of polynomial required to fit the data. We see that although only low orders are required, the values of ρ_D are still very crude. The best we can say is that $\rho_D=0.05 \pm 0.05$. Far better data (an order of magnitude increase in accuracy), particularly at back angles, are required in order to obtain ρ_D to an interesting accuracy. At present the data are sparse, seldom go to angles larger than 150°, and are inconsistent among different experiments. Some feeling for the quality of the data, the nature of the fit, and the problems of extrapolation can be seen in Fig. 1 where the 10 MeV data for $T_{22}/\sin^2\theta_{\text{c.m.}}$ are shown plotted against $\cos\theta_{\text{c.m.}}$. Also shown is our fit with Legendre polynomials up to $L=3$. For 10 MeV z_p is -1.75.

Better data would warrant more sophisticated analysis that should include mapping techniques,[8] and perhaps removal of next nearest singularities in terms of on-shell nucleon-nucleon quantities.[9] It would also warrant investigation of Coulomb corrections (some of which cancel in ratios like

TABLE I. Values of ρ_D from extrapolation of measured T_{22}. E is the laboratory deuteron kinetic energy and $L_{\max}$ is the maximum Legendre polynomial used in the fit.

E (MeV)	χ^2/deg. of freedom	$L_{\max}$	ρ_D
6 [a]	1.2	2	0.03
8 [a]	1.7	3	0.06
10 [a]	1.3	3	0.05
11.5 [a]	4.3	3	0.08
12 [b]	1.5	3	0.15
14 [b]	1.2	4	−0.04
16 [b]	2.9	3	0.19

[a] R. E. White, W. Grüebler, V. König, R. Risler, A. Ruh, P. A. Schmelzbach, and P. Marmier, Nucl. Phys. A197, 273 (1972).

[b] G. G. Ohlsen and W. Grüebler, unpublished measurement from Los Alamos cited in *Proceedings of the Fourth International Symposium on Polarization Phenomena in Nuclear Physics*, edited by W. Grüebler and V. König (Birkhäuser, Basel, 1976), p. 485.

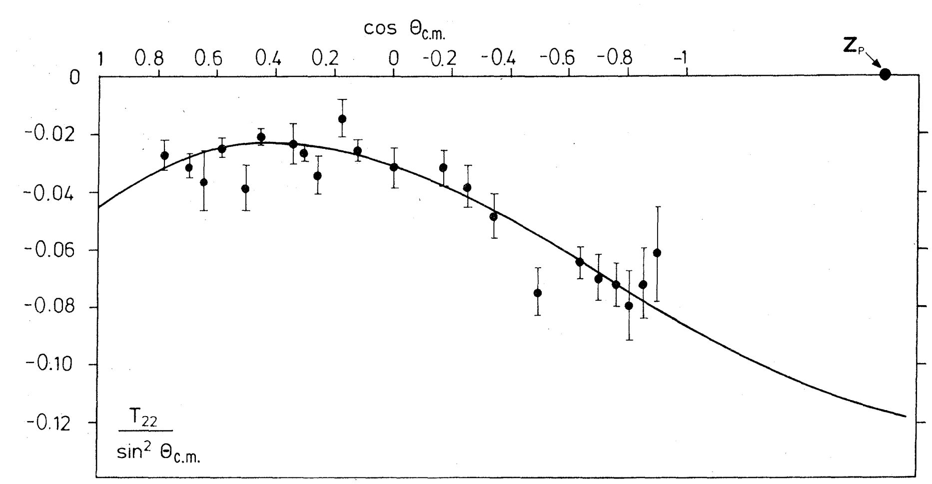

FIG. 1. $T_{22}/\sin^2\theta_{\text{c.m.}}$ for $E=10$ MeV (from the data of R. E. White *et al.* [Nucl. Phys. A197, 273 (1972)]) plotted against $\cos\theta_{\text{c.m.}}$. Also shown as a solid line is the best fit with $L_{\max}=3$ extrapolated to $z_p=-1.75$. The connection between the extrapolated values of $T_{22}/\sin^2\theta_{\text{c.m.}}$ and ρ_D is $(T_{22}/\sin^2\theta_{\text{c.m.}})_{\text{pole}}=E\,\rho_D 0.2495$ with E in MeV.

$T_{2\nu}$). The test of Coulomb corrections, of course, is comparison with n-d data. Coulomb interference effects can also be used to extract amplitudes to be used in fixed angle energy dispersion relations. These are capable, in general, of higher accuracy for ρ_D than the angle extrapolations.[8] Better data and more sophisticated analysis can also make use of T_{21} and T_{22}. We believe the challenge of determining a heretofore unmeasured deuteron property should stimulate the difficult experiments required, as well as the theoretical analysis that must accompany them.

In conclusion, we advocate that the ratio of D-wave to S-wave asymptotic normalization, ρ_D, in the deuteron be determined experimentally as a model independent measure of the D state. We show how this can be done by extrapolating polarization measurements. Present data are only good enough to indicate the feasibility of the method. We hope that our suggestion will generate interest in obtaining much better data, and in doing far more sophisticated analyses for a wide class of deuteron induced elastic scattering and reaction processes in order to obtain ρ_D.

ACKNOWLEDGMENTS

One of us (R.D.A.) is grateful to Dr. F. Lenz for arranging a most pleasant visit at the Swiss Institute for Nuclear Research and to Dr. Willem Hesselink for instruction in the use of numerical "least squares" methods.

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*Permanent address: Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania 19174. Supported in part by the NSF.

[1]For a review of the deuteron see D. W. L. Sprung in *Few Body Problem in Nuclear and Particle Physics*, edited by R. J. Slobodrian, B. Cujec, and K. Ramevataram (Les Presses de l'Université Laval, Québec, 1975, pp. 475–493.

[2]D. W. L. Sprung, in *Few Body Problem in Nuclear and Particle Physics* (see Ref. 1), Table II.

[3]The only fault we find with Sprung's otherwise excellent review is that he lists P_D under measured deuteron properties and ρ_D (called there AD/AS) under calculated. We would have them reversed.

[4]D. Y. Wong, Phys. Rev. Lett. 2, 409 (1959).

[5]L. D. Knutson and W. Häberli, Phys. Rev. Lett. 35, 558 (1975).

[6]In terms of the usual r space wave functions for the deuteron (1) corresponds to $u(r) \rightarrow \gamma_S e^{-x}$, $w(r) \rightarrow \gamma_S \rho_D \times k_2(x)$ as $x \rightarrow \infty$ for $x = \alpha r$ with $\alpha = (MB)^{1/2}$.

[7]M. Simonius, in *Lecture Notes in Physics, No. 30, Polarization Nuclear Physics*, edited by D. Fick (Springer Verlag, Berlin, 1974), pp. 38–87.

[8]For a review of these techniques and their accuracy see M. P. Locher and T. Mizutani, J. Phys. G. 4, No. 2 (1978).

[9]M. G. Fuda, Phys. Rev. C 14, 1336 (1976).