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Effective two-body interactions and energy spectra in $N = 29$ nuclei*

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By assuming a two-range central-plus-tensor potential, the effective residual interaction is determined by a least-squares fit with the low-lying energy spectra of $N = 29$, $21 \leq Z \leq 28$ nuclei. All the strengths are found to be almost the same for the proton-neutron and proton-proton interactions. The strengths are compared with Schiffer's results. The two-body matrix elements are in good agreement with the previous results carried out with a nonlocal potential and the Hamada-Johnston potential. The importance of these potentials seems to be overestimated.

[NUCLEAR STRUCTURE Calculated energy levels and effective interaction, shell model.]

I. INTRODUCTION

There are essentially three approaches in which the residual two-body interaction between valence nucleons has been studied. The first approach is to try and take into account the effect of truncation of the infinite set of basis states, which is caused from the infinite number of degrees of freedom of the valence nucleons, by using the reaction matrix approach.[1] The second approach is to use a phenomenological force for the residual interaction between nucleons.[2,3] Both approaches have their "good points" and their "bad points" and use several parameters which are adjusted to give a best fit to the experimental data. The third approach, that is applicable in a few restricted regions of the Periodic Table, is to do a least-squares fit to a selected set of experimental data, which are assumed to belong within one configuration.[4,5] In this approach, it is usual to treat the matrix elements of the residual interaction as free parameters which are adjusted to give best fit. Schiffer and his collaboraters[6-8] proposed a new and very suggestive method to further pursue this idea. In analyzing the energy spectra for nuclei in the immediate vicinity of closed shells, Anantaraman and Schiffer[8] found that the normalized matrix elements of the residual interaction are to a large extent independent of the configuration, i.e., of the nucleus. The similarity in different multiplets, manifest in the "angular distributions" and the multipole coefficients, suggests a kind of universality of the residual interaction all over the nuclear chart. Since then there have been several attempts to fit the energy levels by a unified form of residual interaction. The multipole analysis of Moinester, Schiffer, and Alford[6] suggests that an interaction potential with a long range should be considered in addition to the short-range component. Molinari *et al.*[9] tried to analyze the low-lying spectra of some simple nuclei in terms of a force with two components, a long-range core-mediate component and a short-range component of the δ-function type. These authors concluded that it is possible to build a simple residual effective interaction which is valid over the entire nuclear chart for nuclei with two particles outside closed shells. Recently, Schiffer and True[10] have tried to investigate systematically the relative importance of each force component with a two-range central-plus-tensor plus spin-orbit interaction. It is found that the inclusion of the two-body spin-orbit interaction does not improve the fitting. It is also revealed that the quality of fitting does not depend on the combinations of the specific values of the range very sensitively. The works mentioned above are restricted to the low-lying energy spectra for the nuclei in the vicinity of doubly-closed shells. Since the configuration space is restricted to one shell only, the diagonal two-body matrix elements of the residual interaction are the only ones needed to be included in their least-squares fit. One can hope that if a single residual interaction is a valid concept, then this interaction should also be applicable to those diagonal and off-diagonal matrix elements where considerable configuration mixing occurs. It would be interesting to investigate whether the two-range force suggested by Schiffer and True[10] could be used to systematically analyze the level spectra for the off closed shell nuclei. The configuration mixings for these states are expected to be important. Since one can use the experi-

mental excitation energies of the states with configuration mixings as well as those of the pure states in the least-squares fit, the nondiagonal matrix elements should be calculated in the same way as the diagonal ones.

It will be a good example to investigate the Schiffer potential with the nuclei of $N=29$ and $Z=21-28$. This is because the number of the protons increases linearly for $N=29$ nuclei, and, thus for the low-lying spectra of these nuclei, one needs the interactions for both the nonidentical and identical nucleons. In the conventional shell-model calculations of these nuclei ^{40}Ca, ^{48}Ca, and ^{56}Ni are usually to be assumed as inert cores. The ^{48}Ca inert core plays a slightly different role from the other two, because the numbers of protons and neutrons in ^{48}Ca are different from each other and, therefore, the protons and neutrons outside the ^{48}Ca core occupy different orbits. Hence, the problem of considering the nuclei with $N=29$ and $Z=21-28$, with ^{48}Ca being an assumed inert core, provides us the tool to investigate the correlation between the protons and the neutrons in different orbits. For the calculations of these nuclei, it is usual to neglect the components of the excitation of the protons in the $\pi f_{7/2}$ shell; thus, the $Z-20$ protons are restricted to the $\pi f_{7/2}$ shell, with the extra neutron in one of the $\nu p_{3/2}$, $\nu p_{1/2}$, and $\nu f_{5/2}$ shells.

For many years a great deal of effort has been expended attempting to obtain a clearer understanding of the $1f$-$2p$ shell nuclei. Wells[11] and Ohnuma and Sasaki[12] investigated the odd nuclei ^{50}Sc and ^{56}Co by assuming central forces between the proton and neutron. Ramavataram[13] applied the unified model to the nuclei ^{51}Ti, ^{53}Cr, and ^{55}Fe, in which the odd neutron-phonon interaction is considered as a variable parameter. Maxwell and Parkinson[14] calculated these three nuclei using a shell-model with only the central forces between the protons and neutron being considered. Benson and Johnstone[15] assumed ^{56}Ni as an inert core and calculated the low-lying neutron-hole states of ^{53}Cr and ^{55}Fe using truncation based on a modified weak-coupling model. The spectra of these two nuclei are also reproduced from the lowest shell-model configuration by Carola and Ohnuma.[16] Ohnuma[17] calculated the spectra of the nuclei with $N=29$, $22\leq Z\leq 26$ using the central p-n interactions. Horie and Ogawa[18] investigated the effective p-n interaction of the nuclei with $N=29$, $20\leq Z\leq 28$ using the matrix elements of the effective interaction between the $1f_{7/2}$ protons and $2p_{3/2}$, $2p_{1/2}$, or $1f_{5/2}$ neutrons as free parameters in the least-squares fit. Such a method was also employed by Vervier,[19] but in his investigation the neutron is restricted to the $2p_{3/2}$ orbit only.

In the works mentioned above, an oversimplified function is adopted for the effective p-n interaction. It is revealed in the calculation of Maxwell *et al.* that the single-neutron spectrum in ^{57}Ni is impossible to understand using only the central force. In Horie's calculation, 20 two-body matrix elements are treated as free parameters which are determined by a least-squares fit to the 38 observed energy levels. It seems that there are too many free parameters in view of the experimental data. Furthermore, in all the previous works, the two-body matrix elements of the effective interactions between protons, which are assumed to be restricted on the $\pi f_{7/2}$ shell, are directly taken from the experimental spectra of the nuclei with same value of Z, but with $N=28$. Hence, the details of the interaction are somewhat ambiguous. In addition to these unsatisfactory aspects, the experimental data for $N=29$ nuclei and $21\leq Z\leq 28$ have become more abundant in the past few years. It is, therefore, worthwhile now to investigate these nuclei in detail with a rather different treatment.

In the present work, we analyze the effective interaction by investigating the low-lying states of nuclei with $N=29$ and $21\leq Z\leq 28$. The calculations can be divided into two parts. In the first step, the form of the residual p-n interaction is assumed to be similar to that used by Schiffer *et al.*[10] except that the two-body spin-orbit component is excluded. For the radial dependence of the interaction potential, we have adopted two different types, the Gaussian form and the Yukawa form. Harmonic-oscillator wave functions are used for the single-particle wave functions, and the matrix elements of the residual p-p interaction are directly taken from the experimental data. In the second step, the effective p-n interactions obtained in the first step are then used to determine the effective p-p interaction. The p-p interaction is also assumed to be similar to that used by Schiffer *et al.*[10] Both Gaussian and Yukawa forms are again employed for the radial dependence of the p-p interaction with the same interaction ranges as in the p-n interaction. By a least-squares fit with the observed energy, the effective p-p interaction can be obtained in a similar manner as in the calculation of the first step. These effective p-n and p-p interactions are then used to reproduce the two-body matrix elements. The results are compared with those deduced from a nonlocal potential[20] and those obtained by Kuo.[21]

II. METHOD OF CALCULATION

As mentioned above, ^{48}Ca is assumed to be an inert core. The $Z-20$ protons are restricted to

the $1f_{7/2}$ shell and the extra active neutron is restricted to the $2p_{3/2}$, $2p_{1/2}$, and $1f_{5/2}$ shells. However, there is experimental evidence that these assumptions are too restrictive. The justification of the limited configuration space has been discussed in detail by Horie *et al.*[18] and by McGrory[22] in the calculation of $N=30$ nuclei. We hope that the effect of the restriction to these shells can be partly compensated or accounted for by the use of a more complicated effective potential.

The eigenvalue for each state in this configuration can be obtained by diagonalizing the Hamiltonian:

$$H = H_{\text{s.p.}} + \sum_{k<l} V_{kl} , \tag{1}$$

where $H_{\text{s.p.}}$ is the Hamiltonian of the single particle in the effective field of the core and V_{kl} represents the two-body interaction between nucleons outside the core. The single-particle energies of orbits $2p_{3/2}$, $2p_{1/2}$, and $1f_{5/2}$ are taken directly from the experimental values observed in ^{49}Ca:

$$\begin{aligned} \epsilon_n(2p_{3/2}) &= -5.144 \text{ MeV}, \\ \epsilon_n(2p_{1/2}) &= -3.116 \text{ MeV}, \\ \epsilon_n(1f_{5/2}) &= -1.186 \text{ MeV}. \end{aligned} \tag{2}$$

The matrix elements of the two-body interaction $\sum_{k<l} V_{kl}$ can be separated into two parts: one for p-n interaction and another for p-p interaction:

$$\begin{aligned} \left\langle j_p^n \alpha_1 J_1, j_n \middle| \sum_{k<l} V_{kl} \middle| j_p^n \alpha_1' J_1', j_n' \right\rangle_J \\ = \langle j_p^n \alpha_1 J_1, j_n | V_{pn} | j_p^n \alpha_1' J_1', j_n' \rangle_J \\ + \delta(J_1, J_1') \delta(j_n, j_n') \langle j_p^n \alpha_1 | V_{pp} | j_p^n \alpha_1' \rangle_{J_1} , \end{aligned} \tag{3}$$

where $j_p = 1f_{7/2}$; $j_n, j_n' = 2p_{3/2}$, $2p_{1/2}$, and $1f_{5/2}$; and

$$\begin{aligned} \langle j_p^n \alpha_1 J_1, j_n | V_{pn} | j_p^n \alpha_1' J_1', j_n' \rangle_J \\ = n \sum_{\alpha_2 J_2} \langle j_p^n \alpha_1 J_1 \{ | j_p^{n-1} \alpha_2 J_2, j_p \rangle \langle j_p^{n-1} \alpha_2 J_2, j_p | \} j_p^n \alpha_1' J_1' \rangle \\ \times \sum_{J_3} \langle J_2 j_p (J_1) j_n; J | J_2, j_p j_n (J_3); J \rangle \langle J_2, j_p j_n (J_3); J | J_2 j_p (J_1') j_n; J \rangle \langle j_p j_n | V_{pn} | j_p j_n \rangle_{J_3} . \end{aligned} \tag{4}$$

Since the proton is in one specific orbit while the neutron is in another, the two-body matrix element in the right-hand side of Eq. (4) can be further expanded as

$$\langle j_p j_n | V_{pn} | j_p j_n \rangle_J = \tfrac{1}{2} \{ \langle j_p j_n | V | j_p j_n \rangle_{J,0} + \langle j_p j_n | V | j_p j_n \rangle_{J,1} \} , \tag{5}$$

the second subscripts 0 and 1 are referred to the isospins. For the p-p interaction, the interaction matrix element can be written as

$$\langle j_p^n \alpha_1 | V_{pp} | j_p^n \alpha_1' \rangle_{J_1} = \tfrac{1}{2} [n(n-1)] \sum_{\alpha_2 J_2 J_3} \langle j_p^n \alpha_1 J_1 \{ | j_p^{n-2} \alpha_2 J_2, j_p^2 J_3 \rangle \langle j_p^{n-2} \alpha_2 J_2, j_p^2 J_3 | \} j_p^n \alpha_1' J_1 \rangle \langle j_p^2 | V_{pp} | j_p^2 \rangle_{J_3} , \tag{6}$$

with the maximum isospin on both sides.

Following Schiffer *et al.*,[10] the two-body matrix element $\langle j_1 j_2 | V_k | j_1' j_2' \rangle_{J,T}$ is assumed to be a two-range central-plus-tensor part. Both ranges of central force contain singlet-odd (CSO) and triplet-even (CTE) components for $T=0$, and singlet-even (CSE) and triplet-odd (CTO) components for $T=1$. The tensor force includes tensor-even (TTE) and tensor-odd (TTO) components. In the practical calculations, it was found that the inclusion of the TTE and TTO components of the long range does not make any significant improvement in the least-squares fitting. Thus, their strengths are set to be zero. Therefore, there are six components for short-range interactions and four components for long-range interactions. The strengths of these 10 components are treated as parameters in the least-squares fit calculation for the low-lying states of the $N=29$ nuclei with $21 \leq Z \leq 28$.

The present calculations are divided into two parts. In the first step, the matrix elements of the effective p-p interaction [i.e., the second term of the right-hand side of Eq. (3)] are taken directly from the experimental spectra of the nucleus with same Z but $N=28$. The strengths for each force component of the p-n interaction [i.e., the first term of the right-hand side of Eq. (3)] are treated as free parameters in the least-squares fitting for 45 observed energies of the low-lying states of $N=29$ nuclei with $21 \leq Z \leq 28$. Eight of the 45 observed energies are the binding energies of the ground states relative to that of the $N=28$ nucleus with same value of Z (i.e., the neutron separation energies) while the other 37 observed energies are the excitation energies from the ground states. In the practical calculations, the harmonic-oscillator wave functions are employed with the oscillator constant being fixed to $\nu = 0.96A^{-1/3}$ fm^{-2}, where $A=50$. The matrix elements were calculated first with a Gaussian interaction with ranges of r_1

TABLE I. The level energies (in MeV) used in the least-squares fit. In columns 6 ($G2$) and 7 ($Y2$), the states marked with asterisks are excluded in the least-squares fit. The experimental energy levels marked with daggers contains large components of neutron $f_{5/2}$ configuration.

Nucleus	J	E_{exp}	E_{cal} ($G1$)	E_{cal} ($Y1$)	E_{cal} ($G2$)	E_{cal} ($Y2$)
^{50}Sc	$5^+_{g.s.}$	−6.060	−6.075	−6.101	0.0*	0.0*
	2^+_1	0.255	0.195	0.248	0.196	0.244*
	3^+_1	0.325	0.138	0.169	0.138	0.168*
	4^+_1	0.755	0.612	0.679	0.611	0.675*
	1^+_1	1.850†	1.884	2.090	1.883	2.087*
^{51}Ti	$\frac{3}{2}^-_{g.s.}$	−6.379	−6.612	−6.614	0.0*	0.0*
	$\frac{1}{2}^-_1$	1.160	1.225	1.238	1.155	1.161
	$\frac{7}{2}^-_1$	1.429	1.543	1.539	1.334	1.345
	$\frac{5}{2}^-_1$	1.559	1.421	1.398	1.271	1.236
	$\frac{5}{2}^-_2$	2.136	2.361	2.368	2.091	2.178
	$\frac{3}{2}^-_2$	2.189	1.884	1.936	1.734	1.780
^{52}V	$3^+_{g.s.}$	−7.309	−7.495	−7.503	0.0*	0.0*
	2^+_1	0.017	0.123	0.148	0.138	0.162
	5^+_1	0.023	0.045	0.046	−0.022	0.018
	1^+_1	0.142	0.337	0.342	0.491	0.412
	4^+_1	0.148	0.174	0.184	0.225	0.208
	2^+_1	0.437	0.476	0.517	0.643	0.615
	3^+_2	0.794	0.730	0.718	0.824	0.758
	4^+_2	0.846	0.712	0.726	0.751	0.732
^{53}Cr	$\frac{3}{2}^-_{g.s.}$	−7.941	−8.020	−8.020	0.0*	0.0*
	$\frac{1}{2}^-_1$	0.565	0.834	0.846	0.865	0.846
	$\frac{5}{2}^-_1$	1.007†	1.380	1.402	1.376	1.373
	$\frac{7}{2}^-_1$	1.286	1.338	1.337	1.328	1.324
^{54}Mn	$3^+_{g.s.}$	−8.941	−8.897	−8.921	0.0*	0.0*
	2^+_1	0.056	0.269	0.294	0.274	0.296
	4^+_1	0.156	0.169	0.227	0.126	0.217
	5^+_1	0.368	0.231	0.248	0.160	0.237
	3^+_2	0.408	0.482	0.469	0.492	0.472
	4^+_2	0.839	0.939	0.930	0.938	0.931
	3^+_3	1.008†	1.185	1.191	1.181	1.169
	2^+_2	1.372	1.157	1.271	1.159	1.267
^{55}Fe	$\frac{3}{2}^-_{g.s.}$	−9.299	−9.257	−9.260	0.0*	0.0*
	$\frac{1}{2}^-_1$	0.411	0.760	0.716	0.764	0.709
	$\frac{5}{2}^-_1$	0.931†	1.093	1.067	1.090	1.061
	$\frac{7}{2}^-_1$	1.317	1.341	1.351	1.310	1.323

TABLE I. (*Continued*)

Nucleus	J	E_{exp}	E_{cal} ($G1$)	E_{cal} ($Y1$)	E_{cal} ($G2$)	E_{cal} ($Y2$)
^{56}Co	$4^+_{g.s.}$	−10.088	−9.962	−9.913	0.0*	0.0*
	3^+_1	0.158	0.105	−0.026	0.107	−0.028
	5^+_1	0.577	0.259	0.196	0.260	0.194
	4^+_2	0.830†	0.925	0.914	0.929	0.913
	2^+_1	0.970	0.730	0.817	0.734	0.812
	5^+_2	1.009†	0.702	0.591	0.705	0.593
	3^+_2	1.115†	0.794	0.760	0.797	0.756
^{57}Ni	$\frac{3}{2}^-_{g.s.}$	−10.257	−10.288	−10.288	0.0*	0.0*
	$\frac{5}{2}^-$	0.768†	0.513	0.521	0.513	0.521
	$\frac{1}{2}^-$	1.112	1.407	1.381	1.407	1.381
rms deviation			0.197	0.192	0.218	0.215

= 1.0 fm for the short range and $r_2 = 3.0$ fm for the long range (henceforth referred to as $G1$). These adopted interaction ranges have about the same values as those of Schiffer *et al.* in their original work.[8] A Yukawa interaction of radial dependence with $r_1 = 1.415$ and $r_2 = 2.0$ fm is then repeated in a later calculation (referred to as $Y1$). These interaction ranges also have the same values used by Schiffer *et al.* in their recent calculation.[10] In the second step, the effective p-p interaction is then assumed to be a two-range central-plus-tensor potential which is also similar to that used by Schiffer *et al.*[10] with the p-n interaction determined in the first step; the strengths of each force component of the p-p interaction are thus obtained in a least-squares fitting for the same 33 observed excited energies of the low-lying states of $N = 29$ nuclei with $21 < Z \leq 28$. In this step, the radial dependence of the effective p-p interaction is again assumed to be the Gaussian form first (referred to as $G2$). A Yukawa form (referred to as $Y2$) is then assumed in a later calculation. Both interactions ($G2$ and $Y2$) are assumed to have the same interaction ranges as those of $G1$ and $Y1$. Since we are interested in the effective interaction, we fit only the excited levels of the observed values in order to avoid the effect due to the core polarization.

In the investigation of the effective p-n interaction, it is found that the long-range part of the force is strongly correlated with the short-range part. Therefore, in the calculation of the effective p-p interaction, the strengths of the long-range components of the p-p interaction are fixed to the values of $G1$ and $Y1$ and adjust only those of the short range. Therefore, this leaves us with three parameters (CSE, CTO, and TTO components of short range) to fit the 33 observed excited levels. It is found that the strength parameters obtained for $G2$ and $Y2$ are quite similar to those for $G1$ and $Y1$.

III. LEVEL SPECTRA

In the present calculations, we include for each nucleus all the available low-lying states with reliable J^π assignments up to the point where the first level with an uncertain J^π assignment appears (except the second $\frac{7}{2}^-$ states in ^{53}Cr and ^{55}Fe). For the $\frac{7}{2}^-$ doublet in ^{53}Cr, the recent experiment of the ^{54}Cr(d, t) ^{53}Cr reaction by Borsaru *et al.*[23] shows that although the first $\frac{7}{2}^-$ state is weakly excited, the second $\frac{7}{2}^-$ level is strongly excited with a large spectroscopic factor for $l_n = 3$. The result of a neutron pickup reaction suggests that the main component is the two-particle one-hole state of the neutron system. In ^{55}Fe, the second $\frac{7}{2}^-$ state is also strongly excited in a pickup reaction on ^{56}Fe; thus, it can be interpreted as a hole state coupled to a ^{56}Fe core. Fortunately, these $\frac{7}{2}^-$ doublet states have a limited mixing of particle and hole configurations confirmed by the small branching ratio for the $(\frac{7}{2}^-)_2 \rightarrow (\frac{7}{2}^-)_1$ transition.[24] Therefore, it is quite reasonable to exclude the second $\frac{7}{2}^-$ states in ^{53}Cr and ^{55}Fe in the least-squares calculation.

The results of least-squares fit to the energy level data are shown in Table I. The experimental

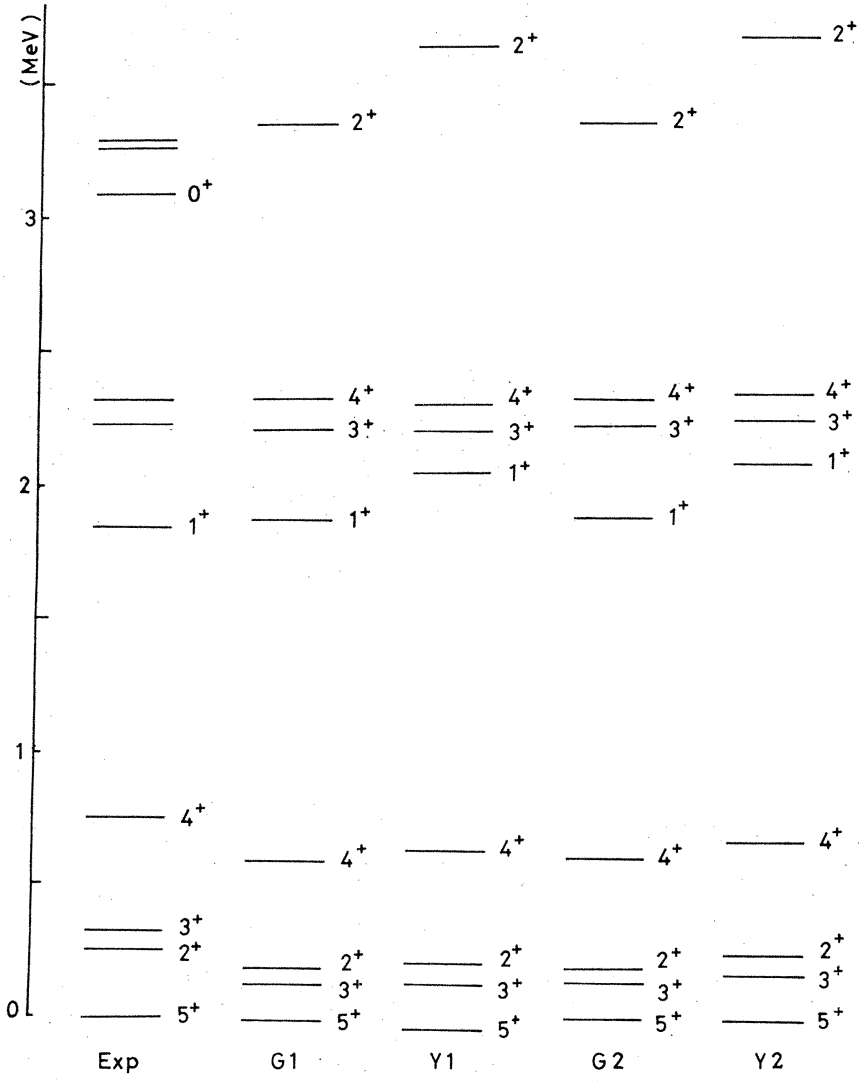

FIG. 1. Experimental and calculated energy spectra for ^{50}Sc.

energy E_k^{exp} represents an excitation energy if the state k is excited and represents the binding energy relative to that of the $N=28$ nucleus with the same Z if k is the ground state. In the columns of $G2$ and $Y2$, the values 0.0 of the ground state marked with an asterisk are those states not selected in the least-squares fit as explained in Sec. II. The last row gives the root-mean-square deviation between the calculated and the experimental energies for each set.

In Figs. 1–8, the calculated energy-level spectra are compared with the experimental values. The 0^+ states in ^{50}Sc and ^{56}Co, which cannot be explained in our model space, perhaps arise from the excitation of the $1f_{7/2}$ neutron with the same coexistence properties of hole and particle configuration as the second $\frac{7}{2}^-$ states do in ^{53}Cr and ^{55}Fe. The lower value in excitation energy for the 0^+ state in ^{56}Co is due to the fact that the $\nu f_{5/2}$ level is lowered rapidly as the $\pi f_{7/2}$ orbit fills.

For odd-mass nuclei, the agreement for levels which lie in the energy region from 0–2 MeV is quite satisfactory except for the $\frac{7}{2}^-$ doublet states in ^{55}Fe. For the ^{51}Ti, ^{53}Cr, and ^{55}Fe nuclei, the ground states are the states with spin $J=\frac{3}{2}^-$, and $J=\frac{1}{2}^-$, $\frac{5}{2}^-$, and $\frac{7}{2}^-$ excitation states are just above these ground states. All of these states are included in the least-squares fit. The results of our calculation show that the lowest $J=\frac{3}{2}^-$ and $\frac{7}{2}^-$ levels are reproduced as states of almost single-par-

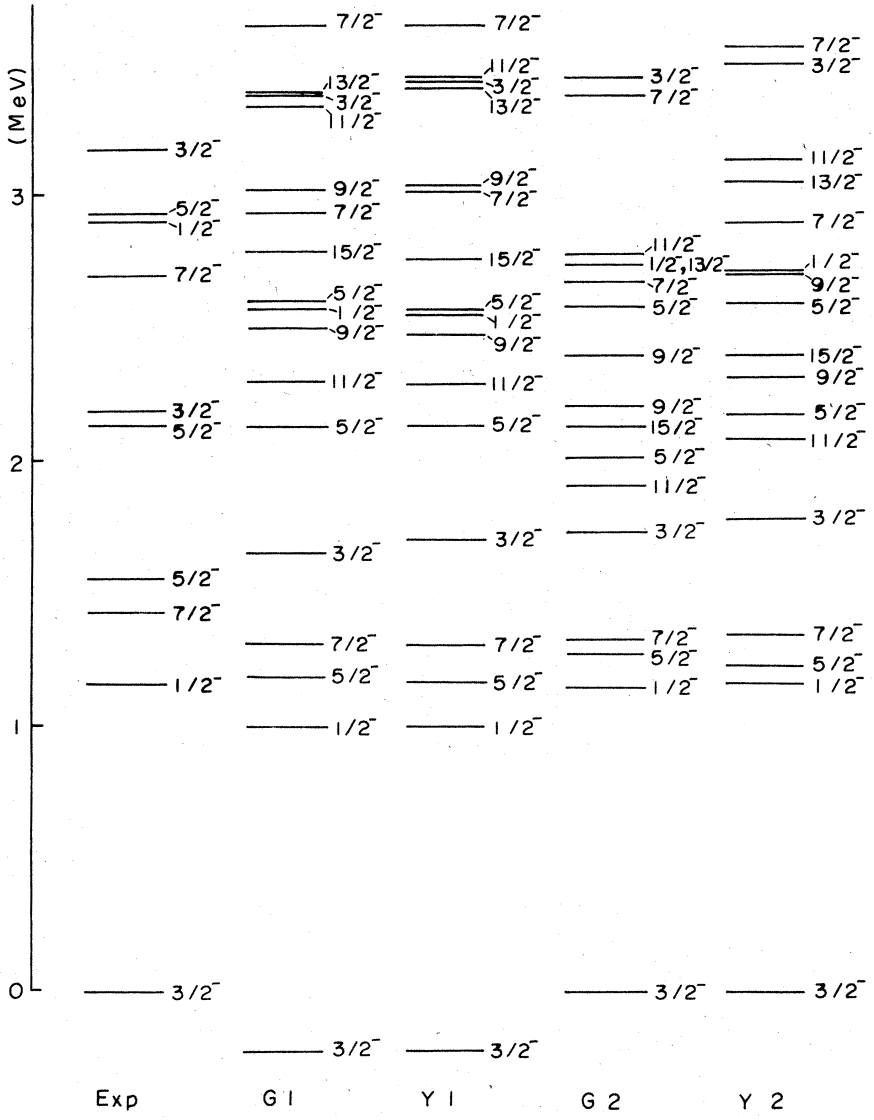

FIG. 2. Experimental and calculated energy spectra for ^{51}Ti.

ticle nature, i.e., the wave functions have the predominant states of $|J_p=0\times p_{3/2};\ J=\frac{3}{2}\rangle$ and $|v_p=2;\ J_p=2\times p_{3/2};\ J=\frac{7}{2}\rangle$, where v_p means the seniority. However, the wave functions of the lowest $\frac{1}{2}^-$ levels are composed mainly of the admixture of the components $|J_p=0\times p_{1/2};\ J=\frac{1}{2}\rangle$ and $|v_p=2,\ J_p=2\times p_{3/2};\ J=\frac{1}{2}\rangle$. This is due to the fact that the relative single-particle energies of $\epsilon_n(2p_{1/2})$ and $\epsilon_n(2p_{3/2})$ are comparable to the excitation energies of the first 2^+ states in the nuclei of the same Z but $N=28$, and the nondiagonal matrix elements of the p-n interaction are not negligible. In the lowest $\frac{5}{2}^-$ states, the wave function contains a predominant component of $|J_p=2\times p_{3/2};\ J=\frac{5}{2}\rangle$ in ^{51}Ti, but shifts to the $|J_p=0\times f_{5/2};\ J=\frac{5}{2}\rangle$ component in ^{55}Fe. This is due to the decrease of the p-n interaction (increase in the absolute value) between $\pi f_{7/2}$ and $\nu f_{5/2}$ as the $\pi f_{7/2}$ orbit fills.

For even-mass nuclei, the low-lying levels do not possess this similarity as in the case of the odd-mass nuclei. The lowest $J=2^+$, 3^+, 4^+, and 5^+ states in ^{50}Sc are almost the pure states of $|\pi f_{7/2}\times\nu p_{3/2};\ J\rangle$. It is interesting to note here that the $J=1^+$, 1.85 MeV state of ^{50}Sc shows very good agreement with the observed energy. This is because the value of the two-body matrix element

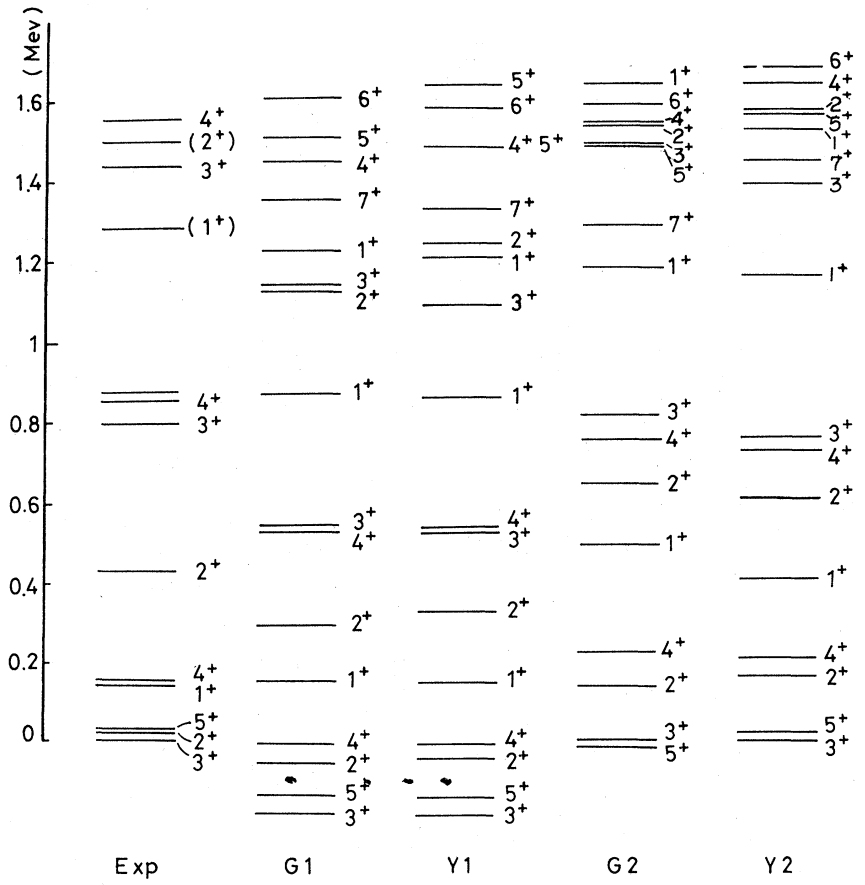

FIG. 3. Experimental and calculated energy spectra for ^{52}V.

$\langle \pi f_{7/2} \nu f_{5/2} | V | \pi f_{7/2} \nu f_{5/2} \rangle_{J=1}$ is very large in the present calculation (see Table III). For the energy range 0–3 MeV, the results we obtained are very similar to those of Hughes and Soga[25] in which the two-body matrix elements are assumed in both the Soper and Rosenfeld mixtures. Figure 3 shows the calculated energy levels of ^{52}V. They are not reproduced so satisfactorily as the other nuclei, but the 8 states with J^π assignments below 1 MeV can still be explained. From our calculation, the J^π of the 0.88 MeV state with an uncertain J^π assignment seems to be 1^+. Furthermore, the wave functions of these low-lying states possess rather spread out components. Only the lowest $J=5^+$ and the ground state have some pure state nature. The intensity of the $|J_p=\frac{7}{2}\times p_{3/2};\ J=5\rangle$ component for the lowest $J=5^+$ state is more than 90% and the $|J_p=\frac{7}{2}\times p_{3/2};\ J=3\rangle$ component for the ground state is about 70%.

For ^{54}Mn, the lowest five states are in good agreement with the observed energies and the energy gap between the fifth and the sixth is also reproduced quite satisfactorily. The components of the wave functions for states of $J\leq 3$ spread over a considerable range. The components $|J_p=\frac{7}{2}\times p_{3/2};\ J=4\rangle$ and $|J_p=\frac{7}{2}\times p_{3/2};\ J=5\rangle$ for the states of $J=4^+$ and 5^+ have an intensity of 70% and 90%, respectively. The calculated excitation energy of the lowest $J=6^+$ is around 1.8 MeV which predicts a large shift from the observed value of 1.08 MeV. The dominant strength in this state is $|J_p=\frac{11}{2}\times p_{3/2};\ J=6\rangle$ which has an intensity above 80% in our calculation.

For ^{56}Co, considerable good agreement exists between the calculated lowest seven states and the corresponding experimental ones. The dominant strength in the lowest $J=2_1^+$, 3_1^+, 4_1^+, and 5_1^+ states is $|(f_{7/2}{}^{-1})_p(p_{3/2})_n;\ J\rangle$ which has an intensity of 90% for all states except the state of $J=2_1^+$ which has an intensity of 70% only. The next $J=3_2^+$, 4_2^+, and 5_2^+ are almost pure $|(f_{7/2}{}^{-1})_p(f_{5/2})_n;\ J\rangle$ with an intensity of more than 90%. Our model space cannot explain the $J=0^+$ state at 1.45 MeV, but near this energy we have obtained a $J=2^+$ state, which is

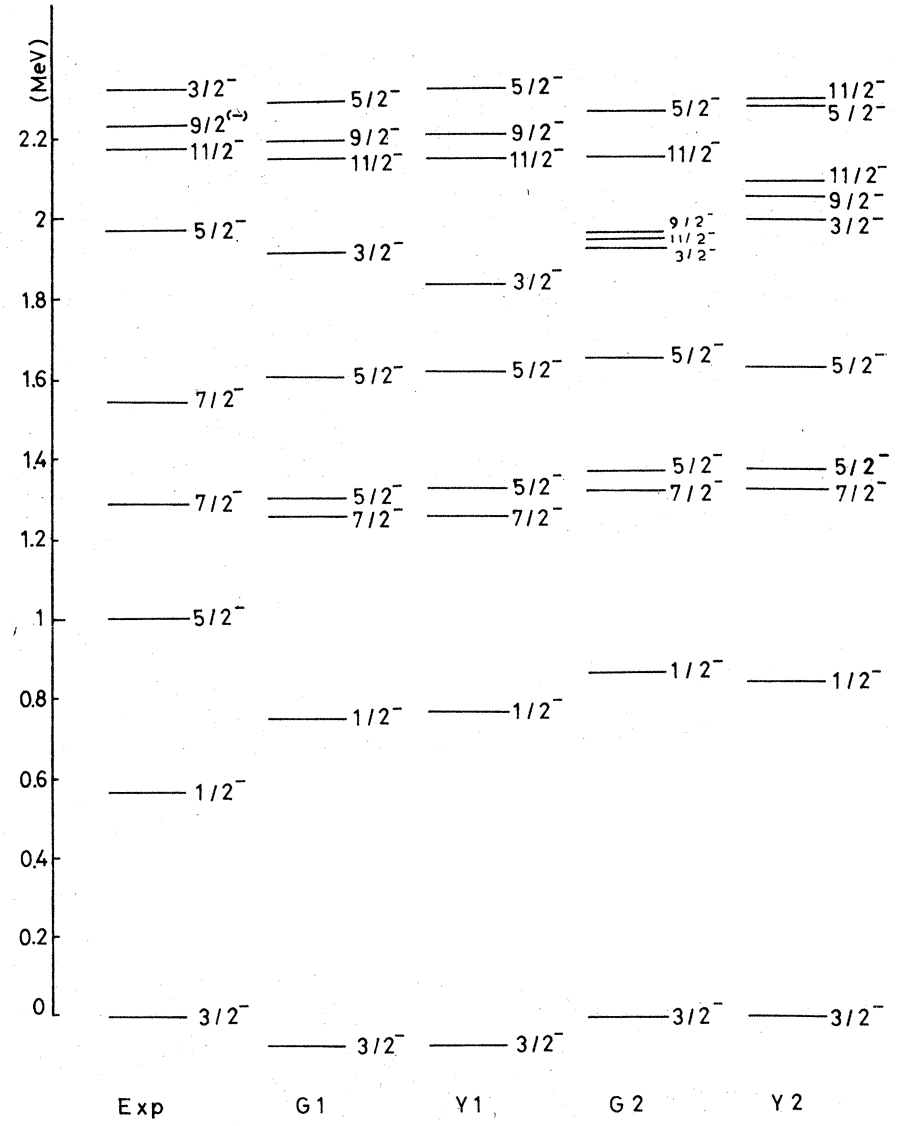

FIG. 4. Experimental and calculated energy spectra for ^{53}Cr.

very sensitive to the choice of different interactions. The $J=1^+$ state at 1.72 MeV is not included in our least-squares fitting, but the result seems very satisfactory.

IV. EFFECTIVE INTERACTIONS

A. Proton-neutron interactions

As explained in Sec. II, the columns $G1$ and $Y1$ in Table II represent the strengths of the p-n interactions in the Gaussian and Yukawa types, respectively. The strengths of the long-range force are found to be very small in the case of Gaussian form; this is because the interaction range for the long-range force is chosen to be three times that for the short-range force. Furthermore, only the CTE interaction strengths have opposite signs in the short- and long-range components and the strengths of the tensor force are rather large.

For the Yukawa form, the strengths of the short- and the long-range forces all have opposite signs. This result is the same as that obtained by Schiffer and True.[10] Furthermore, the magnitudes of the strengths we obtained are very similar to theirs except for the TTO component. The strengths of TTO in $G1$ and $Y1$ are all positive and large. This may be related to the fact that the strengths of our CSE and CTO in the two ranges are more attractive

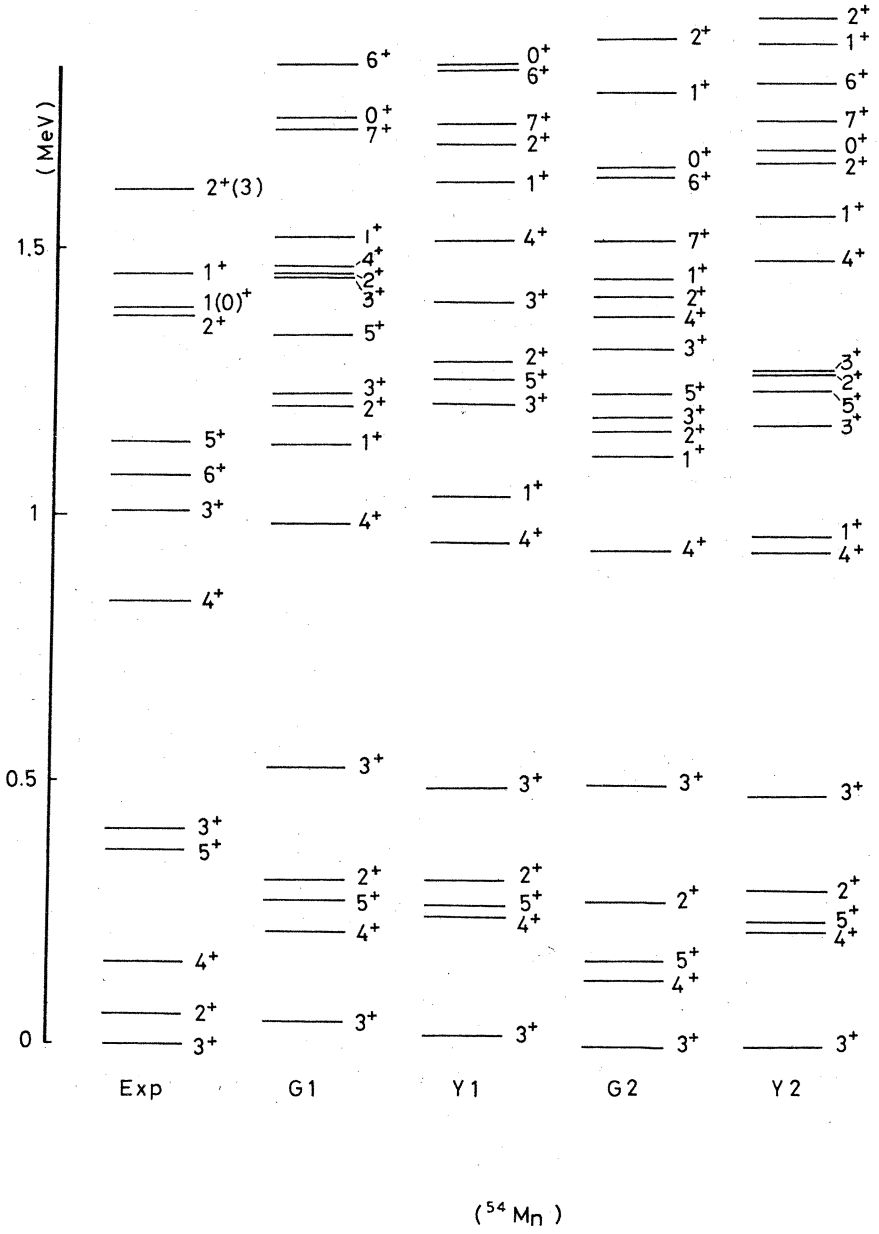

FIG. 5. Experimental and calculated energy spectra for ^{54}Mn.

than those of Schiffer and True.

In order to estimate the relation between the model space and the interactions we obtained, a calculation including only the neutron $p_{3/2}$ and $p_{1/2}$ orbits is performed. The eight experimental energy levels in Table I marked with daggers are excluded from the least-squares fitting. This is because they contain a major component of the neutron $f_{5/2}$ configuration and thus cannot be accounted for by the $(p_{1/2}, p_{3/2})$ configuration only. The intensities of the neutron $f_{5/2}$ components of these levels are more than 90% except the $(\frac{5}{2})_1^-$ level in ^{52}Cr and the 3_3^+ level in ^{54}Mn which have about 40% and 80%, respectively. The rms deviations for Gaussian and Yukawa interactions are 0.187 and 0.179 MeV, respectively. The strengths of these interactions are shown in columns $G1'$ and $Y1'$ of Table II. These strengths are almost the same as those in columns $G1$ and $Y1$. This is obviously due to the fact that the configuration mixing for the

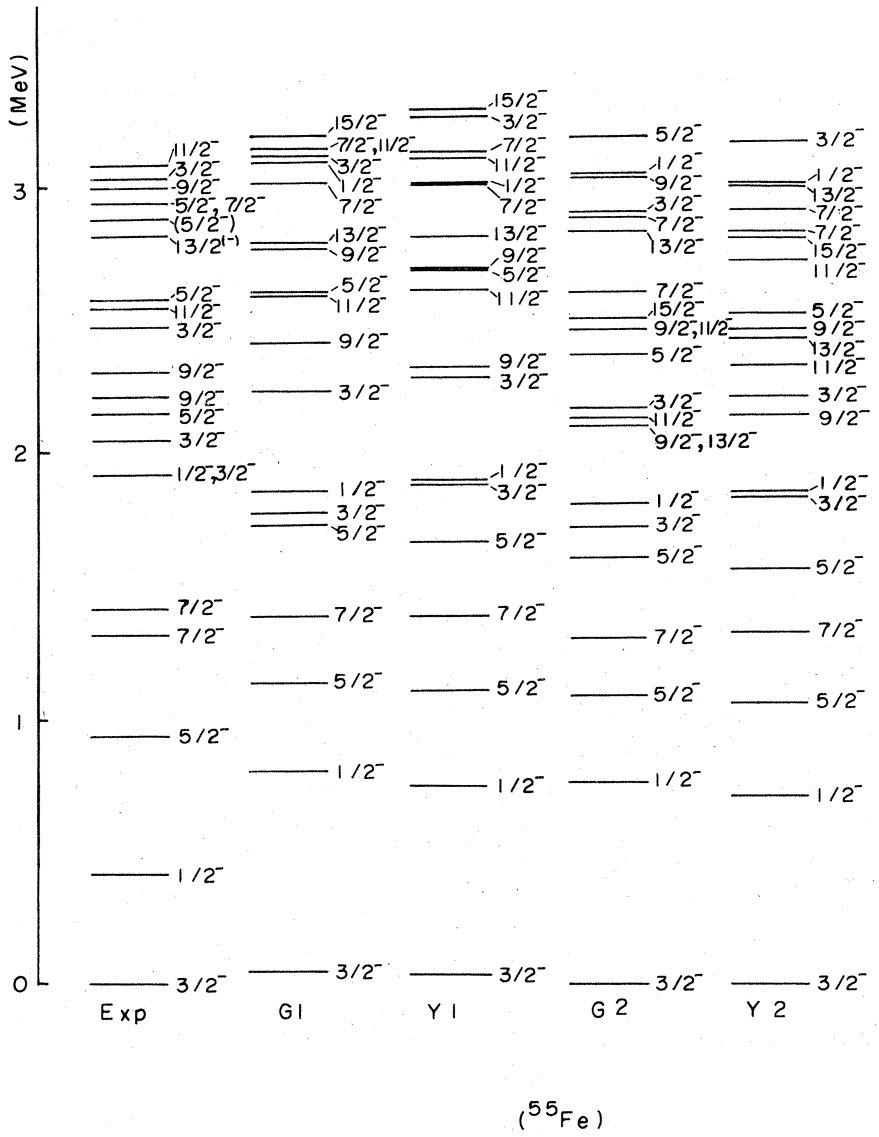

FIG. 6. Experimental and calculated energy spectra for ^{55}Fe.

neutron $f_{5/2}$ orbit is quite small for the low-lying levels in $N=29$ nuclei.

We use the result obtained in the least-squares fit to calculate the p-n two-body matrix elements. The diagonal elements are listed in Table III. This table also contains the matrix elements obtained by Oberlechner and Richert,[20] Kuo,[21] and Horie and Ogawa[18] for comparison. The matrix elements of Oberlechner *et al.* are deduced from a nonlocal potential which reproduces the nucleon-nucleon scattering data while those of Kuo are calculated from the Hamada-Johnston potential. Table III shows that our results for the isospin independence of the two-body matrix elements are very close to Kuo's. Remarkable similarities exist between our results and those of Oberlechner *et al.* and of Horie *et al.* except the one $\langle \pi f_{7/2}\nu f_{5/2}|V|\pi f_{7/2}\nu f_{5/2}\rangle_{J=6}$ in the former case and the one $\langle \pi f_{7/2}\nu f_{5/2}|V|\pi f_{7/2}\nu f_{5/2}\rangle_{J=5}$ in the latter case, respectively. In view of the above results, the importance of the nonlocal and Hamada-Johnston potentials seems to be overestimated.

Figure 9 shows the angular distribution of TBME (two-body matrix elements) of the Yukawa potential. Almost all the points lie on one curve except the two points $\langle \pi f_{7/2}\nu p_{3/2}|V|\nu f_{7/2}\nu p_{3/2}\rangle_{J=2,5}$. Schiffer *et al.*[10] analyzed the angular distribution of the two-body matrix elements and concluded that the TBME

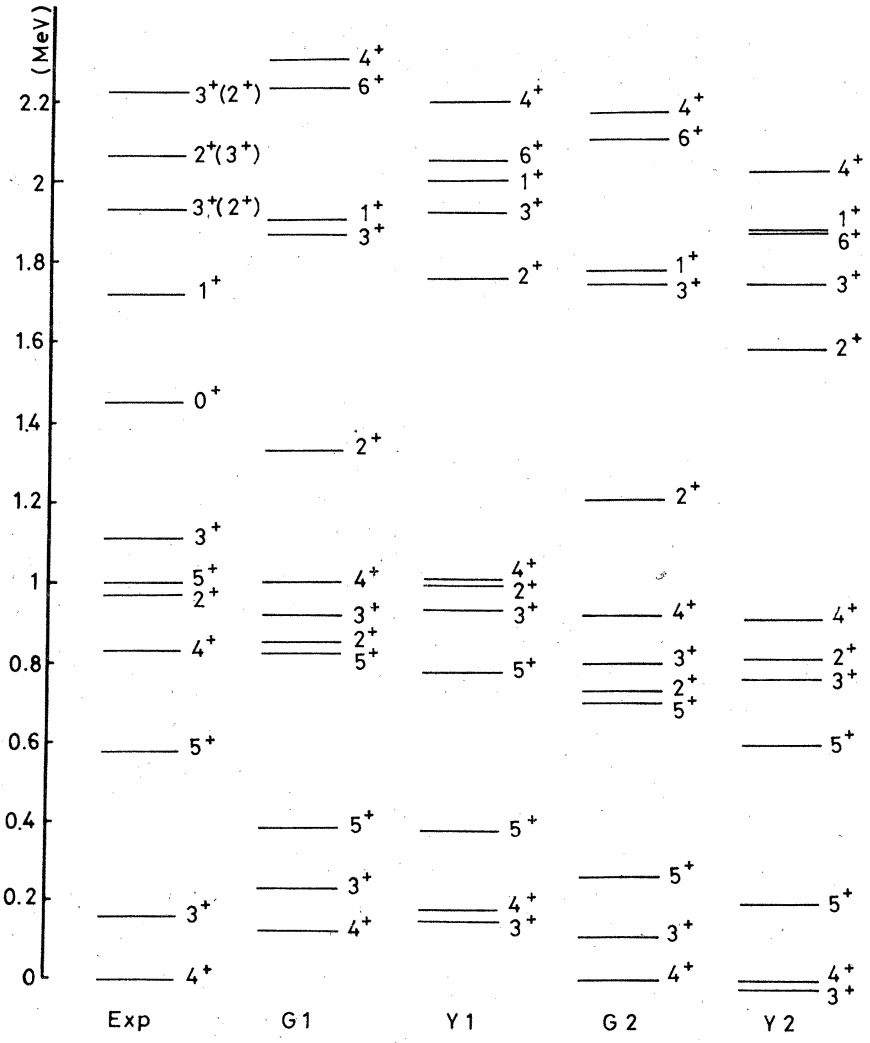

FIG. 7. Experimental and calculated energy spectra for ^{56}Co.

of $T=0$ and $T=1$ are distributed on two curves separately. The form of our curve is very similar to their $T=0$ case.

In order to study the contribution of different interaction ranges to TBME, the angular distribution of $\langle \pi f_{7/2} \nu f_{5/2} | V | \pi f_{7/2} \nu f_{5/2} \rangle_J$ for the Yukawa type is plotted and shown in Fig. 10. The contribution of the short range to the TBME is attractive and the absolute values of both ends are quite large while the contribution to the long range is repulsive with its shape just opposite to that of short-range force. It is also illustrated in Fig. 10 that the combined two-range force gives a very small curvature. The curvature shows the reason why two different interaction ranges must be taken into account.

B. Proton-proton interactions

The strengths of the force components of the p-p interactions are given in the columns $G2$ and $Y2$ of Table II for the Gaussian and Yukawa potentials, respectively. The components of CSE and CTO in $Y2$ are more attractive for the short-range force and are more repulsive for the long-range force than those of Schiffer and True.[10] The reason for this may be that we restrict the protons to $1f_{7/2}$ only and neglect the effect of the excitations from $1f_{7/2}$ to $2p_{3/2}$, $2p_{1/2}$, and $1f_{5/2}$ orbits.

The results of the TBME $\langle f_{7/2} f_{7/2} | V | f_{7/2} f_{7/2} \rangle_J$ are presented in Table IV. The values listed in the columns of ^{42}Sc and ^{50}Ti are taken from the experi-

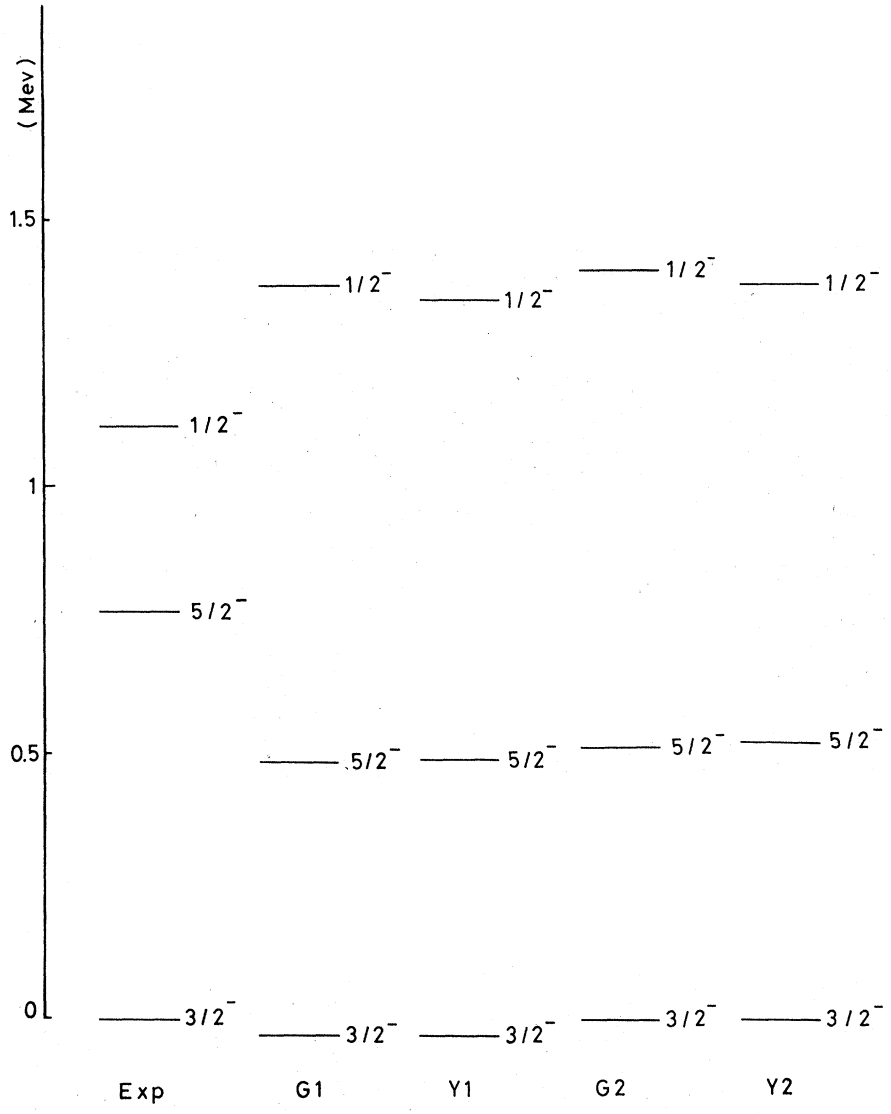

FIG. 8. Experimental and calculated energy spectra for ^{57}Ni.

TABLE II. Interaction strengths (in MeV) compared with those of Schiffer and True (ST).

		$G1$	$G1'$	$G2$	$Y1$	$Y1'$	$Y2$	ST
Short range	CSO	98.696	94.258	•••	137.861	145.226	•••	125.53
	CTE	−128.616	−129.826	•••	−74.458	−76.304	•••	−118.09
	CSE	−78.020	−81.687	−70.077	−68.475	−74.916	−61.475	−49.32
	CTO	−209.395	−204.567	−189.090	−197.143	−194.147	−195.401	−155.82
	TTE	−235.891	−248.495	•••	−35.686	−43.084	•••	−42.52
	TTO	177.197	190.260	−16.820	84.002	88.474	21.220	−6.10
Long range	CSO	6.301	5.564	•••	−44.020	−41.297	•••	−44.37
	CTE	8.374	6.847	•••	29.686	29.580	•••	27.27
	CSE	−6.816	−7.345	−6.816	11.775	9.543	11.775	15.47
	CTO	−9.271	−7.040	−9.271	60.320	60.851	60.320	62.06

TABLE III. Two-body matrix elements (in MeV) for p-n interactions compared with those of Oberlechner and Richert (OR), Horie and Ogawa (HO), and Kuo.

j	J	$G1$	$Y1$	OR	HO	Kuo
$p_{3/2}$	2	−0.699	−0.682	−0.736	−0.787	−0.449
	3	−0.587	−0.634	−0.655	−0.444	−0.447
	4	−0.306	−0.248	−0.273	−0.141	−0.161
	5	−0.929	−0.955	−1.249	−1.026	−1.097
$p_{1/2}$	3	−0.903	−0.870	−0.817	−0.760	−0.799
	4	−0.579	−0.610	−0.755	−0.695	−0.577
$f_{5/2}$	1	−3.004	−2.825	−2.103	−2.544	−2.062
	2	−1.556	−1.251	−1.221	−1.430	−1.455
	3	−0.942	−0.957	−0.831	−0.592	−0.653
	4	−0.884	−0.667	−0.908	−1.336	−1.077
	5	−0.531	−0.613	−0.464	−0.005	−0.315
	6	−1.104	−1.331	−2.053	−1.345	−1.790

ment[26] and the column entitled MWH presents the results of McGrory, Wildenthal, and Halbert[27] which were obtained from a modified Kuo-Brown interaction. Our TBME are more attractive than the experimental values and those of MWH, especially for large J. As mentioned above, this is because our TBME absorb the effect of the excitations of proton from the $1f_{7/2}$ orbit. The excitation energies are large for large J levels in ^{42}Sc and ^{50}Ti; thus, the configuration mixing becomes more important and our results are expected.

In the calculations of the $E2$ properties of ^{48}Ti, Lesser *et al.*[28] found that if one included the configuration of excitation of one proton from $(f_{7/2})^n$ to the $(1f\text{-}2p)$ shell, the results obtained would be much better than the case without including this effect. Therefore, it would be interesting to investigate the effect of this proton excitation on the p-p interaction. In the shell-model calculations the basic states will increase abruptly when more orbits are taken into account and this makes the calculations much more complicated. But if we permit only one proton to be excited, then the dimension of the calculation will not be too large for certain mass number A and spin J.

The angular distributions of the TBME obtained for the Gaussian and Yukawa interactions are shown in Fig. 11. Although we present only the

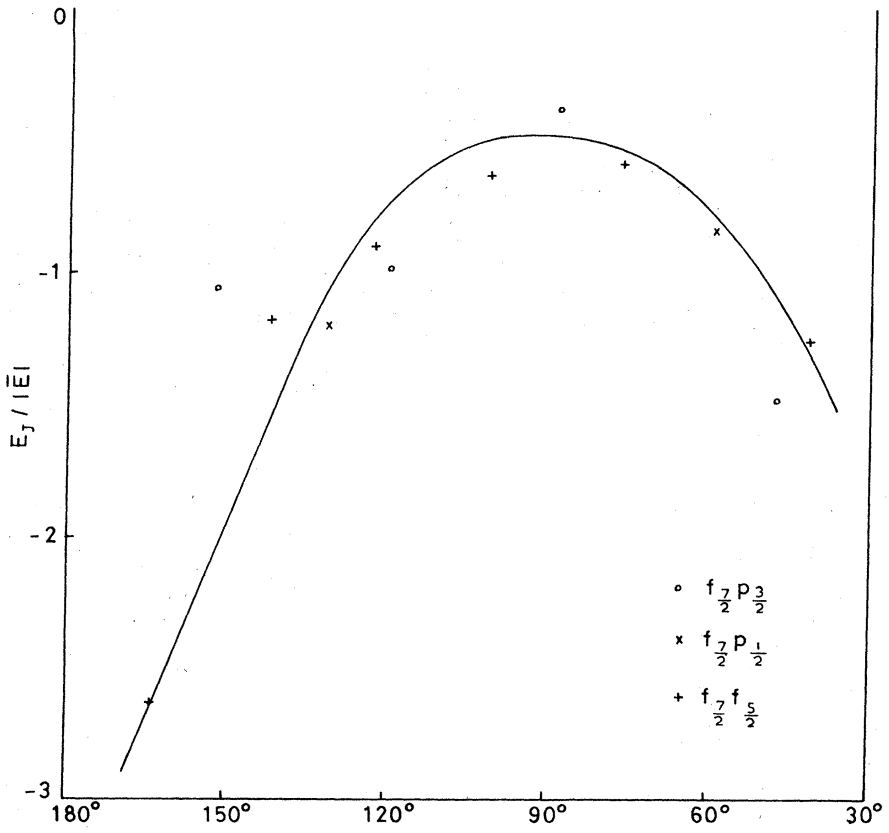

FIG. 9. Angular distribution of two-body matrix elements for the p-n interactions of the Yukawa potential.

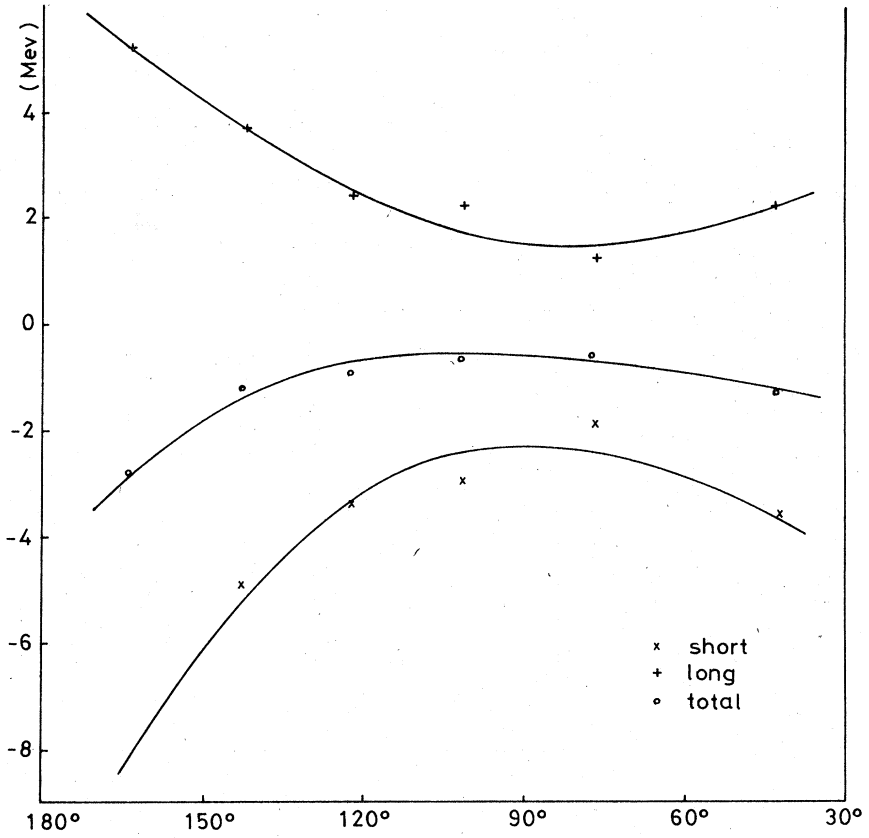

FIG. 10. Decomposed contributions of the short- and long-range components of the matrix elements $\langle \pi f_{7/2} \nu f_{5/2} | V | \pi f_{7/2} \nu f_{5/2} \rangle_J$ for the Yukawa potential.

TBME of $\langle f_{7/2} f_{7/2} | V | f_{7/2} f_{7/2} \rangle_J$, it is instructive to determine the main aspect. The shapes of the curves we obtained look very like the ones for $T = 1$ of Schiffer *et al.*[8]

The decomposed angular distributions of the short- and long-range components of $\langle \pi f_{7/2} \pi f_{7/2} | V | \pi f_{7/2} \pi f_{7/2} \rangle_J$ for the Yukawa type are shown in Fig. 12. The contribution of the short range to the TBME is attractive while that of the long range is repulsive. It is found that the absolute values of both ends are not so large as compared with Fig. 10 and, thus, the curves are smoother. This is because the TTO component for the p-p interaction is not as repulsive as that for the p-n interaction. It is also instructive to see that the combined curve for the two-range forces has a very small curvature and, hence, again exhibits the necessity of the two-range forces.

V. CONCLUSION

To investigate the effect of the potential suggested by Schiffer[7] on the actual shell-model calculation, we assume a two-range central-plus-tensor potential to calculate the low-lying levels of $N = 29$, $21 \leq Z \leq 28$ nuclei by least-squares fitting. The results of Schiffer *et al.*[10] show that the two-body spin-orbit interaction does not help much. Therefore, we also neglect this interaction to simplify the calculation.

For the radial dependence of the interaction, we

TABLE IV. Comparison of experimental and theoretical two-body matrix elements (in MeV) for p-p interactions with those of McGrory, Wildenthal, and Halbert (MWH).

	^{42}Sc [a]	^{50}Ti [b]	$G2$	$Y2$	MWH
0^+	−3.18	−3.18	−3.68	−3.41	−2.11
2^+	−1.59	−1.63	−2.28	−2.01	−1.11
4^+	−0.36	−0.50	−1.53	−1.12	−0.10
6^+	0.06	0.02	−1.37	−0.79	+0.23

[a] Values obtained by assuming that single states in ^{45}Sc represent the $(f_{7/2}^2)$ configuration with $E_0 = 3.18$ (Ref. 14).
[b] The same as footnote a for ^{50}Ti.

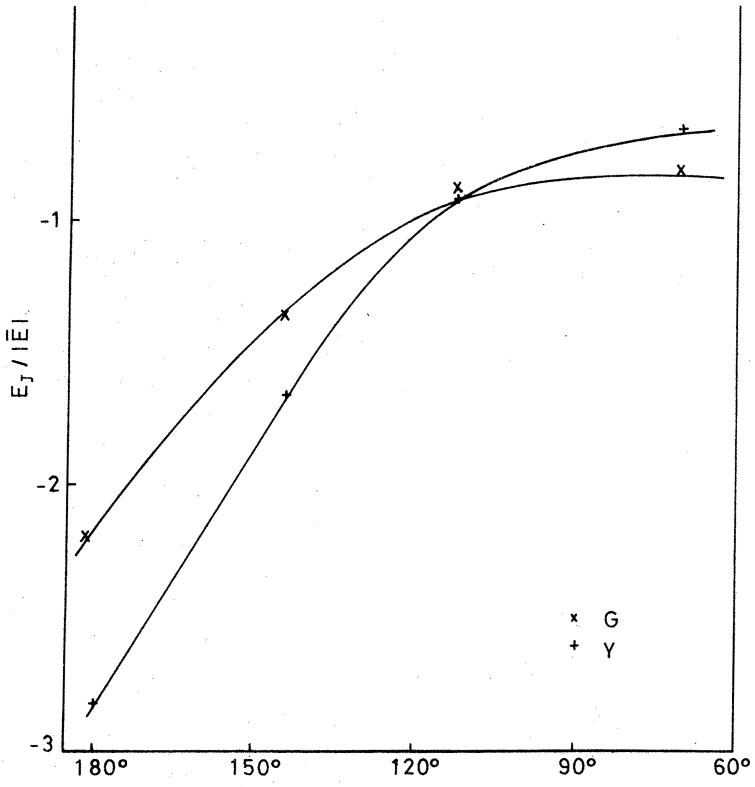

FIG. 11. Angular distribution of two-body matrix elements for p–p interactions of the Gaussian and Yukawa potentials.

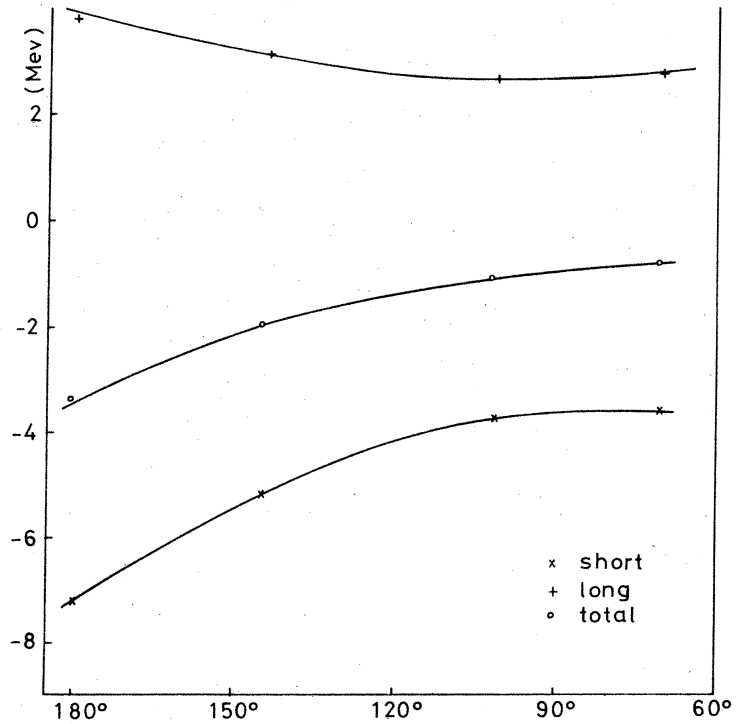

FIG. 12. Decomposed contributions of the short- and long-range components of the matrix elements $\langle \pi f_{7/2} \pi f_{7/2} | V | \pi f_{7/2} \pi f_{7/2} \rangle_J$ for the Yukawa potential.

have adopted two different types, the Gaussian form and the Yukawa form. For the Gaussian type the strengths of the force components are not too sensitive in the χ^2 fitting and, thus, may lead to some uncertainty in the results, while for the Yukawa type the force components are quite sensitive to χ^2 fitting and, hence, the minimum obtained in the Yukawa case is more realistic than the Gaussian case.

The calculations of Schiffer *et al.*[10] show that opposite signs exist between the short- and long-range strengths. In our case this opposite sign behavior is exhibited only in the Yukawa case. This behavior of opposite signs contributes to an easier understanding of the contributions of each interaction range and the purpose of adopting two different interaction ranges.

The force component strengths obtained by the Yukawa radial dependence are very similar to those of Schiffer *et al.*[10] except for component TTO. In our calculations, the interaction strengths for p-n and p-p are almost the same except for component TTO. The difference in TTO strengths may arise from the restriction on the active model space.

By assuming an inert core of ^{40}Ca, McGrory *et al.*[27] made a calculation on the low-lying levels of the calcium isotopes and obtained satisfactory results. Our results show that it is worthwhile to reconsider the above calculation with the potential suggested by Schiffer.[7] Since the low-lying states of $N=28$, $22 \leq Z \leq 28$ nuclei are very similar to those of calcium isotopes, calculations with it should explain the energy spectra if almost the same force strengths are used. Lips and McEllistrem[29] calculate the energy spectra of these nuclei but only consider one proton excited from the $1f_{7/2}$ shell and neglect the $2p_{1/2}$ orbit.

Although our calculations consider only the $N=29$ nuclei, the results, when compared with those of Schiffer *et al.*,[10] show that the nucleon-nucleon interaction for any orbit can be represented by an effective interaction with the same interaction strengths provided that enough model spaces are taken into account. Therefore, we conclude that as far as the low-lying energy levels of nuclei are concerned, the nucleon-nucleon interaction can be described by a single effective interaction but two different ranges are required. Furthermore, our results also show that the interaction of Yukawa type of radial dependence is more adequate for describing such a problem than the Gaussian form.

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