PHYSICAL REVIEW C VOLUME 17, NUMBER 1 JANUARY 1978

High energy collisions between nuclei and correlations*

Girish K. Varma

Serin Physics Laboratory, Rutgers–The State University, Freylinghuysen Road, Piscataway, New Jersey 08854

(Received 22 June 1977)

We investigate the effects of correlations on high energy collisions between nuclei. New types of multiple scattering can occur in such collisions and their contributions to cross sections are shown to depend significantly on the presence (or absence) of short range correlations. Nucleus-nucleus collisions may, therefore, be able to probe such correlations. Calculations are performed for ^{12}C-^{12}C and ^{16}O-^{16}O elastic scattering, using correlation lengths obtained from nuclear matter calculations. The effects of short range correlations are found to be significant and, in the case of collisions between heavy nuclei, their presence prevents the higher order corrections to the optical phase shift function from taking unphysical values at small impact parameters.

[NUCLEAR REACTIONS Nucleus-nucleus scattering, $E=2.1$ GeV/n; correlation effects.]

One of the major objectives of nuclear scattering measurements has been to learn about the density distributions of nucleons in nuclei. One-body proton densities are directly measured by electron scattering. Information on neutron distributions can also be extracted from high energy proton scattering.[1] However, very little information is available on two-body densities or pair-correlation functions. Nuclear correlations have relatively small effects on small angle proton-nucleus scattering[2-5] where a first order optical potential describes quite well the scattering from nuclei heavier than helium. In high energy nucleus-nucleus collisions, on the other hand, a first order optical potential is inadequate[6-9] even at small angles and higher order corrections are needed to reproduce accurately the processes in which one or more nucleons of either nucleus may undergo multiple collisions. These higher order processes, due to their geometry, may be quite sensitive to short range dynamical correlations between bound nucleons and thus nucleus-nucleus collisions may provide information on such correlations. In this paper we examine the effects of short range correlations on nucleus-nucleus elastic scattering cross sections. The correlation functions are chosen to reproduce the correlation lengths obtained from nuclear matter calculations using realistic nucleon-nucleon interactions. Our model calculations indicate that it should be possible to see the effects of such correlations in nucleus-nucleus scattering.

Most of the existing information on two-body densities in nuclei comes from nuclear matter studies (where, for example, the binding energy requires the expectation value of the potential which is a two-body operator). The short range part of the two-body density can be calculated in the independent pair approximation for nuclear matter.[10] (As is well known, this procedure is justified because the pair wave function heals to a noninteracting value in distances small compared with the average internucleon separation.) It is convenient to write the two-body density as

$$\rho^{(2)}(\vec{r}_1, \vec{r}_2) = N\rho(\vec{r}_1)\rho(\vec{r}_2)g(|\vec{r}_1 - \vec{r}_2|)\,, \tag{1}$$

where N is a normalization constant, and define the correlation length l_c as

$$l_c = \int_0^\infty [1 - g(r)]\, dr\,. \tag{2}$$

Equation (1) assumes a correlation function which is translationally invariant and is not suitable for long range correlations. We shall use it to describe short range correlations for which this form is reasonable if the correlation lengths are small compared with the nuclear sizes. (The long range correlations can be added on separately.) Because of the repulsive component which is present in nucleon-nucleon interactions, the correlation function is required to satisfy the constraint $g(r) \to 0$ as $r \to 0$ and also $g(r)$ approaches unity for distances greater than the correlation length.

At high energies the nucleus-nucleus collisions can be described by simple extensions[6,11-14] of the Glauber approximation.[15] The elastic scattering amplitude for collisions between nuclei of mass numbers A_1 and A_2 can be written as[9,16]

$$F(\vec{q}) = \frac{ik}{2\pi} \int d^2b\, e^{i\vec{q}\cdot\vec{b}}(1 - e^{i\chi_{\rm opt}(\vec{b})})\,, \tag{3}$$

where $\hbar k$ is the incident momentum, $\hbar\vec{q}$ is the momentum transfer, and $\vec{b}$ is the impact parameter vector. The optical phase shift function χ_{opt} can be expanded in a series

$$\chi_{\text{opt}} = \chi_1 + \chi_2 + \cdots . \tag{4}$$

In this work we shall restrict ourselves to small momentum transfers $[\hbar^2 q^2 \lesssim 0.14\ (\text{GeV}/c)^2]$ where it is sufficient to retain terms up to χ_2.[9] The phase shift functions χ_i can be expressed in terms of nucleon-nucleon scattering amplitudes $f(q)$ by

$$i\chi_1(\vec{b}) = -\frac{A_1 A_2}{2\pi i k_N} \int d^2q\, e^{-i\vec{q}\cdot\vec{b}} f(q) S_1(\vec{q}) S_2(-\vec{q}) K(\vec{q}) , \tag{5}$$

$$\begin{aligned} i\chi_2(\vec{b}) = \frac{A_1 A_2}{2(2\pi i k_N)^2} \int d^2q\, e^{-i\vec{q}\cdot\vec{b}} f(\vec{q}) & \\ \times \int d^2q' e^{-i\vec{q}'\cdot\vec{b}} f(\vec{q}') \{ & [(A_1-1)(A_2-1) S_1(\vec{q},\vec{q}') S_2(-\vec{q},-\vec{q}') + (A_2-1) S_1(0,\vec{q}+\vec{q}') S_2(-\vec{q},-\vec{q}') \\ & + (A_1-1) S_1(\vec{q},\vec{q}') S_2(0,-\vec{q}-\vec{q}')]\, K(\vec{q}+\vec{q}') \\ & - A_1 A_2 S_1(\vec{q}) S_2(-\vec{q}) S_1(\vec{q}') S_2(-\vec{q}') K(\vec{q}) K(\vec{q}') \} , \end{aligned} \tag{6}$$

where $K(q)$ is the center-of-mass (c.m.) correlation function (assuming that the c.m. and intrinsic wave functions factorize) and the one- and two-body form factors are defined by

$$\begin{aligned} S_i(\vec{q}) &= \int d^3r\, e^{i\vec{q}\cdot\vec{r}} \rho_i^{(1)}(\vec{r}) , \\ S_i(\vec{q},\vec{q}') &= \int d^3r d^3r' e^{i(\vec{q}\cdot\vec{r}+\vec{q}'\cdot\vec{r}')} \rho_i^{(2)}(\vec{r},\vec{r}') . \end{aligned} \tag{7}$$

In order to be able to make some general statements about the effect of correlations, let us first consider the extreme case where the nucleon-nucleon (NN) amplitude is assumed to have a range shorter than the correlation length (in detailed calculations, this assumption will not be made). For comparison, let us first consider the scattering of a single incident particle from a nucleus. In this case

$$i\chi_1(\vec{b}) \approx -A\tilde{f}\rho(\vec{b}), \quad \tilde{f} = \frac{2\pi f(0)}{ik_N} , \quad \rho(\vec{b}) = \int_{-\infty}^{\infty} dz\, \rho(\vec{r}) . \tag{8}$$

In the absence of correlations the second order term $i\chi_2$ is also negative and the ratio χ_2/χ_1 decreases rapidly as A becomes large.[6,8]

In the presence of short range correlations, we have

$$\begin{aligned} i\chi_2(\vec{b}) &\approx -A^2\tilde{f}^2 l_c \int_{-\infty}^{\infty} dz\, [\rho(\vec{r})]^2 , \\ \chi_2/\chi_1 &\sim A \frac{\tilde{f}}{R^2} \left(\frac{l_c}{R}\right) \frac{\rho(b)}{\rho(0)} , \end{aligned} \tag{9}$$

where R is the nuclear size. Since $R \sim r_0 A^{1/3}$, the ratio χ_2/χ_1 is independent of A and decreases rapidly with b. However, the absorption in the central region due to χ_1 is already large for large A and hence short range correlations have little effect on the cross sections. Let us now consider the case of nucleus-nucleus scattering. We obtain, from Eqs. (5) and (6) (assuming A_1, A_2 large so that the c.m. correlations can be neglected),

$$i\chi_1(\vec{b}) \approx -A_1 A_2 \tilde{f} \int d^2s\, \rho_1(\vec{s}) \rho_2(\vec{s}-\vec{b}) , \tag{10}$$

and in absence of correlations,

$$\begin{aligned} i\chi_2(\vec{b}) \approx \tfrac{1}{2}(A_1 A_2)\tilde{f}^2 \Big\{ & (1 - A_1 - A_2) \Big[\int d^2s\, \rho_1(\vec{s}) \rho_2(\vec{s}-\vec{b}) \Big]^2 \\ & + \int d^2s\, \rho_1(\vec{s}) \rho_2(\vec{s}-\vec{b}) [(A_1-1)\rho_1(\vec{s}) + (A_2-1)\rho_2(\vec{s}-\vec{b})] \Big\} . \end{aligned} \tag{11}$$

The last two terms in χ_2 arise due to the processes in which one nucleon in either nucleus can undergo double collisions (these terms are absent in χ_{opt} for particle-nucleus scattering). $\text{Re}(i\chi_2)$ now gives a positive contribution and, at small b, the ratio χ_2/χ_1 grows as $\sim -A^{1/3}$. In most nuclei χ_2 leads to a significant reduction in the size of χ_1. For heavy nuclei $\chi_2 > \chi_1$ at small b and χ_{opt} can take unphysical values if truncated at χ_2.[6-8] This can happen at small b whenever the phase shift series (4) is truncated at even order (the series always converges at large b as the corrections involve higher powers of densities and decrease rapidly with increasing b).

In the presence of short range correlations Eq. (6) becomes

$$i\chi_2(b)=\tfrac{1}{2}(A_1A_2)\tilde{f}^2\Big\{N_1N_2\int d^3r_i d^3r_k dz_j dz_l\,\rho_1(\vec{r}_i)\rho_1(\vec{r}_k)\rho_2(\vec{s}_i-\vec{b},z_j)\rho_2(\vec{s}_k-\vec{b},z_l)$$
$$\times[A_1A_2(g_1(|\vec{r}_i-\vec{r}_k|)g_2(|(\vec{s}_i-\vec{s}_k,z_j-z_l)|)-1)$$
$$+(1-A_1-A_2)g_1(|\vec{r}_i-\vec{r}_k|)g_2(|(\vec{s}_i-\vec{s}_k,z_j-z_l)]$$
$$+N_2(A_2-1)\int d^2s_i dz_j dz_l\,\rho_1(\vec{s}_i)\rho_2(\vec{s}_i-\vec{b},z_j)\rho_2(\vec{s}_i-\vec{b},z_l)g_2(|z_j-z_l|)+(1\leftrightarrow 2)\Big\}\,. \tag{12}$$

For repulsive correlations $[g(r)-1]$ is negative and the leading contributions to $\mathrm{Re}(i\chi_2)$ are now negative. For small b, one can show that the ratio χ_2/χ_1 (for large $A_1=A_2=A$) behaves as $\sim A(l_c/R)(\tilde{f}/R^2)$. With increasing b many terms in $i\chi_2$ compete with each other and one should use the full expression (6) for $i\chi_2$. Nevertheless, we can see that since the correlations produce a change in sign of $i\chi_2$, they prevent χ_{opt} from taking nonunitary values.

In realistic cases, however, one cannot make a short range approximation for NN interactions as it generally leads to too large a correlation effect. In order to evaluate the full expressions (5) and (6) for χ_1 and χ_2 accurately we shall restrict our selves to light nuclei where the densities can be approximated by Gaussians (suitably chosen so that both the one- and two-body densities reproduce the correct rms radii). This approximation is reasonable since we only intend to compare theoretical expressions. The correlation functions needed in two-body densities can be calculated[3,10] from the solution of the Bethe-Goldstone equation employing the "standard hard core potential" of Moszkowski and Scott[17] which has an attractive exponential part outside a hard core of radius 0.4 fm. The hard core interactions lead to a correlation length $l_c=0.85$ fm. In addition one also has Pauli correlations which give $l_c=0.35$ fm. The effective correlation function is the statistical average of Pauli and interacting pair-correlation functions and yields[3] an effective correlation length $l_c=0.74$ fm. The elastic scattering cross sections are quite insensitive to the shape of the correlation function and depend mostly upon the correlation length.[3] It is therefore convenient to parametrize the correlation function by

$$g(r)=1-\exp(-\beta r^2)\,, \tag{13}$$

which satisfies the appropriate requirements at small and large distances. The parameter β is related to the correlation length by means of Eq. (2). The form (13) for g simplifies the calculations considerably and is fairly accurate for our purposes (e.g., it leads to errors $\lesssim 2\%$ for p-nucleus scattering[3]).

In real nuclei one also has longer range correlations. For example, center-of-mass correlations (which have a range of the order of the nuclear radius) are very important.[9,16] The effect of c.m. correlation is to pull the nucleons away from each other and their inclusion significantly improves the convergence of the optical phase shift series at larger impact parameters. These correlations are included in Eqs. (5) and (6) where the c.m. correlation function, for Gaussian (or harmonic oscillator) densities, is given by $K(q)=\exp[q^2(\langle r_1^2\rangle/A_1+\langle r_2^2\rangle/A_2)/6]$, $\langle r_i^2\rangle^{1/2}$ being the rms radii of the two nuclei (obtained from electron scattering measurements after correcting for finite proton size and the center-of-mass motion). For NN amplitudes we use the usual high energy parametrization

$$f(q)=\frac{k_N\sigma(i+\rho)}{4\pi}\,e^{-1/2aq^2}\,. \tag{14}$$

The results of our full calculation for ^{12}C-^{12}C and ^{16}O-^{16}O collisions at 2.1 GeV/n are shown in Fig. 1. For ^{12}C-^{12}C scattering, correlations increase the cross sections by ~15–20% near the second maximum and by ~28–36% near the third maximum. In ^{16}O-^{16}O scattering the effects are roughly ~13–20% and ~25–35%, respectively, at the two maxima. As expected from earlier rough estimates, the effects are smaller in α-α collisions, being ~12–14% near the second maximum. Also shown in Fig. 1 are ^{12}C-^{12}C cross sections neglecting c.m. correlations. The effects of c.m. correlations are quite large and clearly must always be included in any realistic analysis. The effects of short range correlations increase with increasing

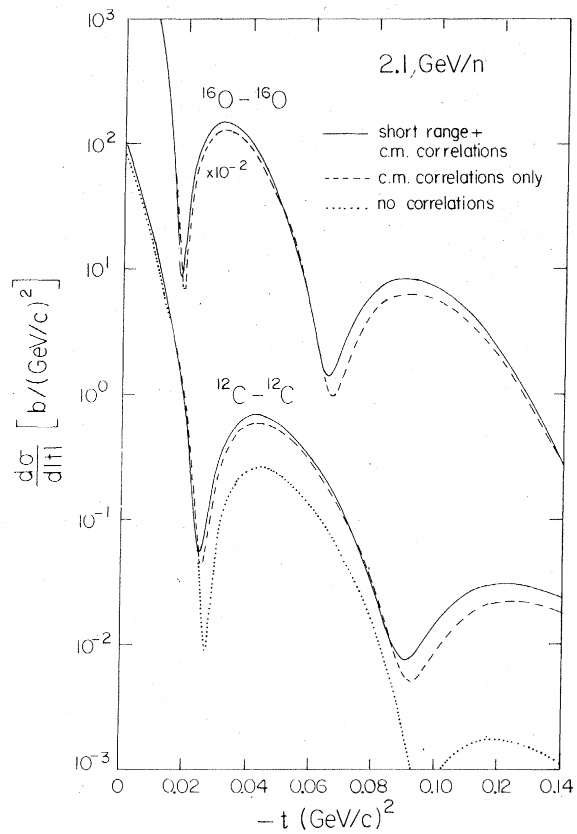

FIG. 1. ^{16}O-^{16}O and ^{12}C-^{12}C elastic scattering at 2.1 GeV/*n* with and without short range correlations. The dotted curve shows the effects of neglecting center-of-mass correlations in ^{12}C. The *NN* amplitude parameters used are $\sigma = 42.7$ mb, $a = 6.2$ (GeV/c)$^{-2}$, and $\rho = -0.28$.

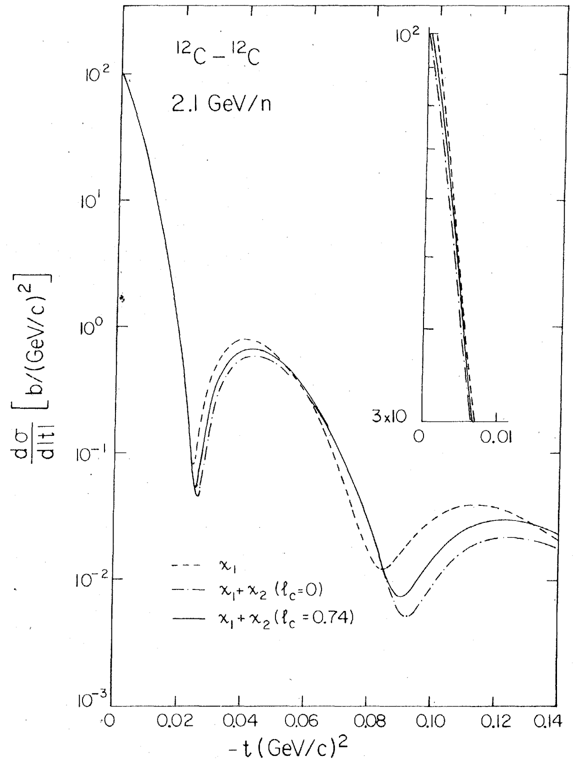

FIG. 2. ^{12}C-^{12}C elastic scattering intensities obtained by truncating the series for the optical phase shift function at χ_1, and at $\chi_1 + \chi_2$ with and without short range correlations.

momentum transfers but then one also has to include the correlation effects in third and higher order corrections to the optical phase shift function. From earlier crude estimates we also found that the size of χ_2 should change very significantly near small b for heavy nuclei due to the presence of correlations. When the full expressions (5) and (6) are evaluated for ^{208}Pb-^{208}Pb collisions, we find that near $b \approx 0$, $\mathrm{Re}(i\chi_1) \sim -740$ and $\mathrm{Re}(i\chi_2) \sim 770$ in the absence of correlations. With a correlation length $l_c = 0.74$ fm, $\mathrm{Re}(i\chi_2)$ becomes ~ -440 near $b \sim 0$.

We have seen that short range repulsive correlations play a significant role in preventing the optical phase shift function from taking unphysical values (for example, $|e^{i\chi_{\mathrm{opt}}}| > 1$) at small impact parameters for collisions between heavy nuclei (even in the absence of dynamical correlations, an effective repulsion is always present due to Pauli principle). How do the short range correlations affect the convergence of phase shift series for light nuclei? In order to answer this question we plot in Fig. 2 ^{12}C-^{12}C elastic scattering cross sections obtained from χ_1 and from $\chi_1 + \chi_2$ with and without correlations. In the momentum transfer region shown the series (4) for χ_{opt} (in the absence of short range correlations) has converged at $\chi_1 + \chi_2$ [9] (i.e., addition of χ_3 and χ_4 will not change the cross sections). Since the general effect of correlations is to inhibit the higher order multiple scattering processes, one might expect that these correlations will make the series converge more quickly. This is borne out by Fig. 2 where the cross sections due to $\chi_1 + \chi_2$ (in the presence of correlations) are closer to the cross sections obtained from χ_1 compared with the cross sections obtained from $\chi_1 + \chi_2$ (without correlations). We thus see that it may be possible to observe the effects of short range correlations in nucleus-nucleus collisions. In order to avoid unnecessary approximations and still keep the calculations tractable we have used relatively simple nuclear densities. However, our conclusions regarding the qualitative effects of correlations are not likely to change if more realistic densities are used.

We should also point out that the use of nuclear matter correlation functions is not very realistic in the exterior region of the nuclei. The calculation can be improved by using density dependent correlation lengths. In particle-nucleus collisions, at least, this results in a slight increase in the effects of correlations.[3] [Alternatively, Eq. (1) can also be used without referring to nuclear matter approximations. A short range repulsion can be introduced in the many-particle wave function by introducing correlations of Jastrow type.[18] A cluster expansion, when truncated at second order, will then yield results quite similar to Eq. (1).[5] Furthermore, before an attempt to extract correlations from experimental data is made one should examine the effects of spin dependence of *NN* amplitudes which may not be negligible at intermediate energies. Pauli correlations can also be added in a more accurate manner (for example, by constructing wave functions from Slater determinants of single particle orbitals).

It is perhaps also worthwhile to clarify the relationship of our results with those of lower energy heavy-ion collisions. The Pauli correlations we have talked about, arise from the antisymmetrization of the projectile and target wave functions separately. The antisymmetrization of the two nuclei (which leads to exchange amplitudes at low energies[19]) becomes increasingly unimportant as the energy goes up and most of the scattering moves to smaller angles. The multiple scattering corrections to the first order optical potential (the "double folding model"[20,21] in the terminology of low energy scattering) involve higher powers of densities, and become much less important at lower energies because the overlap between the two densities is quite small. The scattering is determined by the tails of the heavy-ion potentials (at the so called "strong absorption radius"[21]). Due to the dominance of Coulomb interactions at low energies, this radius is 2–3 fm larger than the sum of the rms radii of the two nuclei.[21] With increasing incident energy, greater penetration of one nucleus into the other becomes possible and the surface and interior regions of nuclei play a more important role. The sensitivity of high energy nucleus-nucleus cross sections to higher order optical potential corrections (and to correlations) is a consequence of this.

We would like to thank Professor D. R. Harrington and Professor L. Zamick for useful comments.

*Supported in part by the National Science Foundation.

[1]G. K. Varma and L. Zamick, Phys. Rev. C 16, 308 (1977); G. D. Alkhazov *et al.*, Nucl. Phys. A274, 443 (1976).

[2]For recent reviews on the subject see, for example, R. D. Viollier and J. D. Walecka, Acta Phys. Pol. B8, 25 (1977); O. Kofoed-Hansen, in *Proceedings of the International Conference on Few Body Problems in Nuclear and Particle Physics, Quebec, 1974,* edited by R. J. Slobodrian *et al.* (Les Presses de l'Universite Laval, Quebec, 1975), p. 630.

[3]E. Moniz and G. D. Nixon, Ann. Phys. (N.Y.) 67, 58 (1971), Phys. Lett. 30B, 393 (1969); R. D. Viollier, Ann. Phys. (N.Y.) 93, 335 (1975).

[4]H. Feshbach, A. Gal, and J. Hufner, Ann. Phys. (N.Y.) 66, 20 (1971); E. Lambert and H. Feshbach, *ibid.* 76, 80 (1972).

[5]R. Guardiola and E. Oset, Nucl. Phys. A234, 458 (1974).

[6]G. K. Varma, Ph.D.Thesis, 1976 (unpublished).

[7]V. Franco, Phys. Lett. 64B, 13 (1976); 61B, 444 (1976).

[8]V. Franco and G. K. Varma, Phys. Rev. C 15, 1375 (1977).

[9]V. Franco and G. K. Varma, report (unpublished).

[10]G. Dahl, E. Ostgaard, and B. Brandow, Nucl. Phys. A124, 481 (1969); A. L. Fetter and J. D. Walecka, *Quantum Theory of Many-Particle Systems* (McGraw-Hill, New York, 1971).

[11]V. Franco, Phys. Rev. 175, 1376 (1968).

[12]W. Czyż and L. C. Maximon, Ann. Phys. (N.Y.) 52, 59 (1969).

[13]D. R. Harrington and A. Pagnamenta, Phys. Rev. 184, 1908 (1969); 173, 1599 (1968); O. Kofoed-Hansen, Nuovo Cimento 60A, 621 (1977); J. Formanek, Nucl. Phys. B12, 441 (1969).

[14]G. K. Varma and V. Franco, Phys. Rev. C 15, 813 (1977).

[15]R. J. Glauber, in *Lectures in Theoretical Physics,* edited by W. E. Brittin and L. G. Dunham (Wiley-Interscience, New York, 1959), Vol. I, p. 315; V. Franco and R. J. Glauber, Phys. Rev. 142, 1195 (1966).

[16]V. Franco and A. Tekou, report (unpublished).

[17]S. A. Moszkowski and B. L. Scott, Ann. Phys. (N.Y.) 11, 65 (1960).

[18]R. J. Jastrow, Phys. Rev. 98, 1479 (1955).

[19]H. R. Jaqaman and A. Z. Mekjian, Nucl. Phys. A259, 157 (1976); M. Golin, F. Petrovich, and D. Robson, Phys. Lett. 64B, 253 (1976).

[20]J. P. Vary and C. B. Dover, Phys. Rev. Lett. 31, 1510 (1973).

[21]G. R. Satchler, in *Proceedings of the International Conference on Reactions Between Complex Nuclei, Tennessee, 1974,* edited by R. L. Robinson *et al.* (North-Holland, Amsterdam, 1974), p. 171.