Alpha particle transfer in the interacting boson approximation

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The interacting boson approximation of Iachello and Arima has been applied to the calculation of α particle transfer spectroscopic factors for nuclei from ^{44}Ti to ^{68}Zn. The results are compared with the results of (^{6}Li,d) experiments in the same mass region.

[NUCLEAR STRUCTURE Calculated α spectroscopic factors; compared with experimental (^{6}Li,d) S values.]

In view of the attractiveness of the interacting boson approximation (IBA) of Arima and Iachello in providing a unified description of nuclear properties for vibrational, rotational, and transitional nuclei,[1] it is interesting to investigate to what extent multinucleon transfer spectroscopic factors can be treated within the IBA framework. In the case of two-nucleon transfer, it has been found[2] that Pauli principle blocking effects are important for a proper description of two-nucleon transfer spectroscopic factors. Systematic information on α particle transfer spectroscopic factors found via the (^{6}Li, d) reaction,[3] will be compared here with IBA predictions for four-nucleon transfer.

In extension of the IBA treatment of two-nucleon transfer,[2] it is assumed that the α-particle creation operator can be expressed in terms of s- and d-boson creation and destruction operators as

$$\begin{aligned} A^\dagger_\alpha(L=0) &= c_1 \mathcal{S}^\dagger_\pi \mathcal{S}^\dagger_\nu + c_3(\mathfrak{D}^\dagger_\pi \times \mathfrak{D}^\dagger_\nu)^0 , \\ A^\dagger_\alpha(L=2) &= c_2(\mathcal{S}^\dagger_\pi \mathfrak{D}^\dagger_\nu + \mathfrak{D}^\dagger_\pi \mathcal{S}^\dagger_\nu)^2 + c_3(\mathfrak{D}^\dagger_\pi \times \mathfrak{D}^\dagger_\nu)^2 , \\ A^\dagger_\alpha(L=4) &= c_3(\mathfrak{D}^\dagger_\pi \times \mathfrak{D}^\dagger_\nu)^4 , \end{aligned} \tag{1}$$

where

$$\mathcal{S}^\dagger \equiv S^\dagger \left(\frac{\Omega - S^\dagger S - 2D^\dagger D}{\Omega} \right)^{1/2} ,$$

$$\mathfrak{D}^\dagger \equiv D^\dagger \left(\frac{(\Omega - S^\dagger S - 2D^\dagger D)(\Omega - S^\dagger S - 2D^\dagger D - 1)}{(\Omega - 2D^\dagger D)(\Omega - 2D^\dagger D - 1)} \right)^{1/2}$$

$S^\dagger$ and $D^\dagger$ are the s- and d-boson creation operators, and the subscripts π and ν refer to protons or neutrons.

The α-particle destruction operators appropriate for α pickup are the adjoints of the operators in (1).

The square root factors are present to take into account the Pauli exclusion principle, since the bosons of the IBA are in reality composed of fermions distributed among a set of shell-model orbits similar to those of the favored pair states considered by Hecht *et al.*[4] The Ω's represent the "boson capacity" of the underlying active shell-model orbits. For example, a single j orbit can hold $2j+1$ identical fermions or $j+\frac{1}{2}$ "bosons," so $\Omega_j = j+\frac{1}{2}$.

In justification of the form chosen for the α-creation operator, the value of the reduced matrix element of the normalized two-particle creation operator between j^n states, labeled in Racah's seniority scheme $\psi(n, v)$, where n is the number of particles and v is the seniority, is given by[5]

$$\langle \psi(n+2, v) \| A^\dagger(n=2, v=0) \| \psi(n, v) \rangle = \left[\left(\frac{n-v}{2} + 1 \right) \left(\frac{2}{2j+1} \right) \left(\frac{2j+1-v-n}{2} \right) \right]^{1/2} .$$

With the identifications

$j+\frac{1}{2} = \Omega$ = boson capacity,

$\frac{1}{2}n = N = n_s + n_d$ = total number of s and d bosons,

$\frac{1}{2}v = n_d$ = number of d bosons,

this matrix element becomes $[(n_s+1)(\Omega - n_s - 2n_d)/\Omega]^{1/2}$. Since the matrix element of the s-boson creation operator acting on a state of n_s s bosons gives a factor of $(n_s+1)^{1/2}$, the remainder of this term represents the Pauli blocking factor.

Similarly for the case of d bosons, the corresponding reduced matrix element in the seniority scheme is

$$\begin{aligned} &\langle \psi(n+2, v+2) \| A^\dagger(n=2, v=2) \| \psi(n, v) \rangle \\ &\quad = \langle \psi(v+2, v+2) \| A^\dagger(n=2, v=2) \| \psi(v, v) \rangle \\ &\quad \times \left(\frac{[\Omega - v - (n-v)/2][\Omega - v - (n-v)/2 - 1]}{(\Omega - v)(\Omega - v - 1)} \right)^{1/2} . \end{aligned}$$

Although the seniority scheme may easily be extended to include several j orbits if they are degenerate, in order to handle nondegenerate cases a more complicated treatment such as the BCS theory may be needed. In order to have simple forms for the transition rates, the operators (1) have been chosen.

Transitions from the ground states of the Ca, Ti, Cr, Fe, and Ni isotopes to the 0_1^+, 2_1^+, 0_2^+, 2_2^+, and 4_1^+ states of the corresponding residual nuclei will be considered. The only assumptions made about the structure of the states involved are that they are based on a ^{56}Ni core and have simple boson structure, that is, the ground states are assumed to have no d bosons, the first excited 2^+ states have one d boson, and the 0_2^+, 2_2^+, and 4_1^+ states have two d bosons.[6] Since only these states will be considered, they may be expressed unambiguously in terms of basis states labeled by

$$|n_{s\pi}, n_{d\pi}, n_{s\nu}, n_{d\nu}, J, M\rangle ,$$

specifying, respectively, the number of s- and d-proton bosons, the number of s- and d-neutron bosons, the total angular momentum, and its z component. These states are assumed to be totally symmetric in the $N_\pi = n_{s\pi} + n_{d\pi}$ proton bosons as well as the $N_\nu = n_{s\nu} + n_{d\nu}$ neutron bosons. Furthermore, they are assumed to be totally symmetric in all $N_\pi + N_\nu$ bosons. Quantities referring to boson hole states in the ^{56}Ni core will be distinguished by placing a bar over them. Some typical states are represented as

$$^{58}\mathrm{Ni}_{g.s.} = |0, 0, 1, 0, 0, 0\rangle ,$$

$$^{54}\mathrm{Fe}_{g.s.} = |\bar{1}, 0, 0, 0, 0, 0\rangle ,$$

$$^{44}\mathrm{Ti}_{g.s.} = |\bar{3}, 0, \bar{3}, 0, 0, 0\rangle ,$$

$$^{44}\mathrm{Ti}(2_1^+)_M = \frac{1}{\sqrt{2}}\left(|\bar{3}, 1, \bar{3}, 0, 2, M\rangle + |\bar{3}, 0, \bar{3}, 1, 2, M\rangle\right) .$$

The last example is representative of the structure assumed for the first excited 2^+ states, viz, they are taken to be symmetric in protons and neutrons.

In the IBA, the α-particle spectroscopic factor is defined as

$$S^L_{\mathrm{IBA}}(\alpha) = \langle n'_{s\pi} n'_{d\pi} n'_{s\nu} n'_{d\nu}, J_f \| A^\dagger_\alpha(L) \| n_{s\pi}, n_{d\pi}, n_{s\nu}, n_{d\nu}, J_i\rangle^2 .$$

The values of S for the cases to be considered (having $J_i = 0, n_{d\pi} = 0, n_{d\nu} = 0$) are given by

$$S(\mathrm{g.s.} \to \mathrm{g.s.}) = c_1^2 (n_{s\pi} + 1)\left(\frac{\Omega_\pi - n_{s\pi}}{\Omega_\pi}\right)(n_{s\nu} + 1)\left(\frac{\Omega_\nu - n_{s\nu}}{\Omega_\nu}\right) ,$$

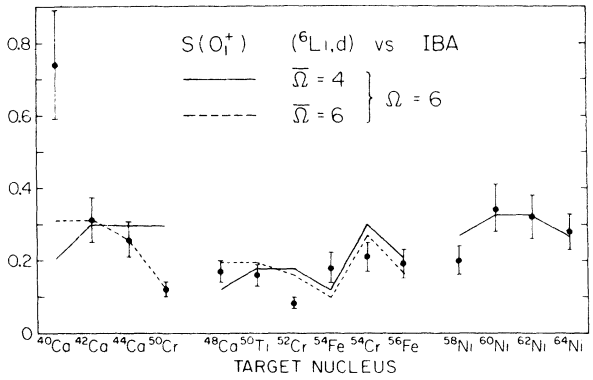

FIG. 1. The systematics of the ground state α-particle S values are compared with IBA results. The points are labeled according to target nucleus. The point for ^{64}Ni is from Ref. 10. The experimental values have 20% uncertainties, not including uncertainties due to the nature of the DWBA analysis used to reduce the data (Ref. 3).

$$\frac{S(\mathrm{g.s.} \to 2_1^+)}{S(\mathrm{g.s.} \to \mathrm{g.s.})} = \frac{c_2^2}{2c_1^2}\left[\left(\frac{(\Omega_\nu - n_{s\nu} - 1)}{(n_{s\nu}+1)(\Omega_\nu - 1)}\right)^{1/2} + \left(\frac{(\Omega_\pi - n_{s\pi} - 1)}{(n_{s\pi}+1)(\Omega_\pi - 1)}\right)^{1/2}\right]^2 ,$$

$$\frac{S(\mathrm{g.s.} \to 0_2^+, 2_2^+, 4_1^+)}{S(\mathrm{g.s.} \to \mathrm{g.s.})} = \left(\frac{c_3}{c_1}\right)^2 \frac{(\Omega_\pi - n_{s\pi} - 1)(\Omega_\nu - n_{s\nu} - 1)}{(n_{s\pi}+1)(\Omega_\pi - 1)(n_{s\nu}+1)(\Omega_\nu - 1)} .$$

These values are compared in Figs. 1–3 with the S values of Ref. 3, which are simply the ratio

$$S_{\mathrm{exp}} \equiv \frac{d\sigma(^6\mathrm{Li}, d)}{d\sigma_{\mathrm{DWBA}}} .$$

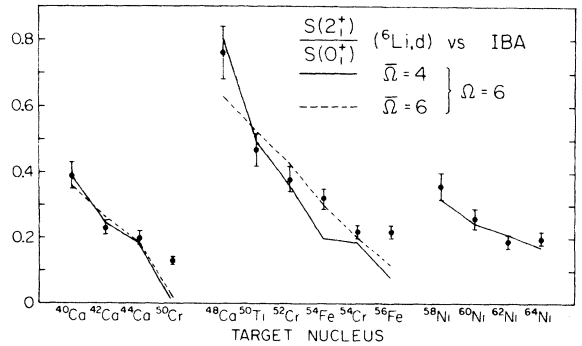

FIG. 2. The S values for the first excited 2^+ states relative to the ground state S value are compared with IBA predictions. The experimental uncertainties drawn are 10%. The point for ^{64}Ni is from Ref. 10.

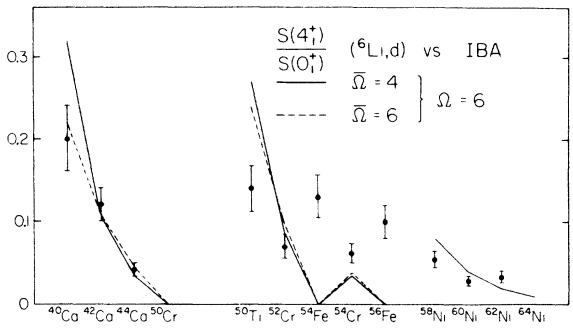

FIG. 3. The S values for the first excited 4^+ state relative to the ground state S values are compared with IBA predictions. The experimental uncertainties drawn are 20%.

In Figs. 1 to 3 the data are shown divided into three separate regions: The first region corresponds to cases in which both the proton and neutron pairs are filling boson holes in the ^{56}Ni core, the second to those cases in which the proton pair fills a boson-hole state and the neutron pair goes into a boson-particle state above the closed $f_{7/2}$ orbit, and the third (Ni targets) to the cases where the proton and neutron pairs are both going into boson-particle states. For all boson-particle states the value $\Omega = 6$ has been assumed, corresponding to active $p_{3/2}$, $p_{1/2}$, and $f_{5/2}$ orbits.

In each of the three regions, the normalization constants c_1, c_2, and c_3 have been adjusted independently to fit the data; their values are listed in Table I. The dotted curves represent the results of calculations with $\bar{\Omega}$, the boson capacity of the hole states in ^{56}Ni, taken to be 4. This value corresponds to having only the $f_{7/2}$ orbit active for the Ca isotopes and ^{50}Cr. The dashed curves result when $\bar{\Omega} = 6$ is assumed; it corresponds to having both the $f_{7/2}$ and $d_{3/2}$ orbits active below the ^{56}Ni core. These curves are included to illustrate possible core polarization effects in the Ca isotopes. [However, the DWBA calculations[3] were made assuming $(fp)^4$ transfer.] In neither case is the large S value for the ^{40}Ca to ^{44}Ti g.s. transition reproduced. The pairing-vibration model,[7] which corresponds to the limiting case $\bar{\Omega} \to \infty$, predicts a linear rise of the S values for the targets ^{44}Ca, ^{42}Ca, and ^{40}Ca. The trend agrees with the data, but the agreement is reached by ignoring the Pauli principle. It is possible that core polarization effects in ^{40}Ca and ^{44}Ti, not included in the IBA model, are acting to enhance the cross section for ^{40}Ca(^{6}Li, d)^{44}Ti. Except for the latter case, the overall agreement of the observed ground state transition strengths with those predicted by IBA theory is good, especially in the Ni region.

TABLE I. Normalization constants for α-transfer operators.

Region	c_1^2		$(c_2/c_1)^2$		$(c_3/c_1)^2$	
	$\bar{\Omega}=4$	$\bar{\Omega}=6$	$\bar{\Omega}=4$	$\bar{\Omega}=6$	$\bar{\Omega}=4$	$\bar{\Omega}=6$
Ca	0.20	0.08	0.20	0.90	0.32	5.5
Middle	0.12	0.10	0.40	0.60	0.80	2.4
Ni	0.16		0.24		0.20	

These results are perhaps related to the previously successful description of the Ni isotopes in terms of generalized seniority.[8]

The pairing vibration model with isospin[9] was also successful in describing the systematics of the ground state strengths, perhaps because Pauli blocking effects are implicitly taken into account by using two-nucleon transfer data to renormalize the strengths predicted for the four-nucleon transfer; however, the enhanced strength for ^{40}Ca to ^{44}Ti g.s. was not reproduced.

The general agreement seen in Fig. 2 between the observed $S(2_1^+)/S_{g.s.}$ ratios and the corresponding predictions of the IBA is striking. The agreement seen (Fig. 3) between the observed values of $S(4_1^+)/S_{g.s.}$ and IBA predictions is not good in the central region. Three cases are predicted to have zero strength, since d-boson transfer which closes the $f_{7/2}$ shell is forbidden.

The data for the 2_2^+ and 0_2^+ states, not so extensive as those for the lower-lying states, are similar in character to the 4_1^+ results. For a few cases, notably ^{44}Ti and ^{66}Zn, the 2_2^+, 0_2^+, and 4_1^+ S values are nearly equal, as they would be if those states are all populated by two d-boson transfers differing only in their angular momentum coupling.

The present simple form of the theory produces for the 0_1^+ and 2_1^+ states a fairly good agreement with the trends of the experimental values for a large number of nuclei in the fp shell. This agreement in trends was displayed here, for each of the regions involved, after arbitrary normalization to the experimental data. A more detailed treatment is being considered in which better contact is made with the underlying microscopic structure.

A preliminary account of this work has been presented earlier.[11]

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[1]A. Arima and F. Iachello, Ann. Phys. 99 (N.Y.) 253 (1976).
[2]A. Arima and F. Iachello, Phys. Rev. C 16, 2085 (1977).
[3]H. W. Fulbright, C. L. Bennett, R. A. Lindgreen, R. G. Markham, S. C. McGuire, G. C. Morrison, U. Strohbusch, and J. Tōke, Nucl. Phys. A284, 329 (1977).
[4]K. T. Hecht, J. B. McGrory and J. P. Draayer, Nucl. Phys A197, 369 (1972).
[5]J. B. French, in *Many-Body Description of Nuclear Structure and Reactions*, in *Proceedings of the International School of Physics, "Enrico Fermi," Course XXXVI, 1966* (Academic Press, Inc., New York, 1966), p. 278.
[6]In those cases where proton and neutron bosons are occupying identical orbits, the assumption of such simple boson structure is inconsistent with the goodness of isopin. For example, in the $f_{7/2}$ shell with S, D, G, and I standing for pairs of $f_{7/2}$ particles coupled to 0, 2, 4, and 6, the linear combination $0.5\,S_\pi S_\nu - 0.37\,D_\pi D_\nu - 0.5\,G_\pi G_\nu - 0.6\,I_\pi I_\nu$ has $J=0$ $T=2$; all other $J=0$ orthogonal states have $T=0$. $S_\pi S_\nu$ is thus 25% $T=2$. The only $J=0$ state composed of S and D bosons that has $T=0$ is the linear combination $0.6\,S_\pi S_\nu + 0.8\,D_\pi D_\nu$. In view of this, the present assumptions for the structure of the states considered are an oversimplification.
[7]A. Bohr, in *International Symposium Nuclear Structure, Dubna, 1968* (International Atomic Energy Agency, Vienna, Austria, 1969), p. 179.
[8]I. Talmi, Nucl. Phys. A172, 1 (1971).
[9]R. R. Betts, Phys. Rev. C 16, 1617 (1977).
[10]R. G. Markham, A. M. Bastawros, C. L. Bennett, H. W. Fulbright, private communication.
[11]H. W. Fulbright and C. L. Bennett, Bull. Am. Phys. Soc. 22, 1012 (1977).