Inhibited electric-quadrupole transitions in odd-neutron spherical nuclei

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A number of inhibited electric-quadrupole transitions have been found in odd-N nuclei, and their lifetimes have been measured. The pairing-plus-quadrupole model predicts a correlation between these lifetimes and the shell-model occupation numbers which can be obtained from pickup and stripping reactions. The correlation observed is about that expected from the accuracy of the data on occupation numbers.

[NUCLEAR REACTIONS d, α, ^3He, $E_\alpha = 12$ MeV, pulsed beam, measured $E2$ transition rates in ^{63}Ni, ^{65}Ni, ^{99}Mo, ^{103}Ru, ^{105}Ru, ^{107}Pd, ^{108}Pd, ^{109}Cd, ^{111}Sn, ^{115}Sn compared to prediction pairing-plus-quadrupole model.]

We report here the results of lifetime measurements on a number of isomeric states in odd-neutron nuclei that decay by $E2$ radiation with a rate much below the single-particle estimate. We have compared our results and other available data with a semiempirical prediction based on the shell model with a pairing-plus-quadrupole residual interaction. Our treatment of the data is based on this model as presented by Kisslinger and Sorensen.[1,2] As they demonstrated, the model leads to the prediction that in odd-A nuclei, $E2$ transition rates between one-quasiparticle levels will be reduced by a factor $(U_iU_f - V_iV_f)^2$ relative to the single-particle rate. The quantities V_i^2 and V_f^2 are the occupation probabilities of the single-particle levels in the ground state of the neighboring even-even nucleus and $U_i^2 + V_i^2 = 1$. The quantities U_i^2 and V_i^2 can be obtained from pickup and stripping reactions.[3] Adding a quadrupole force to the residual interaction mixes the one-quasiparticle states with more complicated states involving quadrupole excitations. Although this results in much higher $E2$ transition rates among low-lying levels, some very inhibited transitions are also predicted and some are observed to occur.

Transition rates for this pairing-plus-quadrupole model have been calculated by Sorensen[4] and by Reehal and Sorensen,[5] who assumed the Kisslinger-Sorensen single-particle energies. These calculations, however, do not include many of the levels considered here. We have used a somewhat different approach in which we avoided assuming single-particle energies and attempted to obtain the parameters necessary to predict the lifetimes directly from experimental data. These parameters include the occupation probabilities V_j^2 and U_j^2 (from pickup and stripping reactions), the energy and $B(E2)$ from the first 2^+ state in the adjacent even-even nucleus with $(N-1)$ neutrons [for ^{68}Ge and ^{110}Sn, $B(E2)$ was estimated[6]], and the quadrupole-coupling parameter χ, which was taken from the analysis of Uher and Sorensen.[7] These parameters were then used with Sorensen's expression to calculate the lifetimes, which were compared with the experimental values.

For a pair of levels with the same parity and with $|j_1 - j_2| = 2$, only the $E2$ partial lifetime is important in determining the observed lifetime. Single-particle estimates of the lifetime of an $E2$ transition between levels separated by 100 keV are of the order of 0.5 μsec, and lifetimes for inhibited transitions are expected to be in the microsecond region. We have found inhibited $E2$ transitions in odd-neutron nuclei in two regions of the Periodic Table. One group appears in the region from nickel to germanium and corresponds to transitions between $p_{1/2}$ and $f_{5/2}$ quasiparticle levels. A second group appears in the region from molybdenum to tin and corresponds to $s_{1/2} \to d_{5/2}$ transitions. Such inhibited transitions are not nearly so common in odd-Z nuclei.

The measurements were made with a pulsed beam[8] from the Argonne tandem Van de Graaff. In some cases, care was required in order to relate the observed spectrum of delayed γ rays to the decay of a particular level in a particular nucleus. Three additional types of observations were used to reduce this uncertainty. (1) As an aid in identifying the nucleus, we observed both those combinations of target and projectile (p, d, ^3He, or ^4He) that could excite the isomer and those that could not. (2) The time dependence of the γ-ray intensities was measured during the pulse as well as after it in order to identify those γ rays originating at the isomeric level. (3) We required that energies of the observed γ rays be consistent with particle-reaction data, and this identification also provided information on the

TABLE I. Mean lives and ratios of the reduced transition rate to the single-particle estimate for $E2$ transitions in odd-neutron nuclei. The pairing-force reduction factor $(U_iU_f - V_iV_f)^2$ for each nucleus was obtained from the experimental papers cited.

	j_i	j_f	E_γ (keV)	τ_{obs} (μsec)	$\frac{B(E2)}{B(\text{s.p.})}$	$(U_iU_f - V_iF_f)^2$
^{63}Ni	$\frac{5}{2}-$	$\frac{1}{2}-$	87.1 87.2	2.43 ± 0.05 2.48 ± 0.04[b]	2.25	0.032[a]
^{65}Ni	$\frac{1}{2}-$	$\frac{5}{2}-$	62.5	96 ± 5	0.047	0.082[a]
^{99}Mo	$\frac{5}{2}+$	$\frac{1}{2}+$	98.1	24.5 ± 2.0	0.053	0.18[c]
^{103}Ru	$\frac{1}{2}+$	$\frac{5}{2}+$	172.2	12.8 ± 1.5	0.0042	0.0016[d]
^{105}Ru	$\frac{1}{2}+$	$\frac{5}{2}+$	143.4	0.08 ± 0.01	1.36	0.0065[d,e]
^{107}Pd	$\frac{1}{2}+$	$\frac{5}{2}+$	115.6 115.6	1.48 ± 0.66 1.23 ± 0.14[g]	0.159	0.013[e,f]
^{109}Pd	$\frac{1}{2}+$	$\frac{5}{2}+$	113.4	0.54 ± 0.04	0.46	0.00088[f]
^{109}Cd	$\frac{1}{2}+$	$\frac{5}{2}+$	59.5	17.5 ± 1.0 17.3 ± 2.6[i]	0.063	...[h]
^{111}Sn	$\frac{1}{2}+$	$\frac{5}{2}+$	100.2 100.0	18.0 ± 1.5 18.0 ± 1.0[k]	0.016	0.01[j]
^{115}Sn	$\frac{7}{2}+$	$\frac{3}{2}+$	115.5 120	5.5 ± 0.5 4.7 ± 1.2[l]	0.064	0.015[j]

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[h] Data not available for occupation numbers.
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multipolarity of the transition.

The decay rate of the isomer was determined by using a small (6 cm^3) Ge(Li) detector with a resolution of 2.5 keV for γ rays of 100 keV. A large core memory stored a two-dimensional array of time and pulse-height data from the detector, and from this the lifetime could be extracted. The time was measured by a digital clock with a basic period of 0.05 μsec. A pulser, triggered at random times, inserted a peak that could be used to correct for dead-time and counting-rate effects which sometimes became important because of the high rate during the beam pulse. γ energies were easily measured with a precision better than 0.5 keV.

Table I lists $E2$ isomers for which we have observed lifetimes in the microsecond region. In a few cases, a second line gives the transition energy and lifetime obtained in a recent measurement by others. The internal-conversion coefficients from the tables of Hager and Seltzer[9] were used to get the partial $E2$ mean life $\tau(E2)$, and hence $B(E2)$, from τ_{obs}. The single-particle estimate $B(\text{s.p.})$ in the ratio of reduced transition rates $B(E2)/B(\text{s.p.})$ is that given by Moszkowski.[10] The quantity $(U_iU_f - V_iV_f)^2$ in the last column was determined by the procedure described below.

In order to compare these lifetimes with the pairing-plus-quadrupole model, we have examined the literature in order to obtain the occupation probabilities. In those cases in which the authors presented tables of U_j^2 or V_j^2, these were taken as given. In most cases in which both (d,p) and (d,t) data were available for the same target nucleus, the experimental values did not satisfy $U_j^2+V_j^2=1$, especially for single-particle levels far from the Fermi level. Usually $(U_j^2+V_j^2)$ deviated from 1 by less than 20% in either direction, although in a few cases the deviation was larger than this. We recalculated V_j^2 by using

$$V_j^2=\frac{V_j^2(\exp)}{V_j^2(\exp)+U_j^2(\exp)}\,.$$

This is equivalent to assuming that the same fraction of the single-particle strength is missed in the (d,p) and (d,t) reactions. The resulting V_j^2 were further normalized by requiring

$$\sum(2j+1)\,V_j^2=N=\text{number of valence neutrons.}$$

In cases in which the (d,p) and (d,t) data were available, this second normalization produced changes of only a few percent in V_j^2. Where (d,p) and (d,t) data were not both available, this normalization usually produced much larger changes in V_j^2. From the V_j^2 and U_j^2 obtained in this way, we formed the factor $(U_iU_f-V_iV_f)^2$ given in the last column of Table I.

Consider now the residual interaction given by Sorensen,[4] namely,

$$\langle 0|\alpha_j|H_{\text{int}}|(B^+\alpha_{j'}^+)_j|0\rangle = -\bar{\chi}(5/4\pi)^{1/2}\langle j|r^2|j'\rangle C_{0\frac{1}{2}\frac{1}{2}}^{2jj'} \times(-1)^{j-j'}(U_jU_{j'}-V_jV_{j'}). \quad (1)$$

This interaction mixes a one-quasiparticle state $\alpha_j^+|0\rangle$ with one-quasiparticle, one-phonon states $(B^+\alpha_{j'}^+)|0\rangle$, and ignores terms involving more complicated states. The wave function in Sorensen's notation now becomes

$$\psi_j=C_{j00}^{\;j}\,\alpha_j^+|0\rangle+\sum_{j'}C_{j'12}^{\;j}\,(B^+\alpha_{j'}^+)_j|0\rangle+\cdots. \quad (2)$$

The coefficients $C_{j'12}^{\;j}$ should be obtained by diagonalizing the system Hamiltonian. For this one needs to have all quantities V_j for the shell under consideration. We chose instead to compute the coefficients $C_{j'12}^{\;j}$ assuming first-order perturbation theory. One then finds

$$\begin{aligned}C_{j'12}^{\;j}&=(5/4\pi)^{-1/2}\bar{\chi}\langle j|r^2|j'\rangle \times C_{0\frac{1}{2}\frac{1}{2}}^{2jj'}(U_jU_{j'}-V_jV_{j'})/\Delta E\\&=(-1)^{j-1/2}\bar{\chi}[B(\text{s.p.};j-j')]^{1/2}\\&\quad\times(U_jU_{j'}-V_jV_{j'})/\Delta E\,,\end{aligned} \quad (3)$$

where $B(\text{s.p.};j\to j')$ is the single-particle $B(E2)$ for a transition $j\to j'$, ΔE is approximately the phonon energy which we have taken from the energy of the first 2^+ level in the adjacent even-even nucleus, and $\bar{\chi}$ is an effective quadrupole-interaction strength given in Ref. 1.

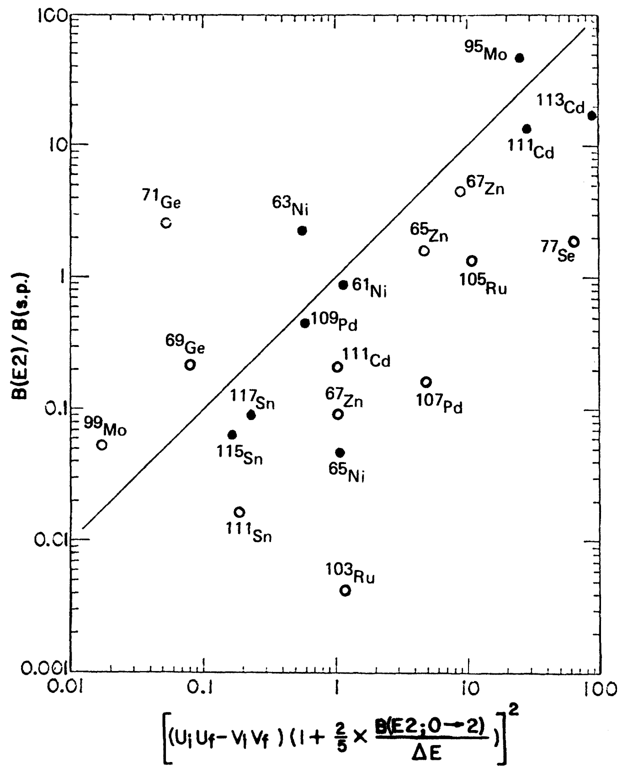

FIG. 1. A plot of the ratio of the observed reduced transition rate $B(E2)$ to the single-particle $B(\text{s.p.})$ versus a parameter, derived from the pairing-plus-quadrupole model, which was obtained from experimentally determined values. These were the occupation numbers V_j^2 from (d,p) and (d,t) reactions, the energy ΔE and $B(E2,\ 0\to2)$ of the transition in the adjacent even-even isotope, and the quadrupole-coupling constant χ. The diagonal line is given by Eq. (4). A closed circle was plotted for transitions for which both (d,p) and (d,t) data exist for determining V_j and U_j. An open circle was plotted for transitions in which the data are more fragmentary.

Now we can write Sorensen's expression[4] for the $E2$ reduced transition rate as

$$B(E2;j_i\to j_f)=B(\text{s.p.};j_i\to j_f)\left(1+\tfrac{2}{5}\chi\,\frac{B(E2;0\to2)}{\Delta E}\right)^2 \times(U_iU_f-V_iV_f)^2, \quad (4)$$

where we have considered only transitions between one-quasiparticle states and between one-quasiparticle-one-phonon and one-quasiparticle-0-phonon states. In this expression for $B(E2;j_i\to j_f)$, we have approximated $\bar{\chi}$ by $\chi[B(E2;2\to0)]^{1/2}$. Here $B(E2;2\to0)$ is the reduced transition rate for the first $2^+ \to 0^+$ transition in the adjacent even-even nucleus and χ is the quadrupole-coupling constant, which we have taken the same for neutrons and protons. In addition, we have assumed that the neutron and proton effective charges are each equal to e. Uher and Sorensen[7] have evaluated χ throughout the Periodic Table from the energy of the first 2^+ state in even-even nuclei. In evaluating Eq. (4) we have used their value of χ, which is a smooth function of nuclear mass.

Figure 1 compares experimental data with the predictions of Eq. (4) for odd-neutron transitions. Included in this figure are the data from Table I as well as additional transitions for which the partial $E2$ lifetimes or $B(E2)$ values are known from direct lifetime measurement or Coulomb-excitation measurements. Electronic measurements of transition rates were used for the $\frac{1}{2}\to\frac{5}{2}$ transition in ^{65}Zn,[11] the $\frac{1}{2}\to\frac{5}{2}$ and $\frac{5}{2}\to\frac{3}{2}$ in ^{67}Zn,[12] the $\frac{1}{2}\to\frac{5}{2}$ in ^{69}Ge,[13] the $\frac{5}{2}\to\frac{1}{2}$ in ^{71}Ge,[14] the $\frac{5}{2}\to\frac{1}{2}$ in ^{77}Se,[15] and the $\frac{5}{2}\to\frac{1}{2}$ in ^{111}Cd.[16] Coulomb-excitation measurements of transition rates were used for the $\frac{3}{2}\to\frac{5}{2}$ in ^{61}Ni,[17] the $\frac{5}{2}\to\frac{3}{2}$ in ^{67}Zn,[12] the $\frac{5}{2}\to\frac{3}{2}$ in ^{95}Mo,[18] the $\frac{1}{2}\to\frac{3}{2}$ in ^{111}Cd,[19] the $\frac{1}{2}\to\frac{3}{2}$ in ^{113}Cd,[20] and the $\frac{1}{2}\to\frac{3}{2}$ in ^{117}Sn.[21] For the determination of V_i, V_f, U_i, and U_f, the data not included in Table I come from pickup and stripping reactions on zinc,[22] germanium,[23] selenium,[24] and cadmium.[25] Although most of the transitions we studied are inhibited relative to the single-particle estimate, many of the other transitions in Fig. 1 are not inhibited. The range of transition rates relative to the single-particle estimate varies over four orders of magnitude. The errors in the vertical direction are small (usually < 10%). Errors in the horizontal direction arise principally in the determination of $(U_iU_f - V_iV_f)^2$. The relative error in the experimental determination of this quantity becomes very large for $(U_iU_f - V_iV_f)^2$ small; for points on the left side of the figure, the error is probably about 10 times the abscissa.

If the relationship (4) accurately represented the data, then one would expect the points to cluster about the diagonal line in Fig. 1. However, the errors in determining the occupation probabilities V_j^2 are large and one must expect considerable scatter of the points. As an alternative treatment, we plotted $B(E2)/B(\text{s.p.})$ vs $(U_iU_f - V_iV_f)^2$; this gave a somewhat larger scatter and a poorer fit to the data. In addition, Fig. 1 shows a noticeable systematic displacement of the points below the line represented by Eq. (4). Although this could arise from a systematic error in the quadrupole coupling constant χ, such an effect could also be produced by our use of perturbation theory.

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