Decoupled Yakubovski equation for a four-body system and its physical implications

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The Yakubovski equation for a four-body system is decoupled into a tractable expression suitable for the treatment of reactions. Because of two-body interactions, some channel wave functions collapse in the course of collision processes. A part of the decoupled Yakubovski wave function justifies the coupled cluster approximation. The optical potential is defined by the L^2 operator in the four-body space. The lowest order amplitude is the impulse approximation.

[NUCLEAR REACTIONS Four-body Yakubovski equation decoupled to handle reactions. Cluster approximation. Relation to optical model and impulse approximation.]

I. INTRODUCTION

The Yakubovski equation[1] is a coupled set of equations for N-body problems, as a generalization of the Faddeev equation[2] for three-body problems. The Yakubovski equation has desirable properties: (i) The Yakubovski equation satisfies the original Schrödinger equation without redundancy. (ii) The wave function has a tree structure with respect to successive interactions. This means that the wave function is constructed following simple hierarchies. (iii) The kernel of the equation is fully connected. (iv) There is no spurious bound state. On the other hand, difficulties in the Yakubovski equation are as follows: (i) The number of coupled equations is too large. In fact, 18 (130) equations are coupled in four- (five-) body problems. (ii) The Yakubovski equation is not designed for use in a reaction theory of many-body systems. For example, in a four-body system, the number of binary partitions directly related to two-body channels is seven. Then the problem arises how to make "eighteen" relate to "seven." These difficulties prevent the Yakubovski equation from being a vital reaction theory. These difficulties are partly removed by recombinations of the Yakubovski wave functions.[3] It has been shown that all amplitudes resulting from the recombined wave functions are brought into the well-known expressions,[4] if we make use of the fact that the final state is on the four-body energy shell.[3,5] Also, it was shown explicitly for three- and four-body systems that this wave function covers the whole Hilbert space [Eq. (55) of Ref. 5] and that the closed channel vanishes in the high energy limit.

Nevertheless, one difficulty still prohibits practical applications of the Yakubovski equation: its lack of an intuitive description of multistep processes. If each step in the multiple scattering expansion is expressed in terms of channel wave functions, we may call it intuitive. Naturally, the channel wave functions require special decoupling of the Yakubovski components. The Yakubovski equation has been constructed with care for full connectivity to two-body interactions. That makes it impossible to express each step of the multiple scattering in terms of channel wave functions. Of course, since the scattering processes in the collision complex take place under the two-body interactions, the whole process need not be described only by the channel wave functions: Some part of the channel wave functions may be collapsed. Nevertheless, it is desirable that at least some part of the process be described by the channel wave functions.

The present paper is a study of this problem. In fact, one of the motivations of the present paper was the question as to whether the Yakubovski equation can be brought to amenable and intuitive form compatible with the contemporary theory of nuclear reactions,[6] which has been developed without care to the structure of N-body Hilbert space, nor the connectivity of kernels, but which has been considered to be more practical in use than the Faddeev-Yakubovski equations. (A criticism to the contemporary theory of nuclear reactions was given in Ref. 5.)

In the present paper, we find that the decoupled Yakubovski equation can in fact be brought to amenable form. For instance, the lowest order term is the impulse approximation; we can find a satisfactory operator which serves as optical potential for the elastic scattering, etc. The results are summarized at the end of the present paper. The author now believes that the decoupled Yakubovski equation does not involve any fundamental difficulty, either in principle or for practical use.

Previous papers by the author are the background of the present paper. References 3 and 5 will be referred to as I and II, respectively. For example, Eq. (10) of Ref. 3 will be quoted as Eq. (I.10).

II. BACKGROUND OF THE CURRENT PROBLEM

The four-body wave function is expressed in two ways. We designate by ϕ_{ij} the wave function in which the pair ij interacts in the final state. Since there are six interactions, the total wave function is expressed as (I.1)

$$\psi=\phi_{12}+\phi_{13}+\phi_{14}+\phi_{23}+\phi_{24}+\phi_{34}. \tag{1}$$

Alternatively, we define the cluster wave function ϕ^{123} to represent a state in which 1, 2, and 3 interact while the particle 4 is in the continuum in the final state. In terms of the cluster wave functions, the total wave function is expressed as (I.9)

$$\psi=\phi^{123}+\phi^{124}+\phi^{134}+\phi^{234}+\phi^{12,34}+\phi^{13,24}+\phi^{14,23}. \tag{2}$$

For the binary partitions of the system, the use of the cluster wave functions ϕ^{123}, $\phi^{12,34}$, etc., is useful, while for the breakup process (ternary and quarterly partitions), the use of ϕ_{12}, etc., is more useful.[5] Anyway, ϕ_{12}, etc., and ϕ^{123}, etc., are obtained by a decoupling of the Yakubovski components.[3,5] ϕ^{123}, $\phi^{12,34}$, etc., are related to ϕ_{12}, etc., by the following relation. To be specific we assume that the particle 4 impinges on the bound 123 in the initial state. We denote the wave function of the initial state as $\varphi_0^{(123)}f_{4,p_0}$. Then ϕ^{123} and $\phi^{12,34}$ are expressed as (I.40)

$$\phi^{123}=\varphi_0^{(123)}f_{4,p_0}+G_0W^{123}(\phi_{14}+\phi_{24}+\phi_{34}) \tag{3}$$

and [cf. Eq. (I.27)]

$$\phi^{12,34}=G_0W^{12,34}(\phi_{13}+\phi_{14}+\phi_{23}+\phi_{24}) \tag{4}$$

with

$$W^{12,34}=W_{12}^{12,34}+W_{34}^{12,34}.$$

The amplitude leading to the bound 123 system and a free particle 4 is obtained from Eq. (3) as [(I. 73), (II. 35)]

$$T_n^{(123)}=\langle\varphi_n^{(123)}|\langle f_{4,p_n}|V_{12}+V_{13}+V_{23}|\phi_{14}+\phi_{24}+\phi_{34}\rangle, \tag{5}$$

where $\varphi_n^{(123)}$ denotes the wave function of the nth state of the bound 123. Similarly, the amplitude leading to the final state $\varphi_n^{(12)}\varphi_m^{(34)}f_{12\text{-}34;nm}$ is obtained from Eq. (4) as [cf. Eq. (I.81)]

$$T_{nm}^{(12)(34)}=\langle\varphi_n^{(12)}|\langle\varphi_m^{(34)}| \times\langle f_{12\text{-}34;nm}|V_{12}+V_{34}|\phi_{13}+\phi_{14}+\phi_{23}+\phi_{24}\rangle. \tag{6}$$

The amplitude for the ternary partition, in which a pair 12 is in a bound state, is expressed as [II.51, II.52, and II.53]

$$T_n^{(12)}=\langle\varphi_n^{(12)}|\langle f_{3,p_3}|\langle f_{4,p_4}|V_{12}|\phi_{13}^{123}+\phi_{23}^{123}+G_0(t_{13}+t_{23})(\phi_{14}+\phi_{24}+\phi_{34})+\phi_{14}^{124}+\phi_{24}^{124}+G_0(t_{14}+t_{24})(\phi_{13}+\phi_{23}+\phi_{34}) + \phi_{34}^{12,34}+G_0t_{34}(\phi_{13}+\phi_{23}+\phi_{14}+\phi_{24})\rangle. \tag{7}$$

$$\phi_{13}^{123}=G_0t_{13}(\phi_{12}+\phi_{23}), \tag{8}$$

$$\phi_{34}^{12,34}=G_0t_{34}\phi_{12}. \tag{9}$$

Now the problem with these expressions is the following: These expressions are described in terms of wave functions such as $\phi_{14}+\phi_{24}+\phi_{34}$ and $\phi_{13}+\phi_{14}+\phi_{23}+\phi_{24}$, etc. Clearly, both for intuitive understanding of the scattering processes and for easy handling of the multiple scatterings, it is desirable that the wave functions $\phi_{14}+\phi_{24}+\phi_{34}$, etc. be expressed in terms of the cluster wave functions ϕ^{123}, etc. Equations (I.44) and (I.49) were found for this purpose. However, the denominator

$$1+G_0(t_{12}+t_{13}+t_{23})[1+G_0(t_{12}+t_{34})]$$

in Eq. (I.44) [Eq. (I.49)] is unfamiliar.

Another way of expressing the amplitudes in terms of the channel wave functions is to first bring Eqs. (5)–(7) to the usual form [(I.73), (I.81), and (II.54)]

$$T_n^{(123)}=\langle\varphi_n^{(123)}|\langle f_{4,p_n}|V_{14}+V_{24}+V_{34}|\psi\rangle, \tag{10}$$

$$T_{nm}^{(12)(34)}=\langle\varphi_n^{(12)}|\langle\varphi_m^{(34)}| \times\langle f_{12\text{-}34;n,m}|V_{13}+V_{14}+V_{23}+V_{24}|\psi\rangle, \tag{11}$$

and

$$T_n^{(12)}=\langle\varphi_n^{(12)}|\langle f_{3,p_3}| \times\langle f_{4,p_4}|V_{13}+V_{14}+V_{23}+V_{24}+V_{34}|\psi\rangle, \tag{12}$$

and then use Eq. (2) for ψ. By this procedure, the first order term is described by the cluster wave functions. For handling the second and higher order terms, we must employ Eqs. (3) and (4). Then the rightmost function becomes again $\phi_{14}+\phi_{24}+\phi_{34}$, etc. For getting the "physical expression" of the amplitude, we must have equations in which ϕ_{12}, etc. are expressed by some combinations of cluster wave functions ϕ^{123}, etc. This kind of equation has not been found so far. The purpose of the present paper is then to give equations of this kind and to see how useful these equations are.

III. NEW SET OF EQUATIONS

Each of the wave functions ϕ_{12}, ϕ^{123}, and $\phi^{12,34}$ consists of the Yakubovski components [(I.6), (I.7), and (I.8)]

$$\phi_{12}=\phi_{12}^{123}+\phi_{12}^{124}+\phi_{12}^{12,34}, \tag{13}$$

$$\phi^{123}=\phi_{12}^{123}+\phi_{13}^{123}+\phi_{23}^{123}, \tag{14}$$

$$\phi^{12,34}=\phi_{12}^{12,34}+\phi_{34}^{12,34}. \tag{15}$$

The wave function ϕ_{12} satisfies the equation [(I.4)]

$$\phi_{12}=G_0t_{12}(\phi_{13}+\phi_{14}+\phi_{23}+\phi_{24}+\phi_{34}). \tag{16}$$

If we use Eqs. (13)–(15), Eq. (16) is brought into the form

$$\phi_{12}=G_0t_{12}(\phi_{13}^{123}+\phi_{23}^{123}+\phi_{24}^{124}+\phi_{14}^{124}+\phi_{34}^{12,34}+\phi^{134}+\phi^{234}+\phi^{13,24}+\phi^{14,23}). \tag{17}$$

The Yakubovski components are expressed as [see the equation following (II.43)]

$$\begin{aligned}\phi_{12}^{123}&=\varphi_{12,0}^{(123)}f_{4,p_0}+G_0W_{12}^{123}(\phi_{14}+\phi_{24}+\phi_{34}),\\ \phi_{12}^{124}&=G_0W_{12}^{124}(\phi_{13}+\phi_{23}+\phi_{34}),\\ \phi_{12}^{12,34}&=G_0W_{12}^{12,34}(\phi_{13}+\phi_{23}+\phi_{14}+\phi_{24}).\end{aligned} \tag{18}$$

Here $\varphi_{12,0}^{(123)}$ represents the 12 component of $\varphi_0^{(123)}$ [cf. Eq. (14)]. The kernel $G_0W_{12}^{123}$ ($G_0W_{12}^{12,34}$) denotes the 12 component of the kernel $G_0W_{12}^{123}$ ($G_0W^{12,34}$) which is fully connected with respect to 123 (12 and 34). The kernel $G_0W_{12}^{123}$ was given in Eq. (I.29) [Eq. (I.34)]. Making use of Eq. (18), we can express e.g. ϕ_{14} in Eq. (5) in a fully connected manner as

$$\begin{aligned}\phi_{14}=G_0t_{14}[&G_0(W_{12}^{124}+W_{24}^{124})(\phi_{13}+\phi_{23}+\phi_{34})\\&+G_0(W_{13}^{134}+W_{34}^{134})(\phi_{12}+\phi_{23}+\phi_{24})\\&+G_0W_{23}^{14,23}(\phi_{12}+\phi_{13}+\phi_{24}+\phi_{34})\\&+\phi^{123}+\phi^{234}+\phi^{12,34}+\phi^{13,24}].\end{aligned} \tag{19}$$

By solving these six equations for ϕ_{ij}, we can express each ϕ_{ij} by combinations of ϕ^{ijk} and $\phi^{ij,kl}$. Therefore, Eq. (19) and similar equations for ϕ_{ij} are equations we wanted to have.

The important features in Eq. (19) are as follows: (i) ϕ_{14}, in general ϕ_{ij}, consists of "cluster terms," $\phi^{123}+\cdots+\phi^{13,24}$ and "collapsed terms" [first three terms on the right-hand side of Eq. (19)]. It may be interpreted that some cluster wave functions [ϕ^{124}, ϕ^{134}, and $\phi^{14,23}$ in Eq. (19)] collapse in the course of collision. (ii) The cluster terms are more important than the collapsed terms, because the cluster terms couple to ϕ_{ij} by the lower order interactions than the collapsed terms do. Thus, we may approximate ϕ_{14} by

$$\phi_{14}\simeq G_0t_{14}(\phi^{123}+\phi^{234}+\phi^{12,34}+\phi^{13,24}). \tag{20}$$

Let us call the approximation (20) the *coupled cluster approximation* (CCA).

IV. COUPLED CLUSTER APPROXIMATION (CCA)

In this section, we investigate the consequences of CCA. Under this approximation, ϕ_{24} and ϕ_{34} read

$$\phi_{24}\simeq G_0t_{24}(\phi^{123}+\phi^{134}+\phi^{12,34}+\phi^{14,23}), \tag{21}$$

$$\phi_{34}\simeq G_0t_{34}(\phi^{123}+\phi^{124}+\phi^{13,24}+\phi^{14,23}). \tag{22}$$

If we put Eqs. (20)–(22) in Eqs. (3) and (4), we obtain

$$\begin{aligned}\phi^{123}\simeq\varphi_0^{(123)}f_{4,p_0}&+G_0W^{123}G_0[t_{14}(\phi^{123}+\phi^{234}+\phi^{12,34}+\phi^{13,24})\\&+t_{24}(\phi^{123}+\phi^{134}+\phi^{12,34}+\phi^{14,23})\\&+t_{34}(\phi^{123}+\phi^{124}+\phi^{13,24}+\phi^{14,23})]\end{aligned} \tag{23}$$

and

$$\begin{aligned}\phi^{12,34}=G_0W^{12,34}G_0[&t_{13}(\phi^{124}+\phi^{234}+\phi^{12,34}+\phi^{14,23})\\&+t_{23}(\phi^{124}+\phi^{134}+\phi^{12,34}+\phi^{13,24})\\&+t_{14}(\phi^{123}+\phi^{234}+\phi^{12,34}+\phi^{13,24})\\&+t_{24}(\phi^{123}+\phi^{134}+\phi^{12,34}+\phi^{14,23})].\end{aligned} \tag{24}$$

These seven coupled integral equations are well defined since all kernels are connected. The correct and intuitive multiple scattering series are obtained by iterating these equations. Further, we can draw the following consequences from these equations.

A. Optical potential

By Eqs. (II.32) and (II.33), if we neglect contributions from breakup and closed channels, the operator G_0W^{123} is approximated by

$$G_0W^{123}=\sum_n|\varphi_n^{(123)}\rangle\frac{1}{E-K_4+|E_{123,n}|+i\epsilon}\langle\varphi_n^{(123)}|[V_{12}G_0(t_{13}+t_{23})+\text{c.p. of }123]. \tag{25}$$

Here c.p. denotes "the cyclic permutation." Equation (23) then tells that the distorting potential (optical potential) $U_{4,n}$ for the elastically scattered particle 4 from the nth bound state of 123 is given by

$$U_{4,n}=\langle\varphi_n^{(123)}|\,[V_{12}G_0(t_{13}+t_{23})+\text{c.p. of }123]G_0(t_{14}+t_{24}+t_{34})\,|\varphi_n^{(123)}\rangle\,. \tag{26}$$

Similarly, the optical potential $U_{12\text{-}34;nm}$ for the elastically scattered wave between the nth bound state of 12 and the mth bound state of 34 is given by

$$U_{12\text{-}34;nm}=\langle\varphi_n^{(12)}|\langle\varphi_m^{(34)}|(V_{12}G_0t_{34}+V_{34}G_0t_{12})G_0(t_{13}+t_{23}+t_{14}+t_{24})\,|\varphi_n^{(12)}\rangle\,|\varphi_m^{(34)}\rangle. \tag{27}$$

Some papers in the past[7] have suggested that the optical potential for the particle 4 is given by

$$\tilde{U}_{4,n}=\langle\varphi_n^{(123)}|t_{14}+t_{24}+t_{34}|\varphi_n^{(123)}\rangle\,. \tag{28}$$

Comparing Eq. (26) with Eq. (28) we see that the factor $[V_{12}G_0(t_{13}+t_{23})+\text{c.p. of }123]$ is new. Thanks to the presence of this factor, the operator $[V_{12}G_0(t_{13}+t_{23})+\text{c.p. of }123]G_0(t_{14}+t_{24}+t_{34})$ is the L^2 operator in the four-body space. Namely, it is eligible to be a transition operator in the four-body space. The situation is the same in Eq. (27).

B. Amplitudes

Under CCA, the amplitude $T_n^{(123)}$ [Eq. (5)] reads

$$T_n^{(123)}=\langle\varphi_n^{(123)}|\langle f_{4,p_n}|(V_{12}+V_{13}+V_{23})G_0(t_{14}|\phi^{123}+\phi^{234}+\phi^{12,34}+\phi^{13,24}\rangle+\text{c.p. of }123). \tag{29}$$

Since the final state $|\varphi_n^{(123)}\rangle|f_{4,p_n}\rangle$ is on the four-body energy shell, satisfying

$$[E-H_0-(V_{12}+V_{13}+V_{23})]\,|\varphi_n^{(123)}\rangle|f_{4,p_n}\rangle=0, \tag{30}$$

Eq. (29) is reduced to

$$T_n^{(123)}=\langle\varphi_n^{(123)}|\langle f_{4,p_n}|(t_{14}|\phi^{123}+\phi^{234}+\phi^{12,34}+\phi^{13,24}\rangle+t_{24}|\phi^{123}+\phi^{134}+\phi^{12,34}+\phi^{14,23}\rangle$$
$$+t_{34}|\phi^{123}+\phi^{124}+\phi^{13,24}+\phi^{14,23}\rangle). \tag{31}$$

For other processes, the amplitudes are given, e.g., by

$$T_n^{(124)}=\langle\varphi_n^{(124)}|\langle f_{3,p_n}|V_{12}+V_{14}+V_{24}|\phi_{13}+\phi_{23}+\phi_{34}\rangle$$
$$\underset{\text{CCA}}{\longrightarrow}\langle\varphi_n^{(124)}|\langle f_{3,p_n}|(t_{13}|\phi^{124}+\phi^{234}+\phi^{12,34}+\phi^{14,23}\rangle+t_{23}|\phi^{124}+\phi^{134}+\phi^{12,34}+\phi^{13,24}\rangle$$
$$+t_{34}|\phi^{124}+\phi^{123}+\phi^{13,24}+\phi^{14,23}\rangle). \tag{32}$$

$$T_{nm}^{(12)(34)}\ [\text{see Eq. (6)}]\ \underset{\text{CCA}}{\longrightarrow}\langle\varphi_n^{(12)}|\langle\varphi_m^{(34)}|\langle f_{12\text{-}34;nm}|(t_{13}|\phi^{124}+\phi^{234}+\phi^{12,34}+\phi^{14,23}\rangle+t_{23}|\phi^{124}+\phi^{134}+\phi^{12,34}+\phi^{13,24}\rangle$$
$$+t_{14}|\phi^{123}+\phi^{234}+\phi^{12,34}+\phi^{13,24}\rangle+t_{24}|\phi^{123}+\phi^{134}+\phi^{12,34}+\phi^{14,23}\rangle). \tag{33}$$

The breakup amplitude Eq. (7) is a little complicated. In CCA we must neglect ϕ_{13}^{123}, ϕ_{23}^{123}, ϕ_{14}^{124}, ϕ_{24}^{124}, and $\phi_{34}^{12,34}$ because these wave functions are parts of cluster wave functions ϕ^{123}, ϕ^{124}, and $\phi^{12,34}$. As a result,

$$T_n^{(12)}\underset{\text{CCA}}{\longrightarrow}\langle\varphi_n^{(12)}|\langle f_{3,p_3}|\langle f_{4,p_4}|V_{12}[G_0(t_{13}+t_{23})\,|\phi_{14}+\phi_{24}+\phi_{34}\rangle$$
$$+G_0(t_{14}+t_{24})\,|\phi_{13}+\phi_{23}+\phi_{34}\rangle+G_0t_{34}\,|\phi_{13}+\phi_{23}+\phi_{14}+\phi_{24}\rangle]$$
$$\underset{\text{CCA}}{\longrightarrow}\langle\varphi_n^{(12)}|\langle f_{3,p_3}|\langle f_{4,p_4}|V_{12}|[G_0(t_{13}+t_{23})G_0(t_{14}|\phi^{123}+\phi^{234}+\phi^{12,34}+\phi^{13,24}\rangle+\text{c.p. of }123)$$
$$+G_0(t_{14}+t_{24})G_0(t_{13}|\phi^{124}+\phi^{234}+\phi^{12,34}+\phi^{14,23}\rangle+\text{c.p. of }124)$$
$$+G_0t_{34}G_0(t_{13}|\phi^{124}+\phi^{234}+\phi^{12,34}+\phi^{14,23}\rangle$$
$$+t_{23}|\phi^{124}+\phi^{134}+\phi^{12,34}+\phi^{13,24}\rangle+t_{14}|\phi^{123}+\phi^{234}+\phi^{12,34}+\phi^{13,24}\rangle$$
$$+t_{24}|\phi^{123}+\phi^{134}+\phi^{12,34}+\phi^{14,23}\rangle)]$$
$$=\langle\phi_n^{(12)}|\langle f_{3,p_3}|\langle f_{4,p_4}|\{(t_{13}+t_{23})G_0(t_{14}|\phi^{123}+\phi^{234}+\phi^{12,34}+\phi^{13,24}\rangle+\text{cp of }123)$$
$$+(t_{14}+t_{24})G_0(t_{13}|\phi^{124}+\phi^{234}+\phi^{12,34}+\phi^{14,23}\rangle+\text{c.p. of }124)$$
$$+t_{34}G_0\{(t_{13}|\phi^{124}+\phi^{234}+\phi^{12,34}+\phi^{14,23}\rangle+(3\rightleftharpoons 4))$$
$$+[t_{23}|\phi^{124}+\phi^{134}+\phi^{12,34}+\phi^{13,24}\rangle+(3\rightleftharpoons 4)]\}. \tag{34}$$

C. Impulse approximation

Now we retain only the lowest order term in each amplitude. We denote by $T_n^{(123)IA}$ the amplitude thus obtained from $T_n^{(123)}$, etc. The results are

$$T_n^{(123)IA}=\langle\varphi_n^{(123)}|\langle f_{4,p_n}|(t_{14}+t_{24}+t_{34})|\varphi_0^{(123)}\rangle|f_{4,p_0}\rangle, \tag{35}$$

$$T_n^{(123)IA}=\langle\varphi_n^{(124)}|\langle f_{3,p_n}|t_{34}|\varphi_0^{(123)}\rangle|f_{4,p_0}\rangle, \tag{36}$$

$$T_{nm}^{(12)(34)IA}=\langle\varphi_n^{(12)}|\langle\varphi_m^{(34)}|\langle f_{12-34;nm}|(t_{14}+t_{24})|\varphi_0^{(123)}\rangle|f_{4,p_0}\rangle, \tag{37}$$

$$T_n^{(12)IA}=\langle\varphi_n^{(12)}|\langle f_{3,p_3}|\langle f_{4,p_4}|[(t_{13}+t_{23})G_0(t_{14}+t_{24}+t_{34}) + (t_{14}+t_{24})G_0t_{34}+t_{34}G_0(t_{14}+t_{24})]|\varphi_0^{(123)}\rangle|f_{4,p_0}\rangle. \tag{38}$$

Equations (35)–(37) are the *impulse* approximation. The lowest order terms of the breakup process are of the second order [Eq. (38)]. This explains why the cluster transfer reactions are more important than the breakup process.

In conclusion, we have shown the following for the decoupled Yakubovski equation:

(I) In the previous paper,

(i) it covers the whole four-body space,[5]

(ii) the closed channel vanishes in the high energy limit,[5]

(iii) the scattering amplitudes are brought into the well-known expressions.[3,5]

(II) In the present paper,

(i) the waves in the intermediate stage consist of the cluster wave function and the collapsed wave functions [Eq. (19)]. The number of equations we have to solve is six [Eq. (19)].

(ii) If we adopt the coupled cluster approximation [Eq. (20)], we can define the optical potential [Eqs. (26) and (27)], in which the transition operator is fully connected with respect to four-particles. We have to solve seven equations [Eqs. (23) and (24)].

(iii) The lowest order terms of cluster transfer reactions are impulse approximations [Eqs. (35), (36), and (37)].

(iv) The cluster transfer reactions are realized more easily than the breakup process [Eqs. (35)–(37) and Eq. (38)].

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