Nuclear muon capture: Hyperfine effects in nuclear spin and isospin $[1/2^{\pm},1/2]\rightarrow[1/2^{\pm},1/2]$ and $[1^{+},0]\rightarrow[0^{+},1]$ transitions*

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Formulas for angular correlations and capture rates in muon-capture transitions between initial and final nuclear states characterized by spin and isospin $[1/2^{\pm},1/2]_{\rm ini}$ and $[1/2^{\pm},1/2]_{\rm fin}$, and $[1^{+},0]_{\rm ini}$ and $[0^{+},1]_{\rm fin}$ are derived and applied to $\mu^{-}p\rightarrow\nu_{\mu}n$, $\mu^{-}\,{}^{3}\text{He}\rightarrow\nu_{\mu}\,{}^{3}\text{H}$, and $\mu^{-}\,{}^{6}\text{Li}\rightarrow\nu_{\mu}\,{}^{6}\text{He}$ in a model-independent (elementary-particle) approach. The results indicate that the angular correlations between the momenta of the recoiling final nuclei and the polarizations of the μ^{-} and/or the initial nuclei are sensitive to the value of the nuclear pseudoscalar form factors $F_P(q^2)$. The analysis also indicates that inconsistencies among the available data on $\mu^{-}\,{}^{6}\text{Li}\rightarrow\nu_{\mu}\,{}^{6}\text{He}$ are most likely of experimental origin.

[RADIOACTIVITY $\mu^{-}p\rightarrow\nu_{\mu}n$, $\mu^{-}\,{}^{3}\text{He}\rightarrow\nu_{\mu}\,{}^{3}\text{H}$, $\mu^{-}\,{}^{6}\text{Li}\rightarrow\nu_{\mu}\,{}^{6}\text{He}$; angular correlations and capture rates. Hyperfine effects are properly included.]

I. INTRODUCTION

Though the muon capture processes $\mu^{-}p\rightarrow\nu_{\mu}n$, $\mu^{-}\,{}^{3}\text{He}\rightarrow\nu_{\mu}\,{}^{3}\text{H}$, and $\mu^{-}\,{}^{6}\text{Li}\rightarrow\nu_{\mu}\,{}^{6}\text{He}$ have been investigated by many authors,[1-9] several interesting quantities associated with these processes have not been calculated, e.g., the angular correlations between the momenta of the recoiling final nuclei and the polarizations of the muon and/or the initial nuclei. In view of this, we discuss in the present paper the various correlations characteristic of the processes in question together with the corresponding capture rates always taking properly into account the effects of the initial total-spin configurations ("hyperfine" effects).

II. MUON CAPTURE IN THE NUCLEAR SPIN AND ISOSPIN DOUBLETS

With $V_\lambda(x)$ and $A_\lambda(x)$ the hadronic weak polar and axial currents, we adopt the following definitions for the various nuclear form factors in the nuclear spin and isospin $[\frac{1}{2}^{\pm},\frac{1}{2}]\rightarrow[\frac{1}{2}^{\pm},\frac{1}{2}]$ transitions[1,10]:

$$\langle N_f(p^{(f)},s^{(f)})|V_\lambda(0)|N_i(p^{(i)},s^{(i)})\rangle=\bar{u}^{(f)}(p^{(f)},s^{(f)})\left[F_V(q^2)\gamma_\lambda-F_M(q^2)\frac{\sigma_{\lambda\eta}q_\eta}{2m_p}\right]u^{(i)}(p^{(i)},s^{(i)}),\tag{1a}$$

$$\langle N_f(p^{(f)},s^{(f)})|A_\lambda(0)|N_i(p^{(i)},s^{(i)})\rangle=\bar{u}^{(f)}(p^{(f)},s^{(f)})\left[F_A(q^2)\gamma_\lambda\gamma_5+F_P(q^2)i\,\frac{2Mq_\lambda\gamma_5}{m_\pi^2}\right]u^{(i)}(p^{(i)},s^{(i)}),\tag{1b}$$

where

$$\bar{u}\equiv i\,u^{\dagger}\gamma_4,\quad q_\lambda\equiv(p^{(f)}-p^{(i)})_\lambda,\quad Q_\lambda\equiv(p^{(f)}+p^{(i)})_\lambda,$$

$$M\equiv\tfrac{1}{2}(M_f+M_i)\cong M_f\cong M_i,$$

and

$$M_{f,i}=[-(p^{(f),(i)})^2]^{1/2}$$

($\Delta\equiv M_f-M_i$ is neglected consistently below), and where $F_{V,M,A,P}(q^2)$ are, respectively, vector, weak magnetism, axial, and pseudoscalar nuclear form factors. In terms of these form factors, the transition amplitude $\mathcal{T}$ for muon capture

$$\mu^{-}(p^{(\mu)},s^{(\mu)})+N_i(p^{(i)},s^{(i)})\rightarrow\nu_\mu(p^{(\nu)},s^{(\nu)})+N_f(p^{(f)},s^{(f)})\tag{2}$$

is given by[10,1]

$$\begin{aligned}\mathcal{T}&=\frac{G}{\sqrt{2}}\langle N_f(p^{(f)},s^{(f)})|[V_\lambda(0)+A_\lambda(0)]|N_i(p^{(i)},s^{(i)})\rangle\,\bar{u}^{(\nu)}(p^{(\nu)},s^{(\nu)})\gamma_\lambda(1+\gamma_5)u^{(\mu)}(p^{(\mu)},s^{(\mu)})\\&=\frac{G}{\sqrt{2}}\left\{\bar{u}^{(f)}(p^{(f)},s^{(f)})\left[F_V(q^2)\gamma_\lambda-F_M(q^2)\frac{\sigma_{\lambda\eta}q_\eta}{2mp}+F_A(q^2)\gamma_\lambda\gamma_5+F_P(q^2)i\,\frac{2Mq_\lambda\gamma_5}{m_\pi^2}\right]u^{(i)}(p^{(i)},s^{(i)})\right\}\\&\quad\times\bar{u}^{(\nu)}(p^{(\nu)},s^{(\nu)})\gamma_\lambda(1+\gamma_5)u^{(\mu)}(p^{(\mu)},s^{(\mu)})\end{aligned}\tag{3}$$

which, to a sufficient approximation, reduces to[1]

$$\mathcal{T}(\hat{s}^{(f)}; S, S_z) = \frac{G}{\sqrt{2}} \frac{1}{\sqrt{2}} \left(\frac{E^{(f)}+M_f}{2E^{(f)}}\right)^{1/2} (v^{(f)} v^{(\nu)})^\dagger (1 - \vec{\sigma}^{(L)} \cdot \hat{\nu}) [G_V + G_A \vec{\sigma}^{(L)} \cdot \vec{\sigma}^{(N)} + G_P \vec{\sigma}^{(N)} \cdot \hat{\nu}] (v^{(i)} v^{(\mu)})_{S, S_z};$$

$$\hat{\nu} \equiv \frac{\vec{p}^{(\nu)}}{|\vec{p}^{(\nu)}|} = -\frac{\vec{p}^{(f)}}{|\vec{p}^{(f)}|} \tag{4}$$

with[1]

$$G_V = F_V(q^2)\left(1+\frac{E^{(\nu)}}{2M_f}\right) - \left(F_M(q^2)\frac{m_\mu}{2m_p}\right)\frac{E^{(\nu)}}{2M_f},$$

$$G_A = -F_A(q^2) - F_V(q^2)\frac{E^{(\nu)}}{2M_f} - F_M(q^2)\frac{E^{(\nu)}}{2m_p},$$

$$G_P = F_P(q^2)\frac{m_\mu E^{(\nu)}}{m_\pi^2} + \left[F_A(q^2) - F_V(q^2)\right]\frac{E^{(\nu)}}{2M_f} - F_M(q^2)\frac{E^{(\nu)}}{2m_p}, \tag{5}$$

where $v^{(f)}$, $v^{(i)}$ and $v^{(\nu)}$, $v^{(\mu)}$ are two-component Pauli spinors for N_f, N_i and ν_μ, μ^-, $\vec{\sigma}^{(N)}$ and $\vec{\sigma}^{(L)}$ are two-by-two Pauli matrices to be sandwiched between $v^{(f)\dagger}$, $v^{(i)}$ and $v^{(\nu)\dagger}$, $v^{(\mu)}$, and (S, S_z) characterizes the total-spin configuration of the initial $[\mu^- N_i]$ state. One has $\vec{\sigma}^{(L)} \cdot \hat{\nu}\, v^{(\nu)} = -v^{(\nu)}$ and we take $\vec{\sigma}^{(N)} \cdot \hat{s}^{(f)} v^{(f)} = v^{(f)}$ so that the helicity of ν_μ is -1 and the spin of $N^{(f)}$ is directed along the unit vector $\hat{s}^{(f)}$.

To investigate the hyperfine effects in $\mu^- N_i \to \nu_\mu N_f$, we evaluate the quantity

$$\langle |\mathcal{T}(\hat{s}^{(f)})|^2 \rangle = \sum_{S, S_z} |\mathcal{T}(\hat{s}^{(f)}; S, S_z)|^2 P(S, S_z), \tag{6}$$

where $P(S, S_z)$ is the probability of finding $[\mu^- N_i]$ at the instant of muon capture in the total-spin configuration specified by S, S_z, i.e.,

$$P(1, \pm 1) = \tfrac{1}{4}(1 \pm P_\mu \pm P_{N_i} + P_\mu P_{N_i}),$$

$$P(1, 0) = P(0, 0) = \tfrac{1}{4}(1 - P_\mu P_{N_i}): \tag{7a}$$

no $S = 1(0) \to S = 0(1)$ conversion in a time $\approx \tau(\mu^-$ decay);

$$P(1, \pm 1) = P(1, 0) = 0,$$

$$P(0, 0) = 1\,[P(1, 1) + P(1, 0) + P(1, -1) = 1,\ P(0, 0) = 0]: \tag{7b}$$

complete $S = 1(0) \to S = 0(1)$ conversion in a time $\approx \tau(\mu^-$ decay), $P_\mu \hat{z}$ and $P_{N_i}\hat{z}$ being the μ^- and N_i polarizations at the instant of arrival of the μ^- in the lowest Bohr orbit around the N_i. Making use of the formula[11]

$$(v^{(i)} v^{(\mu)})^\dagger_{S, S_z} (A + B\vec{\sigma}^{(L)} \cdot \vec{\sigma}^{(N)} + \vec{\sigma}^{(L)} \cdot \vec{C}\, \vec{\sigma}^{(N)} \cdot \vec{D} + \vec{\sigma}^{(L)} \cdot E + \vec{\sigma}^{(N)} \cdot \vec{F} + \vec{\sigma}^{(L)} \times \vec{\sigma}^{(N)} \cdot \vec{G})(v^{(i)} v^{(\mu)})_{S, S_z}$$

$$= A + B[2S(S+1) - 3] + \vec{C} \cdot \vec{D}[S(S+1) - S_z^2 - 1] + \vec{C} \cdot \hat{z}\vec{D} \cdot \hat{z}[-S(S+1) + 3S_z^2] + [\vec{E} \cdot \hat{z} + \vec{F} \cdot \hat{z}]S_z \tag{8}$$

we obtain, using Eqs. (4)–(6),

$$\begin{aligned}
\langle |\mathcal{T}(\hat{s}^{(f)})|^2 \rangle = {} & \left(\frac{G}{\sqrt{2}}\right)^2 \frac{1}{2} \left(\frac{E^{(f)}+M_f}{2E^{(f)}}\right) \\
& \times ([(G_V^2 + 3G_A^2 + G_P^2 - 2G_A G_P) + \hat{s}^{(f)} \cdot \nu (2G_A^2 - 2G_V G_A + 2G_V G_P)] \\
& + [P(1, 1) + P(1, 0) + P(1, -1) - 3P(0, 0)] \\
& \times [(-2G_A^2 + 2G_V G_A + 2G_A G_P) + \hat{s}^{(f)} \cdot \nu(-2G_A^2 + 2G_V G_A + 2G_A G_P)] \\
& + [P(1, 0) - P(0, 0)][-2(G_V + G_A)G_P - \hat{s}^{(f)} \cdot \nu((G_V + G_A)^2 + G_P^2)] + [P(1, 1) + P(1, -1) - 2P(1, 0)] \\
& \times [\hat{s}^{(f)} \cdot \hat{z}\hat{\nu} \cdot \hat{z}(-(G_V + G_A)^2 + G_P^2) + (\hat{\nu} \cdot \hat{z})^2(-2(G_V + G_A)G_P - \hat{s}^{(f)} \cdot \hat{\nu}\, 2G_P^2)] \\
& + [P(1, 1) - P(1, -1)]\{\hat{s}^{(f)} \cdot \hat{z}((G_V + G_A)^2 - G_P^2) \\
& + \hat{\nu} \cdot \hat{z}[-(G_V + G_A - G_P)^2 - \hat{s}^{(f)} \cdot \hat{\nu}(2(G_V + G_A)G_P - 2G_P^2)]\}) .
\end{aligned} \tag{9}$$

With $\langle |\mathcal{T}(\hat{s}^{(f)})|^2\rangle$ specified, we first consider the angular correlation between the neutrino momentum (which is opposite to the momentum of the recoiling final nucleus) and the polarization of the μ^- and/or the N_i, viz.:

$$\mathcal{C}(\hat{\nu}\cdot\hat{z})=\frac{\langle|\mathcal{T}(\hat{s}^{(f)})|^2\rangle+\langle|\mathcal{T}(-\hat{s}^{(f)})|^2\rangle}{\int d\Omega^{(\nu)}/4\pi(\langle|\mathcal{T}(\hat{s}^{(f)})|^2\rangle+\langle|\mathcal{T}(-\hat{s}^{(f)})|^2\rangle)}. \tag{10a}$$

This corresponds to the forward-backward asymmetry

$$\mathcal{A}_{\hat{\nu},\hat{z}}\equiv\frac{\mathcal{C}(\hat{\nu}\cdot\hat{z}=1)-\mathcal{C}(\hat{\nu}\cdot\hat{z}=-1)}{\mathcal{C}(\hat{\nu}\cdot\hat{z}=1)+\mathcal{C}(\hat{\nu}\cdot\hat{z}=-1)}=\frac{n}{d},$$

where

$$n=-[P(1,1)-P(1,-1)](G_V+G_A-G_P)^2$$

and

$$\begin{aligned}d=&[G_V^2+3G_A^2+G_P^2-2G_AG_P]+[P(1,1)+P(1,0)+P(1,1)-3P(0,0)][-2G_A^2+2G_VG_A+2G_AG_P]\\&+[P(1,1)-P(1,0)+P(1,-1)-P(0,0)](-2)(G_V+G_A)G_P,\end{aligned} \tag{10b}$$

where we have used

$$\begin{aligned}\langle|\mathcal{T}(\hat{s}^{(f)})|^2\rangle+\langle|\mathcal{T}(-\hat{s}^{(f)})|^2\rangle=&\left(\frac{G}{\sqrt{2}}\right)^2\left(\frac{E^{(f)}+M_f}{2E^{(f)}}\right)\\&\times\{[G_V^2+3G_A^2+G_P^2-2G_AG_P]+[P(1,1)+P(1,0)+P(1,-1)-3P(0,0)]\\&\times(-2G_A^2+2G_VG_A+2G_AG_P)\\&+[P(1,0)-P(0,0)](-2)(G_V+G_A)G_P+[P(1,1)+P(1,-1)-2P(1,0)]\\&\times(\hat{\nu}\cdot\hat{z})^2(-2)(G_V+G_A)G_P+[P(1,1)-P(1,-1)][-(G_V+G_A-G_P)^2]\}.\end{aligned} \tag{10c}$$

Continuing, we define two other angular correlations involving the spin of N_f. Thus, the angular correlation between the neutrino momentum and the spin of N_f is

$$\mathcal{C}(\hat{s}^{(f)}\cdot\hat{\nu})=\frac{\langle|\mathcal{T}(\hat{s}^{(f)})|^2\rangle}{\int d\Omega^{(\nu)}/4\pi[\langle|\mathcal{T}(\hat{s}^{(f)})|^2\rangle+\langle|\mathcal{T}(-\hat{s}^{(f)})|^2\rangle]} \tag{10d}$$

and corresponds to the forward-backward asymmetry

$$\mathcal{A}_{\hat{s}^{(f)},\hat{\nu}}\equiv\frac{\mathcal{C}(\hat{s}^{(f)}\cdot\hat{\nu}=1)-\mathcal{C}(\hat{s}^{(f)}\cdot\hat{\nu}=-1)}{\mathcal{C}(\hat{s}^{(f)}\cdot\hat{\nu}=1)+\mathcal{C}(\hat{s}^{(f)}\cdot\hat{\nu}=-1)}=\frac{n'}{d'},$$

where

$$\begin{aligned}n'=&(2G_A^2-2G_VG_A+2G_VG_P)+[P(1,1)+P(1,0)+P(1,-1)-3P(0,0)](-2G_A^2+2G_VG_A+2G_AG_P)\\&+[P(1,0)-P(0,0)][-(G_V+G_A)^2-G_P^2]+[P(1,1)+P(1,-1)-2P(1,0)](\hat{\nu}\cdot\hat{z})^2[-(G_V+G_A)^2-G_P^2]\\&+[P(1,1)-P(1,-1)]\hat{\nu}\cdot\hat{z}[(G_V+G_A)^2+G_P^2-2(G_V+G_A)G_P]\end{aligned}$$

and

$$\begin{aligned}d'=&(G_V^2+3G_A^2+G_P^2-2G_AG_P)+[P(1,1)+P(1,0)+P(1,-1)-3P(0,0)](-2G_A^2+2G_VG_A+2G_AG_P)\\&+[P(1,0)-P(0,0)](-2)(G_V+G_A)G_P+[P(1,1)+P(1,-1)-2P(1,0)](\hat{\nu}\cdot\hat{z})^2(-2)(G_V+G_A)G_P\\&+[P(1,1)-P(1,-1)]\hat{\nu}\cdot\hat{z}[-(G_V+G_A-G_P)^2].\end{aligned} \tag{10e}$$

The quantity $\mathcal{A}_{\hat{s}^{(f)},\hat{\nu}}$ is just the helicity of the recoiling final nucleus. Also, the angular correlation between the spin of the N_f and the polarization of the μ^- and/or the N_i is

$$\mathcal{C}(\hat{s}^{(f)}\cdot\hat{z})=\frac{\int d\Omega^{(\nu)}/4\pi\langle|\mathcal{T}(\hat{s}^{(f)})|^2\rangle}{\int d\Omega^{(\nu)}/4\pi(\langle|\mathcal{T}(\hat{s}^{(f)})|^2\rangle+\langle|\mathcal{T}(-\hat{s}^{(f)})|^2\rangle)}. \tag{10f}$$

This corresponds to the forward-backward asymmetry

$$\mathcal{A}_{\hat{s}^{(f)},\hat{z}} \equiv \frac{\mathcal{C}(\hat{s}^{(f)}\cdot\hat{z}=1)-\mathcal{C}(\hat{s}^{(f)}\cdot\hat{z}=-1)}{\mathcal{C}(\hat{s}^{(f)}\cdot\hat{z}=1)+\mathcal{C}(\hat{s}^{(f)}\cdot\hat{z}=-1)}$$

$$= \frac{[P(1,1)-P(1,-1)][(G_V+G_A)^2-\frac{2}{3}(G_V+G_A)G_P-\frac{1}{3}G_P^2]}{(G_V^2+3G_A^2+G_P^2-2G_AG_P)+[P(1,1)+P(1,0)+P(1,-1)-3P(0,0)](-2G_A^2+2G_VG_A-\frac{2}{3}G_VG_P+\frac{4}{3}G_AG_P)}, \qquad (10g)$$

where we have used

$$\int\frac{d\Omega^{(\nu)}}{4\pi}\langle|\mathcal{T}(\hat{s}^{(f)})|^2\rangle=\left(\frac{G}{\sqrt{2}}\right)^2\frac{1}{2}\left(\frac{E^{(f)}+M_f}{2E^{(f)}}\right)$$
$$\times\{(G_V^2+3G_A^2+G_P^2-2G_AG_P)+[P(1,1)+P(1,0)+P(1,-1)-3P(0,0)]$$
$$\times(-2G_A^2+2G_VG_A-\tfrac{2}{3}G_VG_P+\tfrac{4}{3}G_AG_P)$$
$$+[P(1,1)-P(1,-1)][(G_V+G_A)^2-\tfrac{2}{3}(G_V+G_A)G_P-\tfrac{1}{3}G_P^2]\,\hat{s}^{(f)}\cdot\hat{z}\}. \qquad (10h)$$

The quantity $\mathcal{A}_{\hat{s}^{(f)},\hat{z}}$ is identical with the quantity called the average polarization of the recoiling final nucleus, P_{av}.

Finally, we recall that the muon capture rate is given by[1]

$$\Gamma(\mu^-N_i\to\nu_\mu N_f)=\Gamma_0\left\{\frac{\int d\Omega^{(\nu)}/4\pi(\langle|\mathcal{T}(\hat{s}^{(f)})|^2\rangle+\langle|\mathcal{T}(-\hat{s}^{(f)})|^2\rangle)}{(G/\sqrt{2})^2[(E^{(f)}+M_f)/2E^{(f)}]}\right\}$$
$$=\Gamma_0\{(G_V^2+3G_A^2+G_P^2-2G_AG_P)$$
$$+[P(1,1)+P(1,0)+P(1,-1)-3P(0,0)](-2G_A^2+2G_VG_A-\tfrac{2}{3}G_VG_P+\tfrac{4}{3}G_AG_P)\}, \qquad (10i)$$

$$\Gamma_0\equiv\frac{G^2m_\mu^5}{2\pi^2}\left(1-2\frac{m_\mu}{m_\mu+M_f}\right)C_i\left(Z_i\alpha\frac{M_i}{m_\mu+M_i}\right)^3;\quad C_i=1.00,\ N_i=P$$
$$=0.96,\ N_i={}^3\mathrm{He},$$

where C_i is a correction factor arising from the non-point-charge distribution of N_i.

We proceed to give numerical values for $\mathcal{A}_{\hat{\nu},\hat{z}}$, $\mathcal{A}_{\hat{s}^{(f)},\hat{\nu}}$, $\mathcal{A}_{\hat{s}^{(f)},\hat{z}}$ and $\Gamma(\mu^-N_i\to\nu_\mu N_f)$ for the various cases of experimental interest appropriate to $\mu^-p\to\nu_\mu n$ and $\mu^-\,{}^3\mathrm{He}\to\nu_\mu\,{}^3\mathrm{H}$. The momentum transfer in these two processes is

$$q^2\equiv(p^{(f)}-p^{(i)})^2=(p^{(\mu)}-p^{(\nu)})^2=-m_\mu^2+2E^{(\nu)}m_\mu$$
$$\cong m_\mu^2-2m_\mu\frac{m_\mu^2}{2M_f}=0.88m_\mu^2,\quad \mu^-p\to\nu_\mu n$$
$$=0.96m_\mu^2,\quad \mu^-\,{}^3\mathrm{He}\to\nu_\mu\,{}^3\mathrm{H} \qquad (11)$$

and all the form factors are to be evaluated at these q^2.

A. $\mu^-p\to\nu_\mu n$

Here, using Ref. 1, we have

$$F_V(0.88m_\mu^2)=F_V(0)\times0.977=0.977,$$
$$F_M(0.88m_\mu^2)=F_M(0)\times0.971=3.598,$$
$$F_A(0.88m_\mu^2)=F_A(0)\times0.978=1.213, \qquad (12)$$
$$F_P(0.88m_\mu^2)\cong-\frac{F_A(0.88m_\mu^2)}{1+0.88m_\mu^2/m_\pi^2}(1+0.02)=-0.822.$$

(a) *No $S=1\to S=0$ conversion: $P(S,S_z)$ as in Eq. (7); μ^- beam stops in low-density gaseous hydrogen.*[1,11] Using Eqs. (10a)–(10i), (11), and (12), we have

$$\mathcal{A}_{\hat{\nu},\hat{z}}=\frac{(-0.04)\frac{1}{2}(P_\mu+P_p)}{5.99-5.95P_\mu P_p}, \qquad (13a)$$

$$\mathfrak{A}_{\hat{s}(f),\hat{\nu}}=\frac{5.95+[-5.40-0.59(\hat{\nu}\cdot\hat{z})^2]P_\mu P_P+(0.04\hat{\nu}\cdot\hat{z})\frac{1}{2}(P_\mu+P_P)}{5.99+[-5.40-0.55(\hat{\nu}\cdot\hat{z})^2]P_\mu P_P-(0.04\hat{\nu}\cdot\hat{z})\frac{1}{2}(P_\mu+P_P)}\,, \tag{13b}$$

$$\mathfrak{A}_{\hat{s}(f),\hat{z}}=\frac{(-0.12)\frac{1}{2}(P_\mu+P_P)}{5.99-5.59P_\mu P_P}\,, \tag{13c}$$

$$\Gamma(\mu^- p\to\nu_\mu n)=\Gamma_0(5.99-5.59P_\mu P_P)=(29.2\ \text{sec}^{-1})(5.99-5.59P_\mu P_P)=(175-163P_\mu P_P)\ \text{sec}^{-1}\,. \tag{13d}$$

We note that all the above values [Eqs. (13a)–(13d)] are very sensitive to the value of $P_\mu P_P$, the value $P_\mu P_P=1$ corresponding to $P(0,0)=0$ [and $P(1,1)=1, P(1,0)=P(1,-1)=0$], and so to muon capture from the triplet ($S=1, S_z=1$) total-spin configuration of $[\mu^- P]$. Unfortunately, however, attainment in practice of anything but very small values of $P_\mu P_P$ appears to be extremely difficult.

(b) *Complete $S=1\to S=0$ conversion: $P(S,S_z)$ as in Eq. (7); μ^- beam stops in medium-density gaseous hydrogen.*[1,11] Using Eqs. (10a)–(10i), (11), and (12), we have

$$\mathfrak{A}_{\hat{\nu},\hat{z}}=0\,, \tag{13e}$$

$$\mathfrak{A}_{\hat{s}(f),\hat{\nu}}=1\,, \tag{13f}$$

$$\mathfrak{A}_{\hat{s}(f),\hat{z}}=0\,, \tag{13g}$$

$$\Gamma(\mu^- p\to\nu_\mu n)=\Gamma_0[5.99-5.59(-3)]=665\ \text{sec}^{-1}\,. \tag{13h}$$

Experimentally, one has

$$[\Gamma(\mu^- p\to\nu_\mu n)]_{\text{exp}}=(651\pm57)\ \text{sec}^{-1}\quad\text{(CERN-Bologna, Ref. 12)}$$
$$=(686\pm88)\ \text{sec}^{-1}\quad\text{(Dubna, Ref. 13)}. \tag{13i}$$

(c) *$\mu^- p\to\nu_\mu n$ from the total-spin $\frac{1}{2}$ ortho-$[P\mu^- P]$ molecule: $P(1,\pm1)=P(1,0)=\frac{1}{12}, P(0,0)=\frac{3}{4}$; μ^- beam stops in high-density gaseous hydrogen or in liquid hydrogen.*[1,11] Using Eqs. (10a)–(10i), (11), and (12), we have

$$\mathfrak{A}_{\hat{\nu},\hat{z}}=0\,, \tag{13j}$$

$$\mathfrak{A}_{\hat{s}(f),\hat{\nu}}=1.00\,, \tag{13k}$$

$$\mathfrak{A}_{\hat{s}(f),\hat{z}}=0\,, \tag{13l}$$

$$\Gamma(\mu^- p\to\nu_\mu n)=\Gamma_0[5.99-5.59(-2)]=501\ \text{sec}^{-1}\,. \tag{13m}$$

Experimentally, one has

$$[\Gamma(\mu^- p\to\nu_\mu n)]_{\text{exp}}=(480\pm50)\ \text{sec}^{-1}\quad\text{(Columbia, Ref. 14)}. \tag{13n}$$

B. $\mu^-\ {}^3\text{He}\to\nu_\mu\ {}^3\text{H}$

Here, using Ref. 2, we have

$$\begin{aligned}F_V(0.96m_\mu^2)&=F_V(0)\times0.82=0.82\,,\\ F_M(0.96m_\mu^2)&=F_M(0)\times0.87=-4.73\,,\\ F_A(0.96m_\mu^2)&=F_A(0)\times0.87=-1.06\,,\\ F_P(0.96m_\mu^2)&\cong-\frac{F_A(0.96m_\mu^2)}{1+0.96m_\mu^2/m_\pi^2}=0.68\,.\end{aligned} \tag{14}$$

Note that F_A in the ${}^3\text{He}\to{}^3\text{H}$ case is negative while F_A in the $p\to n$ case is positive. Also, in $\mu^-\ {}^3\text{He}\to\nu_\mu\ {}^3\text{H}$, $S=0\to S=1$ conversion and $[{}^3\text{He}\,\mu^-\ {}^3\text{He}]$ molecule formation do not take place so that the only situation of practical interest is[11] as follows.

(a) *No $S=0\to S=1$ conversion: $P(S,S_z)$ as in Eq. (7).* Using Eqs. (10a)–(10i), (11), and (14), we have

$$\mathfrak{A}_{\hat{\nu},\hat{z}}=\frac{(-2.31)\frac{1}{2}(P_\mu+P_{^3\text{He}})}{4.60-2.29P_\mu P_{^3\text{He}}}\,, \tag{15a}$$

$$\mathfrak{A}_{\hat{s}(f),\hat{\nu}}=\frac{2.29+[0.42-5.02(\hat{\nu}\cdot\hat{z})^2]P_\mu P_{^3\text{He}}+(2.31\hat{\nu}\cdot\hat{z})\frac{1}{2}(P_\mu+P_{^3\text{He}})}{4.60+[0.42-2.71(\hat{\nu}\cdot\hat{z})^2]P_\mu P_{^3\text{He}}-(2.31\hat{\nu}\cdot\hat{z})\frac{1}{2}(P_\mu+P_{^3\text{He}})}\,, \tag{15b}$$

$$\mathfrak{A}_{\hat{s}(f),\hat{z}}=\frac{3.59\frac{1}{2}(P_\mu+P_{^3\text{He}})}{4.60-0.48P_\mu P_{^3\text{He}}}\,, \tag{15c}$$

$$\Gamma(\mu^-\ {}^3\text{He}\to\nu_\mu\ {}^3\text{H})=\Gamma_0(4.60-0.48P_\mu P_{^3\text{He}})=(326\ \text{sec}^{-1})(4.60-0.48P_\mu P_{^3\text{He}})=(1500-156P_\mu P_{^3\text{He}})\ \text{sec}^{-1}\,. \tag{15d}$$

Experimentally, one has, for $P_{^3\text{He}}=0$,

$$[\Gamma(\mu^-\ {}^3\text{He}\to\nu_\mu\ {}^3\text{H})]_{\text{exp}}=(1505\pm46)\ \text{sec}^{-1}\quad\text{(Berkeley, Ref. 15)}$$
$$=(1465\pm67)\ \text{sec}^{-1}\quad\text{(Carnegie-Mellon, Ref. 16)}. \tag{15e}$$

It is worth pointing out in concluding this section that the value of $\alpha_{\hat{\nu},\hat{z}}$ is rather sensitive to the value assumed for

$$\left[F_P(q^2)\Big/\left(-\frac{F_A(q^2)}{1+q^2/m_\pi^2}\right)\right]_{q^2 \text{ as given by Eq. (11)}} = \xi\,. \tag{15f}$$

Thus, considering the case B(a) with $P_\mu \cong 0$ and $P_{^3\text{He}} \cong 1$ (i.e., an almost completely polarized target), we have, using Eqs. (10b) and (5), $F_{V,M,A}(0.96m_\mu^2)$ as given by Eq. (14), and Eq. (15f)

$$\begin{aligned}\alpha_{\hat{\nu},\hat{z}} &= -0.33, \quad \xi = \tfrac{1}{2},\\ &= -0.25, \quad \xi = 1 \ \ [\text{Eq. (15a)}],\\ &= -0.10, \quad \xi = 2\,.\end{aligned} \tag{15g}$$

Since $\alpha_{\hat{\nu},\hat{z}}$ is relatively easy to measure, this last result should encourage development of techniques to obtain highly polarized ^{3}He targets and so test in a novel way the validity of the PCAC—implied relation between $F_P(q^2)$ and $F_A(q^2)$[Eq. (15f) with $\xi \cong 1$].

III. MUON CAPTURE BY ^{6}Li

We proceed to investigate the process $\mu^-\,{}^6\text{Li} \to \nu_\mu\,{}^6\text{He}$ [^{6}He ≡ ^{6}He (ground state)], with particular emphasis on the hyperfine effects; this process is a typical example of a nuclear spin and isospin $[1^+,0]\to[0^+,1]$ transition. We begin with the standard definitions of the various nuclear form factors[10]:

$$\langle {}^6\text{He}(p^{(f)})|V_\lambda(0)|{}^6\text{Li}(p^{(i)},\xi)\rangle = -\sqrt{2}\,\epsilon_{\lambda\kappa\rho\eta}\xi_\kappa \frac{q_\rho}{2m_p}\frac{Q_\eta}{2M}F_M(q^2)\,, \tag{16a}$$

$$\langle {}^6\text{He}(p^{(f)})|A_\lambda(0)|{}^6\text{Li}(p^{(i)},\xi)\rangle = \sqrt{2}\left(\xi_\lambda F_A(q^2) + q_\lambda \frac{q\cdot\xi}{m_\pi^2}F_p(q^2) - \frac{Q_\lambda}{2M}\frac{q\cdot\xi}{2m_p}F_E(q^2)\right), \tag{16b}$$

$$\langle {}^6\text{Li}^*(p^{(f)})|J_\lambda^{\text{em}}(0)|{}^6\text{Li}(p^{(i)},\xi)\rangle = -\sqrt{2}\,\epsilon_{\lambda\kappa\rho\eta}\xi_\kappa \frac{q_\rho}{2m_p}\frac{Q_\eta}{2M}\mu(q^2)\,, \tag{16c}$$

where

$$q_\lambda \equiv (p^{(f)} - p^{(i)})_\lambda\,,\ Q_\lambda \equiv (p^{(f)} + p^{(i)})_\lambda\,,\ M \equiv \tfrac{1}{2}(M_f + M_i)\,,$$

and $M_{f,i} = [-(p^{(f),(i)})^2]^{1/2}$, ξ_λ is the polarization four-vector of the spin-one ^{6}Li nucleus, and $F_{M,A,P,E}(q^2)$ and $\mu(q^2)$ are, respectively, the nuclear weak magnetism, axial, pseudoscalar, and weak electricity (or pseudotensor) form factors, and the nuclear magnetic transition form factor. Further, ^{6}Li* is the ($I=1$, $I_z=0$) member of the same isotriplet as the ^{6}He (ground state); the relatively small mass difference between ^{6}He and ^{6}Li* is neglected in Eqs. (16a)–(16c).

In terms of the form factors of Eqs. (16a)–(16c), the transition amplitude $\mathcal{T}$ for muon capture

$$\mu^-(p^{(\mu)},s^{(\mu)}) + {}^6\text{Li}(p^{(i)},\xi) \to \nu_\mu(p^{(\nu)},s^{(\nu)}) + {}^6\text{He}(p^{(f)}) \tag{17}$$

is given by[10]

$$\mathcal{T} = \frac{G}{\sqrt{2}}\langle {}^6\text{He}(p^{(f)})|\,[V_\lambda(0)+A_\lambda(0)]|{}^6\text{Li}(p^{(i)},\xi)\rangle \times \bar{u}^{(\nu)}(p^{(\nu)},s^{(\nu)})\gamma_\lambda(1+\gamma_5)u^{(\mu)}(p^{(\mu)},s^{(\mu)}) \tag{18}$$

which, to a sufficient approximation, reduces to

$$\mathcal{T}(S,S_z) = \frac{G}{\sqrt{2}}v^{(\nu)\dagger}(1-\vec{\sigma}^{(L)}\cdot\hat{\nu})\left\{F_M(q^2)\frac{E^{(\nu)}}{2m_p}i\vec{\sigma}^{(L)}\times\hat{\nu} - F_A(q^2)\vec{\sigma}^{(L)} + \left[F_P(q^2)\frac{m_\mu E^{(\nu)}}{m_\pi^2} - F_E(q^2)\frac{E^{(\nu)}}{2m_p}\right]\hat{\nu}\right\}\cdot(\vec{\xi}v^{(\mu)})_{S,S_z}\,, \tag{19}$$

where $\hat{z}$ is the direction of polarization of the μ^- and/or the ^{6}Li and $\hat{\nu}, v^{(\nu)}, v^{(\mu)}$ are as defined in Eq. (4) *et seq*. Then, using the fact that $\vec{\xi} = \mp(1/\sqrt{2})(\hat{x}\pm i\hat{y})$ $(\equiv\vec{\xi}_{\pm1})$, $\hat{z}(\equiv\vec{\xi}_0)$ correspond, respectively, to the $s_z^{(^6\text{Li})} = \pm1, 0$ substates, we have

$$\begin{aligned}(\vec{\xi}v^{(\mu)})_{3/2,3/2} &= \vec{\xi}_{+1}\tbinom{1}{0}\,,\\ (\vec{\xi}v^{(\mu)})_{3/2,1/2} &= (1/\sqrt{3})\vec{\xi}_{+1}\tbinom{0}{1} + \sqrt{\tfrac{2}{3}}\,\vec{\xi}_0\tbinom{1}{0}\,,\\ (\vec{\xi}v^{(\mu)})_{3/2,-1/2} &= \sqrt{\tfrac{2}{3}}\,\vec{\xi}_0\tbinom{0}{1} + (1/\sqrt{3})\vec{\xi}_{-1}\tbinom{1}{0}\,,\\ (\vec{\xi}v^{(\mu)})_{3/2,-3/2} &= \vec{\xi}_{-1}\tbinom{0}{1}\,,\\ (\vec{\xi}v^{(\mu)})_{1/2,1/2} &= \sqrt{\tfrac{2}{3}}\,\vec{\xi}_{+1}\tbinom{0}{1} - (1/\sqrt{3})\vec{\xi}_0\tbinom{1}{0}\,,\\ (\vec{\xi}v^{(\mu)})_{1/2,-1/2} &= (1/\sqrt{3})\vec{\xi}_0\tbinom{0}{1} - \sqrt{\tfrac{2}{3}}\,\vec{\xi}_{-1}\tbinom{1}{0}\,;\end{aligned} \tag{20}$$

$$|\mathcal{T}(\tfrac{3}{2},\pm\tfrac{3}{2})|^2 = \left(\frac{G}{\sqrt{2}}\right)^2 G_P^{\,2}(1\mp\hat{\nu}\cdot\hat{z})[1-(\hat{\nu}\cdot\hat{z})^2]\,, \tag{21a}$$

$$|\mathcal{T}(\tfrac{3}{2},\pm\tfrac{1}{2})|^2 = \left(\frac{G}{\sqrt{2}}\right)^2 \tfrac{1}{3}G_P^{\,2}(1\mp\hat{\nu}\cdot\hat{z})(1\pm3\hat{\nu}\cdot\hat{z})^2\,, \tag{21b}$$

$$|\mathcal{T}(\tfrac{1}{2},\pm\tfrac{1}{2})|^2 = \left(\frac{G}{\sqrt{2}}\right)^2 \tfrac{2}{3}(3G_A - G_P)^2(1\mp\hat{\nu}\cdot\hat{z})\,, \tag{21c}$$

where

$$G_A = -F_A(q^2) - F_M(q^2)\frac{E^{(\nu)}}{2m_p}\,,$$

$$G_P = F_P(q^2)\frac{m_\mu E^{(\nu)}}{m_\pi{}^2} - F_E(q^2)\frac{E^{(\nu)}}{2m_p} - F_M(q^2)\frac{E^{(\nu)}}{2m_p} \tag{22}$$

with

$$E^{(\nu)} = 100.72 \text{ MeV},$$
$$q^2 = (p^{(f)} - p^{(i)})^2 = (p^{(\mu)} - p^{(\nu)})^2 = -m_\mu{}^2 + 2m_\mu E^{(\nu)} = 0.906 m_\mu{}^2 . \tag{23}$$

At the instant of arrival of the μ^- in the lowest Bohr orbit around the ^{6}Li, the μ^- is characterized by the polarization P_μ while the ^{6}Li is specified by the polarization $P_{^6\text{Li}}$ and the alignment $A_{^6\text{Li}}$. In the case of *no* $S=\frac{3}{2}\to S=\frac{1}{2}$ conversion in a time $\approx\tau(\mu^-$ decay), the probability $P(S, S_z)$ of finding $[\mu^-\,{}^6\text{Li}]$ in the total-spin configuration specified by S, S_z is given by

$$\begin{aligned} P(\tfrac{3}{2}, \pm\tfrac{3}{2}) &= \tfrac{1}{6}(1 + \tfrac{1}{2}A_{^6\text{Li}} \pm P_\mu \pm \tfrac{1}{2}P_\mu A_{^6\text{Li}} \pm \tfrac{3}{2}P_{^6\text{Li}} + \tfrac{3}{2}P_\mu P_{^6\text{Li}}), \\ P(\tfrac{3}{2}, \pm\tfrac{1}{2}) &= \tfrac{1}{6}(1 - \tfrac{1}{2}A_{^6\text{Li}} \pm \tfrac{1}{3}P_\mu \mp \tfrac{5}{6}P_\mu A_{^6\text{Li}} \pm \tfrac{1}{2}P_{^6\text{Li}} - \tfrac{1}{2}P_\mu P_{^6\text{Li}}), \\ P(\tfrac{1}{2}, \pm\tfrac{1}{2}) &= \tfrac{1}{6}(1 \mp \tfrac{1}{3}P_\mu \mp \tfrac{2}{3}P_\mu A_{^6\text{Li}} \pm P_{^6\text{Li}} - P_\mu P_{^6\text{Li}}). \end{aligned} \tag{24}$$

Further, though it is rather unlikely that the $S=\frac{3}{2}\to S=\frac{1}{2}$ conversion takes place in a time $\approx\tau(\mu^-$ decay),[17,18] we append here, as a reference, the $P(S, S_z)$ in the case of *complete* $S=\frac{3}{2}\to S=\frac{1}{2}$ conversion, viz.:

$$\begin{aligned} P(\tfrac{3}{2}, \pm\tfrac{3}{2}) &= P(\tfrac{3}{2}, \pm\tfrac{1}{2}) = 0 , \\ P(\tfrac{1}{2}, \pm\tfrac{1}{2}) &= \tfrac{1}{2}(1 \pm \tfrac{7}{27}P_\mu \mp \tfrac{4}{27}P_\mu A_{^6\text{Li}} \pm \tfrac{8}{9}P_{^6\text{Li}}), \end{aligned} \tag{25}$$

where we have assumed that the interaction H_{conver}, which transforms under rotations like an angular momentum vector, does not induce any transitions between the two $S=\frac{1}{2}$ sublevels. We stress the fact that Eq. (25) does *not* hold for a general conversion mechanism.

To verify whether any $S=\frac{3}{2}\to S=\frac{1}{2}$ conversion actually takes place, we can calculate the angular correlation between the e^- momentum and the polarization of the μ^- and/or ^{6}Li. This is:

No $S=\frac{3}{2}\to S=\frac{1}{2}$ *conversion:* $P(S, S_z)$ *as in Eq. (24):*

$$\mathcal{C}(\hat{p}_e\cdot\hat{z}) = 1 - \tfrac{1}{3}(\hat{p}_e\cdot\hat{z})(\tfrac{11}{27}P_\mu + \tfrac{4}{27}P_\mu A_{^6\text{Li}} + \tfrac{4}{9}P_{^6\text{Li}}),$$

$$\mathcal{A}_{\hat{p}_e,\hat{z}} \equiv \frac{\mathcal{C}(\hat{p}_e\cdot\hat{z}=1) - \mathcal{C}(\hat{p}_e\cdot\hat{z}=-1)}{\mathcal{C}(\hat{p}_e\cdot\hat{z}=1) + \mathcal{C}(\hat{p}_e\cdot\hat{z}=-1)} = -\tfrac{1}{3}(\tfrac{11}{27}P_\mu + \tfrac{4}{27}P_\mu A_{^6\text{Li}} + \tfrac{4}{9}P_{^6\text{Li}}). \tag{26}$$

Complete $S=\frac{3}{2}\to S=\frac{1}{2}$ *conversion:* $P(S, S_z)$ *as in Eq. (25):*

$$\mathcal{C}(\hat{p}_e\cdot\hat{z}) = 1 - \tfrac{1}{3}(\hat{p}_e\cdot\hat{z})(-\tfrac{7}{81}P_\mu + \tfrac{4}{81}P_\mu A_{^6\text{Li}} - \tfrac{8}{27}P_{^6\text{Li}}),$$

$$\mathcal{A}_{\hat{p}_e,\hat{z}} \equiv \frac{\mathcal{C}(\hat{p}_e\cdot\hat{z}=1) - \mathcal{C}(\hat{p}_e\cdot\hat{z}=-1)}{\mathcal{C}(\hat{p}_e\cdot\hat{z}=1) + \mathcal{C}(\hat{p}_e\cdot\hat{z}=-1)} = -\tfrac{1}{3}(-\tfrac{7}{81}P_\mu + \tfrac{4}{81}P_\mu A_{^6\text{Li}} - \tfrac{8}{27}P_{^6\text{Li}}) \tag{27}$$

while, as regards experiment, a measurement of $\mathcal{A}_{\hat{p}_e,\hat{z}}$ yields[17]

$$[\mathcal{A}_{\hat{p}_e,\hat{z}}/(-\tfrac{1}{3}P_\mu)]_{\text{exp}} = 0.45 \pm 0.02, \quad P_{^6\text{Li}} = A_{^6\text{Li}} = 0. \tag{28}$$

Again, as seen from Eqs. (26) and (27), any $S=\frac{3}{2}\to S=\frac{1}{2}$ conversion tends to *decrease* the value of $[\mathcal{A}_{\hat{p}_e,\hat{z}}/(-\frac{1}{3}P_\mu)]_{P_{^6\text{Li}}=A_{^6\text{Li}}=0}$ below $\frac{11}{27}$ (=0.407) so that if the discrepancy between the observed 0.45 ± 0.02 and predicted $\frac{11}{27}$ is real it can be due, e.g., to the fact that a small fraction of the μ^- decay from higher Bohr orbits. In any case, the experimental result in Eq. (28) indicates that no appreciable $S=\frac{3}{2}\to S=\frac{1}{2}$ conversion takes place before the muon is captured.[18]

We note further that the discrepancy between the predicted and measured capture rates [Eqs. (48), (49) below] would be resolved completely if the $S=\frac{3}{2}\to S=\frac{1}{2}$ conversion did occur at a rate

$$\frac{\Gamma(S=\frac{3}{2}\to S=\frac{1}{2}\text{ conversion})}{\Gamma(S=\frac{3}{2}\ \mu^-\text{ capture}) + \Gamma(\mu^-\text{ decay})} \approx (16\pm5)\% \tag{29}$$

[see discussion after Eq. (54c) below]. Equation (29) implies, for $P_{^6\text{Li}}=0$ and $A_{^6\text{Li}}=0$,

$$[\mathcal{A}_{\hat{p}_e,\hat{z}}/(-\tfrac{1}{3}P_\mu)] \approx 0.33 \tag{30}$$

which, in comparison with $[\mathcal{A}_{\hat{p}_e,\hat{z}}/(-\frac{1}{3}P_\mu)]_{\text{exp}} = 0.45 \pm 0.02$, appears as too low. In this connection, it is also worth mentioning that the existence of any $S=\frac{3}{2}\to S=\frac{1}{2}$ conversion may be investigated by measuring the muon capture rate in ^{6}Li metal and in ^{6}LiF ionic crystal; then, if some conversion does take place, more of it will occur in the metal than in the ionic crystal, and the capture rate will be larger in the former than in the latter.[3]

We proceed to set down the formula for the angular correlation between the neutrino momentum (which is opposite to the momentum of the recoiling final ^{6}He) and the polarization of the μ^- and/or the ^{6}Li and, also, the formula for the capture rate in the case of *no* $S=\frac{3}{2}\to S=\frac{1}{2}$ conversion. The angular correlation is proportional to

$$\langle|\mathcal{T}|^2\rangle \equiv \sum_{S,S_z} |\mathcal{T}(S,S_z)|^2 P(S,S_z) \tag{31}$$

which, using Eqs. (21a)–(21c), (24), and (31), is given by

$$\langle |\mathcal{T}|^2\rangle = \left(\frac{G}{\sqrt{2}}\right)^2 \tfrac{4}{9}G_P^{\,2}\{(1+\tfrac{1}{2}P_\mu P_{^6\mathrm{Li}}) - (\tfrac{1}{3}P_\mu + \tfrac{1}{15}P_\mu A_{^6\mathrm{Li}} + \tfrac{1}{2}P_{^6\mathrm{Li}})\hat{\nu}\cdot\hat{z} - (\tfrac{1}{2}A_{^6\mathrm{Li}} + P_\mu P_{^6\mathrm{Li}})[\tfrac{3}{2}(\hat{\nu}\cdot\hat{z})^2 - \tfrac{1}{2}]$$
$$+\tfrac{9}{10}P_\mu A_{^6\mathrm{Li}}[\tfrac{5}{2}(\hat{\nu}\cdot\hat{z})^3 - \tfrac{3}{2}\hat{\nu}\cdot\hat{z}]\}$$
$$+\left(\frac{G}{\sqrt{2}}\right)^2 \tfrac{2}{9}(3G_A - G_P)^2[(1-P_\mu P_{^6\mathrm{Li}}) + (\tfrac{1}{3}P_\mu + \tfrac{2}{3}P_\mu A_{^6\mathrm{Li}} - P_{^6\mathrm{Li}})\hat{\nu}\cdot\hat{z}]. \tag{32}$$

The corresponding forward-backward asymmetry is then[19]

$$\mathcal{A}_{\hat{\nu},\hat{z}} = \frac{\langle|\mathcal{T}|^2\rangle|_{\hat{\nu}\cdot\hat{z}=1} - \langle|\mathcal{T}|^2\rangle|_{\hat{\nu}\cdot\hat{z}=-1}}{\langle|\mathcal{T}|^2\rangle|_{\hat{\nu}\cdot\hat{z}=1} + \langle|\mathcal{T}|^2\rangle|_{\hat{\nu}\cdot\hat{z}=-1}}$$
$$= \frac{1}{3}\,\frac{2G_P^{\,2}(-P_\mu + \tfrac{5}{2}P_\mu A_{^6\mathrm{Li}} - \tfrac{3}{2}P_{^6\mathrm{Li}}) + (3G_A - G_P)^2(P_\mu + 2P_\mu A_{^6\mathrm{Li}} - 3P_{^6\mathrm{Li}})}{2G_P^{\,2}(1 - \tfrac{1}{2}A_{^6\mathrm{Li}} - \tfrac{1}{2}P_\mu P_{^6\mathrm{Li}}) + (3G_A - G_P)^2(1 - P_\mu P_{^6\mathrm{Li}})}. \tag{33}$$

Further, the capture rate is specified by

$$\Gamma(\mu^- {}^6\mathrm{Li} \to \nu_\mu {}^6\mathrm{He})|_{\text{no } S=3/2\to S=1/2 \text{ conversion}} = \Gamma_0[\tfrac{2}{3}G_P^{\,2}(1+\tfrac{1}{2}P_\mu P_{^6\mathrm{Li}}) + \tfrac{1}{3}(3G_A - G_P)^2(1-P_\mu P_{^6\mathrm{Li}})],$$
$$\Gamma_0 = \frac{1}{3}\,\frac{G^2(E^{(\nu)})^2}{\pi^2}\left(1 - \frac{E^{(\nu)}}{m_\mu + M_f}\right)C_i\left(Z_i\alpha\frac{m_\mu M_i}{m_\mu + M_i}\right)^3,\quad C_i = 0.92 \ \ (\text{Ref. 7}). \tag{34a}$$

This value is to be contrasted with the value corresponding to complete $S=\frac{3}{2}\to S=\frac{1}{2}$ conversion:

$$\Gamma(\mu^- {}^6\mathrm{Li} \to \nu_\mu {}^6\mathrm{He})|_{\text{complete } S=3/2\to S=1/2 \text{ conversion}} = \Gamma_0(3G_A - G_P)^2. \tag{34b}$$

We next obtain numerical values of the various nuclear form factors via the following "standard" procedure:

(1). The nuclear axial form factor $F_A(q^2)$ at $q^2=0$ is obtained from the e^--decay rate of ^{6}He[7,10]:

$$\Gamma(^6\mathrm{He}\to {}^6\mathrm{Li}e^-\bar{\nu}_e) = 3\,\frac{G^2\Delta^5}{2\pi^3}\,(f)_{e^-}\left(\frac{m_e}{\Delta}\right)^5[\sqrt{2}F_A(0)]^2\left(1 + \frac{\Delta}{3m_p}\,\frac{F_E(0)}{F_A(0)}\right),$$
$$\Delta \equiv M(^6\mathrm{He}) - M(^6\mathrm{Li}) = 4.019\ \mathrm{MeV}. \tag{35}$$

Then, since

$$\left|\frac{\Delta}{3m_p}\,\frac{F_E(0)}{F_A(0)}\right| \ll 1,$$

$$F_A(0) = \left(\frac{\pi^3 \ln 2}{3G^2 m_e^{\,5}(ft_{1/2})}\right)^{1/2} \tag{36}$$

so that, with $(ft_{1/2}) = 805.6\ \mathrm{sec}^{-1}$ (Ref. 7) and $G = 1.140\times 10^{-11}\ \mathrm{MeV}^{-2}$,

$$F_A(0) = 1.137. \tag{37}$$

(2). The nuclear weak magnetism form factor $F_M(q^2)$ is related to the nuclear magnetic transition form factor $\mu(q^2)$ through conserved vector current (CVC)[10]:

$$F_M(q^2) = \sqrt{2}\mu(q^2). \tag{38}$$

Further, $\mu(0)$ is determined from the γ-decay rate of ^{6}Li*:

$$\Gamma(^6\mathrm{Li}^* \to {}^6\mathrm{Li}\gamma) = 3\,\frac{\alpha}{3}\,\frac{E_\gamma^{\,3}}{m_p^{\,2}}\,|\sqrt{2}\mu(0)|^2, \tag{39}$$

whence, using the experimental values[8]

$$[\Gamma(^6\mathrm{Li}^* \to {}^6\mathrm{Li}\gamma)]_{\exp} = (8.16\pm 0.19)\ \mathrm{eV},$$
$$E_\gamma = 3.562\ \mathrm{MeV}, \tag{40}$$

we obtain

$$F_M(0) = \sqrt{2}\mu(0) = 4.67\pm 0.05. \tag{41}$$

(3). The angular correlation between the electron and neutrino momenta in the e^- decay of ^{6}He, $1 + a\hat{p}_e\cdot\hat{\nu}$, determines the nuclear weak electricity form factor $F_E(0)$ [$F_E(q^2)$ assumed to be predominantly first class] since

$$a = -\tfrac{1}{3}\left(1 - \frac{4\Delta}{3m_p}\,\frac{F_E(0)}{F_A(0)}\right). \tag{42}$$

Thus, using the experimental value[20]

$$(a)_{\exp} = -0.3343\pm 0.0030 \tag{43}$$

we obtain

$$\frac{F_E(0)}{F_A(0)} = -(0.53\pm 1.59). \tag{44}$$

There is no recent theoretical calculation of the value of $F_E(0)/F_A(0)$ in the ^{6}Li $\leftrightarrow$ ^{6}He case[5]; in contrast, we recall that, in the $A=12$ nuclei, $F_E(0)/F_A(0) = 3.6$ from a nuclear-physics calculation[21] and also from the most recent experimental values for the electron momentum—nuclear spin correlation functions.[22-24]

(4) The nuclear pseudoscalar form factor $F_P(q^2)$ is determined through the PCAC-implied relation[10]:

$$\left.\frac{F_P(q^2)}{F_A(q^2)}\right|_{q^2\cong 0.906m_\mu{}^2} \cong -\frac{1}{1+0.906m_\mu{}^2/m_\pi{}^2} = -0.653. \quad (45)$$

(5). We assume that[10]

$$\frac{F_A(q^2)}{F_A(0)} \cong \frac{F_M(q^2)}{F_M(0)} = \frac{\mu(q^2)}{\mu(0)} \quad (46a)$$

and

$$\frac{F_E(q^2)}{F_E(0)} \cong \frac{F_A(q^2)}{F_A(0)} \quad (46b)$$

with the observation that the validity of Eq. (46b) is not at all critical in the calculation of the capture rate. The q^2-dependence of the $\mu(q^2)$ is determined from the inelastic electron scattering data and is given by[6]

$$\frac{\mu(q^2)}{\mu(0)} = e^{-aq^2},$$
$$a=(4.61\pm 0.10)\times 10^{-5}\ \text{MeV}^{-2}; \quad (47)$$
$$\left.\frac{\mu(q^2)}{\mu(0)}\right|_{q^2=0.906m_\mu{}^2} = 0.393\pm 0.004.$$

We further note that, on the basis of the impulse approximation, it is hardly possible to understand a value for

$$\left|1-\left(\frac{F_A(q^2)}{F_A(0)}\right)\bigg/\left(\frac{\mu(q^2)}{\mu(0)}\right)\right|$$

of more than 5%.[5,10]

The numerical values of $F_{M,A,P,E}(q^2)$ [Eqs. (37), (41), (44), (45), (46a), (46b), and (47)] and Eqs. (22), (23) for G_A, G_P in terms of $F_{M,A,P,E}(q^2)$ lead to the following prediction for the capture rate [Eq. (34a)]:

$$\Gamma(\mu^- {}^6\text{Li}\to\nu_\mu {}^6\text{He})\big|_{\text{no } S=3/2\to S=1/2 \text{ conversion}}$$
$$=[1241(\pm 71)-1127(\pm 62)P_\mu P_{^6\text{Li}}]\ \sec^{-1}$$
$$=(1241\pm 71)\ \sec^{-1},\quad P_{^6\text{Li}}=0 \quad (48)$$

which is to be compared with the measured value[25]:

$$[\Gamma(\mu^- {}^6\text{Li}\to\nu_\mu {}^6\text{He})]_{\text{exp}} = (1600^{+330}_{-130})\ \sec^{-1},\quad P_{^6\text{Li}}=0. \quad (49)$$

We are therefore faced with what appears to be a discrepancy between theory and experiment.

Now, a failure of Eq. (46a) as the explanation of this discrepancy is deemed unlikely since, to remove the discrepancy, we require

$$\left|1-\left(\frac{F_A(q^2)}{F_A(0)}\right)\bigg/\left(\frac{\mu(q^2)}{\mu(0)}\right)_{\text{exp}}\right| = (14\pm 6)\%, \quad (50)$$

which is in serious disagreement with what is expected from the impulse approximation. Alternatively, a departure from CVC could also explain the discrepancy; thus, if instead of Eq. (41) we take

$$F_M(0)=1.85[F_M(0)|_{\text{CVC}}=8.64 \quad (51)$$

the predicted capture rate will agree with the measured value. On the other hand, such a departure from CVC cannot be tolerated in view of the general body of evidence in favor of this principle[26,27] so that Eq. (51) cannot be seriously considered even though an independent experimental determination of $F_M(0)$ (from the shape of the electron momentum spectrum in ${}^6\text{He}\to{}^6\text{Li}e^-\bar{\nu}_e$) is lacking. Similarly, while the discrepancy can be removed by assuming

$$\frac{F_E(0)}{F_A(0)} = -(10\pm 3) \quad (52)$$

or

$$\left.\frac{F_P(q^2)}{F_A(q^2)}\right|_{q^2=0.906m_\mu{}^2} = +(0.39\pm 0.23) \quad (53)$$

these assumptions are unacceptable in view of Eqs. (44) and (45). In this connection, it is also interesting to point out that a direct test of the PCAC-implied relation in Eq. (45) can be performed by means of a measurement of $\mathcal{A}_{\hat{\nu},\hat{z}}$ [Eq. (33)]; the sensitivity of the predicted values of $\mathcal{A}_{\hat{\nu},\hat{z}}$ to the value assumed for

$$\left[F_P(q^2)\bigg/\left(-\frac{F_A(q^2)}{1+\dfrac{q^2}{m_\pi{}^2}}\right)\right]_{q^2=0.906m_\mu{}^2} = \xi$$

is illustrated by the expressions:

$$\mathcal{A}_{\hat{\nu},\hat{z}} = \frac{1}{3}\,\frac{0.97P_\mu + 2.07P_\mu A_{^6\text{Li}} - 3.04P_{^6\text{Li}}}{1.03-0.01A_{^6\text{Li}} - 1.01P_\mu P_{^6\text{Li}}},\quad \xi=\tfrac{1}{2}, \quad (54a)$$

$$\mathcal{A}_{\hat{\nu},\hat{z}} = \frac{1}{3}\,\frac{0.93P_\mu + 2.16P_\mu A_{^6\text{Li}} - 3.10P_{^6\text{Li}}}{1.07-0.03A_{^6\text{Li}} - 1.03P_\mu P_{^6\text{Li}}},\quad \xi=1, \quad (54b)$$

$$\mathcal{A}_{\hat{\nu},\hat{z}} = \frac{1}{3}\,\frac{0.77P_\mu + 2.57P_\mu A_{^6\text{Li}} - 3.34P_{^6\text{Li}}}{1.23-0.11A_{^6\text{Li}} - 1.11P_\mu P_{^6\text{Li}}},\quad \xi=2. \quad (54c)$$

In summary, it is rather unlikely that CVC [Eqs. (38)–(41)], or PCAC [Eq. (45)], or the assumption of similar q^2 dependence of the various form factors [Eqs. (46a) and (46b)] is incorrect by an amount sufficiently large to account for the discrepancy between the predicted and the measured capture rates [Eqs. (48) and (49)]. Instead, it is likely that the discrepancy is largely of experimental origin—either the measured capture rate

[Eq. (49)] is too high or the measured forward-backward asymmetry [Eq. (28)] is too large. In the latter case the "correct" value of the forward-backward asymmetry $[\mathcal{A}_{\hat{P}_e,\hat{z}}/(-\frac{1}{3}P_\mu)]\approx 0.33$ [Eq. (30)], together with the measured capture rate $\Gamma(\mu^- {}^6\mathrm{Li}\to\nu_\mu {}^6\mathrm{He})=1600\ \mathrm{sec}^{-1}$ [Eq. (49)], are consistent with about 16% $S=\frac{3}{2}\to S=\frac{1}{2}$ conversion, since, in particular,

$$0.84\Gamma(\mu^- {}^6\mathrm{Li}\to\nu_\mu {}^6\mathrm{He})|_{\mathrm{no}\ S=3/2\to S=1/2\ \mathrm{conversion}}+0.16\Gamma(\mu^- {}^6\mathrm{Li}\to\nu_\mu {}^6\mathrm{He})|_{\mathrm{complete}\ S=3/2\to S=1/2\ \mathrm{conversion}}$$

$$=(0.84\times 1241+0.16\times 3496)\ \mathrm{sec}^{-1}=1602\ \mathrm{sec}^{-1}\ \text{[Eqs. (34a), (48), and (34b)]}.$$

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