Spin-orbit terms in heavy-ion elastic scattering potentials

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A general double folding prescription is used in conjunction with a realistic G matrix interaction to estimate the strength of the elastic spin-orbit potential for several heavy-ion systems. The potentials obtained give a reasonable description of recent elastic asymmetry data obtained with polarized ^{6}Li ions, but they are much smaller than those assumed in recent analyses of some heavy-ion transfer reactions. The calculated spin-orbit potentials have essentially no effect on elastic angular distributions.

[NUCLEAR REACTIONS Heavy-ion spin-orbit potential, double-folding model.]

I. INTRODUCTION

There has been considerable recent interest in the spin-orbit term in the heavy-ion optical potential.[1-7] The elastic asymmetry has been measured for the scattering of polarized ^{6}Li ions from several targets.[1] A reasonable description of these data has been obtained using a theoretical spin-orbit potential based on a single folding cluster model.[1,2] The spin-flip probability for the excitation of the first 2^+ level in ^{58}Ni by ^{13}C has also been measured.[3] Others have speculated[4] that spin-orbit distortion might have important effects on the angular distributions for transfer reactions. In still another recent experiment,[5] significant back angle differences were noted in the elastic differential cross sections for ^{10,11}B+^{27}Al at 50 MeV and ^{12}C+^{27}Al at 55 MeV. These differences were largely attributed[5] to quadrupole terms in the elastic scattering potentials; however, it was suggested that possible effects due to spin-orbit terms should also be investigated.

In Ref. 5 the quadrupole terms in the elastic scattering potentials were estimated using a double folding model[8] that includes single nucleon knock-out exchange[9] (SNKE) and employs a realistic G matrix interaction.[10] This model had previously provided useful estimates of the real central parts of heavy-ion optical potentials.[11,12] Below we extend the model to the calculation of the spin-orbit terms in the heavy-ion optical potentials. The resulting prescription is more general than others which have appeared recently in the literature.[2,6,7] We present results for the spin-orbit potential in the form of equivalent Woods-Saxon potentials and discuss these in light of the theoretical and experimental work cited above.

II. THEORY

We assume a two-body spin-orbit interaction between the projectile and target nucleons that is of the form

$$v_{pt} = [g^0(s) + g^1(s)\vec{\tau}_p \cdot \vec{\tau}_t]\vec{l}_s \cdot (\vec{s}_p + \vec{s}_t), \tag{1a}$$

$$\vec{l}_s = \vec{s} \times \vec{p}_s, \tag{1b}$$

where $\vec{s}$ and $\vec{p}_s$ are the relative nucleon-nucleon separation and momentum, respectively. We introduce the usual double folding coordinates shown in Fig. 1 and note the following decompositions of $\vec{s}$ and $\vec{p}_s$:

$$\vec{s} = \vec{r}_p + \vec{r} - \vec{r}_t, \tag{2a}$$

$$\vec{p}_s = \tfrac{1}{2}\vec{p}_p + \frac{1}{2}\frac{A_p + A_t}{A_p A_t}\vec{p}_r - \tfrac{1}{2}\vec{p}_t \tag{2b}$$

which are exact except for the neglect of recoil in the projectile and target. The heavy-ion spin-orbit potential is then obtained by keeping only the second term on the right in Eq. (2b) and taking the

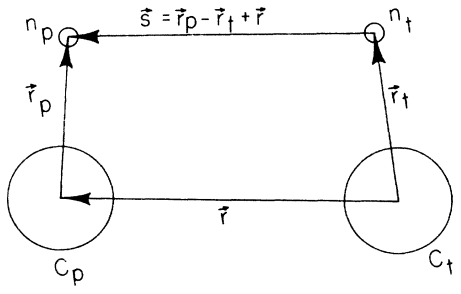

FIG. 1. Coordinates used in double folding calculations.

expectation of $V=\Sigma_{pt} v_{pt}$ over the ground states of the projectile and target. This is most easily done by expanding $g(s)\vec{s}$ in momentum space using the techniques of Refs. 8 and 13.

For the case of a spin zero target we obtain a nucleus-nucleus spin-orbit potential of the form

$$V_{ls}=\frac{1}{r}\frac{d}{dr}V_{ls}(r)\vec{l}_r\cdot\vec{I}_p$$
$$=\frac{1}{r}\frac{d}{dr}V_{ls}(r)(\vec{r}\times\vec{p}_r)\cdot\vec{I}_p \quad (3)$$

with

$$V_{ls}(r)=4\pi\sum_{L_p^n T} w_n^2 C_{L_p}\tilde{g}_1^T(k_n)\rho_t^{0T}(k_n)$$
$$\times F_p^{L_p 11,T}(k_n) j_0(k_n r)\,. \quad (4)$$

Here $\rho_t^{0T}(k_n)$ and $F_p^{L_p 11,T}(k_n)$ are the Bessel transforms of the target ground state density and the projectile dipole densities as defined in Ref. 8, $\vec{I}_p$ is the total spin of the projectile,

$$\tilde{g}_1^T(k_n)=-\frac{4\pi}{k_n}\int_0^\infty j_1(k_n s)g^T(s)s^3 ds \quad (5)$$

and

$$C_0=\sqrt{2}C_2=\frac{1}{8\sqrt{\pi}}\frac{A_p+A_t}{A_p A_t}\frac{1}{[I_p(I_p+1)]^{1/2}}\,. \quad (6)$$

This result is, of course, also valid for the case of a spin zero projectile provided the subscripts p and t are interchanged. When both the projectile and target have nonzero spin the optical potential can contain terms in $\vec{p}_r$ more complicated than $\vec{l}_r\cdot(\vec{I}_p+\vec{I}_t)$. In addition, higher shape multipoles of the densities can contribute to $V_{ls}(r)$ in these cases. The full ground state densities of the projectile and target do not enter the calculation of these additional terms and they are expected to be small.

To include the contribution from SNKE we make use of the fact that the two-body spin-orbit interaction has very short range[10] and appeal to the short range limit where this contribution is known exactly (see discussions in Refs. 14–17). This limit suggests that it is a reasonable approximation to replace $\tilde{g}^T(k_n)$ in Eq. (4) by $2\tilde{g}_{\rm odd}^T(k_n)$ where $\tilde{g}_{\rm odd}^T(k_n)$ is constructed only from the odd part of the two-body spin-orbit interaction. The restriction to odd interaction components also gives $\tilde{g}_{\rm odd}^0(k_n)=3\tilde{g}_{\rm odd}^1(k_n)$.

In the strict short range limit $g_{\rm odd}^T(k_n)$ is a constant κ^T given by

TABLE I. Results for $V_{ls}(r)$ expressed in the form of a Woods-Saxon potential $V_{ls}(r)=V_{ls}\{1+\exp[(r-R_{ls})/a_{ls}]\}^{-1}$, $R_{ls}=r_{ls}(A_p^{1/3}+A_t^{1/3})$ fitted to the folded potentials obtained using the G matrix of Ref. 10.

Projectile	Target	I_p	V_{ls} (MeV fm^2)	r_{ls} (fm)	a_{ls} (fm)	Wave functions	$\mu_{\rm theor}$ (μ_N)	$\mu_{\rm exp}$ [a] (μ_N)
^{6}Li	^{12}C	1	10.138	0.671	0.650	$LS(^3S_1)$	0.880	0.822
	^{16}O		6.812	0.710	0.711			
	^{28}Si		7.166	0.752	0.717			
	^{58}Ni		9.833	0.818	0.683			
^{7}Li	^{24}Mg	$\frac{3}{2}$	2.603	0.778	0.619	$LS(^2P_{3/2})$ [b]	3.13	3.26
^{10}B	^{27}Al	3	2.162	0.700	0.670	$LS(^3D_3)$	1.88	1.80
^{11}B	^{27}Al	$\frac{3}{2}$	1.516	0.687	0.674	c	2.63	2.69
^{13}C	^{40}Ca	$\frac{1}{2}$	−6.993	0.647	0.656	$1p_{1/2}$	0.637	0.702
^{19}F	^{28}Si	$\frac{1}{2}$	8.457	0.654	0.666	$2s_{1/2}$	2.79	2.63
^{27}Al	^{10}B	$\frac{5}{2}$	1.113	0.698	0.574	$1d_{5/2}$	4.79	3.64
	^{11}B		1.224	0.689	0.574			
	^{12}C		1.343	0.680	0.570			
^{31}P	^{16}O	$\frac{1}{2}$	5.317	0.742	0.568	$2s_{1/2}$	2.79	1.13

[a] Reference 18.
[b] The required $^2P_{3/2}$ configuration is the spatially symmetric one.
[c] Reference 19.

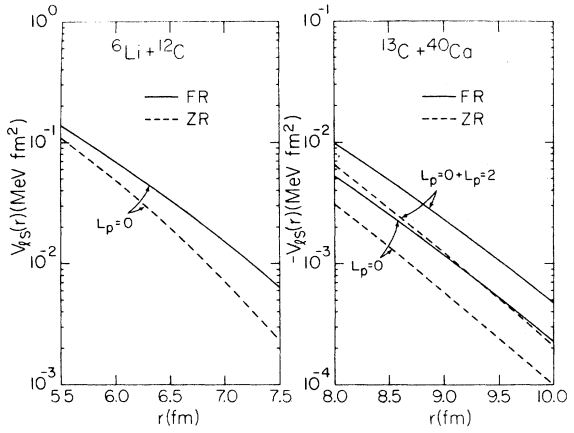

FIG. 2. Graphs of $V_{ls}(r)$ defined in the text over the radial range $1.4-1.7(A_p^{1/3}+A_t^{1/3})$. Dashed curves are short range limit results and solid curves are results with finite range G matrix interaction. Results with and without $L_p=2$ contribution are both shown.

$$\kappa^T = -\frac{4\pi}{3}\int_0^\infty g^T_{\text{odd}}(s)s^4 ds\,. \tag{7}$$

For the G matrix interactions of Ref. 10, $\kappa^0 \approx 160$ MeV fm^5. For the case of a proton projectile $F^{L_p p^{11},T}_p(k_n) = (12\pi)^{1/2}\delta_{L_p 0}$ and we find that in the strict short range limit

$$V_{ls}(r) \approx \sum_T \kappa^T \rho^{0T}_t(r)\,. \tag{8}$$

A direct comparison of Eq. (8) with the spin-orbit terms in phenomenological optical potentials for elastic proton scattering yields $\kappa^0 \approx 168$ MeV fm^5 [14-17] which agrees well with the interactions of Ref. 10.

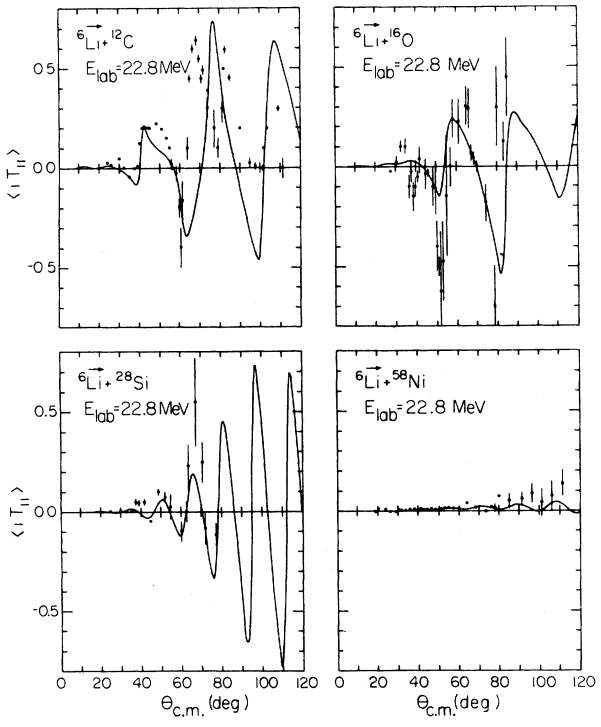

FIG. 3. Fits to the experimental asymmetries of Ref. 1 using spin-orbit potentials from the present work and the central optical potentials of Ref. 1.

III. RESULTS AND CONCLUSION

We have calculated heavy-ion spin-orbit potentials for many systems of interest. The calculations have been made in the strict short range limit and using the two-body spin-orbit interaction fitted to the Elliott matrix elements in Ref. 10. The latter results for $V_{ls}(r)$ are summarized in Table I in the form of Woods-Saxon parameters obtained by fitting the folded potentials in the radial range from $1.4-1.7(A_p^{1/3}+A_t^{1/3})$ which is most important for the scattering. We have also indicated the wave functions assumed in constructing the dipole densities and listed the magnetic moments. The experimental magnetic moments are well reproduced except in the case of ^{27}Al and ^{31}P. The assumed $2s_{1/2}$ single particle state ^{31}P is particularly naive. Oscillator forms have been used for the radial parts of the dipole densities. For the full ground state densities both oscillator and Woods-Saxon forms have been used. Electron scattering results have been used in fixing the parameters of all of these distribution.[20]

Figure 2 contains plots of $V_{ls}(r)$ for the ^{6}Li + ^{12}C and ^{13}C + ^{40}Ca systems. It is clear that the finite range of the two-body spin-orbit interaction contributes about a factor of 2 to the strength of $V_{ls}(r)$ in the important tail region. The nonspherical part of the dipole density is vanishingly small for ^{6}Li and thus there is no $L_p=2$ contribution to $V_{ls}(r)$ for ^{6}Li + ^{12}C. In the case of ^{13}C + ^{40}Ca the $L_p=2$ contribution is comparable to the $L_p=0$ contribution. In all other cases considered the $L_p=2$ contributions were no more than the 20% of the $L_p=0$ contributions. It is also important to note the difference in sign between the ^{6}Li + ^{12}C and ^{13}C + ^{40}Ca potentials. In the extreme single particle picture $V_{ls}(r)>0$ for $j=l+\frac{1}{2}$ and $V_{ls}(r)<0$ for $j=l-\frac{1}{2}$.[7,21]

The finite range result for ^{6}Li + ^{12}C shown in Fig. 2 agrees very well with the cluster model single folding results of Refs. 1 and 2. We have calculated the elastic asymmetries for the ^{6}Li + ^{12}C, ^{16}O, ^{28}Si, and ^{58}Ni systems using the spin-orbit parameters given in Table I and the central optical potential parameters from Ref. 1. The results, which are shown in Fig. 3, are essentially identi-

cal to those shown in Ref. 1. In calculations for the $^{10,11}B + ^{27}Al$ and $^{12}C + ^{27}Al$ systems,[5] no effect was noted at back angles when the spin-orbit terms were included in the optical potential. The $^{19}F + ^{28}Si$ and $^{16}O + ^{31}P$ spin-orbit potentials assumed in the transfer studies of Refs. 4 are 5–100 times stronger in the region of the strong absorption radius ($D_{1/2}$) than those we have estimated here. In other calculations for the $^{40}Ca(^{13}C, ^{14}N)^{39}K$ reaction[4] the $^{13}C + ^{40}Ca$ spin-orbit potential assumed is over 100 times stronger than ours at $D_{1/2}$. Moffa[7] has obtained a spin-orbit potential for $^{13}C + ^{40}Ca$ that is 10 times stronger than ours, but still an order of magnitude smaller than assumed in Ref. 4. He uses a double folding model, but does not include any finite range effects, no $L_p = 2$ contributions, and has fixed the strength of his interaction from a phenomenological $\alpha + ^{9}Be$ spin-orbit potential. The difference in magnitude between our results and his is mainly in the assumed strength of the interaction.

In summary, a general method for estimating the lowest order heavy-ion spin-orbit potential from the nucleon-nucleon spin-orbit interaction has been presented. The theoretical spin-orbit potentials obtained are fairly weak and it is encouraging that they seem consistent with recent asymmetry data for ^{6}Li elastic scattering. The spin-orbit potentials introduced in the recent analyses of some heavy-ion transfer reactions appear to be much too strong. Other effects which are being investigated are possible exchange contributions to the spin-orbit potential from central and tensor components of the nucleon-nucleon interactions[21] and possible higher order shape polarization contributions to the spin-orbit potential. It remains to be seen how these might alter the above conclusions.

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