

Analysis of $^{136}\text{Xe}(p, p_1)$ on analog resonances and the structure of ^{137}Xe

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Analysis of proton inelastic scattering through analog resonances in ^{137}Cs is carried out by the addition of a resonant scattering matrix to a coupled-channel scattering matrix. Neutron coupling to the 2_1^+ excited state of ^{136}Xe in wave functions of the parent states in ^{137}Xe is thus determined. For low lying levels, the parentage coefficients extracted from experimental data are in general agreement with calculations performed using the particle-vibrator coupling model.

[NUCLEAR REACTIONS $^{136}\text{Xe}(p, p')$; $E=10.26-11.82$ MeV; analyzed $\sigma(E_p, \theta)$;
 ^{137}Cs IAR deduced Γ , parentage coefficients; ^{137}Xe calculated neutron parent-
age, levels.]

I. INTRODUCTION

Of central importance to all variants of the nuclear shell model is the question of parentage, that is to what extent a nuclear state can be built up by adding a nucleon in a definite single-particle state to a definite core state. Inelastic proton scattering through analog resonances provides a powerful tool to study the neutron parentage of the low-lying parent analog states. When nonresonant scattering can be neglected, a simple Breit-Wigner data analysis is satisfactory. Usually however, the core states of greatest interest exhibit the strongest nonresonant cross sections. Then an adequate treatment becomes much more difficult but additional information, the signs of the parentage coefficients, can be obtained. Recently a number of analyses to 2_1^+ target levels have been carried out by considering at the same time distorted wave Born approximation (DWBA) and resonant scattering amplitudes.¹⁻¹¹ In this present work a resonant C matrix is added to a coupled channel C matrix using the program JUPITOR.¹² Computationally, our procedure is little more time consuming than the DWBA approach and provides a greater flexibility.

II. EXPERIMENT

The experimental procedure and much of the data are presented in Ref. 13. All of the data are presented in Ref. 14. Angular distributions of $\sigma(\theta)$ were measured on five analog resonances at center of target laboratory energies of 10.26, 10.87, 11.27, 11.57, and 11.82 MeV having spins of $\frac{7}{2}^-$, $\frac{3}{2}^-$, $\frac{1}{2}^-$, $\frac{5}{2}^-$, and $\frac{7}{2}^-$, respectively. An off-resonance angular distribution was taken at 10.64 MeV. The 2_1^+ angular distribution for the $l=3$, 11.82 MeV $\frac{7}{2}^-$ resonance could not be fitted as-

suming a $\frac{5}{2}^-$ spin. The energies of the first three resonances were determined by measuring the maximum yield of the neutron particle hole states,¹³ those of the last two by the maximum yield of the 2_1^+ .

III. ANALYSIS

The isobaric analog state (IAS) can be written in terms of the parent states (PS) as

$$|\text{IAS}; J_\lambda\rangle = (2T_0 + 1)^{-1/2} T^- |\text{PS}; J_\lambda\rangle,$$

where J_λ labels the λ th resonance with angular momentum J , T^- is the nuclear isospin lowering operator, and T_0 is the isospin of the target nucleus.

The neutron parentage coefficients are defined as

$$\theta(lj, I_n; J_\lambda) = (2J_\lambda + 1)^{-1/2} \langle \text{PS}; J_\lambda || a_{lj}^\dagger || I_n \rangle,$$

where the operator a_{lj}^\dagger creates a neutron in orbital (lj) and I_n stands for the n th core state with angular momentum I .

The expression for the cross section in terms of the C matrix is given in Ref. 12. The scattering matrix can be written as a background $S_{\alpha\alpha'}^{JB}$, plus a resonant term $S_{\alpha\alpha'}^{JR}$ with

$$S_{\alpha\alpha'}^{JR} = ie^{i(\phi_\alpha + \phi_{\alpha'})} \sum_\lambda \frac{g(J_\lambda \alpha) g(J_\lambda \alpha')}{E_{J_\lambda} - E - \frac{1}{2} i \Gamma_{J_\lambda}^R}$$

in the R matrix formalism.¹⁵ The C matrix becomes¹²

$$C_{\alpha\alpha'}^J = C_{\alpha\alpha'}^{JB} - \frac{i}{2} \frac{k_\alpha}{k_{\alpha'}} \left(\frac{2l+1}{2l'+1} \right)^{1/2} \times \{ \delta_{\alpha\alpha'} - e^{2i\sigma_{\alpha'}} S_{\alpha\alpha'}^{JR} \},$$

where $C_{\alpha\alpha'}^J$, the background term, is calculated using the program JUPITOR. Here $\alpha = \{ljI_n\}$ and

$\alpha' = \{l'j'I_{\alpha'}\}$ stand, respectively, for the entrance and exit channel quantum numbers; k_{α} and σ_{α} are the wave number and Coulomb phase; ξ_{α} and ζ_{α} are the real and imaginary optical phases; $\phi_{\alpha} = \xi_{\alpha} + \sigma_{\alpha} + \psi_{\alpha}^R$ where ψ_{α}^R is the resonance mixing phase¹⁵; $\Gamma_{J_{\lambda}}^T$ is the total width of the resonance and $E_{J_{\lambda}}$ its energy; E is the incident center of mass energy. The resonance amplitudes $g(J_{\lambda}\alpha)$ are related to the parentage coefficients and partial widths, $\Gamma_{J_{\lambda}\alpha}$ through the relations

$$\theta(lj, I_n; J_{\lambda}) = g(J_{\lambda}\alpha) / g(J_{\lambda}\alpha)^{s,p},$$

$$\Gamma_{J_{\lambda}\alpha} = |g(J_{\lambda}\alpha)|^2,$$

where $g(J_{\lambda}\alpha)^{s,p}$ is the single-particle amplitude in the channel α at the energy of the resonance J_{λ} . Throughout this work we adopt the coupling scheme $\vec{J} = \vec{j} + \vec{I}$ with $\vec{j} = \vec{l} + \vec{s}$ and take the single-particle wave functions to transform under the time reversal operator t as

$$t |ljm\rangle = (-1)^{j-m} |lj - m\rangle.$$

The optical potential employed is Set P of Sen, Riley, and Udagawa¹⁶ with the energy dependence $V = V_0 - 0.32E$ and $W' = W'_0 - 0.25E$. The nuclear deformation parameter $\beta_2^N = 0.064$ for the coupling potential comes from the same reference. The Coulomb deformation parameter used, $\beta_2^C = 0.085$, is that for ^{138}Ba , taken from Ref. 17. Resonance mixing phases and elastic partial widths come from a reanalysis of elastic scattering excitation curves of Ref. 13 using the program ANSPEC.^{18,19} Figure 1 shows the fit to the 159° elastic scattering data. Figure 2 shows the elastic angular distribution data on the resonances together with the angular distributions predicted by the fitted mixing phases and partial widths. We obtained $\psi_{1=3}^R = 1^\circ$ and $\psi_{1=1}^R = 7^\circ$. These phases were then used in the inelastic calculations. Our elastic partial widths are essentially¹⁵ and the Γ_{ij}^A of Ref. 13 multiplied by $e^{-2k_{ij}}$. Single-particle amplitudes are also calculated with ANSPEC. Total widths were

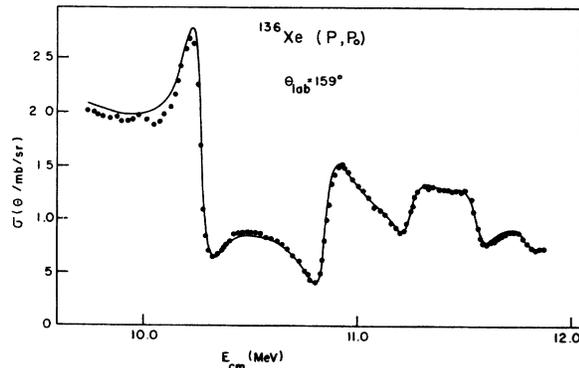


FIG. 1. Fit to 159° elastic excitation function.

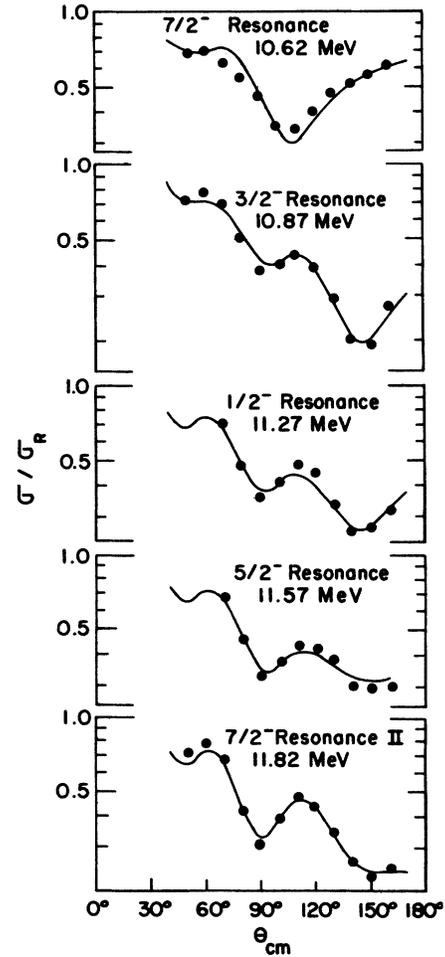


FIG. 2. Elastic angular distributions on analog resonances. The solid lines indicate calculations with coupled channel plus resonance C matrices, using fitted mixing phases and partial widths.

taken from Ref. 13. The only free parameters in the inelastic analysis are the $g(J_{\lambda}\alpha)$.

For the $\frac{7}{2}_1^-$, $\frac{3}{2}_2^-$, and $\frac{7}{2}_2^-$ resonances the initial fitting parameters were obtained by fitting the back angle data in the manner of Hiddleston and Riley²⁰ using the program GRILLE.²¹ In each case one of the multiple GRILLE solutions gave a satisfactory description of the forward angle data and that solution was used as the starting point in the fitting. The calculated wave function was taken as the starting point of the relatively structureless $\frac{5}{2}_1^-$ angular distribution. The resonances were fitted separately but at every step of the fitting procedure the parameters for the other resonances from the previous iteration were included in the background. This process was continued until the values of the resonance parameters stabilized. The $h_{0/2}$ partial widths were not varied in the fitting. Because of its nearness to the $\frac{5}{2}_1^-$

TABLE I. Resonant partial widths. Elastic and inelastic partial widths are denoted as Γ^0 and Γ_{ij}^1 . For phases see Table II.

Res	Γ^0	Γ^1 $f_{7/2}$	Γ^1 $p_{3/2}$	Γ^1 $p_{1/2}$	Γ^1 $f_{5/2}$	Γ^1 $h_{9/2}$
$\frac{7^-}{2_1}$	22.0 ± 1	1.63 ± 0.2	0.55 ± 0.2	...	0.09 ± 0.2	...
$\frac{3^-}{2_1}$	28.9 ± 1	5.86 ± 0.5	3.79 ± 0.5	0.05 ± 0.5	0.13 ± 0.5	...
$\frac{1^-}{2_1}$	21.3 ± 1.5	...	8.18 ± 1.5	...	2.56 ± 1.5	...
$\frac{5^-}{2_1}$	10.9 ± 1.5	3.45 ± 1.5	4.55 ± 1.5	0.72 ± 1.5	1.48 ± 1.5	0.001^a
$\frac{7^-}{2_2}$	4.4 ± 1	11.5 ± 2.5	8.24 ± 2.5	...	0.20 ± 2.5	...

^aDetermined from structure calculation and held fixed.

resonance, the 11.50 MeV $\frac{9^-}{2_1}$ resonance was included in the background using partial widths determined from the calculation, assuming $\Gamma^T = 70$ keV. The $\frac{3^-}{2_2}$ and $\frac{5^-}{2_2}$ resonances were also included as background using partial widths taken from the structure calculation and total widths based on systematics. The $\frac{9^-}{2_2}$ resonance, probably near the $\frac{7^-}{2_2}$ resonance by comparison with ¹³⁹Ba and our calculation, is not included since its location is not known, nor have we included the $\frac{11^-}{2_1}$ resonance which should be very weakly excited. Table I shows the experimental partial widths.

IV. CALCULATION

The low-lying states of ¹³⁷Xe are calculated in the framework of the particle-vibrator coupling model.²² It is assumed that the contribution of neutron particle-hole excitations in building up the vibrational field (¹³⁶Xe core) is negligible and that the total Hamiltonian of the system may be written as

$$H = H_0 + H_{\text{int}},$$

where H_0 represents the energy of the quadrupole vibrational field and the valence neutron, while the particle-vibrator coupling H_{int} is given by

$$H_{\text{int}} = -\frac{\beta_2}{\sqrt{5}} K \sum_{\mu} \{b^{\mu\dagger} + (-)^{\mu} b^{-\mu}\} Y_2^{\mu*}(\theta, \phi),$$

K being the coupling strength and b^{\dagger} (b) the creation (destruction) operator of the phonon field. The eigenstate of the coupled system is a linear combination of the basis vectors $|lj, NR, J_{\lambda}\rangle$, i.e.,

$$|J_{\lambda}\rangle = \sum_{l j N R} \eta(lj, NR; J_{\lambda}) |lj, NR; J_{\lambda}\rangle,$$

where the symbols N and R represent the phonon number and the angular momentum quantum num-

ber of the N -phonon state, respectively. In the foregoing model the wave function amplitudes are related in a trivial way with the parentage coefficients for a core state which may be identified as collective; in particular: $\eta(lj, 00; J_{\lambda}) = \theta(lj, 0_1^+; J_{\lambda})$ and $\eta(lj, 12; J_{\lambda}) = \theta(lj, 2_1^+; J_{\lambda})$. The numerical calculation was carried out taking the interaction strength $\langle K \rangle = 50$ MeV, corresponding to the estimate from Ref. 22; the phonon energy $\hbar\omega = 1.31$ MeV, which is that of the 2_1^+ state

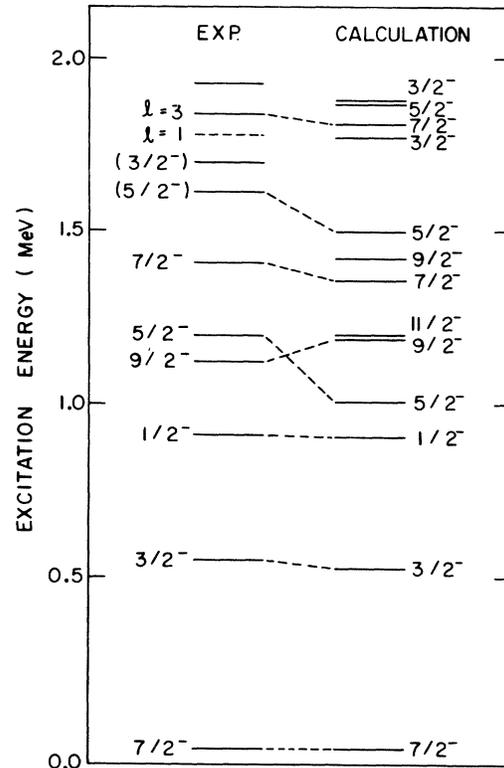


FIG. 3. Calculated negative parity level scheme compared with known negative parity levels.

in $^{136}\text{Xe}^1$; and the single-particle energies $\epsilon(lj)$ from Ref. 23, namely (in MeV): $\epsilon(f_{7/2})=0$, $\epsilon(p_{3/2})=1.09$, $\epsilon(p_{1/2})=1.30$, $\epsilon(h_{9/2})=1.42$, and $\epsilon(f_{5/2})=1.5$. Vibrational states up to $N=3$ quadrupole phonons were considered. Figure 3 shows the calculated negative parity level scheme compared with the known negative parity levels.²⁴

V. RESULTS AND CONCLUSIONS

Table II shows the parentage coefficients extracted from the experimental data together with those from the model calculation. Errors are estimated from the comportment of the partial widths in relation to χ^2 during the fitting. Uncertainties in the $g^{s,p}$ of 10% are not included in the errors. Figure 4 shows the fits to the data and the angular distributions predicted by the calculation. There is an uncertainty of about 20% in the predicted cross sections because of uncertainties in the $g^{s,p}$ and Γ^T .

Although the level scheme of ^{136}Xe above the 2_1^+ state is not that of a quadrupole vibrator, the particle-phonon coupling model appears to explain the main features of the low-lying states of ^{137}Xe . Differences between the data and the fitted angular distributions are comparable to the differences between the data and the renormalized calculated angular distributions. Use of separate β_2^N and β_2^C improves fits and agreement with the model. In addition to the uncertainties involved in the analysis of the data, the theoretical results

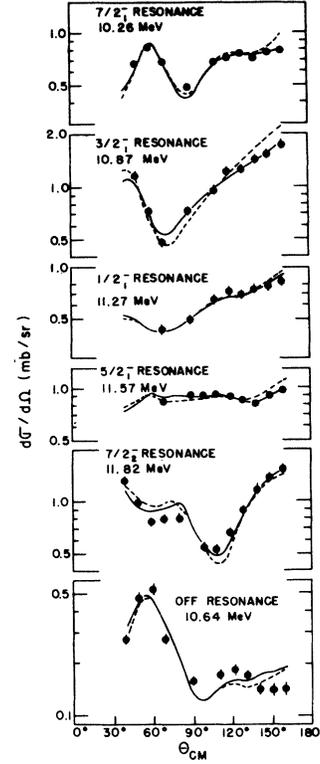


FIG. 4. Inelastic angular distributions. Fits indicated by solid lines; predictions of calculation indicated by dotted lines. The predicted angular distributions are renormalized to the data points to facilitate a visual comparison.

TABLE II. Experimental and calculated parentage coefficients for low-lying states in ^{137}Xe . Experimental excitation energies are taken (Ref. 16) from $^{136}\text{Xe}(d, p)$. We abbreviate $\theta(lj, I_n, J_\lambda)$ as $\theta(lj, I_n)$.

	J_λ	E (MeV)	$\theta(lj, 0_1^+)$	$\theta(f_{7/2}, 2_1^+)$	$\theta(p_{3/2}, 2_1^+)$	$\theta(p_{1/2}, 2_1^+)$	$\theta(f_{5/2}, 2_1^+)$	$\theta(h_{9/2}, 2_1^+)$	$\sum \theta^2$
Exp	$\frac{7}{2}^-$	0.00	0.92 ± 0.02	-0.37 ± 0.03	-0.13 ± 0.03	...	$-0.09 _{-0.1}^{+0.2}$...	1.01
Th		0.00	0.92	-0.31	-0.19	...	-0.05	-0.05	0.98
Exp	$\frac{3}{2}^-$	0.55	0.75 ± 0.01	-0.58 ± 0.02	-0.32 ± 0.02	$-0.04 _{-0.1}^{+0.2}$	$-0.09 _{-0.1}^{+0.2}$...	1.01
Th		0.53	0.70	-0.60	-0.21	-0.17	-0.10	...	0.93
Exp	$\frac{1}{2}^-$	0.91	0.65 ± 0.03	...	$+0.45 \pm 0.04$...	-0.38 ± 0.12	...	0.77
Th		0.91	0.76	...	+0.43	...	-0.35	...	0.89
Exp	$\frac{5}{2}^-$	1.20	0.57 ± 0.04	$+0.36 \pm 0.09$	$+0.32 \pm 0.06$	$-0.13 _{-0.1}^{+0.3}$	$-0.27 _{-0.1}^{+0.3}$	-0.228^a	0.70
Th		1.07	0.71	+0.43	+0.19	-0.25	-0.23	-0.228	0.89
Exp	$\frac{7}{2}^-$	1.41	0.30 ± 0.04	$+0.63 \pm 0.08$	-0.42 ± 0.07	...	-0.09 ± 0.2	...	0.67
Th		1.36	0.31	+0.85	-0.05	...	-0.03	0.00	0.82

^a Taken from calculation; value fixed in fitting.

are rather sensitive to the model parametrization as a consequence of the proximity of the single-particle energies used in the calculation and the phonon energy.

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